How a Physical Definition of Overmodedness Can Explain Local Statistical Non-Compliancy

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***Abstract*—Current understanding of reverberation chambers is firmly funded upon their being operated as overmoded cavities. A proper definition of this condition has been lacking for a long time, while being regarded as automatically satisfied as soon as a large (and unspecified) number of normal modes are set to resonate at the frequency of operation of the chamber. In a recent work, we have introduced in a formal way a proper definition of the overmoded condition, showing how, from a strict mathematical viewpoint, it is not related to any threshold frequency, but rather requirs a statistical framework. In this paper we revisit the most important results from our analysis, and show how our theory can straightforwardly explain the local non-compliancies sometimes observed for field statistics in what are deemed to be overmoded cavities. Partial overlapping between the normal modes of the cavity are shown to lead to an alternative and more physically-based explanation for this phenomenon, without invoking the concept of an ineffective stirring technique, nor the presence of unstirred components. The proposed theory is finally applied for the assessment of the relationship existing between the actual confidence margin based on the ideal properties of a reverberation chamber and those predicted by our model, showing how there is a non-negligible probability of having a large number of samples outside the original confidence margin, i.e., a potentially erroneous estimate of the true variability of a field-related quantity.**

***Index Terms*—Statistics, Cavities, Losses, Error analysis.**

I. Introduction

The way reverberation chambers (RCs) are commonly utilized in EMC radiated tests is based on a strong assumption, that they are capable of providing a diffused field. This property is never directly invoked in RC theory as developed within the EMC community, though it is of fundamental importance in the physics of cavities and the study of room acoustics [1]. Historical reasons are likely to blame for this mismatch, as well as the fact that electrical engineers have long preferred to have a more practical and test-driven approach to the understanding of these facilities. Still, the lack of the clear definition of a diffused field has been an invisible obstacle, in our opinion, to the cross-fertilization that should have existed between the EMC community and the acoustics one. Indeed, EMC engineers could benefit from the deeper understanding and modeling tools developed in acoustics. In this respect, we will regard the concepts of overmoded RC and that of diffused field as synonyms, as both refer to a cavity that ensures, asymptotically, to expect a very specific statistical behavior for field-related quantities.

Yet, both fields lacked a proper physics-based model capable of explaining under what conditions a cavity can be considered to allow a diffused field to establish. Acoustics, too, tends to consider the idea of a threshold frequency [1], often referred to as Lowest Usable Frequency (LUF) in EMC. In a recent paper [2], we have proposed such a model, establishing a direct link between the physics of an RC and the statistical properties of the field it supports. In this paper we recall the most important steps and assumptions that have led to this result, and show how it can explain another phenomenon with an unsatisfying description, that of local statistical non-compliancy of field-related quantities.

Our definition of an ovemoded cavity (and thus of a diffused field) is based on the use of a modal description of the field excited within an RC, coupled to a statistical approach. The interest of this approach is that it allows avoiding making non-physical assumptions on the need for a threshold on the modal density, as apparently assumed on the EMC side, or on the sheer fact that a threshold frequency exists. Of particular importance is the prediction that losses should not be regarded as a limitation or nuisance in the operation of RCs, but that they rather lie at the heart of its functioning. Without losses, an RC would never be capable of supporting a diffused field, independently of the modal density it supports: hence, our regarding the use of the term overmoded as a potentially treacherous habit, as the mere availability of a high modal density is not a sufficient condition. In the following, it will be shown that an overmoded RC is a cavity that supports a large number of overlapped modes, which is not the same as just supporting a large number of modes, as estimated by looking at modal density alone. This fact was unacknowledged in EMC, while well understood in acoustics and, as we will show in this paper, this misinterpretation can be regarded as one of the reasons why the phenomenon of local non-compliancy is still characterized by an unsatisfying explanation.

II. Defining the overmoded condition

The common sense given to the concept of overmoded cavity is tightly linked to the expectation of the following properties: 1) field statistics are independent of the position in space where they are tested, at least in a sub-volume of the cavity; 2) the three orthogonal components of the electric (respectively, magnetic) field behave as independent and identically (iid) distributed random variables; 3) they follow a Gaussian distribution law, with zero mean-value. It has been shown that these hypothesis are met whenever the field distribution can be described by means of a continuous random plane-wave spectrum, with specific statistical properties [3]. Although such type of model well approximates the behavior of real-life RCs in their high-frequency range, it is well-known that there might exist some frequencies where it is incapable of predicting a drift from it [4]. These disagreements are often explained through a stirrer inefficiency or a sub-optimal alignment for the sources [5], though we are quite skeptical of these explanations [6], whereas in the lower frequency range they are usually just fitted to alternative probability distribution functions [7].

It is, in our opinion, by far more interesting and useful to get back to the basics of the physics of cavities, while taking on a more statistical approach from the beginning. To this end, we will consider a modal expansion for the electric field distribution observed at the position $\boldsymbol{r}$ within an RC at the working frequency f [8]

$$\boldsymbol{E}(\boldsymbol{r}, f) = \sum_{i \in \mathscr{M}} \gamma_i(\boldsymbol{r}, f)\psi_i(f)\hat{\boldsymbol{\xi}}_i(\boldsymbol{r}, f) \quad , \tag{1}$$

where the $\gamma_i(\boldsymbol{r}, f)$ reflect the excitation of each mode, depending on the nature of the source and its position relative to the modal topographies, $\psi_i(f)$ is the bell-shaped frequency response of each mode and $\hat{\boldsymbol{\xi}}_i(\boldsymbol{r}, f)$ is the polarization of the electric field contribution provided by each mode. The sum in (1) is taken over the set $\mathscr{M}$ of modes that are effectively excited at the working frequency. A proper justification for this model has been provided in [2]. Rather than attempting to apply (1) in a deterministic way, we will regard all of these modal quantities as random ones, with statistical properties dictated (or suggested) by their physical properties.

In this respect, the modal weights $\{\gamma_i\}$ can be expected to behave as centered random variables, as they are directly related to the scalar projection of the equivalent currents representing a source and the modal topographies, which are standing waves, thus characterized by a pseudo-periodic change in their sign over space. No further assumption is needed on the nature of the probability law for the γ_i. The functions $\{\psi_i(f)\}$ represent the responses of second-order systems, defined as follows

$$\psi_i(f) = \frac{1}{f_i^2(1 + \mathrm{j}/2Q_i)^2 - f^2} \quad , \tag{2}$$

where f_i is the resonance frequency of the i-th mode and Q_i its quality factor. As the presence of a stirring technique implies, from an ideal perspective, that the frequencies of resonance of the modes are thoroughly mixed and swapped about the working frequency, it is reasonable to assume that the variables $\{f_i\}$ be uniformly distributed. Moreover, the quality factors $\{Q_i\}$ can be characterized by means of their average (or composite) value, as will be shown later. This latter quantity can be experimentally assessed. Finally, the polarization unit vectors $\{\hat{\boldsymbol{\xi}}_i\}$ will be assumed to be uniformly distributed over 4π steradian, again as a consequence of the assumption of an ideal stirring technique at work. It is important to notice that assuming a perfect stirring is actually sensible, as we are interested in assessing whether it is possible to explain the eventual non-compliancy of an RC even having enforced an ideal performance on the stirring technique.

With this model at our disposal, we now aim at studying the first two statistical moments of the electric energy density

$$W(\boldsymbol{r}, f) = \epsilon_0 \|\boldsymbol{E}(\boldsymbol{r}, f)\|^2 \quad , \tag{3}$$

with ϵ_0 the dielectric permittivity of the medium filling the cavity. The electric energy density can be described, in the case of an ideally diffused field, as a random variable distributed as a χ_6^2 law, as a consequence of the features recalled at the beginning of this Section. As opposed to this hypothesis, by computing the first two moments of $W(\boldsymbol{r}, f)$ and employing the previously introduced assumptions on the statistics of the modal quantities should allow to check under what conditions the ideal behavior is actually met. The procedure for this calculation has been presented in details in [2], and leads to the following standardized variance

$$\varsigma_W^2 = \left(\frac{\sigma_W}{\mu_W}\right)^2 = \frac{1}{3} + \frac{1}{\pi}\frac{\mu_4}{\mu_2^2}\frac{1}{M_M} \quad , \tag{4}$$

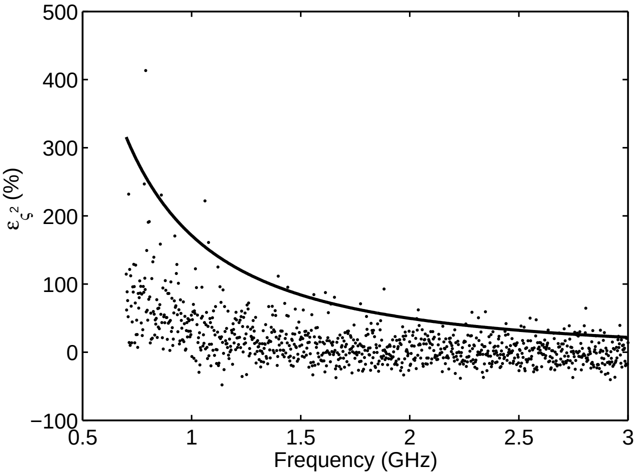
with $M_M = mB_M$ the average number of modes overlapping within the average -3 dB bandwidth $B_M = f/Q$ centered around the working frequency, for a modal density $m(f)$ and an average composite quality factor Q. The terms μ_n refer to the moments of the modal weights $\{\gamma_i\}$

$$\mu_n = \mathsf{E}\left[|\gamma_i|^n\right] \quad , \tag{5}$$

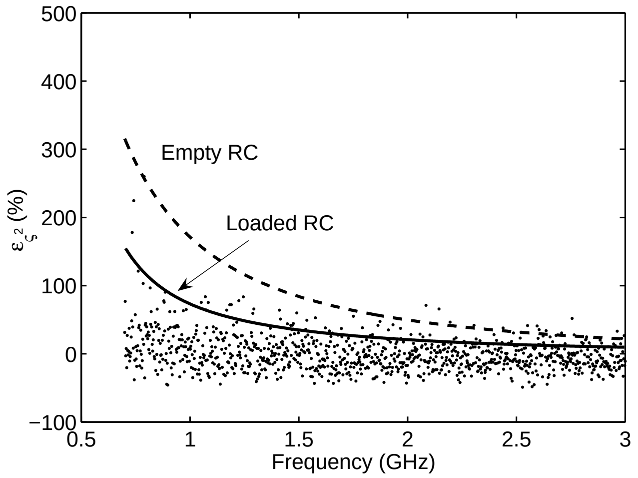
so that $\mu_4/\mu_2^2 = 2$ for the case of normally distributed modal weights, which is the usual assumption applied for these quantities. Other distribution laws do not have much of an effect on this ratio, so that the final result is weakly dependent on any assumption on the nature of the modal weights.

Comparing (4) to the standardized variance $\varsigma_{\chi_6^2}^2 = 1/3$ obtained for the ideal diffused field scenario shows what is needed for an RC to be as close as possible to the ideal case, i.e., presenting a large number of overlapped modes M_M. Whence, it is clear that there is no reason for assuming that any threshold frequency ensures an RC to support a diffused field, as M_M is actually a complex function of frequency, with a non monotonous trend. As a matter of fact, both the composite quality factor Q and the modal density m are non-monotonous; attention must be paid to the fact that the often used Weyl's approximation is indeed a smooth fitting curve to the real $m(f)$, which is generally not known. Local variations around this smooth function can be far from negligible, leading to modal depletion or excess: it can therefore be intuitively understood that M_M follows a similar trend, whose range of variations is made further wider and unpredictable by the dominant role of the composite quality factor Q.

As a result, the standardized variance ς_W^2 can get quite larger than the $1/3$ value expected in the ideal case. This value is attained only asymptotically as $M_M \to \infty$, a condition that



(a)



(b)

Fig. 1. Estimates of the standardized variance of the energy density W, as assessed from experimental results obtained for : (a) an empty cavity (apart for the excitation antenna) and (b) one loaded with a set of 4 pyramidal absorbers about 30 cm high.

can be thought as well approximated as the working frequency increases, since the modal density will increase (on average) quadratically with the frequency, while the composite quality factor has typically a slower rate of growth. In any case, (4) links the rate of convergence of the statistics of W to the average number of overlapped modes M_M: hence, by settling on an acceptable error between the real and ideal ς^2, we can come up with a threshold on the minimum number of modes that should overlap on each other bandwidth, as a way to ensure this maximum error.

In fact, the results here recalled are just sensible and can be understood by recalling that any component of the electric field is actually made up by a discrete superposition of contributions, with a small number of them dominating the final result. If all of these terms are treated as random variables, ideally independent, the central-limit theorem ensures that their sum, even weighted, can be expected to converge to a normally distributed random variable, with a degree of accuracy that broadly increases with the number of terms involved. In other words, a large number of independent degrees of freedom are needed. Here we can see two physical reasons that can put in jeopardy the normal-distribution hypothesis: 1) there is no way of ensuring that the modes are excited as independent random variables by a single source, even when applying a stirring technique; 2) the number of degrees of freedom is actually directly related to the number of modes effectively excited at the working frequency. The first point is hard to be assessed, as the actual excitation of the modes is intimately related to the modal topographies, which are hardly accessible in real-life configurations. We are convinced that a given amount of residual correlation is always present, and that it can lead to surprising results when neglected [6]. Concerning the second point, the advantage of a modal approach is that it makes visible the intrinsical discrete nature of the degrees of freedom actually needed to describe the electromagnetic field within a cavity. Even more importantly, it allows not getting out of sight of a fundamental phenomenon, i.e., the bandwidth of the frequency response of each mode is strongly dependent on the amount of losses associated to each modal topography. In such conditions, the central-limit theorem cannot be fully invoked, so that disagreements with the ideal-scenario requirements should not come as a surprise.

Since $M_M(f)$ can be reasonably expected to broadly increase with the frequency, (4) predicts that the standardized variance of the energy density should converge to the asymptotic value $1/3$ at high frequency. This fact is demonstrated in Fig. 1a, where experimentally determined estimates of ς_W^2 are shown over a wide frequency range, as obtained with a 100-step mechanical stirrer in an RC with a volume of $13.8\ \mathrm{m}^3$. The RC was excited by means of a log-periodic dipole antenna pointed at one corner, while the field samples were measured with an electro-optical probe positioned at the center of the cavity. Fig. 1 confirms that indeed the error $\epsilon_{\varsigma^2} = \varsigma_W^2 - 1/3$ broadly decreases with the frequency, while it is far from negligible in the lower frequency range. The thicker line in Fig. 1 has been obtained by applying (4), using a loose estimate for M_M. This latter quantity was derived by using the average modal density predicted by Weyl's formula and a smooth majorant of an experimentally-derived estimate of the composite quality factor. This operation was aimed at assessing whether (4) is capable of providing a loose upper-bound for the error observed in practice on the standardized variance ς_W^2. Indeed, a good agreement between the prediction of the maximum error yielded by (4) and the experimental results is observed, with an envelop well identified by the former. The case treated in Fig. 1b deals with the introduction of additional losses: in this case too our model predicts in a fairly good way the reduction of the variability of W. Further details about the validity of (4) are provided in [2], particularly about the role of losses in the statistical compliance of the energy density in an RC.

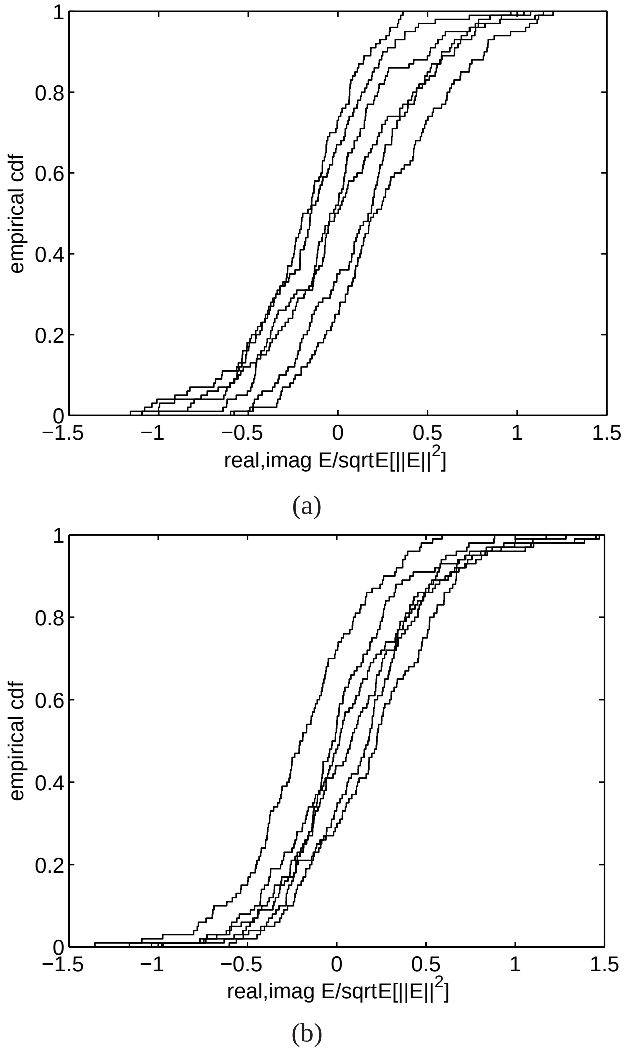


Fig. 2. Empirical cumulative distribution functions for the real and imaginary parts of the three Cartesian components of the electric field, as estimated from 100-sample populations for two frequencies: (a) 1.438 GHz and (b) 1.441 GHz.

III. Statistical compliance

In the present work we are rather more interested in utilizing the physical understanding brought by (4), for interpreting why an RC can present a non-compliant behavior at some specific frequencies. As already recalled, the idea of unstirred field components and ineffective stirring techniques are often invoked in such cases. But another possibility is perhaps more likely: the fact that if at a given frequency the modal overlapping (represented by M_M) is not large enough, the number of degrees of freedom effectively involved in the generation of the electromagnetic field is not high enough to lead to a good approximation of a Gaussian law. Hence, a larger variance for the electric energy density, with respect to that expected in the asymptotic modeling. It is noteworthy to recall that this argument is based on the assumption of an ideal stirring technique, thus involving a random excitation of the modes and a random positioning of their resonance frequencies.

In order to assess the soundness of this explanation, we have given a closer look at the field samples measured for an empty cavity at two frequencies where our RC can be regarded as behaving ideally or not; we have chosen 1.438 GHz and 1.441 GHz to represent this phenomenon, with an error ϵ_{ς^2} about 0 % and 100 %, respectively. These frequencies are almost three times higher than the LUF of our RC, intervening at about 550 MHz, assessed as required in [9]. The stirring paddle is about 5 wavelengths wide, and non-periodic, so that hardly any doubts can subsist on its effectiveness. At the same time, the likeliness of the presence of unstirred components has been minimized by orienting the excitation antenna in such a way as to have its main lobe of radiation pointing towards a corner of the RC, as to be reflected over the stirrer and avoid any direct path towards the probe.

The empirical cumulative distribution functions (cdfs) of the real and imaginary parts of the three Cartesian components of the samples are shown in Fig. 2, for the two frequencies chosen above. The first thing to notice is that for $f = 1.441$ GHz the statistical non-compliancy of our RC cannot be explained by any unstirred component alone. As a matter of fact, it even seems, on a qualitative level, that some field components is actually more biased for $f = 1.438$ GHz, though in this case the energy density presents a very good agreement with a χ_6^2 law. A closer look suggests that the non-compliancy is likely due to the higher kurtosis featured at 1.441 GHz: 3.7 on average against 3.0 at 1.438 GHz, the latter corresponding to the kurtosis of a Gaussian law. This is clearly visible in the longer transitions from the central part of the cdf towards its ends. Recalling that the field is related to the energy density by a square power, the kurtosis is directly linked to the variance of W, while the field variance is rather related to average value of W. As predicted by (4), in the case of a low modal overlapping, the standardized variance is to increase with respect to the asymptotic case: this is what is observed in these results.

Hence, it seems likely that the non-compliancy is better explained by a local limitation of the RC rather than the stirrer or other phenomena. As a matter of fact, it is hard to imagine why a stirrer and the positioning of the sources should work fine over a majority of frequencies, while leading to a bad performance just over a handful of frequencies. On the other hand, the concept of low modal overlapping can easily explain this disturbing phenomenon, as the combination of high composite quality factor and local modal depletion can indeed appear. Moreover, the fact that the insertion of losses yields an improved performance (see Fig. 1), i.e., a lower incidence of non-compliant frequencies, supports this interpretation, as already pointed out in [2].

A higher statistical dispersion has a direct impact on the results of compliancy tests required in international standards [9]. As a matter of fact, the upper-bounds defined for the maximum acceptable spatial variability of the maximum field intensity are based on an asymptotic model [10]. Hence, a higher variability for the field samples implies a higher

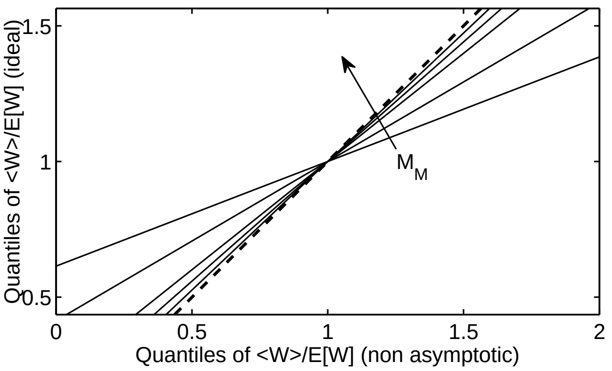


Fig. 3. Theoretical quantile-quantile plot of the asymptotic distribution laws for $\langle W \rangle$ in the asymptotic and non-asymptotic case, as based on (4), for a number of average overlapped modes $M_M = \{1, 3, 10, 20, 50\}$. The dashed line stands for the bisector, which is asymptotically approached as M_M increases. The size of the sample population is not needed in this type of plots, as it only intervenes in setting the confidence interval, with no effect on the relationship between the two quantile distributions.

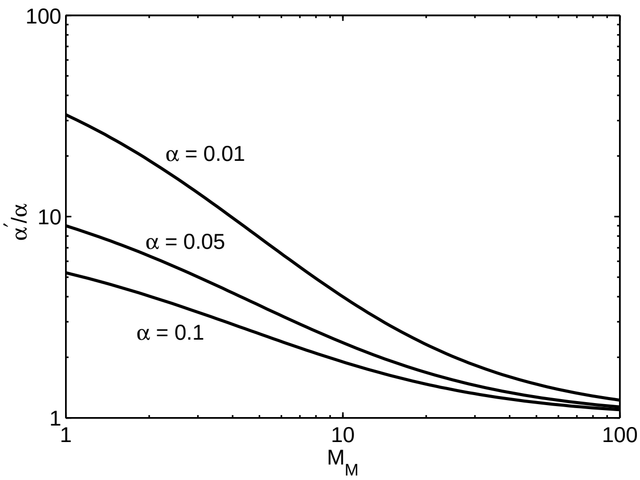


Fig. 4. Apparent significance level α' associated to the one based on asymptotic statistics, α. As the number of overlapped modes M_M decreases, the statistical interpretation of the arithmetic mean of experimental samples can be increasingly misleading, due to a higher variance with respect to the samples mean.

probability for them to get closer to the upper-bound. The context of this paper is not suitable for studying the statistical properties of the extreme values of W and the related squared field components. Still, as shown in [11], there exists a direct relationship between the average value of the energy density components and their maximum value, as estimated from a finite population of samples. A similar reasoning applies for the energy density W, which is our case of interest in this paper.

It is therefore interesting to assess how the increment of variability predicted by (4) could affect the probability of having samples out of a given confidence margin. This latter could be set, e.g., in order to ensure a minimum accuracy for a measurement or, as it is the case for industrial tests, the reproducibility of a test result.

To this end, let us consider a number N of independent samples W_i large enough as to have their arithmetic average $\langle W \rangle$ converging towards a normal law, thanks to the central-limit theorem. We aim at computing how a confidence margin defined for a significance level α under the hypothesis of an asymptotic behavior, translates into another confidence margin whenever M_M happens not to be high enough. A good and intuitive way of assessing the effect of a finite M_M is to check how the quantiles of the asymptotic case relate to those of the modal description we have introduced. As we are dealing with this problem under the approximation of a normally distributed $\langle W \rangle$ (central-limit theorem), the quantiles are given by [12]

$$q_p = \mu + \sigma\sqrt{2}\mathrm{erf}^{-1}(2p - 1) \quad , \tag{6}$$

where $p = P(\langle W \rangle < q_p)$ is the probability associated to the quantile q_p, while μ and σ are the expected mean-value and the standard deviation for $\langle W \rangle$. The central-limit theorem allows to compute a good approximation of the quantiles just by knowing these two moments, for the case of an ideal χ^2_6-distributed W_i and for the more realistic case provided by (4). The result of this operation is presented in Fig. 3, where any confidence margin established on the asymptotic case for a given significance level α is shown to lead to an inevitably larger margin with the same significance level as soon as M_M is found to be finite. It is interesting to notice that the case $M_M = 3$, often regarded as a good compromise for a diffused field in room acoustics, actually provides a more than twofold increase in the original confidence margin, whereas the case $M_M = 20$ suggested as a tentative threshold on M_M in [2] settles to an about 50 % larger interval.

The other way round, choosing the same interval margin for the ideal and non-ideal cases, if this interval corresponds to a significance level α for the former case, this will lead to a corresponding significance level α' in the latter, related as

$$\alpha' = 1 - \mathrm{erf}\left(\frac{q_{1-\alpha/2}}{\sqrt{2}}\sqrt{\frac{M_M}{18/\pi + M_M}}\right) \quad . \tag{7}$$

This function is plotted in Fig. 4, for several values of α. These results provide a direct feeling about the increased probability of incurring into samples falling outside the originally intended confidence margin. The probability α' of this event increases not only when M_M is relatively low, but also when the significance level α is reduced. Such relationship has a potentially harmful impact, as α is typically reduced in order to improve the significance of the results of a statistical test: when expecting asymptotic results from a realistic RC, this risks leading to a higher rate of rejection of eventual hypothesis tests, rather than the opposite. One straightforward way of reducing this eventuality is to increase the average modal overlapping for the RC, as suggested by (4): the most practical approach is likely to slightly increase the average amount of dissipated power, as long as this procedure is not against minimum field intensity constraints.

IV. Conclusions

In this paper we have briefly recalled the reasons of our introducing a physically-motivated definition of the overmoded condition for an RC. The use of a modal approach, though inevitably based on simplifying assumptions, has led to pointing out that there is no threshold frequency for an real RC to behave as an ideal one. Local disagreements for the statistics of field-related quantities can appear at any frequency, depending on eventual configurations featuring a weak overlapping of modes. The probability for these events is intuitively expected to decrease for an increasing frequency, and the proposed model indeed supports this idea. Still, having put the assumption of an ideal stirring technique at the heart of our model, while it admits configurations where an RC can present a non-compliant statistics, it is no more reasonable to infer that statistical non-compliancy is a synonyms of non-ideal stirring techniques. This fact was further discussed in this paper, showing how the excess statistical dispersion found in practical configurations cannot be explained with respect to the stirring techniques, but is more likely due to an intrinsical local limitation of the RC, particularly its finite modal overlapping at certain frequencies. This fact becomes critical as soon as an ideal RC behavior is taken for granted.

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