HAL
open science

Can LR Test Be Helpful in Choosing the Optimal Lag order in the VAR Model When Information Criteria Suggest Different Lag Orders?

Abdulnasser Hatemi-J, Scott Hacker

To cite this version:

Abdulnasser Hatemi-J, Scott Hacker. Can LR Test Be Helpful in Choosing the Optimal Lag order in the VAR Model When Information Criteria Suggest Different Lag Orders?. Applied Economics, 2009, 41 (09), pp.1121-1125. 10.1080/00036840601019273 . hal-00582108

HAL Id: hal-00582108

https://hal.science/hal-00582108

Submitted on 1 Apr 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

For Peer Review

Can LR Test Be Helpful in Choosing the Optimal Lag order in the VAR Model When Information Criteria Suggest Different Lag Orders?

Journal:	*Applied Economics*
Manuscript ID:	APE-06-0201.R1
Journal Selection:	Applied Economics
JEL Code:	C30 - General < C3 - Econometric Methods: Multiple/Simultaneous Equation Models < C - Mathematical and Quantitative Methods, C32 - Time-Series Models < C3 - Econometric Methods: Multiple/Simultaneous Equation Models < C - Mathematical and Quantitative Methods
Keywords:	VAR, Lag length, Information Criteria, Likelihood Ratio Test, Monte Carlo Simulations

powered by ScholarOne
Manuscript Central™

Submitted Manuscript

Can LR Test Be Helpful in Choosing the Optimal Lag order in the VAR Model When Information Criteria Suggest Different Lag Orders?

Abstract

The objective of this simulation study is to investigate whether the likelihood ratio (LR) test can pick the optimal lag order in the vector autoregressive model when the most applied information criteria (i.e. vector *Schwarz-Bayesian, SBC, and vector Hannan-Quinn, HQC) suggest two different lag orders. The results based on the Monte Carlo simulations show that combining the LR test with SBC and HQC results in a significant increase in the success rate of choosing the optimal lag order compared to cases when only SBC or HQC are used. This is true irrespective of homoscedasticity or conditional heteroscedasticity. This improvement in choosing the right lag order also tends to improve the forecasting capability.*

.

Running title: Optimal Choice of the Lag Length in the VAR Model Using LR

JEL Classification: C32, C30

Keywords: VAR, Lag length, Information Criteria, Monte Carlo Simulations, Likelihood Ratio Test

For Peer Review

Editorial Office, Dept of Economics, Warwick University, Coventry CV4 7AL, UK

1. Introduction

One of the most applied models in the empirical economics is the vector autoregressive (VAR) model. In addition to its simplicity regarding estimation and interpretation, and its good forecasting capabilities, the VAR model treats all the variables of interest endogenesously. This property is important since in macroeconomics exogenous variables are rare. The VAR models also provide the possibility to investigate the causal relationship between the variables. It is also possible to transform the VAR model to a vector moving average (VMA) representation in order to trace the effects of the shocks on each variable in the system by calculating the impulse response functions and the variance decompositions. Due to the recent developments in dealing with integrated variables, the VAR model has been proven to be even more useful since it can be used to test for the long-run equilibrium relationship between the variables in combination with the short-run adjustment process. Obviously the VAR model is dynamic which accords with economic theory. However, economic theory is usually not much of help regarding the length of the dynamic process. In the literature several lag choosing criteria have been proposed for this purpose. Three of the most used information criteria are the Akaike (1969) information criterion, (AIC), the Schwarz (1978) Bayesian criterion (SBC) and the Hannan and Quinn (1979) criterion (HQC). These information criteria were originally developed for single equations. But they can be extended in vector form to determine the lag order of systems of equations, i.e. VAR models. However, the choice of these criteria for determining the lag order in the VAR model is usually arbitrary in the applied studies. Sometimes these information criteria do not agree in choosing the lag order. The question is then upon which information criteria one should rely.

Hatemi-J (1999, 2001) suggested using two of these criteria to choose the optimal lag length in the VAR model. If these two criteria choose different lag orders then the author suggested using the likelihood ratio (LR) test to choose between these two lags. It should be expressed that the LR test will be used only once in this case.[1] This means that the problem of mass-significance that occurs when the test is used sequentially can be avoided.[2] According to a simulation study performed by Hacker and Hatemi-J (2005) SBC

[1] For an application of this approach see Hacker and Hatemi-J (2003).
[2] See also Hatemi-J (2003).

For Peer Review

has best performance in many cases. However, HQC has better performance in some cases. Thus, which criteria should be used is dependent on the data generating process for the variables. However, one cannot be sure about this issue when actual data is used because the true data generating process for the actual data cannot be known. This subject is important because inference in the VAR model is dependent on the choice of lag length. The purpose of this article is to evaluate the lag choosing procedure suggested by Hatemi-J. Thus, the purpose of this study is to see whether the LR test can be useful in picking the optimal lag order of the VAR model when SBC and HQC suggest different lag orders. It should be pointed out that we will make use of many combinations of the parameters in the VAR model in the simulations in order to make the results as general as possible.

This paper is organized as follows. In the next section we will describe the VAR model and different criteria that can be used to determine the optimal lag order. Section 3 describes the design of our simulation. Section 4 presents the simulation results and conclusions.

2. The VAR Model and the Lag choosing Criteria

Let us define the following VAR model, consisting of n variables that is characterized by an order less-than or equal to K:

$$Z_t = \Gamma D_t + \sum_{k=1}^{K} \beta_k Z_{t-k} + \varepsilon_t, \qquad t = 1, \cdots, T. \tag{1}$$

where ε_t is a $n\times 1$ vector of disturbance terms that are assumed to be independently identically distributed errors with the distribution $N_n(0, \Omega)$, β_k is a matrix of coefficients for Z_{t-k}, and D_t represents non-stochastic components such as constant terms, linear trend, or seasonal dummies. The initial values, $Z_{1-K}, \cdots, Z_0$, are assumed to be fixed. Our objective is to choose the largest order for the time series, denoted by $k_l \in K$, such that $\beta_{k_l} \neq 0$ and $\beta_j = 0$, $\forall\, j > k_l$. To accomplish choosing the optimal lag length in the VAR model the following general form for an information criterion can be applied:

$$IC = \ln\left(\det \hat{\Omega}_j\right) + j\frac{f(T)}{T}, \qquad j = 0, \cdots, K, \tag{2}$$

For Peer Review

where $\hat{\Omega}_j$ is the maximum likelihood estimate of the variance-covariance matrix Ω when the lag order used in estimation is *j*. Logarithm is denoted by *ln* and *det* represents the determinant of the corresponding matrix.

The objective is to estimate k_l by the *j* that minimizes the above criterion. Schwarz (1978) suggested $f(T) = n^2 \ln T$, whereas Hannan and Quinn (1979) preferred $f(T) = 2n^2 \ln(\ln T)$.[3]

The lag order of the VAR model can also be determined by testing the significance of parameters for each specific lag order. The likelihood ratio (LR) test that can by applied for this purpose, due to Sims (1980), is defined as the following:

$$LR = (T - c)\left|\log|\Omega_1| - \log|\Omega_2|\right| \tag{3}$$

where *T* is the sample size and *c* is the total number of parameters estimated in the VAR model under the alternative hypothesis. Ω_1 is the maximum likelihood estimate of the variance-covariance matrix of the residuals in the VAR model under null hypothesis and Ω_2 is the maximum likelihood estimate of the variance-covariance matrix of the residuals in the VAR model under alternative hypothesis. The LR test is chi-square distributed with the degrees of freedom equal to the number of restriction that are tested.

3. The Simulation Design

We make use of the following bivariate VAR model, which is of order two, in our simulations:

$$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} \beta_{1,11} & \beta_{1,12} \\ \beta_{1,21} & \beta_{1,22} \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} \beta_{2,11} & \beta_{2,12} \\ \beta_{2,21} & \beta_{2,22} \end{bmatrix} \begin{bmatrix} z_{1t-2} \\ z_{2t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{4}$$

The error terms vector $\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$ are designed to be independent or conditionally heteroscedastic. More specifically, the simulations are also run when the variance of the error terms can be described by the following autoregressive conditional heteroscedasticity (ARCH):

$$\begin{bmatrix} \sigma_{1t}^2 \\ \sigma_{2t}^2 \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \end{bmatrix}, \tag{5}$$

For Peer Review

To cancel the effect of starting up values, we generated 100 presample observations. This gives us the possibility to have the same number of observations in estimating the VAR model regardless of the number of lags.

A central issue in a Monte Carlo simulation like the current one is choosing a variety of parameters that as a group has characteristics that fairly represent those of the infinite space of possible parameters. In order to obtain general results we consider all the combinations shown in Table 1 for the coefficient matrices. There are 12500 (5×5×5×5×4×5) possible combinations of the elements in this table. The VAR model is always of the second order since $\beta_{2,22}$ is never zero.

Table 1.

Parameter Values for VAR model of equation (4)

$\beta_{1,11}$	-1	0.5	0	0.5	1
$\beta_{1,22}$	-0.5	-0.25	0	0.25	0.5
$\beta_{1,12} = \beta_{1,21}$	-0.5	-0.1	0	0.1	0.5
$\beta_{2,11}$	-0.8	-0.2	0	0.2	0.8
$\beta_{2,22}$	-0.6	-0.1		0.1	0.6
$\beta_{2,12} = \beta_{2,21}$	-0.5	-0.1	0	0.1	0.5

To assure that we have stable cases we make sure that the modulus (the square root of the summed squares of the real and imaginary eigenvalue components) of each eigenvalue of the following companion matrix (B) is less than one.

$$B = \begin{bmatrix} \beta_{1,11} & \beta_{1,12} & \beta_{2,11} & \beta_{2,12} \\ \beta_{1,21} & \beta_{1,22} & \beta_{2,21} & \beta_{2,22} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{6}$$

We run separate simulations based on small sample sizes ($T = 40$).[4]

[3] Nielsen (2001) shows that SBC and HQC are consistent regardless of the assumption about the characteristic roots in the VAR model. By consistency is meant that the criterion selects the true order of the VAR system with probability one asymptotically.

[4] It should be pointed out that we have run 10000 simulations for each combination of the parameters. This means that the total number of the simulations is 125000000 for each sample size. The simulations are performed by using GAUSS.

For Peer Review

Since VAR models are extensively used for forecasting purposes, we also investigate whether combining the LR test with the two information criteria can result in determining lag orders that can result in more accuracy in forecasts. Assume that $E(z_{i,T+h})$ is the forecast of variable z_i, $i = 1,2$, for h periods into the future that we would make if we know the actual parameters of the VAR model (assuming errors are equal to zero for future periods). This forecast is equivalent to the expected value of $z_{i,T+h}$ based on the information available in the last observed period T. Let us denote $\hat{z}_{i,T+h}$ the forecast of variable z_i for h periods into the future that we make using the estimated parameters based on the lag length chosen by the criterion. Finally, let $\bar{z}_{i,T+h}$ denote the forecast of variable z_i for h periods into the future that is made based on the estimated parameters using always the right lag length of two. Then, the sum of squared errors ratio, *SSER*, can be calculated for a particular case (set of parameters in a scenario) by the following equation:

$$SSER = \frac{\sum_S \left[\hat{z}_{i,T+h} - E\left(z_{i,T+h}\right)\right]^2}{\sum_S \left[\bar{z}_{i,T+h} - E\left(z_{i,T+h}\right)\right]^2}, \qquad (7)$$

where S denotes the set of 1000 simulations for a particular case, so in the numerator and denominator we are summing up over the simulations. In the numerator we show the systematic sum of squared errors of the forecasted variable based on forecasts using lags chosen by the criterion.[5] In the denominator we show the systematic sum of squared errors of a forecasted variable using always the correct lag length. The lower *SSER* is, the better the forecasting is in comparison to forecasts based on knowing the correct lag length. If the chosen criterion does just as well in forecasting as when the correct lag length is always chosen, then this ratio would be one. For any particular scenario we calculate the average *SSER*, i.e. the mean of the *SSER* over all the cases and over the two variables, z_1 and z_2.

4. The Results of the Simulations and the Conclusions

[5] Lütkepohl (1985,1991) handles his presentation of forecast capability in a different fashion. He focuses not on the systematic forecast error, $\hat{z}_{i,T+h} - E(z_{i,T+h})$, but on the overall forecast error $\hat{z}_{i,T+h} - z_{i,T+h}$, where $z_{i,T+h}$ is an actualized outcome of z_i for h periods into the future. He thus also includes in his focus the unsystematic (random) components of the forecast error, $E(z_{i,T+h}) - z_{i,T+h}$, which injects additional randomness that we find undesirable for comparison since it is unsystematic. For his presentation in his 1985 article he normalizes (divides) an approximation of the mean squared overall forecast error with the theoretical variance of $E(z_{i,T+h}) - z_{i,T+h}$.

For Peer Review

In Table 2 we present the frequency distributions in the presence and absence of conditional heteroscedasticity. We can see that the two criteria perform differently and which performance is better depends on the circumstances. However, when we combine these two criteria with the LR test for picking the true lag order in the VAR model the percentage choosing the optimal order increases significantly compared to cases when only one criterion is utilized, particularly for small sample sizes. This is true whether conditional heteroscedasticity is present or not. The same conclusion can be drawn regarding the forecasting properties (see Table 3). Using the LR test combined with SBC and HQC to choose the optimal lag order in the VAR model is going to result in choosing lag orders that are going to result in more precise forecasts compared to cases when only SBC and HQC are used.

Table 2.

Results for picking the optimal lag order based on simulations for a VAR(2) model with and without autoregressive conditional heteroscedasticity (ARCH).

Info criterion & variance situation ↓	Lag length→	0	1	2	3	4	5	6	7
		Frequency distribution of estimated VAR orders, *without ARCH*							
HQC,		0.019	0.031	0.860	0.056	0.018	0.007	0.005	0.004
SBC		0.066	0.070	0.850	0.013	0.001	0.000	0.000	0.000
HQC, SBC and LR		0.021	0.050	0.908	0.018	0.003	0.000	0.000	0.000
		Frequency distribution of estimated VAR orders, *with ARCH*							
HQC,		0.018	0.032	0.833	0.078	0.022	0.008	0.005	0.004
SBC		0.060	0.069	0.846	0.022	0.002	0.000	0.000	0.000
HQC, SBC and LR		0.019	0.051	0.897	0.028	0.004	0.000	0.000	0.000

For Peer Review

Table 3.

Results for forecasting performance (SSER) based on simulations for a VAR(2) model with and without autoregressive conditional heteroscedasticity (ARCH).

Forecasting Period	Forecasting Error for HQC	
Forecasting Period	Without ARCH	With ARCH
1	1.249	1.334
2	1.267	1.422
3	1.240	1.403
4	1.229	1.453
5	1.194	1.877
Forecasting Period	Forecasting Error for SBC	
1	1.283	1.254
2	1.285	1.257
3	1.139	1.149
4	1.111	1.149
5	1.063	1.181
Forecasting Period	Forecasting Error for HQC, SBC and LR	
1	1.141	1.162
2	1.147	1.176
3	1.089	1.131
4	1.073	1.148
5	1.049	1.18

For Peer Review

REFERENCES

Akaike, H. (1969) "Fitting Autoregressions for Prediction", *Annals of the Institute of Statistical Mathematics*, **21**, pp. 243-247.

Hacker, R. S. and Hatemi-J, A. (2003) How Productivity and Domestic Output Are Related to Exports and Foreign Output in the Case of Sweden. *Empirical Economics*, 28(4), 767-782.

Hacker, R. S. and Hatemi-J, A. (2005) "Optimal Lag Length Choice in Stable and Unstable VAR Models under Situations of Homoscedasticity and Heteroscedasticity". *Journal of Applied Statistics*, forthcoming.

Hannan, E. J., and Quinn, B.G. (1979) "The Determination of the Order of an Autoregressive," *Journal of the Royal Statistical Society*, B41, pp. 190-195.

Hatemi-J, A. (1999) "Essays on the Use of VAR Models in Macroeconomics, Thesis in Econometrics, Department of Economics, Lund University.

Hatemi-J, A. (2003) A New method to . A New Method to Choose the Optimal Lag Order in Stable and Unstable VAR Models. *Applied Economics Letters*, 2003, Vol. 10 (3),135-137.

Lütkepohl, H. (1985) "Comparison of Criteria for Estimating the Order of a Vector Autoregressive Process", *Journal of Time Series Analysis*, 6(1), pp. 35-52.

Lütkepohl, H. (1991) "Introduction to Multiple Time Series Analysis", *Springer-Verlag*, Berlin.

Nielsen, B. (2001) "Weak Consistency of criterions of Order Determination in a General Vector Autoregression", Department of Economics, University of Oxford.

Schwarz, G. (1978) "Estimation the Dimension of a Model", *Annals of Statistics*, 6, pp. 461-464.

Sims, C. (1980) "Macroeconomics and reality?", *Econometrica,* 48, pp. 1-18.

For Peer Review

Can LR Test Be Helpful in Choosing the Optimal Lag order in the VAR Model When Information Criteria Suggest Different Lag Orders?

Abstract

The objective of this simulation study is to investigate whether the likelihood ratio (LR) test can pick the optimal lag order in the vector autoregressive model when the most applied information criteria (i.e. vector *Schwarz-Bayesian, SBC, and vector Hannan-Quinn, HQC) suggest two different lag orders. This lag-choosing procedure has been suggested by Hatemi-J (1999). The results based on the Monte Carlo simulations show that combining the LR test with SBC and HQC causes a substantial increase in the success rate of choosing the optimal lag order compared to cases when only SBC or HQC are used. This appears to be the case irrespective of homoscedasticity or conditional heteroscedasticity properties of the error-term in small sample sizes. This improvement in choosing the right lag order also tends to improve the forecasting capability of the underlying model.*

.

Running title: Optimal Choice of the Lag Length in the VAR Model Using LR

JEL Classification: C32, C30

Keywords: VAR, Lag length, Information Criteria, Monte Carlo Simulations, Likelihood Ratio Test

For Peer Review

1. Introduction

One of the most applied models in the empirical economics is the vector autoregressive (VAR) model. In addition to its simplicity regarding estimation and interpretation, and its good forecasting capabilities, the VAR model treats all the variables of interest endogenously. This property is important since in macroeconomics exogenous variables are rare. The VAR models furthermore provide the possibility to investigate the causal relationship between the variables. It is also possible to transform the VAR model to a vector moving average (VMA) representation in order to trace the effects of the shocks on each variable in the system by calculating the impulse response functions and the variance decompositions. Due to the recent developments in dealing with integrated variables, the VAR model has been proven to be even more useful since it can be used to test for the long-run equilibrium relationship between the variables in combination with the short-run adjustment process. Obviously the VAR model is dynamic which accords with economic theory on a variety of topics. However, economic theory is usually not much of help regarding the length of the dynamic process. In the literature several lag choosing criteria have been proposed for this purpose. Three of the most used information criteria are the Akaike (1969) information criterion, (AIC), the Schwarz (1978) Bayesian criterion (SBC) and the Hannan and Quinn (1979) criterion (HQC). These information criteria were originally developed for single equations, but they can be extended in vector form to determine the lag order of systems of equations, i.e. VAR models. However, the choice of which of these criteria to use for determining the lag order in the VAR model is usually arbitrary in applied studies. Sometimes these information criteria do not agree in choosing the lag order. The question is then upon which information criterion one should rely.

Hatemi-J (1999, 2001) suggested using two of these criteria to choose the optimal lag length in the VAR model. If these two criteria choose different lag orders then the author suggested using the likelihood ratio (LR) test to choose between these two lags. It should be expressed that the LR test will be used only once in this case.[1] This means that the problem of mass-significance that occurs when the test is used sequentially can be avoided.[2] According to a simulation study performed by Hacker and Hatemi-J (2005) SBC has best performance in

[1] For an application of this approach see Hacker and Hatemi-J (2003).
[2] See also Hatemi-J (2003, 2006), and Bahmani-Oskooee and Brooks (2003).

For Peer Review

many cases. However, HQC has better performance in some cases. Thus, which criteria should be used is dependent on the data generating process for the variables. However, one cannot be sure about this issue when actual data is used because the true data generating process for the actual data cannot be known. This subject is important because inference in the VAR model is dependent on the choice of lag length. The purpose of this article is to evaluate the lag choosing procedure suggested by Hatemi-J (1999). Thus, the purpose of this study is to see whether the LR test can be useful in picking the optimal lag order of the VAR model when SBC and HQC suggest different lag orders. It should be pointed out that we will make use of many combinations of the parameters in the VAR model in the simulations in order to make the results as general as possible.

This paper is organized as follows. In the next section we will describe the VAR model and different criteria that can be used to determine the optimal lag order. Section 3 describes the design of our simulation. Section 4 presents the simulation results and conclusions.

2. The VAR Model and the Lag Choosing Criteria

Let us define the following VAR model, consisting of n variables that is characterized by an order less-than or equal to K:

$$Z_t = \Gamma D_t + \sum_{k=1}^{K} \beta_k Z_{t-k} + \varepsilon_t, \qquad t = 1, \cdots, T. \tag{1}$$

where ε_t is a $n\times 1$ vector of disturbance terms that are assumed to be independently identically distributed errors with the distribution $N_n(0, \Omega)$, β_k is a matrix of coefficients for Z_{t-k}, and D_t represents non-stochastic components such as constant terms, linear trend, or seasonal dummies. The initial values, $Z_{1-K}, \cdots, Z_0$, are assumed to be fixed. Our objective is to choose the largest order for the time series, denoted by $k_l \in K$, such that $\beta_{k_l} \neq 0$ and $\beta_j = 0$, $\forall\, j > k_l$. To accomplish choosing the optimal lag length in the VAR model the following general form for an information criterion can be applied:

$$IC = \ln\left(\det \hat{\Omega}_j\right) + j\frac{f(T)}{T}, \qquad j = 0, \cdots, K, \tag{2}$$

where $\hat{\Omega}_j$ is the maximum likelihood estimate of the variance-covariance matrix Ω when the lag order used in estimation is j. Logarithm is denoted by *ln* and *det* represents the determinant of the corresponding matrix.

For Peer Review

The objective is to estimate k_l by the j that minimizes the above criterion. Schwarz (1978) suggested $f(T) = n^2 \ln T$, whereas Hannan and Quinn (1979) preferred $f(T) = 2n^2 \ln(\ln T)$.[3]

The lag order of the VAR model can also be determined by testing the significance of parameters for each specific lag order. The likelihood ratio (LR) test that can be applied for this purpose, due to Sims (1980), is defined as the following:

$$LR = (T - c)\left|\log|\Omega_1| - \log|\Omega_2|\right| \tag{3}$$

where T is the sample size and c is the total number of parameters estimated in the VAR model under the alternative hypothesis. Ω_1 is the maximum likelihood estimate of the error-term variance-covariance matrix in the VAR model under the null hypothesis and Ω_2 is the maximum likelihood estimate of the error-term variance-covariance matrix in the VAR model under the alternative hypothesis. Under the null hypothesis the LR test is chi-square distributed with the degrees of freedom equal to the number of restriction that are tested.

3. The Simulation Design

We make use of the following bivariate VAR model, which is of order two, in our simulations:

$$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix} + \begin{bmatrix} \beta_{1,11} & \beta_{1,12} \\ \beta_{1,21} & \beta_{1,22} \end{bmatrix} \begin{bmatrix} z_{1t-1} \\ z_{2t-1} \end{bmatrix} + \begin{bmatrix} \beta_{2,11} & \beta_{2,12} \\ \beta_{2,21} & \beta_{2,22} \end{bmatrix} \begin{bmatrix} z_{1t-2} \\ z_{2t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}, \tag{4}$$

The error terms vector $\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$ are designed to be independent and homoscedastic or conditionally heteroscedastic. In the conditionally heteroscedastic situation the simulations are run when the variance of the error terms can be described by the following autoregressive conditional heteroscedasticity (ARCH)[4]:

[3] Nielsen (2001) shows that SBC and HQC are consistent regardless of the assumption about the characteristic roots in the VAR model. By consistency is meant that the criterion selects the true order of the VAR system with probability one asymptotically.

[4] In this formulation of multivariate ARCH the conditional and unconditional variances are equal to each other asymptotically. This seems to be a necessary condition in order to make sure that the comparison of our simulation results for homoscedastic and conditionally heteroscedastic cases makes sense. A mathematical derivation of equation (5) is provided by Hatemi-J (2004). For a test of multivariate ARCH effects in the VAR model the interested reader is referred to Hacker and Hatemi-J (2005).

For Peer Review

$$\begin{bmatrix} \sigma_{1t}^2 \\ \sigma_{2t}^2 \end{bmatrix} = \begin{bmatrix} 0.5. \\ 0.5 \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} \times \begin{bmatrix} \varepsilon_{1t-1}^2 \\ \varepsilon_{2t-1}^2 \end{bmatrix}, \tag{5}$$

where σ_{it}^2 represents the conditional variance at time t for variable i ($i = 1, 2.$). To cancel the effect of starting up values, we generated 100 presample observations. This gives us the possibility to have the same number of observations in estimating the VAR model regardless of the number of lags.

A central issue in a Monte Carlo simulation like the current one is choosing a variety of parameters that as a group has characteristics that fairly represent those of the infinite space of possible parameters. In order to obtain general results we consider all the combinations shown in Table 1 for the coefficient matrices. There are 12500 (5×5×5×5×4×5) possible combinations of the elements in this table. The VAR model is always of the second order since $\beta_{2,22}$ is never zero.

Table 1.

Parameter Values for VAR model of equation (4)

$\beta_{1,11}$	-1	0.5	0	0.5	1
$\beta_{1,22}$	-0.5	-0.25	0	0.25	0.5
$\beta_{1,12} = \beta_{1,21}$	-0.5	-0.1	0	0.1	0.5
$\beta_{2,11}$	-0.8	-0.2	0	0.2	0.8
$\beta_{2,22}$	-0.6	-0.1		0.1	0.6
$\beta_{2,12} = \beta_{2,21}$	-0.5	-0.1	0	0.1	0.5

To assure that we have stable cases we make sure that the modulus (the square root of the summed squares of the real and imaginary eigenvalue components) of each eigenvalue of the following companion matrix (B) is less than one.

$$B = \begin{bmatrix} \beta_{1,11} & \beta_{1,12} & \beta_{2,11} & \beta_{2,12} \\ \beta_{1,21} & \beta_{1,22} & \beta_{2,21} & \beta_{2,22} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tag{6}$$

We run simulations based on small sample sizes (T = 40). It should be pointed out that we have run 10000 simulations for each combination of the parameters. This means that the total

For Peer Review

number of the simulations is 125000000. The simulations are performed by a program procedure written by the authors in GAUSS.[5]

We investigate whether performance on lag length choice using either criterion can be improved by employing the *LR* test (with 5% significance for null hypothesis rejection) when the two criteria disagree. Since VAR models are extensively used for forecasting purposes, we also investigate whether combining the *LR* test with the two information criteria can result in determining lag orders that can result in more accuracy in forecasts. Assume that $E(z_{i,T+h})$ is the forecast of variable z_i, $i = 1,2$, for h periods into the future that we would make if we know the actual parameters of the VAR model (assuming errors are equal to zero for future periods). This forecast is equivalent to the expected value of $z_{i,T+h}$ based on the information available in the last observed period T. Let us denote $\hat{z}_{i,T+h}$ the forecast of variable z_i for h periods into the future that we make using the estimated parameters based on the lag length chosen by the criterion. Finally, let $\bar{z}_{i,T+h}$ denote the forecast of variable z_i for h periods into the future that is made based on the estimated parameters using always the right lag length of two. Then, the sum of squared errors ratio, *SSER*, can be calculated for a particular case (set of parameters in a scenario) by the following equation:

$$SSER = \frac{\sum_S \left[\hat{z}_{i,T+h} - E\left(z_{i,T+h}\right)\right]^2}{\sum_S \left[\bar{z}_{i,T+h} - E\left(z_{i,T+h}\right)\right]^2}, \qquad (7)$$

where S denotes the set of 1000 simulations for a particular case, so in the numerator and denominator we are summing up over the simulations. In the numerator we show the systematic sum of squared errors of the forecasted variable based on forecasts using lags chosen by the criterion.[6] In the denominator we show the systematic sum of squared errors of

[5] Notice that SBC and HQC choose different lag orders more frequently in small sample sizes. In large sample sizes (asymptotically) both information criteria are expected to choose the same lag order and there will be no need for using the LR test in such cases. That is why we concentrate on small sample sizes in our simulations. However, we also conducted simulations for a sample size of 70. The results, not presented to save space, showed similar qualitative results.

[6] Lütkepohl (1985, 1991) handles his presentation of forecast capability in a different fashion. He focuses not on the systematic forecast error, $\hat{z}_{i,T+h} - E(z_{i,T+h})$, but on the overall forecast error $\hat{z}_{i,T+h} - z_{i,T+h}$, where $z_{i,T+h}$ is an actualized outcome of z_i for h periods into the future. He thus also includes in his focus the unsystematic (random) components of the forecast error, $E(z_{i,T+h}) - z_{i,T+h}$, which injects additional randomness that we find undesirable for comparison since it is unsystematic. For his presentation in his 1985 article he normalizes (divides) an approximation of the mean squared overall forecast error with the theoretical variance of $E(z_{i,T+h}) - z_{i,T+h}$.

For Peer Review

a forecasted variable using always the correct lag length. The lower *SSER* is, the better the forecasting is in comparison to forecasts based on knowing the correct lag length. If the chosen criterion does just as well in forecasting as when the correct lag length is always chosen, then this ratio would be one. For any particular scenario we calculate the average *SSER*, i.e. the mean of the *SSER* over all the cases and over the two variables, z_1 and z_2.

4. The Results of the Simulations and the Conclusions

In Table 2 we present the frequency distributions in the presence and absence of conditional heteroscedasticity. We can see that the two criteria perform differently and which performance is better depends on the circumstances. However, when we combine these two criteria with the *LR* test for picking the true lag order in the VAR model the percentage choosing the optimal order increases substantially compared to cases when only one criterion is utilized, particularly for small sample sizes. This is true whether conditional heteroscedasticity is present or not. The same conclusion can be drawn regarding the forecasting properties (see Table 3). Using the *LR* test combined with SBC and HQC to choose the optimal lag order in the VAR model is going to result in choosing lag orders that are going to result in more precise forecasts compared to cases when only SBC and HQC are used.

Table 2.

Results for picking the optimal lag order based on simulations for a VAR(2) model with and without autoregressive conditional heteroscedasticity (ARCH).

Info criterion & variance situation	Lag length							
	0	1	2	3	4	5	6	7
	Frequency distribution of estimated VAR orders, *without ARCH*							
HQC,	0.019	0.031	0.860	0.056	0.018	0.007	0.005	0.004
SBC	0.066	0.070	0.850	0.013	0.001	0.000	0.000	0.000
HQC, SBC and LR	0.021	0.050	0.908	0.018	0.003	0.000	0.000	0.000
	Frequency distribution of estimated VAR orders, *with ARCH*							
HQC,	0.018	0.032	0.833	0.078	0.022	0.008	0.005	0.004
SBC	0.060	0.069	0.846	0.022	0.002	0.000	0.000	0.000
HQC, SBC and LR	0.019	0.051	0.897	0.028	0.004	0.000	0.000	0.000

For Peer Review

Table 3.

Results for forecasting performance (average *SSER*) based on simulations for a VAR(2) model with and without autoregressive conditional heteroscedasticity (ARCH).

Forecasting Period	Forecasting Error for HQC	
Forecasting Period	Without ARCH	With ARCH
1	1.249	1.334
2	1.267	1.422
3	1.240	1.403
4	1.229	1.453
5	1.194	1.877
Forecasting Period	Forecasting Error for SBC	
1	1.283	1.254
2	1.285	1.257
3	1.139	1.149
4	1.111	1.149
5	1.063	1.181
Forecasting Period	Forecasting Error for HQC, SBC and LR	
1	1.141	1.162
2	1.147	1.176
3	1.089	1.131
4	1.073	1.148
5	1.049	1.18

Acknowledgment

We would like to thank the editor of this journal and an anonymous referee for useful comments and suggestions that resulted in an improvement of this paper. However, the usual disclaimer applies.

For Peer Review

REFERENCES

Akaike, H. (1969) "Fitting Autoregressions for Prediction", *Annals of the Institute of Statistical Mathematics*, 21, pp. 243-247.

Bahmani-Oskooee, M. and Brooks, T. J. (2003) A New Criteria for Selecting the Optimum Lags in Johansen's Cointegration Technique, *Applied Economics*,. 35(8), 875-80.

Hacker, R. S. and Hatemi-J, A. (2003) How Productivity and Domestic Output Are Related to Exports and Foreign Output in the Case of Sweden. *Empirical Economics*, 28(4), 767-782.

Hacker, R. S. and Hatemi-J, A. (2005) A Test for Multivariate ARCH Effects. *Applied Economics Letters*, 12(7), 411-417.

Hacker, R. S. and Hatemi-J, A. (2006) "Optimal Lag Length Choice in Stable and Unstable VAR Models under Situations of Homoscedasticity and Heteroscedasticity". *Journal of Applied Statistics*, forthcoming.

Hannan, E. J., and Quinn, B.G. (1979) "The Determination of the Order of an Autoregressive," *Journal of the Royal Statistical Society*, B41, 190-195.

Hatemi-J, A. (1999) "Essays on the Use of VAR Models in Macroeconomics, Thesis in Econometrics, Department of Economics, Lund University.

Hatemi-J, A. (2001) "Time-Series Econometrics Applied to Macroeconomic Issues", Jonkoping International Business School's Dissertation Series, Nr. 007, Jonkoping University.

Hatemi-J, A. (2003) A New Method to Choose the Optimal Lag Order in Stable and Unstable VAR Models. *Applied Economics Letters*, Vol. 10 (3), 135-137.

Hatemi-J, A. (2004) Multivariate Tests for Autocorrelation in the Stable and Unstable VAR Models. *Economic Modelling*, Vol. 21(4), 661-683.

Hatemi-J, A. (2006) Forecasting Properties of a New Method to Determine the Optimal Lag Order in Stable and Unstable VAR Models. *Applied Economics Letters*, forthcoming.

Lütkepohl, H. (1985) "Comparison of Criteria for Estimating the Order of a Vector Autoregressive Process", *Journal of Time Series Analysis*, 6(1), pp. 35-52.

Lütkepohl, H. (1991) "Introduction to Multiple Time Series Analysis", *Springer-Verlag*, Berlin.

Nielsen, B. (2001) "Weak Consistency of Criterions of Order Determination in a General Vector Autoregression", Department of Economics, University of Oxford.

Schwarz, G. (1978) "Estimation the Dimension of a Model", *Annals of Statistics*, 6, 461-464.

Sims, C. (1980) "Macroeconomics and reality?", *Econometrica,* 48, 1-18.

For Peer Review