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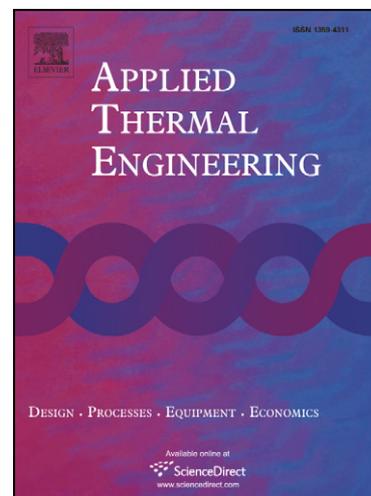
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**THEORETICAL ANALYSIS OF A CO₂-NH₃ CASCADE REFRIGERATION SYSTEM FOR
COOLING APPLICATIONS AT LOW-TEMPERATURES**

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Abstract

As a result of environmental problems related to global warming and depletion of the ozone layer caused by the use of synthetic refrigerants (CFC's, HCFC's and HFC's) experienced over the last decades, the return to the use of natural substances for refrigeration purposes, appears to be the best long-term alternative. In this paper a cascade refrigeration system with CO₂ and NH₃ as working fluids in the low and high temperature stages respectively, has been analyzed. Results of COP and exergetic efficiency versus operating and design parameters have been obtained. In addition, an optimization study based on the optimum CO₂ condensing temperature has been done. Results show that following both method's exergy analysis and energy optimization, an optimum value of condensing CO₂ temperature is obtained. The compressor isentropic efficiency influence on the optimum system COP has been demonstrated. A methodology to obtain relevant diagrams and correlations to serve as a guideline for design and optimization of this type of systems has been developed and it is presented in the paper.

Keywords: Cascade refrigeration / Compression system / CO₂-NH₃ / Natural refrigerants / Optimization

Nomenclature

A	Area	m^2
COP	Coefficient of performance	
DT	Temperature difference in the cascade heat exchanger	K
h	Specific enthalpy	$kJ \cdot kg^{-1}$
\dot{m}	Mass flow rate	$kg \cdot s^{-1}$
P	Pressure	kPa
\dot{Q}	Heat transfer rate	kW
RC	Compressor pressure ratio (discharge/suction)	
T	Temperature	K
U	Overall heat transfer coefficient	$kW \cdot m^{-2} \cdot K^{-1}$
\dot{W}	Power	kW
\dot{X}	Exergy lost rate	kW
Greek symbols		
η	Efficiency	
η_{II}	Exergetic efficiency	
ψ	Stream specific exergy	$kJ \cdot kg^{-1}$
Subscripts		
Act	Actual	
CHE	Cascade heat exchanger	
CO_2	Carbon dioxide	
$Comp$	Compressor	
$Cond$	Condenser, condensation	
$Elec$	Electrical	
$Evap$	Evaporator, evaporation	
Exp	Expansion device	
F	Cooling space	

<i>Max</i>	Maximum
<i>Mec</i>	Mechanical
<i>NH3</i>	Ammonia
<i>Opt</i>	Optimum
<i>Rev</i>	Reversible
<i>s</i>	Isentropic
<i>0</i>	Ambient

ACCEPTED MANUSCRIPT

1. INTRODUCTION

As a result of environmental problems related to global warming and depletion of the ozone layer caused by the use of synthetic refrigerants (CFC's, HCFC's and HFC's) experienced over the last decades, the return to the use of natural substances for refrigeration purposes appears to be sound practice. It must be a better solution to use naturally existing and environmentally harmless substances as alternatives refrigerants in refrigeration systems.

Amongst the natural refrigerants, Lorentzen and Petterson [1] suggested the use of carbon dioxide (CO_2) and seems to be the most promising one especially as the natural refrigerant [1-6]. The key advantages of CO_2 include the fact that is not explosive, non-toxic, easily available, environmental friendly and has excellent thermo-physical properties.

On the other hand, researches in Norway in 1993 showed that the refrigerant leakages coming from the commercial sector were 30% of the annual total [7]. In this research, the use of a cascade system using CO_2 in the low temperature stage and NH_3 in the high temperature stage turned out to be an excellent alternative for cooling applications at very low temperatures [8-10]. Researches from Eggen and Aflekt [11], Pearson and Cable [12] and Van Riessen [13] show practical examples of the use of a cascade system of CO_2/NH_3 for cooling in supermarkets. Eggen and Aflekt [11] developed research based on a prototype of a cooling system built in Norway. Pearson and Cable [12] showed data from a cooling system used in a Scottish supermarket line, (ASDA), and Van Riessen [13] carried out technical energy and economic research of a cooling system used in a Dutch supermarket.

In the same way, different researches about the performance of different cooling systems involving CO_2 have been carried out together with its reuse as a refrigerant fluid. Lorentzen and Petterson [1] evaluated the possibility of the use of a heat exchanger in a CO_2 transcritical system. Hwang et al. [6] showed experimental results and simulation research including expanders and double stage cycles. Groll et al. [14] carried out a numerical analysis of a double stage cycle changing the compression ratio of each compression stage.

Bhattacharyya et al. [15] showed an optimization research of the $\text{CO}_2/\text{C}_3\text{H}_8$ cascade system for cooling and heating. Kim et al. [16] summed up the improvements in the performance of systems based in CO_2 and their applications. They provided a critical review of literature, and discussed important trends and characteristics in the development of CO_2 technology in refrigeration applications.

Recently, studies including a wide range of possibilities in the use of CO_2 as refrigerant have been published. Deng et al. [17] described a theoretical analysis of a transcritical CO_2 ejector expansion refrigeration cycle, using an ejector as the main expansion device instead of an expansion valve. Youngming et al. [18] constructed and tested a wet-compression absorption carbon dioxide refrigeration system. Fernández-Seara et al. [19] analyzed a compression-absorption cascade refrigeration system considering CO_2 and NH_3 as refrigerants in the compression stage and the pair $\text{NH}_3\text{-H}_2\text{O}$ in the absorption stage and evaluated the possibilities of powering the cascade refrigeration system by means of a cogeneration system. Lee et al. [20] carried out a thermodynamic analysis of optimal condensing temperature of cascade-condenser in CO_2/NH_3 cascade refrigeration systems. In this work, effects of isentropic compressors efficiencies and practical limit of the compressors discharge temperatures were not taking into account and the exergetic efficiency was not evaluated.

The scope of the present research is focused on the analysis of the parameters of design and operation of a CO_2/NH_3 cascade cooling system and their influence over the system's COP and exergetic efficiency. The statistical significance of each of the parameters evaluated has been analysed. Moreover, optimization research of these parameters has been included in order to show highest COP. Finally, a discussion about the effect of the compressors' isentropic efficiency on the optimum system COP is presented.

2. SYSTEM DESCRIPTION

A schematic diagram of the cascade system is shown in figure 1. The cascade refrigeration system is constituted by two single stage systems connected by a heat exchanger (cascade heat exchanger). The low temperature system with CO_2 as refrigerant is used for

cooling. The high temperature system with NH_3 as refrigerant is used to condensate the CO_2 of the low temperature system.

In the evaporator, the CO_2 at the evaporating temperature absorbs the cooling duty $\dot{Q}_{\text{Evap CO}_2}$ from the cooling space (at T_F temperature), then is compressed in the CO_2 compressor and condensed in the cascade heat exchanger at a condensing temperature of $T_{\text{Cond CO}_2}$, and then sent to the expensor from which the evaporator is supplied.

In the condenser, the heat flow $\dot{Q}_{\text{Cond NH}_3}$ is removed from the NH_3 at condensing temperature of $T_{\text{Cond NH}_3}$ to condensing medium (at T_0 temperature). The NH_3 is expanded, then evaporated at an evaporating temperature of $T_{\text{Evap NH}_3}$ in the cascade heat exchanger, and then compressed in the NH_3 compressor and discharged into the condenser.

Figure 2 shows the process evolutions for both the CO_2 and NH_3 cycles in a logP-h diagram. Saturation lines are included.

3. MATHEMATICAL MODEL

3.1. Model

To calculate the compressors' powers, heat transfer rates and energetic and exergetic efficiencies, each cascade system component is considered as a control volume at stationary flow. Mass balance, energy balance and exergy balance have been performed.

The following assumptions are made in the analysis:

- Refrigerants at the cascade heat exchanger outlets, condenser outlet and evaporator outlet, are saturated.
- Pressure losses in connecting pipes and heat exchangers have been neglected.
- Cascade heat exchanger and pipes are perfectly isolated.
- The dead state (ambient) is $T_0 = 25 \text{ }^\circ\text{C}$ and $P_0 = 1 \text{ atm}$.
- The difference between the refrigerated space temperature (T_F) and the

evaporation temperature ($T_{Evap\ CO_2}$) is constant and equal to 5 °C.

Taking into account the assumptions previously made, the mass, energy and exergy balances are given by Eqs. 1, 2 and 3, respectively.

Mass balance

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \quad (1)$$

Energy balance

$$\dot{Q} - \dot{W} = \sum_{out} \dot{m} \cdot h - \sum_{in} \dot{m} \cdot h \quad (2)$$

Exergy balance

$$\dot{X}_{Lost} = \sum_{out} \left(1 - \frac{T_0}{T_j} \right) \cdot \dot{Q}_j - \dot{W} + \sum_{in} \dot{m} \cdot \psi - \sum_{out} \dot{m} \cdot \psi \quad (3)$$

In Table 1 specifics equations for each system's components are summarized.

Compressor isentropic efficiency was calculated using the Eqs. 4 and 5 for each case.

$$\eta_{s\ Comp\ CO_2} = \frac{(h_{2s} - h_1)}{(h_2 - h_1)} \quad (4)$$

$$\eta_{s\ Comp\ NH_3} = \frac{(h_{6s} - h_5)}{(h_6 - h_5)} \quad (5)$$

Overall compressor's efficiency has been determined with the Eq. 6.

$$\eta_{Comp} = \eta_s \cdot \eta_{Mec} \cdot \eta_{Elec} \quad (6)$$

The lowest temperature difference (DT) in the cascade heat exchanger is formulated by Eq. 7.

$$DT = T_{Cond\ CO_2} - T_{Evap\ NH_3} \quad (7)$$

The system's COP has been calculated by the Eq. 8.

$$COP = \frac{\dot{Q}_{Evap\ CO_2}}{\dot{W}_{Comp\ CO_2} + \dot{W}_{Comp\ NH_3}} \quad (8)$$

The system's exergetic efficiency is given by Eq. 9.

$$\eta_{II} = \frac{\dot{W}_{Rev}}{\dot{W}_{Act}} \quad (9)$$

where \dot{W}_{Rev} and \dot{W}_{Act} are the system's reversible power input and actual power input, respectively [21].

The system's reversible power input is given by Eq. 10.

$$\dot{W}_{Rev} = \dot{Q}_{Evap\ CO_2} \cdot \left(\frac{T_0}{T_{Evap\ CO_2}} - 1 \right) \quad (10)$$

3.2. Parametric study

The equations of the mathematical model reveal that both the system's COP and its exergetic efficiency can be expressed as a function of six design/operating parameters, as shown in Eq. 11.

$$(COP, \eta_{II}) = f(T_{Evap\ CO_2}, T_{Cond\ NH_3}, T_{Cond\ CO_2}, DT, \eta_{Comp\ CO_2}, \eta_{Comp\ NH_3}) \quad (11)$$

To determine the influence of each design/operating parameter on the system's COP and its exergetic efficiency a parametric study was done. The ranges of values for each parameter comprise the specific intervals of interest. $T_{Evap\ CO_2}$, $T_{Cond\ NH_3}$ and $T_{Cond\ CO_2}$ were varied from -55 to -30 °C, from 25 to 50 °C and from -25 to 5 °C, respectively, at intervals of 5 °C. The temperature difference in the cascade heat exchanger was varied from 3 to 6 °C. Moreover, a prototype cascade refrigeration system (in construction at this moment in our laboratory) was taken into account as reference. This prototype has been supplied with a semi-hermetic compressor to compress the CO_2 and an open type compressor was selected to compress the NH_3 . Due to the compressors selection, were considered 0.9 and 0.95 the values of mechanical and electrical efficiency (η_{Mec} , η_{Elec}) to the CO_2 compressor and 0.9 for both efficiencies (η_{Mec} , η_{Elec}) to the NH_3 compressor.

The isentropic efficiency of each compressor is considered equal to the volumetric efficiency and it is estimated according to Eq. 12.

$$\eta_s = 1 - 0.04 \cdot RC \quad (12)$$

To solve each case derived from the parametric study specific software developed by Sieres and Fernández-Seara [22] has been employed. The thermodynamic properties of CO₂ and NH₃ were calculated from REFPROP [23].

3.3. Model validation

Reference [24] reports experimental data from a NH₃/CO₂ cascade system prototype for refrigeration in supermarkets. This prototype includes a CO₂ circuit at medium temperature, thus, in this case the COPs of the low and high stages are compared separately. The experimental data for the CO₂ stage are: $T_{\text{Evap CO}_2} = -26$ °C with 7 °C superheat at the evaporator outlet, $T_{\text{Cond CO}_2} = -9$ °C assuming no subcooling, $\eta_s = 0.21$ due to a deficient compressor operation, $\eta_{\text{Mec}} = 0.93$ and $\eta_{\text{Elec}} = 0.8$. Experimental data for the NH₃ stage are: $T_{\text{Evap NH}_3} = -11$ °C, $T_{\text{Cond NH}_3} = 32$ °C, no superheating and subcooling are assumed, $\eta_s = 0.76$, $\eta_{\text{Mec}} = 0.93$ and $\eta_{\text{Elec}} = 0.8$. The cooling duty at the medium temperature level is considered at this stage. The model results provide a relative error with respect to the experimental COPs of 1.8% and 7.1% for the CO₂ and NH₃ stages, respectively. The results of the model validation with the experimental data are considered satisfactory.

4. RESULTS AND DISCUSSION

4.1. Exergy losses

To evaluate the exergy losses of each system's components and the exergy loss rate of the whole system a parametric study was applied at prototype specified design conditions: the system's cooling capacity is 9 kW, the CO₂ evaporating temperature ($T_{\text{Evap CO}_2}$) -50 °C, the NH₃ condensing temperature ($T_{\text{Cond NH}_3}$) 40 °C and the cascade heat exchanger temperature difference (DT) 5 °C. Figure 3 shows the exergy loss rate of each system's components (left axe) and the exergy loss rate of the whole system (right axe). It can be clearly appreciated that the highest exergy loss rate occurs at the lowest CO₂ condensing temperature, and decreasing trends are present in all the NH₃ system's components. In the CO₂ compressor case, the CO₂ throttle and the cascade heat exchanger present an increasing tendency with $T_{\text{Cond CO}_2}$

increments. The exergy loss rate in the CO₂ evaporator tends to be constant. In some of the system's components the exergy loss rate increases whilst in others the exergy loss rate decreases with T_{Cond CO2} increments. This behaviour has been observed after evaluating the whole system whereby there existed initially decreasing trends up to a minimum exergy loss rate at T_{Cond CO2} -5 °C approximately, and then increasing trends appear. Therefore an optimum value of T_{Cond CO2} is found.

4.2. System COP and exergetic efficiency trends

To evaluate the influence of the operating parameters on both the system's COP and exergetic efficiency, a statistical procedure has been used to analyse the parametric study results obtained considering the ranges of values indicated in section 3.2. This statistical procedure is the Anova Multifactorial [25]. In Table 2 the results of the "P-Value" statistical factor can be observed. This factor tests the statistical significance of each of the operating parameters evaluated. Since all P-values are less than 0.05, all the evaluated parameters have a statistically significant effect at the 95% confidence level. Therefore all the parameters considered in Eq. (11) should be included in the analysis and none can be discarded.

Figures 4 and 5 depict the system's COP and exergetic efficiency average values and the ranges of values resulting from the parametric studies for each one of the evaluated parameters. Therefore, these figures show the maximum and minimum values of the COP and exergetic efficiency for the entire region analysed, as well as their average value for each parameter when varying all others simultaneously. Thus, the results and conclusions obtained from this analysis have applicability over the entire region studied.

In figure 4, the system's COP and the exergetic efficiency trends with increases in T_{Evap CO2} (a), T_{Cond NH3} (b) and DT (c) are represented. In the first case (a) the COP average value increases 70% within the T_{Evap CO2} range due to the increase of the COP at the CO₂ stage. Moreover, as the T_{Evap CO2} increases, the influence of the other parameters on the COP also increases, as can be seen in figure 4a. However, in the other two cases a decreasing tendency is observed caused by the COP decrease at the NH₃ stage. The COP average value diminishes 45% and 9% within the ranges of T_{Cond NH3} and DT analyzed, respectively. Thus, the results

shown in figure 4 reveal that the $T_{\text{Evap CO}_2}$ has a greater effect than $T_{\text{Cond NH}_3}$ on the system COP; meanwhile the influence of DT is small. In figure 4 the exergetic efficiency trends are also depicted. There is not a clear overall tendency in the case of $T_{\text{Evap CO}_2}$. (a). In this case there is an initial increasing tendency up to -40 °C then the change tends to decrease for average values, while maximum values increase and minimum values decrease continuously. However, the exergetic efficiencies show a decreasing tendency with increases in $T_{\text{Cond NH}_3}$ (b) and DT (c), around 45% and 9%, respectively. The influence of DT on the U·A product in the cascade heat exchanger is also included in figure 4c. The results are given as increments in percentage with respect to the case of $DT=5$ °C. If a specific type of heat exchanger is selected, these results will evaluate the increase in size, and consequently in cost, of the cascade heat exchanger.

Figure 5 shows both COP and exergetic efficiency behaviour versus $T_{\text{Cond CO}_2}$ variations. In both cases increasing trends are present initially up to -5 °C, after which the change tends to decrease. These trends occur for the average, maximum and minimum values. Thus, these results indicate that there is a $T_{\text{Cond CO}_2}$ optimal value. The COP of the NH_3 stage increases with $T_{\text{Cond CO}_2}$ whereas the COP of the CO_2 stage decreases. Consequently, a maximum COP exists that corresponds to the $T_{\text{Cond CO}_2}$ optimal value. This optimal value also corresponds to the minimum found in section 4.1 for the exergy loss rate of the whole system, as can be seen in figure 3.

4.3. Optimization

From the results obtained a clear viewpoint of the method to obtain a maximum COP value is proven.

The CO_2 evaporating temperature has to be as high as possible in order to obtain the highest COP. This temperature depends on the environmental temperature (T_F) that has to be cooled (system operating parameter) and the difference between temperatures established as a design parameter in the evaporator, between the refrigerant fluid and the environmentally cooled temperature. It is therefore clear that for a specific application, optimization has to be the maximum reduction of this temperature difference.

On the other hand, the NH_3 condensing temperature has to be as low as possible to

increase the COP to the maximum. This temperature is a function of the condensing environment and the difference of the temperatures between the condensing environment (T_0) and the NH_3 temperature established as a design parameter of the condenser. If the environmental condensing temperature can not be modified because the system is subjected to a particular situation, the only remedy is to decrease the temperature differences in order to obtain the higher COP increase.

As for temperature differences in the cascade exchanger, DT, results always show that for a lower DT a higher COP is obtained.

Differing from the previous parameter, the CO_2 optimum condensing temperature does not present a continuous trend (decreasing or increasing). There exists a value for this temperature that defines the highest COP, so we are talking about a “ CO_2 optimum condensing temperature ($T_{\text{Cond CO}_2 \text{ Opt}}$)”. Another temperature value entailed a lower COP.

So for a specific value in the DT parameter design, the CO_2 optimum condensing temperature value could be established as a function depending only on the CO_2 evaporating temperature and the NH_3 condensing temperature. As a result the maximum COP value is set as a function of the established parameters.

Through a regressive analysis over the obtained results, two correlations have been established, Eqs. 13 and 14, the optimum values for $T_{\text{Cond CO}_2}$ and maximum COP as a function of CO_2 evaporating temperature, NH_3 condensing temperature and the DT temperature differences, can be determined.

$$T_{\text{Cond CO}_2 \text{ Opt}} = -218.78 + 0.3965 \cdot T_{\text{Evap CO}_2} + 0.39064 \cdot T_{\text{Cond NH}_3} + 0.60747 \cdot DT \quad (13)$$

$$\text{COP}_{\text{Max}} = \left(\frac{-27.76 + 0.15944 \cdot T_{\text{Evap NH}_3}}{-15.33 + 0.06438 \cdot T_{\text{Cond CO}_2}} \right) - 0.03279 \cdot DT - 0.3422 \quad (14)$$

The proposed correlations (Eqs. 13 and 14) explain 99.45% and 99.21% of the variability (R^2 statistic) in $T_{\text{Cond CO}_2 \text{ Opt}}$ and COP, respectively. The mean absolute errors between the raw data and the predictions are 0.3 and 0.025 for the $T_{\text{Cond CO}_2 \text{ Opt}}$ and COP, respectively.

Eqs. 13 and 14 contribute important information for a cascade system such as the one studied. It is important to emphasize that Eq. 13 gives the optimum value of the CO₂ condensing temperature for specific values of the CO₂ evaporating temperature and the NH₃ condensing temperature which can be replaced (previously defining the values of the temperature differences in the evaporator and condenser) by the cooled environment temperature and the condensed environment temperature, respectively. On the other hand, the maximum possible COP that the system can reach, operating per a specific situation, is determined by Eq. (14). This maximum value of COP obviously corresponds to a CO₂ optimum condensing temperature value. Using Eqs. 13 and 14, the lines corresponding to the iso-values of the CO₂ optimum condensing temperature and the system's maximum COP have been drawn, for DT = 5 °C, figures 6 and 7, respectively. A shadowed area is included in both figures, which corresponds to the operating system's conditions where the CO₂ or the NH₃ compressors discharge temperature is higher than 120 °C. If the discharge temperature is above this practical limit, usability and stability of the associated lubricant oil may become a severe problem after a long-term operation [26].

From figure 6, it is obvious that the CO₂ optimum condensing temperature increases with the CO₂ evaporating temperature and the NH₃ condensing temperature increases. In figure 7 it can be observed that the system's maximum COP increases with the CO₂ evaporating temperature increase, and the NH₃ condensing temperature decrease.

4.4. Effect of the compressor isentropic efficiency on system COP

For all the cases previously studied, the compressor isentropic efficiency was considered as a function of the compression ratio of each one using Eq. 12. Different empirical correlations to calculate the compressor's isentropic efficiency can be found in existing literature. Eqs. 15, 16 and 17; Eqs. 18 and 20; and Eq. 19 were obtained from [27], [20] and [28], respectively. Moreover, if a specific installation or prototype is considered the isentropic efficiency equation for each compressor will be determined from the manufacturer data.

For a CO₂ compressor

$$a. \eta_s = 1.003 - 0.121 \cdot RC \quad (15)$$

$$b. \eta_s = 0.9343 - 0.04478 \cdot RC \quad (16)$$

$$c. \eta_s = 0.815 + 0.022 \cdot RC - 0.0041 \cdot RC^2 + 0.0001 \cdot RC^3 \quad (17)$$

$$d. \eta_s = 0.89810 - 0.09238 \cdot RC + 0.00476 \cdot RC^2 \quad (18)$$

For an NH₃ compressor

$$m. \eta_s = 0.976695 - 0.0366432 \cdot RC + 0.0013378 \cdot RC^2 \quad (19)$$

$$n. \eta_s = 0.83955 - 0.01026 \cdot RC - 0.00097 \cdot RC^2 \quad (20)$$

In figure 8 the system's COP evolution versus $T_{\text{Cond CO}_2}$ depicts the use of different correlation combinations to calculate the isentropic efficiency of each compressor. Again, a shadowed area is included to represent the operating system's conditions when the CO₂ or the NH₃ compressors discharge temperature is higher than 120 °C. It can be appreciated that in each case, a different value of CO₂ optimum condensing temperature exists. These values are included in a range from -25 °C (a-m correlations combination case) up to -5 °C (initial correlation considered -Eq. 12- in both compressors). Results in figure 8 clearly show the significant influence of the compressors isentropic efficiency on the optimal CO₂ condensing temperature. Therefore, it is concluded that these efficiencies should be evaluated as accurately as possible and taken into account in the analysis of a cascade refrigeration system.

5. CONCLUSIONS

In this paper, the analysis of the parameters of design and operation of a CO₂/NH₃ cascade cooling system and their influence over the system's COP and exergetic efficiency is reported. The analysis was carried out based on a general mathematical model that was validated using experimental data found in the literature. The system's COP and its exergetic efficiency can be expressed as a function of six design/operating parameters. To determine the influence of each parameter a parametric study was done considering the specific intervals of interest. A statistical procedure has been used to analyse the parametric study results obtained. The analysis reveals that all the evaluated parameters have a statistically significant effect and should be taken into account.

The results show that the COP increases 70% when the $T_{\text{Evap CO}_2}$ varies from $-55\text{ }^{\circ}\text{C}$ to $-30\text{ }^{\circ}\text{C}$. As $T_{\text{Evap CO}_2}$ increases, the influence of the other parameters on the COP also increases. The COP diminishes 45% when the $T_{\text{Cond NH}_3}$ increases from $25\text{ }^{\circ}\text{C}$ to $50\text{ }^{\circ}\text{C}$. The system COP diminishes 9% when DT varies from $3\text{ }^{\circ}\text{C}$ to $6\text{ }^{\circ}\text{C}$. The exergetic efficiency decreases around 45% and 9% with the increases indicated above in $T_{\text{Cond NH}_3}$ and DT, respectively. In the case of $T_{\text{Evap CO}_2}$ there is an initial increasing tendency up to $-40\text{ }^{\circ}\text{C}$ then the change tends to decrease.

For a specific system and operating conditions, results show that following both, exergy analysis and energy optimization methods, an optimum value of condensing CO_2 temperature is obtained. The CO_2 optimum condensing temperature value increases when CO_2 evaporating and/or NH_3 condensing temperatures increase. Diagrams and correlations to demonstrate the evaluations of the maximum COP and CO_2 optimum condensing temperatures for the specific conditions considered in the analysis have been developed. On the other hand, it was established that the isentropic compressors efficiency influences the maximum COP and the CO_2 optimum condensing temperature calculations. Different empirical correlations to calculate the compressor's isentropic efficiency that can be found in existing literature were used to demonstrate the influence of these parameters on the maximum COP and CO_2 optimum condensing temperature. Based on the results shown in the paper, it is concluded that, if a specific installation or prototype is considered, the isentropic efficiency for each compressor in the cascade system should be determined as accurately as possible from the manufacturer or experimental data in order to obtain reliable values for the optimum CO_2 condensing temperature and maximum COP:

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Figure captions

Figure 1. Schematic diagram of the CO₂ /NH₃ cascade refrigeration system.

Figure 2. LogP-h diagram of the CO₂ and the NH₃ thermodynamic cycles.

Figure 3. Exergy lost rates of each system's components (left axe) and of the whole system (right axe) as function of T_{Cond CO2}.

Figure 4. Average value and range of values of the system's COP and exergetic efficiency as function of (a) T_{Evap CO2}, (b) T_{Cond NH3} and (c) DT. Figure (c) also includes the relative U·A increments in the cascade heat exchanger as function of DT.

Figure 5. Average value and range of values of the system's COP and exergetic efficiency as function of T_{Cond CO2}.

Figure 6. Iso-T_{Cond CO2 Opt} contours plotted on T_{Evap CO2} - T_{Cond NH3} plane.

Figure 7. Iso-COP_{Max} contours plotted on T_{Evap CO2} - T_{Cond NH3} plane.

Figure 8. System COP vs. T_{Cond CO2} using different combinations of correlations to calculate the compressors' isentropic efficiency.

Tables

Table 1. Balance equations for each system component.

Component	Mass	Energy	Exergy
CO ₂ -Compressor	$\dot{m}_2 = \dot{m}_1$	$\dot{W}_{Comp CO_2} = \frac{\dot{m}_1 \cdot (h_{2s} - h_1)}{\eta_{Comp CO_2}}$	$\dot{X}_{Lost Comp CO_2} = \dot{W}_{Comp CO_2} - \dot{m}_1 \cdot (\psi_2 - \psi_1)$
NH ₃ -Compressor	$\dot{m}_6 = \dot{m}_5$	$\dot{W}_{Comp NH_3} = \frac{\dot{m}_5 \cdot (h_{6s} - h_5)}{\eta_{Comp NH_3}}$	$\dot{X}_{Lost Comp NH_3} = \dot{W}_{Comp NH_3} - \dot{m}_5 \cdot (\psi_6 - \psi_5)$
CO ₂ Exp. Device	$\dot{m}_4 = \dot{m}_3$	$h_4 = h_3$	$\dot{X}_{Lost CO_2 Exp} = \dot{m}_1 \cdot (\psi_3 - \psi_4)$
NH ₃ Exp. Device	$\dot{m}_8 = \dot{m}_7$	$h_8 = h_7$	$\dot{X}_{Lost NH_3 Exp} = \dot{m}_5 \cdot (\psi_7 - \psi_8)$
Evaporator (CO ₂)	$\dot{m}_1 = \dot{m}_4$	$\dot{Q}_{Evap CO_2} = \dot{m}_1 \cdot (h_1 - h_4)$	$\dot{X}_{Lost Evap CO_2} = \left(1 - \frac{T_0}{T_F}\right) \cdot \dot{Q}_{Evap CO_2} + \dot{m}_1 \cdot (\psi_4 - \psi_1)$
Condenser (NH ₃)	$\dot{m}_7 = \dot{m}_6$	$\dot{Q}_{Cond NH_3} = \dot{m}_5 \cdot (h_7 - h_6)$	$\dot{X}_{Lost Cond NH_3} = \dot{m}_5 \cdot (\psi_6 - \psi_7)$
Cascade heat exchanger	$\dot{m}_3 = \dot{m}_2$ $\dot{m}_5 = \dot{m}_8$	$\dot{m}_1 \cdot (h_3 - h_2) = \dot{m}_5 \cdot (h_5 - h_8)$	$\dot{X}_{Lost CHE} = \dot{m}_5 \cdot (\psi_8 - \psi_5) - \dot{m}_1 \cdot (\psi_3 - \psi_2)$

Table 2. ANOVA Results of COP and Exergetic Efficiency.

Source	Sum of Squares		Df		Mean Square		F-Ratio		P-Value	
	COP	η_{II}	COP	η_{II}	COP	η_{II}	COP	η_{II}	COP	η_{II}
A: T _{Evap CO2}	55.404	0.06310	5	5	11.0808	0.01262	824.79	23.41	0.000	0.000
B: T _{Cond NH3}	71.291	5.67083	5	5	14.2583	1.13417	1061.30	2103.82	0.000	0.000
C: DT	2.120	0.16968	3	3	0.7066	0.0566	52.60	104.92	0.000	0.000
D: T _{Cond CO2}	26.939	2.03396	6	6	4.48981	0.3390	334.19	628.82	0.000	0.000
RESIDUAL	13.274	0.53263	988	988	0.0134	0.00054				
TOTAL				100						
(CORR.)	169.03	8.47019	100	7						

Figures

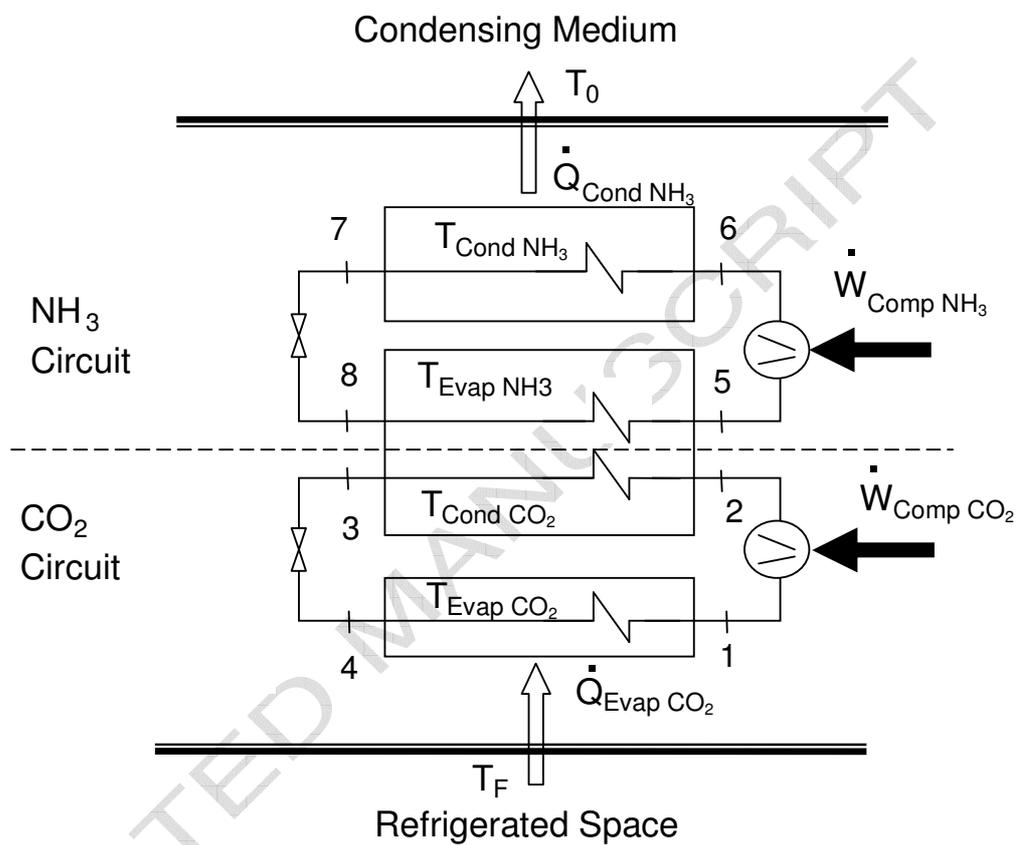


Figure 1

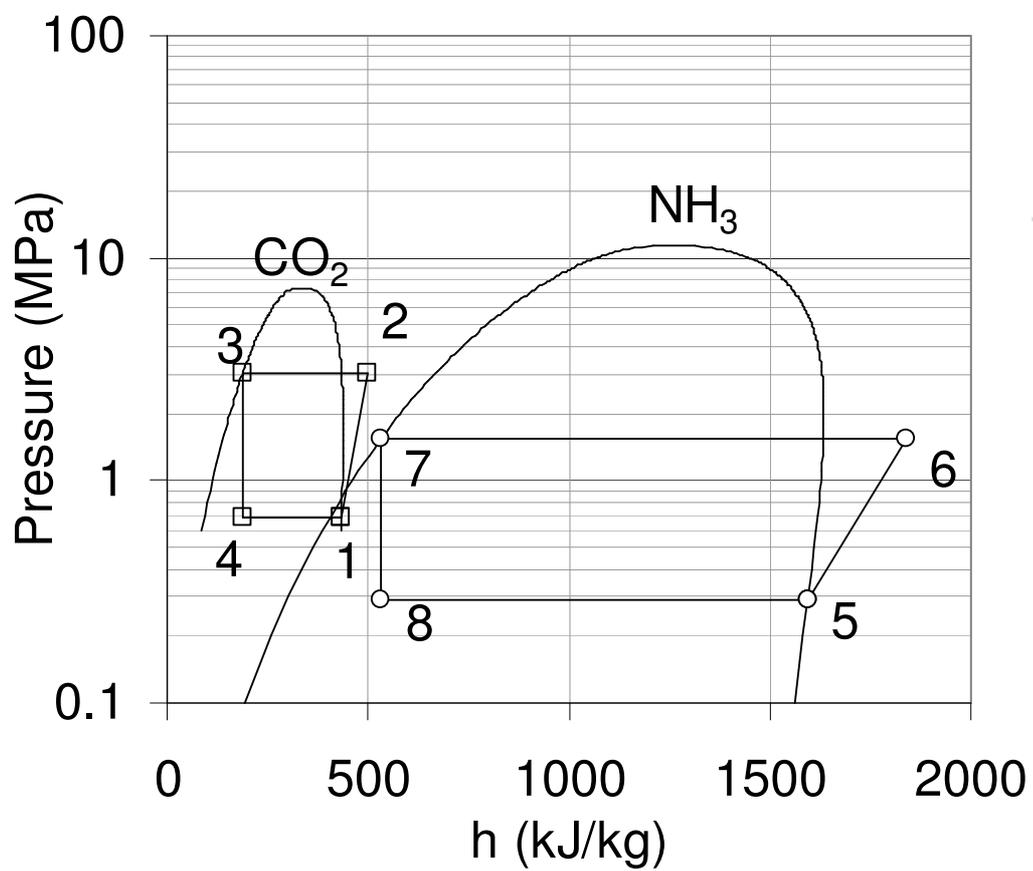


Figure 2

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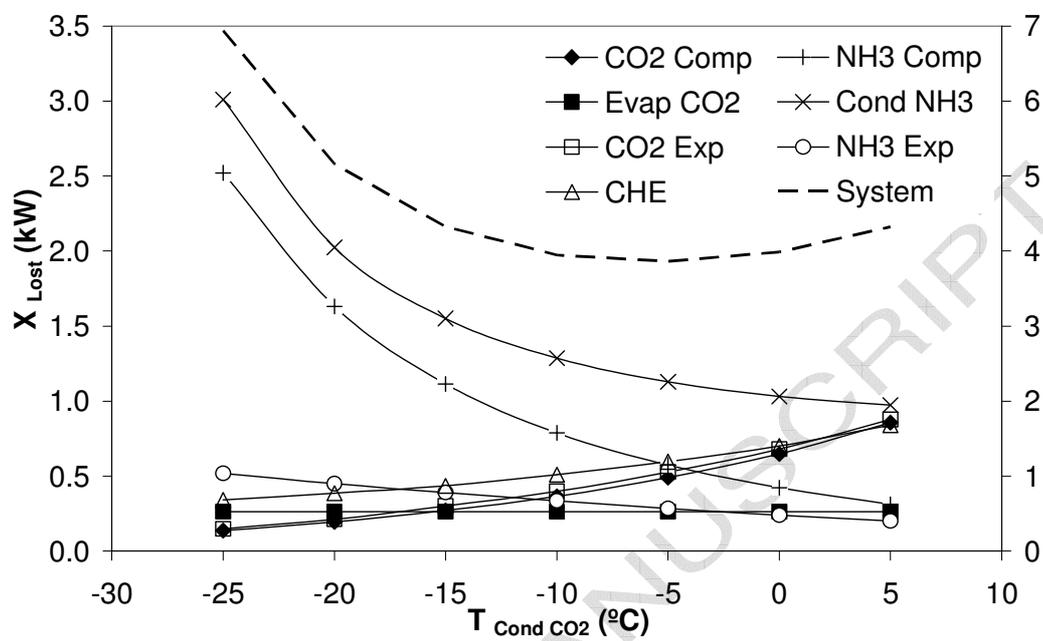
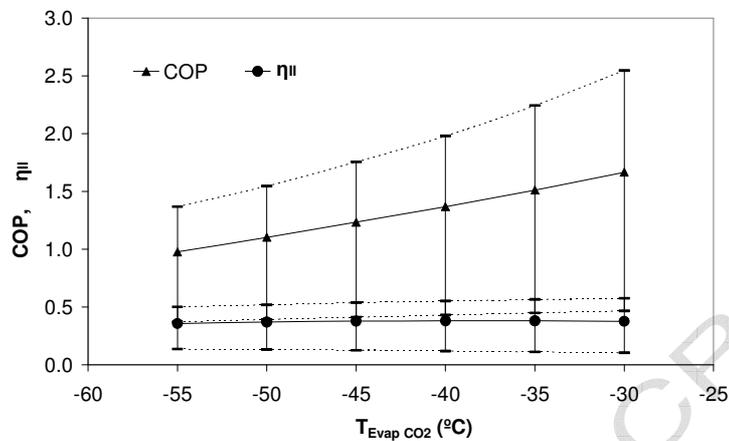
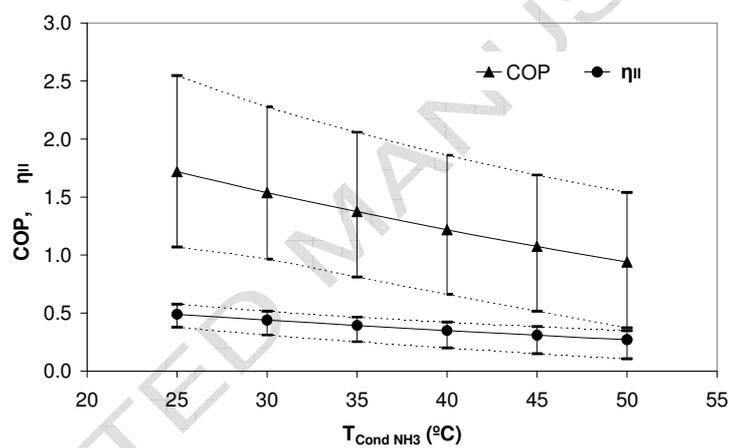


Figure 3

a)



b)



c)

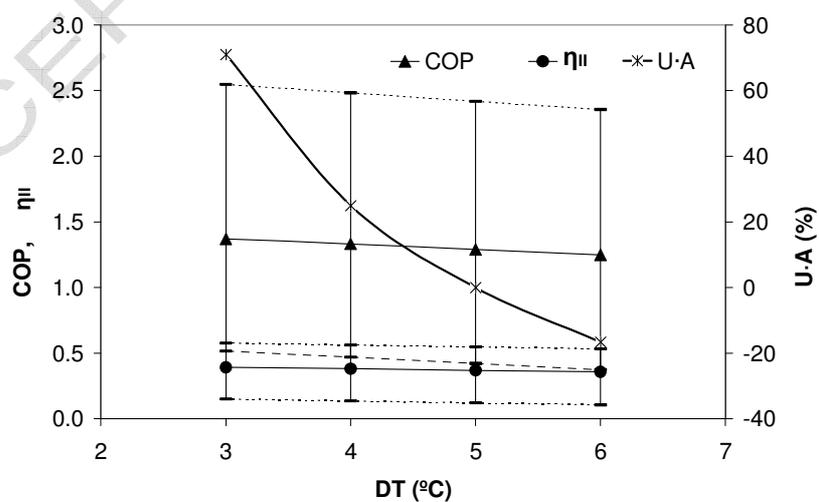


Figure 4

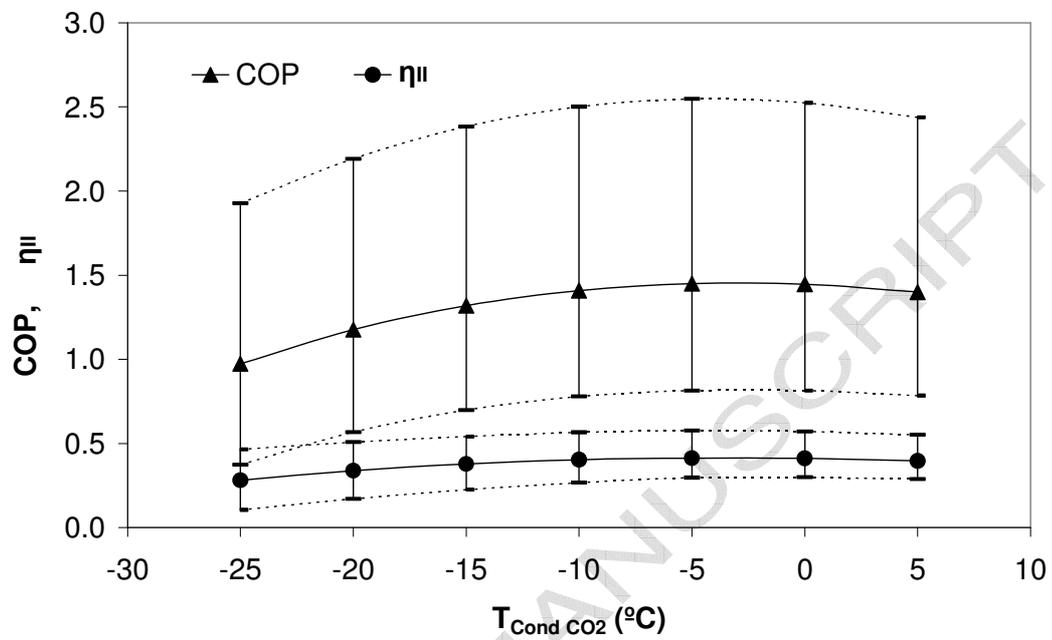


Figure 5

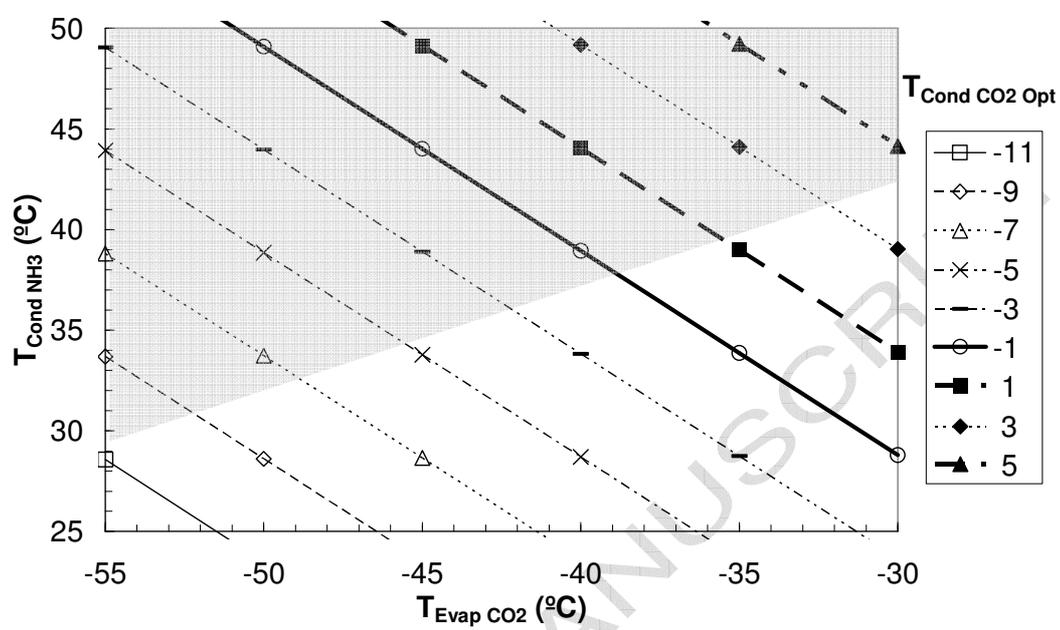


Figure 6

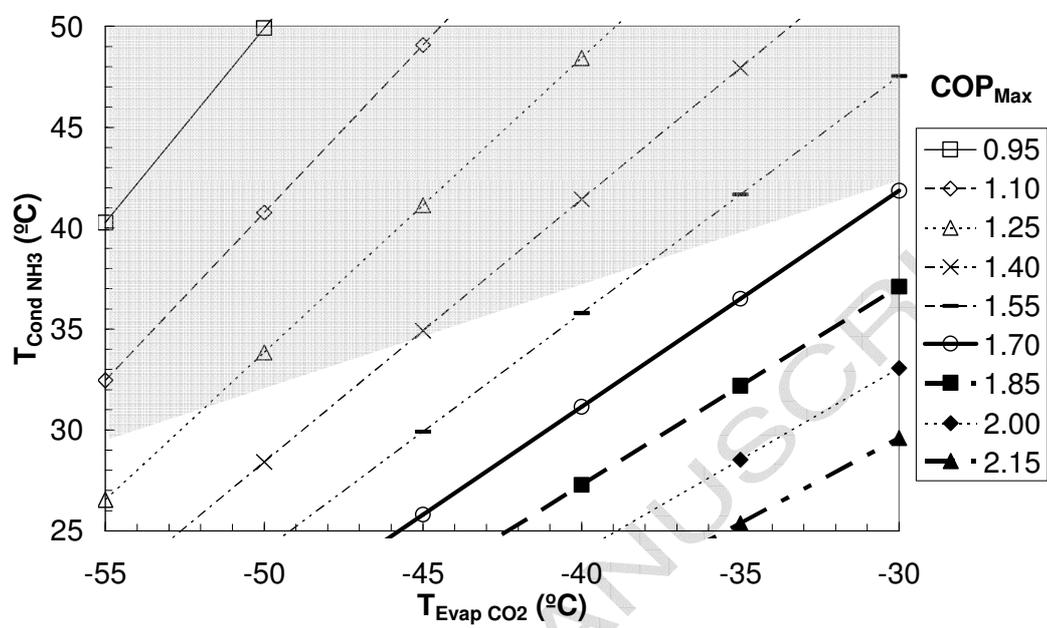


Figure 7

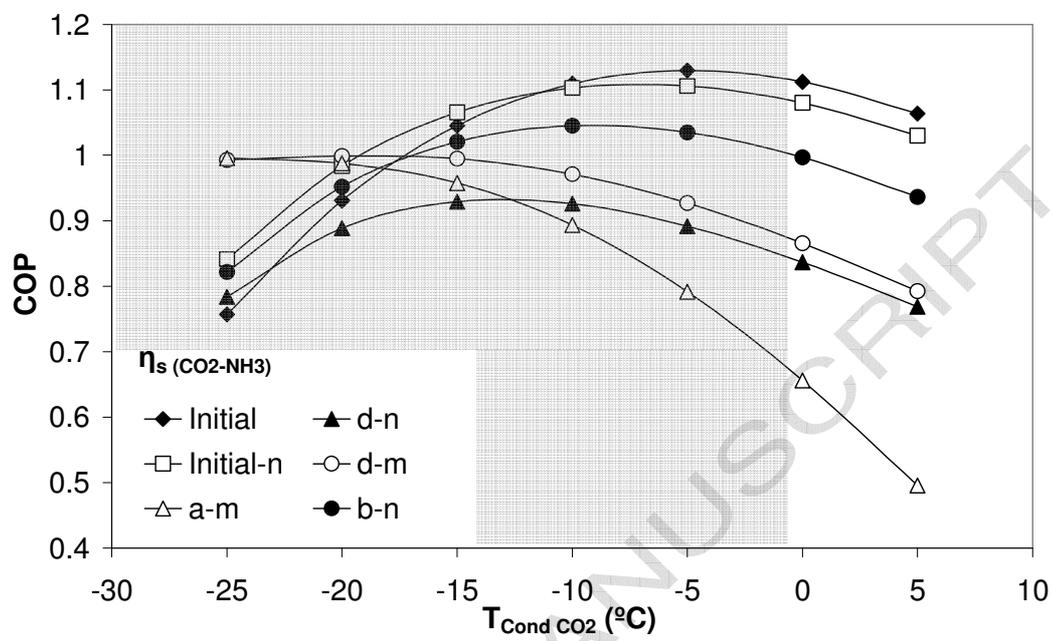


Figure 8