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by S.H. Saker

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Rebuttal of “Compatibility of local and global stability conditions for some discrete population models”, by S. H. Saker

Eduardo Liz

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Paper [1] contains two main results (Theorems 2.1 and 2.3), and a corollary (labelled as Theorem 2.2). The author claims that these results are new, and establishes a comparison with some previous papers. The aim of this rebuttal is not only to clarify that this claim is false (that is, these results are not new), but also to show that the proofs in [1] are incorrect.

Regarding the novelty of the results, the reader can check that the statement of Theorem 2.1 in [1] is proved in [2, Theorem 2], and the statement of Theorem 2.3 in [1] was previously demonstrated in [3, Example 2.3]. We notice that Theorem 2.2 in [1] is a direct consequence of both Theorem 2.1 and Theorem 2.3.

Amazingly, paper [3] is included in the list of references of [1], but the author does not recognize in his discussion that Theorem 2.3 was already proven there; rather than this, he choose to compare his result with other papers, that were already cited in [3].

The note [2] is not included in the bibliography of paper [1], but I suspect that Dr. Saker knows the paper. For instance, he uses exactly the same motivation and references as in [2].

Secondly, it should be noticed that the techniques used in the proofs of Theorems 2.1 and 2.3 in [1] are different from the ones employed in [2, 3]. However, as I show below, Saker’s proofs are wrong. To justify my assertion, I focus my discussion on the first result, which establishes a global stability condition for the positive

equilibrium of the first-order difference equation

$$x_{n+1} = f(x_n), n = 0, 1, \dots \quad (1)$$

where

$$f(x) = \alpha x + \frac{\beta x}{1 + x^\gamma}. \quad (2)$$

Here, α, β, γ are real constants satisfying $0 \leq \alpha < 1 < \alpha + \beta$, $\gamma > 0$. Equation (1) has a unique positive equilibrium $\bar{x}$, which is locally asymptotically stable if the following condition holds:

$$\gamma < \frac{2\beta}{(1 - \alpha)(\alpha + \beta - 1)}. \quad (3)$$

Theorem 2.1 in [1] states that condition (3) ensures the global asymptotic stability of the positive equilibrium $\bar{x}$, that is, all solutions of (1) with positive initial condition converge to $\bar{x}$. For it, the author claims that the map $V(x) = (x - \bar{x})^2$ is a (strict) Lyapunov function for (1). The same idea is used in the proof of Theorem 2.3.

If one looks closely to the proof of Theorem 2.1 in [1], it becomes apparent that there are some gaps; namely, it is proved that function $\Delta V(x) := V(f(x)) - V(x)$ has a local maximum at $x = \bar{x}$, with $\Delta V(\bar{x}) = 0$, but existence of other possible critical points is not excluded. This gap does really matter. Actually, function ΔV can possess other local maxima reaching positive values, so in that case V is not a Lyapunov function for (1), in contradiction with the claim in [1].

For example, take $\alpha = 1/2$, $\beta = 9$, and $\gamma = 4$ in (2). It can be easily verified that inequality (3) holds, but $V(x) = (x - \bar{x})^2$ is not a Lyapunov function for (1). See Figure 1, where the graph of $\Delta V(x)$ is represented.

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Fig. 1 Graph of the function $\Delta V(x)$. Clearly, it does not reach a global maximum at $x = \bar{x}$.

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