Acoustic properties of a hollow sphere for gravitational wave detectors

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Abstract. We report on experimental work on a small prototype of a hollow sphere, aiming at assessing the feasibility of such a resonator as a third generation g.w. resonant detector. We measured the resonant frequencies and quality factors of the spheroidal quadrupolar modes of a welded hollow sphere. The eigenfrequencies are found where predicted by the theory, and the quality factors were degraded from a minimum of 20 % to a maximum of 60 % with respect to the bulk sphere.

1. Introduction

Resonant detectors of gravitational waves (g.w.) are reliable machines that have proven capable to operate for long uninterrupted periods of time. Cryogenic and ultracryogenic bars have indeed carried out long term observation with meaningful sensitivity. Spherical antennas were proposed [1] as next generation of such detectors, as they can offer, with respect to a cylinder of the same linear dimension, larger mass (and therefore cross section) and omnidirectionality: the miniGRAIL [2] group at Leiden University (NL) has been carrying out, over the last several years, most of the pioneering experimental work for the feasibility of a large scale spherical antenna, and the Mario SCHENBERG project in São Paulo (BR) pursues a similar design with parametric readout [3]. Indeed, an omnidirectional resonant detector could complement observations by large interferometers, adding information about a possible scalar component of g.w., predicted in some metric theories [5]. Two choices are given, in this respect: explore higher frequencies than those where interferometers are most sensitive or insist on the same frequency band ($f \leq 1kHz$) to correlate interferometer information with data gathered on a different physical principle. The resonant frequency of the first spheroidal quadrupolar modes is roughly [4] $v_s/4R$, where v_s is the sound velocity of the material and R the sphere radius: for a 3m diameter sphere of Al5056 or CuAl this results in about 1 kHz. In some cases, one may want a lower frequency of sensitivity without dealing with an unpractically large resonator. A hollow sphere [7] was proposed as an interesting solution: the resonant frequency of its lowest quadrupolar modes can indeed be chosen with more flexibility with respect to the dimensions, simply by selecting appropriate values of internal and external diameter. Recent theoretical and numerical work [8] on the sensitivity of a hollow sphere confirms the interest in this kind of detector.

Further advantages of a hollow sphere are:

- A hollow sphere can be more easily fabricated in large dimensions, by welding together either thick, curved plates or superimposed rings of the chosen material: the fabrication is then reduced to a quasi-two-dimensional process. The loss in mass is marginal ($\leq 25\%$) as long as the thickness $t \equiv R_{ext} - R_{int}$ exceeds one third of the outer radius. In this paper we shall use, as natural expansion parameter, the fractional thickness of the sphere wall: $\xi \equiv t/R_{ext}$. The above requirement therefore reads $\xi > 0.3$.
- A hollow sphere can be cooled more easily: the shell thickness $t < R_{ext}$ is cooled, e.g. by exchange gas, at the same time from the external and the internal surface: in this way we effectively reduce the thermal resistance and consequently the cooling time, at least down to 4.2 K.
- The second quadrupolar modes $\{n = 2; l = 2; -2 \leq m \leq 2\}$ of a spherical resonator have a significant sensitivity to g.w. In a hollow sphere their resonant frequency, and their cross section can be selected by proper choice on the ratio ξ. It is then possible to position two different spectral windows of observation at frequencies of interest or choose an "aspect ratio" such as to provide equal cross section at the two frequencies. In all cases this corresponds to an effective widening of the bandwidth of observation of a resonant detector.

Although most experimental aspects of a spherical detectors have been analyzed and dealt with in recent years [2], a hollow sphere proposes several new practical challenges and problems that need to be studied and question that need to be answered. In this work we have tried to address the following issues:

- A hollow sphere cannot be suspended by its center of mass: will a surface suspension affect the mechanical properties of the resonator ?
- Do welded joints in a solid body affect its normal modes of vibration ? Does the welding discontinuity represent an obstacle for the elastic waves ? and if it does not:
- To what extent do these welded joints increase the mechanical losses for the vibrational modes of interest ? This last point is relevant because, as well known, a high quality factor of the observed modes is required to achieve best sensitivity in ultracrogenic detectors.

In order to find answers to these questions, we undertook experimenting with small metal spheres: after some preliminary tests with spheres of Aluminum alloy, $R_{ext} = 125\,mm$, we focused on a sphere made of CuAl 6% , the same material of the Minigrail detector, with $R_{ext} = 75\,mm$. Interesting enough, despite the different dimensions, the two samples have comparable mass (about 14 kg) and resonant frequency (about 13 kHz), due to the higher density and lower speed of sound of CuAl ($\rho_{CuAl} = 8 \cdot 10^3 kg/m^3 \simeq 3\rho_{Al}$; $v_s = 3860\ m/s$,).

We briefly recall here that the eigenmodes of a sphere represent a classical problem in elasticity theory, that had already been tackled by Love [11]. The solution for a bulk sphere boils down to a rather simple equation for its first quadrupolar modes[6] at

$$f_{\{n=1,l=2\}} = 1.62 v_s/2\pi R; \qquad f_{\{n=2,l=2\}} = 3.12 v_s/2\pi R; \qquad (1)$$

on the other hand, for a hollow sphere the calculation is more complex: the issue has been recently developed further[7], yielding what is friendly called the "Lobo solution": the eigenfrequencies of the spheroidal modes, given by the roots of a 4 x 4 determinant involving spherical Bessel functions and their derivatives, were numerically computed. These results are summarized in the plot of fig.(1) where the eigenvalues kR_{ext} for the $n = 1$ and $n = 2$ quadrupolar modes are plotted vs the fractional radial thickness ξ; here $k = 2\pi f_{n,2}/v_s\sqrt{2(1+\sigma)}$ and σ is the Poisson ratio [9]

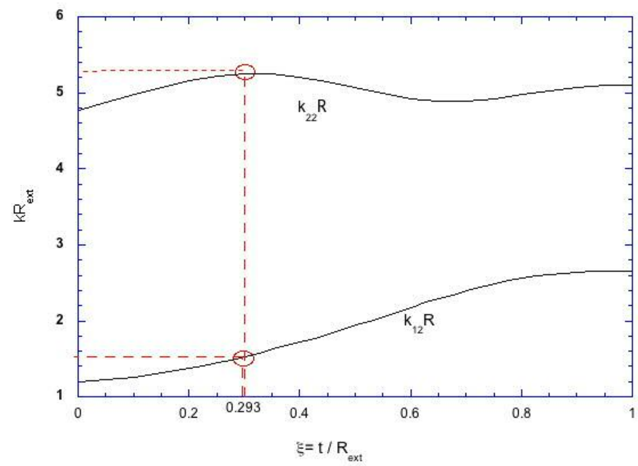

Figure 1. Eigenvalues of the first and second quadrupolar modes of a hollow sphere vs the filling factor $\xi = t/R_{ext}$ (adapted from ref. [7]) The points corresponding to our experimental configuration ($\xi = .293$) are marked. The bulk sphere limit ($\xi = 1$) is found at the right edge. See text for the relation between kR_{ext} and the resonant frequency

2. The benchmark: measurement on a bulk sphere

A hollow sphere can be produced in at least two ways: by fusion or by welding parts together. Fusion with a melting core is probably the most straightforward method, as it produces a resonator without seams, and represents, for a small sample, no serious technical challenge. Nevertheless we chose to investigate on a sample obtained by welding two half spheres, for a twofold reason:

First, we were interested in exploring the feasibility of a large sphere, that would most probably be fabricated out of plates, as discussed above. Moreover, in this way we are able to compare the internal dissipation (measured by the Q of the oscillator) of a hollow sphere with that measured, before machining, on the bulk sample of the same material, size and instrumentation: in this way we can extract information about the Q reduction due to re-assembly.

We started out with characterizing the bulk resonator from which we eventually derived the hollow sphere: it is a sphere in CuAl 6% (94 % Copper, 6% Alluminum), 15 cm in diameter, that was previously used and characterized in Leiden University [10]. These measurements, as all those reported here, were made using two piezoelectric ceramics (PZT) glued on the sphere surface, at polar angles $\theta = \pi/4; \phi = \pm\pi/4$. Alternatively, we also used small accelerometers and impulsive excitation provided by a coil activated “hammer”. As these auxiliary sensors and actuators cannot be used in cryogenic enviroment (due to excessive heat dissipation), the relevant data were in the end taken by exciting with one PZT and reading out with the other.

Figure 2. An image of the sphere hanging from the cold plate of the cryostat vibration isolation system. Note the suspension cable

Several suspension methods were tested, including 4 points loaded cantelivers, a V-shaped cable passing through a ring fastened to a sphere pole and a Λ shaped cable hooked to two "ears" machined at the ends of an equatorial diameter. We describe here in detail the one that was eventually chosen, as giving the highest Q and the best isolation from external noise. We bored a hole across the sphere diameter, with two different sizes (see fig 3): 5 mm diameter from one pole to its center, 6 mm diameter from the center to the other pole. The hole was tapped M7 at the wider end and M6 near the center, making it possible to be hung via a suspension cable fastened either near the sphere center of mass or on its surface. We used stainless steel cables, 1mm in diameter and 300 mm long, terminated by two silver soldered brass cylinders, threaded M6 at one end, to be fastened to the cold plate of the cryostat, and M6 or M7 at the other end in order to suspend the sphere.

Eigenfrequency and Q measurements, carried out at room temperature in both configurations and shown in fig.3, show no degradation due to surface suspension; actually, the Q of the third quadrupolar modes was even larger (by 20%) than when suspended near the center of mass. We did not extend this comparison down to low temperature: our results closely replicate those obtained at 300K by by the Minigrail group [10] with a similar suspension. Their measurements show, at all temperatures down to 20 millikelvin, no difference in Q due to the suspension, up to within the 10^7 range.

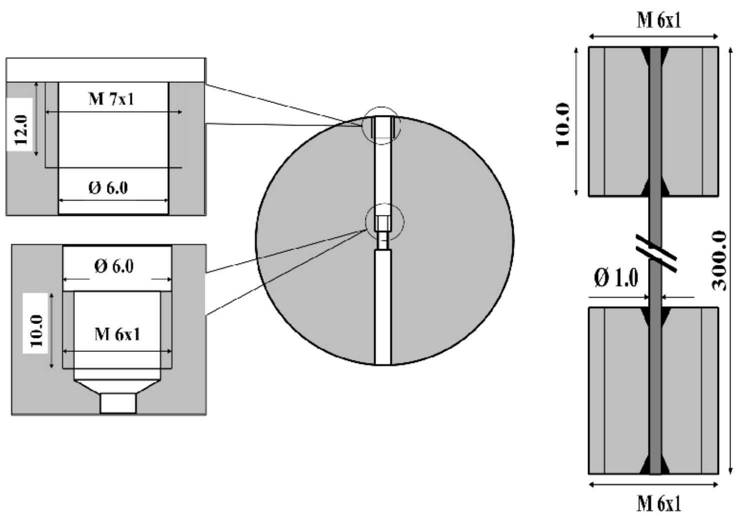

Figure 3. Schematic of the sphere suspension: the hole is tapped both near the center and at one end, so that the sphere can be tested, via one of the suspension cables (scketched on the right) with either a center of mass or a surface connection.

This resonator was then cooled using the surface suspension, and its elastic properties were measured to serve as a benchmark for the hollow sphere. Eq.(1) predicts a value $f_0 = 13270 Hz$, fully compatible with our experimentally measured (at 4.2 K) values, ranging from 13161 to $14593 Hz$. The spread of the five resonant frequencies is due to the removal of degeneracy due to gravity, suspension, transducers etc. The Q

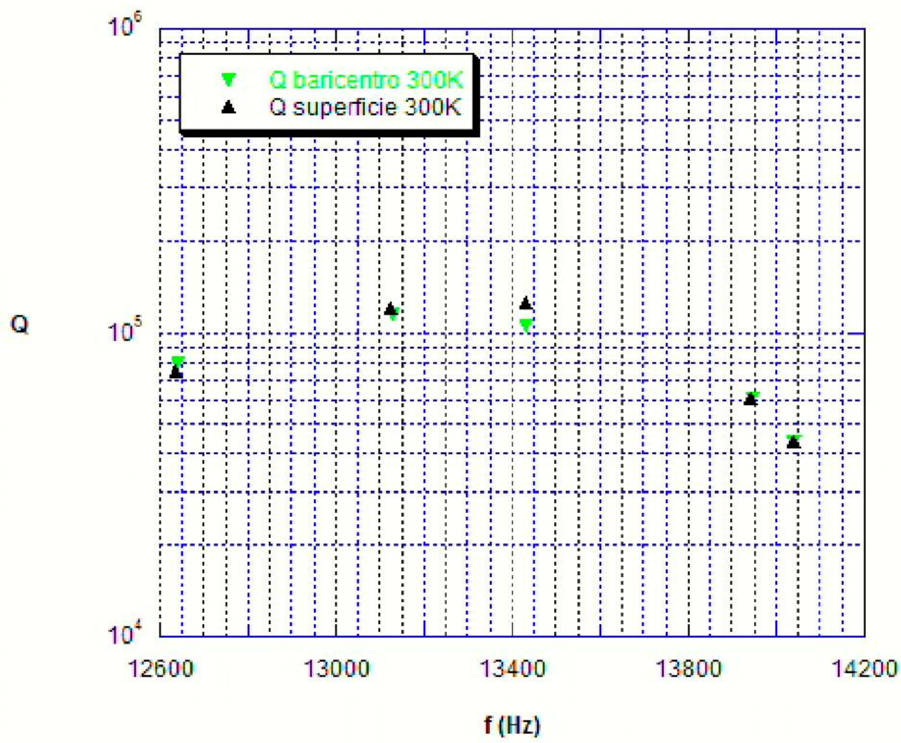

Figure 4. Q values, at room temperature, of the quadrupolar modes of the bulk sphere with center of mass and with surface suspension. There is no remarkable difference on the influence that the two suspension methods have on the quality factor of the quadrupolar modes of a bulk sphere.

values we recorded, consistent with previous measurement done in Leiden, were spread around 10^5 at room temperature and just below 10^6 at liquid Helium temperature (see table 4.1). Larger resonators, like MiniGRAIL, have shown Q values up to ten times larger [12]: these lower values can be due to several possible causes: insufficient isolation of the cryostat, larger surface to volume ratio and heavier influence of the surface suspension and transducers on a smaller oscillator, small metallurgical differences in the alloy or in the aging history of the sample.

3. Fabrication of a hollow sphere

Once the bulk sphere was properly characterized, we split it in two equal parts with wire EDM machining, that removed the smallest amount of material (the cut was about 0.5 mm wide), in order to minimize deviations from sphericity after reassembling the two halves. Both hemispheres were then carved out and reduced to shells with 22 mm thickness, i.e. $\xi = 0.293$.

We considered three methods for reassembling the two half carved spheres:

- *Electron Beam Welding (EBW)*: although high penetration welding is possible and common on pure Copper, CuAl did not perform as well: a test weld on a small sample proved to be brittle, uneven, full of cracks and with a penetration of few millimeters. Besides, we were unable to spot a company that could perform joints thicker than a few cm; this technique is not therefore exportable to a large size ($R_{ext} > 1m$) sphere, where a penetration depth $t \geq 0.3m$ or larger would be required.

- *Diffusion Welding*: it is a special soldering technique, where no foreign material is interposed between two mating surfaces. The native alloy, brought to a temperature close to the melting point, diffuses across the boundary, hopefully recreating the same metallurgical bonds of the bulk. This approach appears promising with respect to preserving the Q of the resonator, as no discontinuity is met by the elastic wave traveling in the solid.
  We carried out some tests on a CuAl cylinder, 224 mm long and 56 mm in diameter. The cylinder was machined hollow to the same radial thickness $t = 22mm$ we had planned for the hollow sphere: on this simpler geometry it is straightforward to verify whether the bond between two shorter cylinders recreates one long elastic cylinder: the first longitudinal mode is simply related to the length ($f_1 = v_s/2L$) and can be identified unambiguously. The two parts were kept for 1 hour under mechanical pressure at 1020 ^oC, i.e. 50 degrees below the melting point of CuAl, and then allowed to slowly cool in vacuum.

sample	**resonant frequency of mode 1L (Hz)**	**Q value**
bulk cylinder	8335 Hz	$2.6 \cdot 10^5$
hollow cylinder (uncut)	8312 Hz	$2.6 \cdot 10^5$
2 halves diffusion welded	8500 Hz	$1.9 \cdot 10^5$
2 halves diffusion welded (after thermal cycle)	8500 Hz	$1.1 \cdot 10^5$
2 halves silver brazed	8419 Hz	$2.4 \cdot 10^5$
2 halves silver brazed (after thermal cycle)	8419 Hz	$2.4 \cdot 10^5$

Table 1. Resonant frequencies and Q values of the first longitudinal mode of vibration of the hollow cylinder used to investigate bonding techniques. The increase in resonant frequency of the re-assembled cylinder is due to the loss in length (2 mm out of 228) caused by cutting. Measurement errors were about 2 Hz for freqencies and $5 \cdot 10^3$ for Qs

Results at room temperature were encouraging for this test: the processes of cutting and welding did not affect the resonant frequency, while Q of the first longitudinal mode was reduced by one fourth (see table 1). At first, we considered this an acceptable loss. However, when the sample was shock-cooled in liquid Nitrogen, its Q was irreversibly degraded, most probably due to the insurgence of a crack in the weld. This was probably a rougher test than required: a gentler and slower cooling (as it would undergo under vacuum) could, in principle, produce a less disruptive result; however, in the spirit of the feasibility test for a larger sphere, we applied the most severe conditions, in order to avoid qualifying a risky procedure that could yield a faulty resonator. In the light of the subsequent tests, described below, where the Q value did not change appreciably even after thermal cycling, the immediate (i.e. before cooling) drop of 27% from the uncut value appears a posteriori as indicator of an unsuitable joint.

- *Silver brazing*: we therefore turned to the traditional technique of oven brazing with layer of silver based filler ((61.5% Ag, 24% Cu, 14.5% In). The filler was shaped into a thin ($\sim 50\mu m$) washer and squeezed between the two short cylinders. The cylinder was then hard brazed for an hour at $750\,^oC$ under vacuum. The sample regained its initial Q and no effect was seen after shock cooling; as these tests were passed with full satisfaction, so we proceeded to brazing the two spherical shells.

4. Experimental Results

4.1. Resonant frequencies

The hollow sphere so produced was extensively tested both at room temperature and in cryogenic conditions. Low temperature tests were carried out in the laboratory of Tor Vergata University, in a cryogenic facility specially built to test mechanical resonators down to 4.2 K, with due attention to isolation from external mechanical disturbances.

For our sample, with $\xi = 0.293$, the plot of fig.(1) we read $k_{12}R_{ext} = 1.54$ yields (with $\sigma = 0.3$) $f_{\{n=1,l=2\}} = 7823Hz$. Analogously, for the second quadrupolar modes, we get the value $k_{22}R_{ext} = 5.24$, i.e. $f_{\{n=2,l=2\}} = 26.6kHz$.

We can summarize the experimental results as follows:

The quadrupolar modes of vibration of the sphere can be easily identified and are found clustered around the expected frequency. The second quadrupolar modes were also found at the expected frequencies. Unfortunately, our measuring apparatus was not meant to operate at frequencies as high as $f_{\{2,2\}} \sim 27kHz$ and so detailed information could not be obtained.

From these measurements we deduce three important points:

(i) The elastic waves propagate across the bonding discontinuity of the resonator without measurable effect on the resonances; although we had no provision for visualizing the mode shapes, we can infer that the quadrupolar modes were not affected by the presence of a layer of bonding material: in other words, brazing reconstructs an elastic sphere.

(ii) We have found the five quadrupolar modes at $7072 \leq f_{\{1,2,m\}} \leq 7602\,Hz$ at room temperature, to be compared with a predicted value of 7823 Hz: this is well within the experimental uncertitudes on the values of v_s and σ. We have therefore validated the “Lobo solution” [7] that had never been tested.

(iii) By carving a large internal cavity we removed about one third of the mass (from 14.4 to 9.4 kg). This would bear little consequence on the sensitivity of a real detector. However the resonant frequency of operation is lowered to about half (57% in our case) of the initial value, showing that this is an effective way to tune the resonator to a given frequency. In a bulk sphere, one would need to double the diameter to obtain the same result.

Bulk sphere		**Hollow sphere**	
Resonant frequency (Hz)	**Q**	**Resonant frequency** (Hz)	**Q**
13161.3	$6.20\ 10^5$	7370.2	$8.0\ 10^4$
13668.3	$6.44\ 10^5$	7759.3	$1.82\ 10^5$
13976.8	$8.78\ 10^5$	7782.3	$2.92\ 10^5$
14503.3	5.4710^5	7846.4	$3.48\ 10^5$
14593.5	$8.71\ 10^5$	7909.0	$1.54\ 10^5$

Table 2. Resonant frequencies and Q values of the five quadrupolar modes of a bulk (left) and a hollow (right) sphere, both measured at 4.2 K using the surface suspension. Experimental errors are 0.1 Hz on the frequencies and $2 \cdot 10^3$ (about 1%) on the Qs

We also performed a detailed finite element analysis of both the bulk and the hollow sphere, where the suspension was modeled and taken into account. The resulting computed eigenfrequencies very well agree (see fig.5) with the measured values, and give a satisfactory quantitative account of the splitting of four modes out of the five considered.

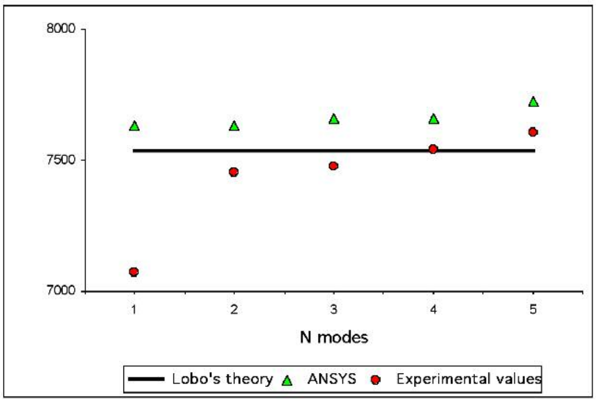

Figure 5. The resonant frequencies of the five lowest quadrupolar modes at room temperature: the solid (black) line represents the value predicted by the theory of ref. [7], where the degeneracy is not removed and no splitting is predicted. The triangles above it (green) show the prediction of the Ansys finite element calculation while the (red) circles are the measured values

4.2. Internal friction and quality factors

Q measurements were carried out on the quadrupolar modes at temperatures between 300K and 4.2K. The results, showing the typical increase in Q values with decreasing temperature, are shown in fig.6.

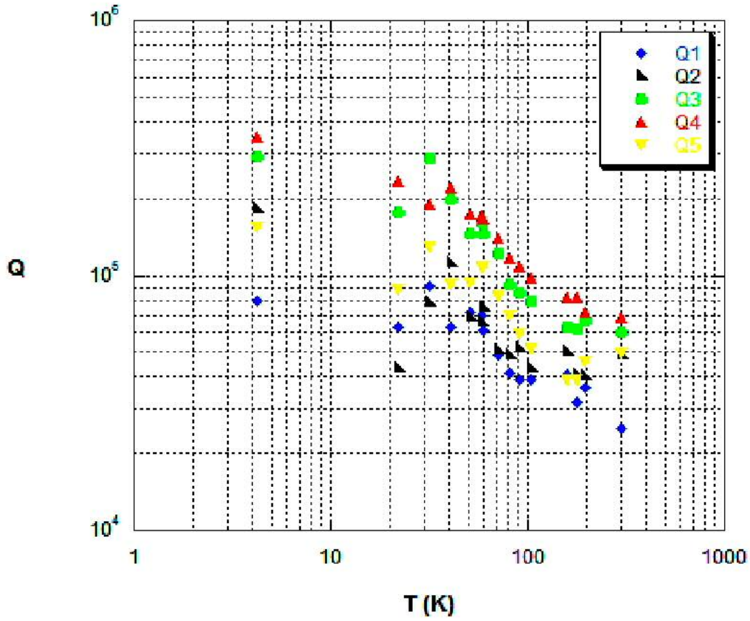

Figure 6. Quality factors of the quadrupolar ($n = 1, l = 2$) modes vs. temperature.

The measured Qs are consistently lower than those previously measured on the bulk sphere: in fig.(7) we compare of the values measured at 300 K, 77 K and at 4.2 K.

We observe that at room temperature, as well as at 77 K, the Q loss is within a factor 2, with a typical reduction, averaged among the 5 modes, to about 65 % of the values of the bulk sphere. At liquid Helium temperature the effect is more relevant: the measured values are between $0.8 - 3.5 \cdot 10^5$, with an average reduction to one third of the bulk values. Unlike most similar experiments on metallic resonators, the quality factors only marginally increase from 77 to 4 K, an indication that some dissipation mechanism limits them at the 10^5 level.

4.3. Discussion of the results

In this final parargaph we switch to a more convenient notation: we introduce the loss angle[13] $\phi \equiv 1/Q$; this is advantageous in considering various contributions, because loss angles due to different causes are simply summed, while, when dealing with Qs, an harmonic sum is needed. We use superscript to indicate the hollow (**h**) or bulk (**b**) configuration, and subscript to mark the different physical processes generating the losses.

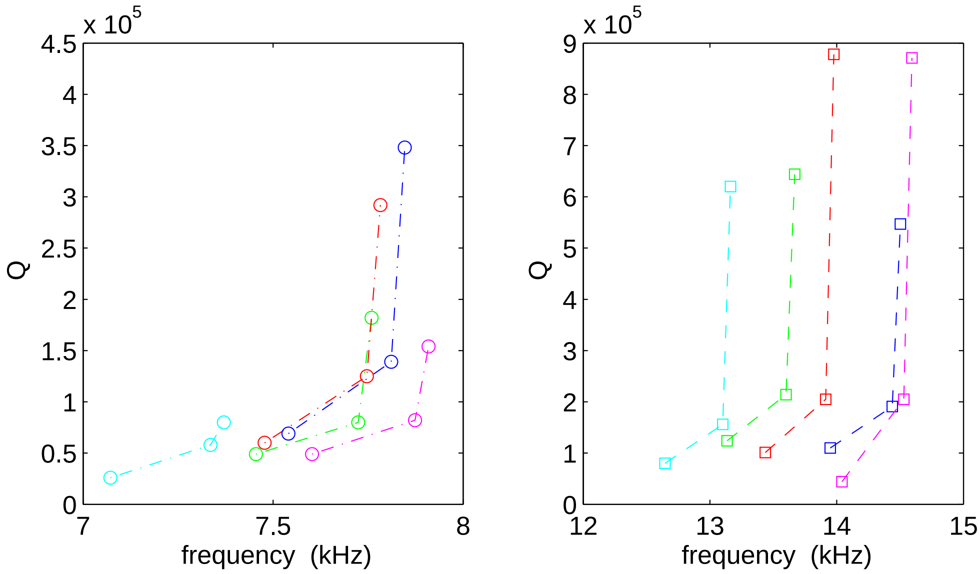

Figure 7. Q vs. frequency behavior for the five quadrupolar modes of the hollow (left) and bulk(right) sphere. The values reported are taken at room temperature (lowest), 77 K (middle) and 4.2 K (highest). Note the Q in the hollow sphere are consistently lower (by about a factor 2) than in the bulk one. Note also that the spread of the mode frequencies in the bulk sample is much wider than in the hollow one

Three obvious candidates are considered as causes of this increased dissipation: the different geometry of the resonator, the welded bond and the suspension.
We sum up all contributions to the losses of a hollow sphere:

$$\phi^{hollow} = \phi_0 + \phi^{\mathbf{h}}_{susp} + \phi^{\mathbf{h}}_{geom} + \phi_{bond} \tag{2}$$

where in ϕ_0 we have summarized all the losses that are identical in both configurations (material defects, transducers, residual gas...) and ϕ_{bond} contains all the losses involved in the machining and assembling the two halves (machine hardening, welding, roughness of the internal surface near the joint, different thermal contraction of the filler etc.). Analogously, we write the losses of the bulk sphere:

$$\phi^{bulk} = \phi_0 + \phi^{\mathbf{b}}_{susp} + \phi^{\mathbf{b}}_{geom} \tag{3}$$

With regard to the influence of surface suspension, we notice that the five quadrupolar modes have, at all temperatures, smaller dissipation in the bulk sphere than in the hollow one, despite an identical suspension. Although the suspended mass is somewhat different (thus marginally changing the load), we can safely state that the losses measured on the hollow sphere were larger than the limits set by the suspension. In any case, being the suspension identical in both cases, we shall absorb its contribution ϕ_{susp} into ϕ_0.

Concerning the new geometry, we recall that the spheroidal vibrations we are interested in belong to the class of volumetric waves [11] where no shear motion, that involves frictional shifts of material layers, takes place in the bulk of the elastic body:

all motion is radial and therefore we can adopt the usual approximation of considering the elastic energy to be stored in the bulk and dissipated at the surface, where the wave is scattered. By introducing a phenomenological "dissipation depth" δ that summarizes all loss mechanisms of this species, it is customary to write: $\phi_{geom} = (Area \cdot \delta)/Volume$ From the data we gathered on the bulk sphere we can estimate an upper limit: $\delta \leq \phi^{bulk} R_{ext}/3 = 30 \div 45\ nm$

We then expect, for any two solid bodies, the surface losses purely due to surface effect to be related by the geometrical ratio:

$$G_{2/1} \equiv \left[\frac{\phi^2}{\phi^1}\right]_{geom} = \left(\frac{\delta \cdot Surface}{Volume}\right)_2 \left(\frac{Volume}{\delta \cdot Surface}\right)_1 \tag{4}$$

When the ϕ values are scaled according to this factor, the remaining, unaccounted for, difference in internal losses can be imputed to other, non geometrical causes like, e.g. the welded joint or the lower resonant frequency (indeed, $\phi = 1/\pi f \tau$, and the decay time τ is essentially unchanged in the two configurations). For a hollow vs bulk sphere, the ratio of geometrical factors is

$$G_{h/b} = \frac{2-\xi}{1-\xi(1-\xi)} \tag{5}$$

For our $\xi = 0.293$, this ratio takes the value $G_{h/b} = 2.15$, consistent with the intuitive notion that, as mentioned above, a hollow sphere has about twice the surface and roughly the same volume than the bulk one. If we consider the ratio of the lowest ϕ values achieved in the bulk (mode m=5) and hollow (m=4) configuration: $(\phi_{min})^{hollow}/(\phi_{min})^{bulk} = 2.5$: we can conclude that the change of geometry accounts for most (86%) of the increase in losses. The remaining is probably due to some cause related to the welded joint; the large spread in the Qs of the hollow sphere is an evidence of that, as different modes have different amplitude of motion at the equatorial plane, where the weld is. So we can finally rewrite eq. (2) as:

$$\phi^{hollow} = \phi_0 + G_{h/b} \cdot \phi^{\mathbf{b}}_{geom} + \phi_{bond} \tag{6}$$

The two relations (3) and (6) are obviously unsufficient to determine the three unknown: $\phi^{\mathbf{b}}_{geom}$, ϕ_0 and ϕ_{bond}. However, a couple of interesting limiting cases can be discussed:

- assume $\phi_{geom} >> \phi_0$, i.e. all the losses in the bulk sphere be due to surface effect. Then
  $\phi_{bond} = \phi^{hollow} - G_{h/b} \cdot \phi^{bulk} = 4 \cdot 10^{-7}$
- assume instead $\phi_{geom} << \phi_0$, i.e. no influence of the surface on the dissipation (all the losses in the bulk sphere are due, e.g., to the suspension). In this case $\phi_{bond} = \phi^{hollow} - \phi^{bulk} = 1.7 \cdot 10^{-6}$

We can safely presume that the real situation lays somewhere in between these two extremes, and therefore state that the dissipation due to the welding joint would limit the Q of the resonator at a level that somewhere in the range $6 \cdot 10^5 < Q_{bond} < 2.5 \cdot 10^6$.

A further geometrical consideration can be made: ϕ_{bond} is proportional to the fraction of volume that is interested by the bond: this volume is $2\pi R_{ext}td$, where the tangential height d is characteristic of the bonding technique and will not grow with the dimension of the sphere. Therefore the ratio of this "lossy" volume involved in the weld to the total volume, scaling as $V_{bond}/V = 3d\xi/2R_{ext}(1-\xi^3)$ will decrease in a larger sphere, as required for real detector, making the bonding losses less relevant. Besides, as the resonator size increases, also losses due to transducers and suspension are bound to have a smaller impact on the overall Q.

5. Conclusions and perspectives

We have described some experimental tests regarding construction and operation of a hollow sphere, as this geometry is interesting for third generation resonant g.w. detectors. We have verified that, in a bulk sphere, surface suspension does not degrade the quality factor of the first quadrupolar modes. We tested various methods of bonding two carved half shells to create the hollow resonator. In our tests the traditional silver brazing gave the best result, and we used it to fabricate our resonator. The resonant frequency of the first quadrupolar modes of oscillation was found exactly where the theory (that had never been verified before) predicts it, thus showing that a welded bond does not interfere with the propagation of elastic waves. We have measured the quality factors of the quadrupolar modes from room temperature down to 4.2 K and found them to be limited to about 10^5 and consistently lower than those of the bulk sphere. The cause of this extra source of dissipation were investigated and, although no quantitative conclusion can be drawn, it appears that the welded joint does not affect these losses up to a value of at least $Q > 6 \cdot 10^5$

Future work might include a systematic search of the source of extra dissipation. A Q measurement on a hollow, seamless sphere (that can be obtained by fusion) would give us the possibility to distinguish the geometrical losses from those due to the joint. We also plan to further investigate the technique of diffusion welding, that appears in principle the most promising: the problems encountered in the first tests could be overcome by modifying the procedure, e.g. by increasing the pressure or the oven time, or the temperature even closer to the melting point, in order to improve the bonding.

The results of our tests show that a fabricated hollow sphere, proposed years ago as a versatile resonant g.w. detector, is a viable solution

6. Acknowledgments

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[9] We have found convenient the following third order fit for the first quadrupolare eigenfrequency: $k_{12}R_{ext} = 2.658 + 0.3396\,\xi - 5.159\,\xi^2 + 3.385\,\xi^3$; for k_{22} a fifth order polynomial is needed.

[10] A. de Waard, PhD Dissertation - Leiden University 2003 (unpublished).

[11] A.E.H. Love: *A treatise on the mathematical theory of elasticity* . Chapter XII “Vibrations of spheres and Cylinders”, Cambridge University Press, 1927 (fourth edition) pag 286 and references therein.

[12] A. de Waard, L. Gottardi and G. Frossati: Proc. of the 9th M. Grossmann Meeting on General Relativity (Rome, Italy, 2000) - V. Gurzadyan, R. T. Jantzen and R. Ruffini editors, World Scientific, 2002.

[13] the loss angle is so called because it represents the ratio between the real (dissipative) and imaginary, reactive part of the mechanical impedance