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ANALYTICAL METHODS FOR VELOCITY DISTRIBUTION AND DIP-PHENOMENON IN NARROW OPEN-CHANNEL FLOWS

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Abstract. *In this study, we present analytical tools for velocity distribution in steady uniform open-channel flows. Simple dip-modified laws are presented. An ordinary differential equation based on an analysis of the Reynolds-averaged Navier-Stokes (RANS) equations is proposed. Comparisons of predicted velocity profiles with experimental data show good agreement. It is well known that dip-phenomenon depends on aspect ratio Ar (ratio of the channel width b to the water depth h). Results show that it should depend also on channel slope and friction velocity.*

1 INTRODUCTION

In open-channel flows, the vertical velocity profile is well described by the classical log law in the inner region ($\xi < 0.2$, where ξ is the ratio of the distance from the bed y to the water depth h). However, in the outer region ($\xi > 0.2$) this law deviates from experimental data. This deviation is accounted for by adding the Coles' wake function. In 2D open-channel flows, the log-wake law appears to be the most reasonable extension of the log law [5]. However, in narrow open-channels (i.e. aspect ratio *Ar* smaller than 5, where *Ar* is the ratio of the channel width b to the water depth h) the maximum velocity appears below the free surface (i.e. the velocity dip phenomenon) involving deviation from the log-wake law. This phenomenon is related to secondary currents which appear in 3D open-channel flows. Coles' wake function is unable to represent this behavior since it predicts a velocity which increases with distance from the bed.

In order to predict the velocity dip phenomenon, empirical relations were proposed. However, these empirical laws need a certain number of parameters such as dip position. Based on an analysis of the Reynolds-averaged Navier-Stokes (RANS) equations, Yang et al. [6] proposed a dip-modified log law which consists of two logarithmic distances, one from the bed (i.e. the log law), and the other from the free surface. This law presents the advantage that it contains only one parameter of dip-correction α. The dip-modified log law reverts the classical log law for $\alpha = 0$.

In our study, we will point out that even if the dip-modified log law [6] predicts dip-phenomenon, it presents important difference with experimental data. In most cases, it is not possible to improve velocity profiles by adjusting the parameter α. The aim of this study is to improve the prediction of velocity distribution with dip phenomenon in open channel flows.

The shortcoming of the dip-modified log law will be analyzed and a simple dip-modified log-wake law, which allows improving predicted velocity profiles, will be presented. An ordinary differential equation, based on an analysis of the Reynolds-averaged Navier-Stokes (RANS) equations, will be proposed. Predicted profiles will be compared with experimental data of measured velocities in open-channels.

2 MODEL EQUATIONS

For steady uniform open-channel flows, using the continuity equation, the Reynolds-averaged Navier-Stokes (RANS) momentum equation becomes in the streamwise direction x

$$\frac{\partial\, U\, V}{\partial\, y}+\frac{\partial\, U\, W}{\partial\, z}=\nu\frac{\partial^2 U}{\partial\, y^2}+\nu\frac{\partial^2 U}{\partial\, z^2}+\frac{\partial -\overline{u\,v}}{\partial\, y}+\frac{\partial -\overline{u\,w}}{\partial\, z}+g\, sin\theta \qquad (1)$$

where x, y and z are respectively the streamwise, vertical and lateral directions and U, V and W the three corresponding mean velocities (u, v and w are the turbulent fluctuations), ν the fluid kinematic viscosity, g the gravitational acceleration, and θ is the angle of the channel bed to the horizontal axis [1] [2].

3 DIP-MODIFIED LAWS

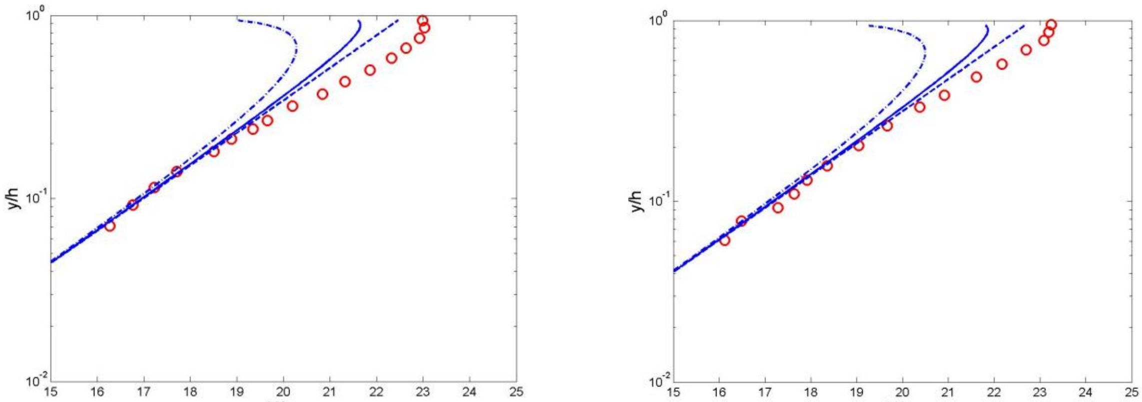
3.1 Dip-modified log law

From an analysis of the Reynolds-averaged Navier-Stokes (RANS) equations and by assuming a parabolic profile for the eddy viscosity, Yang et al. [6] proposed the following law

$$\frac{U}{u_*} = \frac{1}{\kappa}\left[ln\left(\frac{y}{y_0}\right) + \alpha\, ln\left(1 - \frac{y}{h}\right)\right] \tag{2}$$

where u_* is the friction velocity, h the water depth, κ the von Karman constant (≈ 0.41) and y_0 the distance at which the velocity is hypothetically equal to zero. This law is referred as the dip-modified log law. It presents the advantage that it contains only one parameter α and it reverts the classical log law for $\alpha = 0$.

Figure (1) presents comparisons of predicted velocity profiles obtained by dip-modified log law (Eq. 2) for different values of coefficient α with experimental data [4]. This figure shows that Eq. (2) with α given by empirical formula $\alpha = 1.3 \exp(-Ar/2)$ proposed by Yang et al. (solid lines) is able to predict a small deviation from log law profile (dip phenomenon). We notice that, it is not possible to improve predicted velocity profiles by adjusting the parameter α; since by increasing α, the deviation from experimental data increases (dash-dotted lines, with α = 0:5) and by decreasing the value of α predicted profiles approach log law profiles (dashed lines).



a) b)

Figure 1. Comparison of dip-modified log law profiles (Eq. 2) [6] for different values of coefficient α with experimental data; dashed lines: α = 0 log law profile, solid lines: $\alpha = 1.3 \exp(-Ar/2)$ proposed by Yang et al. [6], dash-dotted lines: α = 0.5; Symbols: measurements [4] for (a) C4, $Ar = 4.68$ and $S = 0.00401$, (b) C3, $Ar = 4.68$ and $S = 0.00296$.

3.2 A simple dip-modified log-wake law

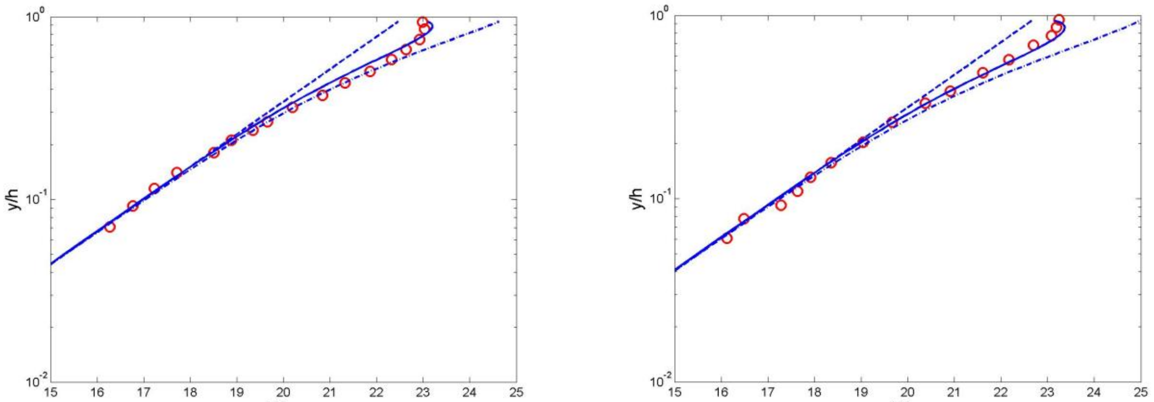
Figure (1) shows that log law profiles (dashed lines) are valid only in the inner region ($\xi = y/h < 0.2$). In the outer region ($\xi > 0.2$) log law deviates from experimental data. In 2D open-channel flows, this deviation is accounted for by adding the Coles wake function as

$$\frac{U}{u_*} = \frac{1}{\kappa}\left[ln\left(\frac{y}{y_0}\right) + 2\,\Pi\, sin^2\left(\frac{\pi\, y}{2\, h}\right)\right] \tag{3}$$

where Π is the Coles parameter expressing the strength of the wake function. It was found experimentally that Π increases with the Reynolds number Re in zero-pressure-gradient boundary layers, attaining an asymptotic value of $\Pi = 0.55$ at high Re. Examination of the log-wake law in the outer layer of 2D open-channel flows [5], shows that Π increases from zero with the friction Reynolds number Re_* and becomes nearly constant $\Pi \approx 0.2$, for $Re_* > 2000$. In 2D open-channel flows, the log-wake law appears to be the most reasonable extension of the log law.

However, in 3D open-channel flows with secondary currents, the log-wake law is unable to predict velocity dip-phenomenon (figure 2). A suitable simple law is possible by adding to log law both Coles' wake function and the term linearly proportional to the logarithmic distance from the free surface as

$$\frac{U}{u_*} = \frac{1}{\kappa}\left[ln\left(\frac{y}{y_0}\right) + 2\,\Pi\, sin^2\left(\frac{\pi\, y}{2\, h}\right) + \alpha\, ln\left(1 - \frac{y}{h}\right)\right] \tag{4}$$



a) b)

Figure 2. Comparison of our simple dip-modified log-wake law profiles (Eq. 4, solid lines) with experimental data; Symbols: measurements [4], dashed lines: log law profile ($\alpha = 0$ and $\Pi = 0$), dash-dotted lines: log-wake law profiles ($\alpha = 0$ and $\Pi = 0.2$), solid lines: simple dip-modified log-wake law (Eq. 4) with (a) $\alpha = 0.22$ and $\Pi = 0.2$ (C4 for *Ar* = 4.68 and S = 0.00401), (b) $\alpha = 0.27$ and $\Pi = 0.2$ (C3 for *Ar* = 4.68 and S = 0.00296).

Figure (2) presents comparisons of predicted velocity profiles obtained by our simple dip-modified log-wake law (Eq. 4) with experimental data of Lyn [4] for an aspect ration *Ar* equal to 4.68 but for different channel slope *S* = 0.00296 for C3 (figure 2.a) and *S* = 0.00401 for C4 (figure 2.b). Our profiles (solid lines) are also compared to log law profiles (dashed lines, $\alpha = 0$ and $\Pi = 0$) and log-wake law profiles (dash-dotted lines, $\alpha = 0$ and $\Pi = 0.2$). Comparison with experimental data shows reasonable agreement. Velocity profiles (solid lines) are obtained by dip-modified log-wake law with $\Pi = 0.2$ but with different coefficients α. This indicates that coefficient α is not dependent only on aspect ration *Ar*, as proposed by Yang et al. [6], but should depend also on channel slope S.

3.2 Modified log-wake law

Guo and Julien (2008) proposed a modified log-wake law as

$$\frac{U}{u_*} = \left[\frac{1}{\kappa}\, ln\left(\frac{y\, u_*}{\nu}\right) + B\right] + \frac{2\,\Pi}{\kappa}\, sin^2\left(\frac{\pi\, y}{2\,\delta}\right) - \frac{1}{3\,\kappa}\left(\frac{y}{\delta}\right)^3 \tag{5}$$

where δ is the dip distance from the bed. Eq. (5) fits better experimental data. However, it needs a certain number of parameters such as δ the dip position. This seems to exclude the possibility of its adoption in predictive and practical applications.

4 AN ORDINARY DIFFERENTIAL EQUATION

From the analysis of the Reynolds-averaged Navier-Stokes (RANS) equations and by using a more appropriate approximation for the eddy viscosity in accordance to log-wake law, we obtain the following ordinary differential equation (ODE) [2]

$$\frac{d\,U_a}{d\,\xi} = \frac{1}{\kappa}\left(1 - \alpha\,\frac{\xi}{1-\xi}\right)\,\left[\frac{1}{\xi} + \pi\,\Pi\, sin\,(\pi\,\xi)\right] \tag{6}$$

where $\xi = y/h$, $U_a = U/u_*$. Eq. (6) shows the interest that for $\alpha = 0$ and $\Pi = 0$ it gives the log law and for $\alpha = 0$ it gives the log-wake law.

Figure (3) presents comparisons of predicted velocity profiles obtained from the proposed ODE (Eq. 6, solid lines) with experimental data of Lyn (1986) [4] (fig. 3.a, 3.b) for an aspect ration *Ar* = 4.68 and Sarma et al. [7] (fig. 3.c, 3.d) for an aspect ration *Ar* = 4.98. In these figures, α and Π are chosen to give the best fit. Predicted profiles show good agreement. This figure confirms that the parameter α is not only a function of aspect ratio but should include effect of channel slope and friction velocity.

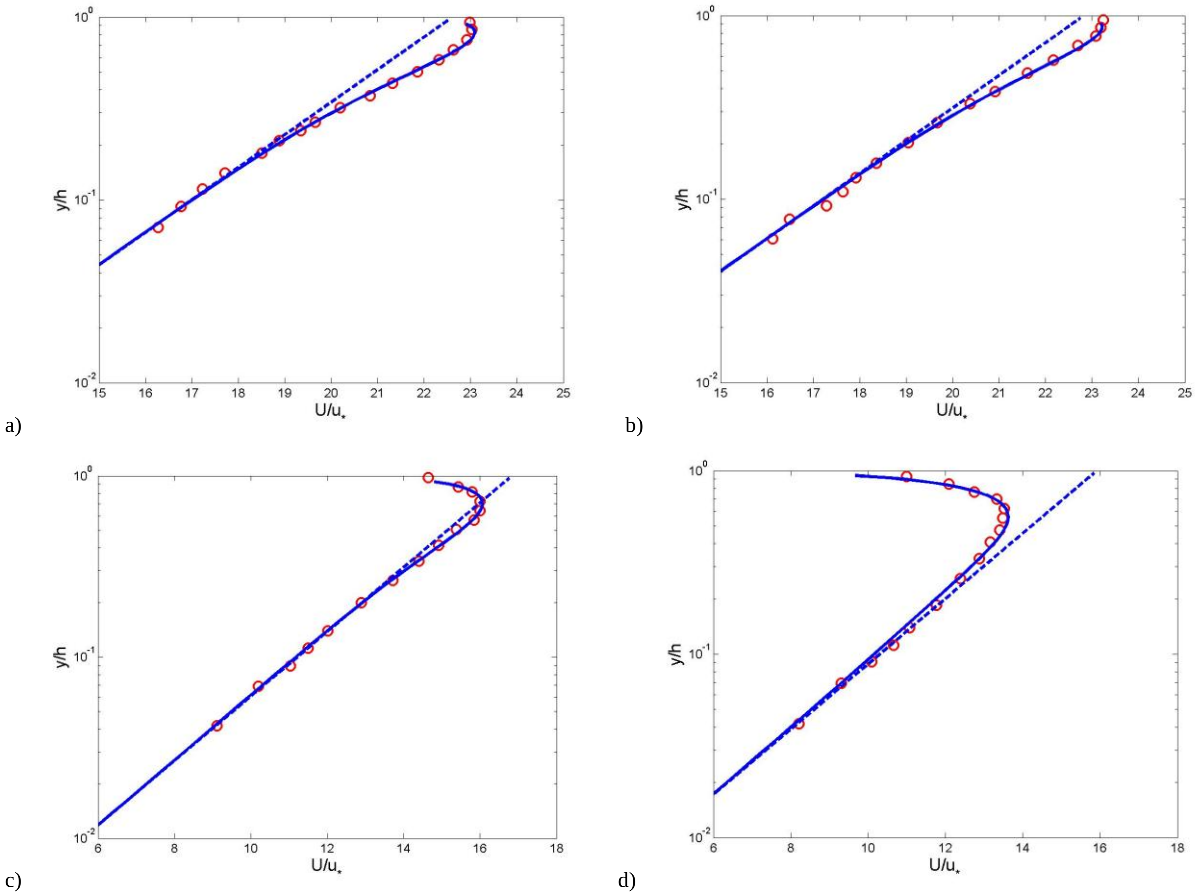


Figure 3. Comparison of velocity profiles obtained from ODE (Eq. 6) with experimental data [4] [7]: Symbols; dashed lines: log law profiles ($\alpha = 0$ and $\Pi = 0$); solid lines: solution of ODE (Eq. 6) with: (a) $\alpha = 0.21$ and $\Pi = 0.55$ (C4 for *Ar* = 4.68 and S = 0.00401 [4]), (b) $\alpha = 0.15$ and $\Pi = 0.4$ (C3 for *Ar* = 4.68 and S = 0.00296 [4]), (c) α = = 0.42 and $\Pi = 0.55$ (C4 for *Ar* = 4.98 and u_*=0.1688m/s [7]), (d) $\alpha = 0.78$ and $\Pi = 0.45$ (C3 for *Ar* = 4.98 and u_*=0.1244m/s [7]).

5 CONCLUSION

In this study, we proposed an ordinary differential equation based on an analysis of the Reynolds-averaged Navier-Stokes (RANS) equations. Comparisons of predicted velocity profiles with experimental data show good agreement. Our results show that coefficient α is not dependent only on aspect ration *Ar*, but should depend also on channel slope S and friction velocity.

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