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Piping flow erosion in water retaining structures: inferring erosion rates from hole erosion tests and quantifying the failure time

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Abstract
The piping flow erosion process, involving the enlargement of a continuous tunnel between upstream and downstream, is a major cause of water retaining structures. Such a pipe can be imputed to roots or burrows. The coefficient of erosion must be known in order to estimate the remaining time to failure and to downstream flood. The Hole Erosion test is a laboratory experiment especially suited to estimate a priori this geotechnical parameter. We propose therefore simplified expressions for the remaining time to breaching accounting for this erosion parameter. We established that the radius evolution of the pipe follows a two-parameters scaling law. The first parameter is the critical stress. The second parameter is the characteristic time of piping erosion, which is a function of the initial hydraulic gradient and the coefficient of erosion. We establish here new mechanically based relations for water retaining structures. The time to failure and the peak flow are related to the two basic parameters of piping failure: the coefficient of erosion, and the maximum pipe diameter prior to roof collapse and breaching. Orders of magnitude of the coefficient of erosion and the erosion rate are finally inferred from 18 case studies.

Introduction
The two most common failure modes of water retaining structures (earth-dams, dykes, levees) result from overtopping and piping. The breach due to failure generates a flood wave that propagates downstream the valley below the structure. Historically, the emphasis in dam safety has been on floods and overtopping. This case is fairly well documented [25]. However, the statistics of failure of embankment dams indicates that improvement in the understanding of piping is a significant concern of dam engineers. A comprehensive review of published literature on soil piping phenomena is presented by Richards and Reddy [17]. Piping accounts for 43% of all embankment dam failures, 54% for dams constructed after 1950 [10]. These statistics are based on the information reported in dam databases. Statistics concerning piping failures in dykes and levees are not yet available. The term “piping” is usually applied to a process that starts at the exit point of seepage and in which a continuous passage or pipe is developed in the soil by backward erosion, and enlarged by piping erosion. Evaluating the erodibility of a soil, both in terms of threshold of erosion (initiation) and rate of erosion (progression) are critical when evaluating the safety of a water retaining structure. Different soils erode at different rate. However, the relationship between the erosion parameters and the geotechnical and chemical properties of the soils remain unknown [7]. An overview of the research work on the erodibility of soils is presented in [23] and [24].

The present study concerns the progression phase of the piping process: the enlargement of a tunnel. The hole erosion test appears to be an efficient and simple means of quantifying the erosion parameters. The experience acquired on more than 200 tests on several soils has confirmed what an excellent tool this test can be for quantifying the rate of piping erosion in a soil, and for finding the critical shear stress corresponding to initiation of piping erosion. A model for interpreting the hole erosion test with a constant pressure drop was developed in [2] and [3]. A characteristic time of the internal erosion process was proposed, as a function of the initial hydraulic gradient, and the coefficient of erosion. It was shown that the product of the coefficient of erosion and the flow velocity is a significant adimensional number: when this number is small, the kinetic of erosion is low, and the concentration does not have any influence on the flow. This situation covers the main part of the available test results. Theoretical and experimental evidence was presented to the effect that the radius evolution of the pipe during erosion with constant pressure drop follows a scaling exponential law.

When piping erosion is suspected of occurring or has already been detected in situ, the rate of development is difficult to predict. In a growing number of cases, the location of population centers near the structure makes accurate prediction of breach parameters (namely the time to failure and the peak flow) crucial to the analysis. A critical analysis of the existing relationship was presented by Wahl [22]. These empirical relations are mostly straightforward regression relations that give the breach parameters as a function of various dam and reservoir parameters. It is questionable whether such relationships related to the reservoir storage, but not related to the rate of erosion, can be applied to estimate breach parameters for piping failure...
scenarios. However, concerning piping failures, few investigators have attempted to relate the breach parameters to basic parameters. This paper aimed to establish new mechanically based relations relating the time to failure and the peak flow to the two basic parameters of piping failure: the coefficient of erosion, and the maximum pipe diameter prior to roof collapse. These relations make possible to infer orders of magnitude of the coefficient of erosion from field data.

Piping flow erosion

Piping flow erosion in cohesive soils

Piping occurs if \( P_i > \tau_c \) where \( P_i \) is the driving pressure, equal to the tangential shear stress exerted by the piping flow on the soil, and \( \tau_c \) is the critical stress. The radius evolution of the pipe during erosion with constant pressure drop follows a scaling exponential law [3]:

\[
R(t) = R_0 \left[ \frac{\tau_c}{P_0} + \left( 1 - \frac{\tau_c}{P_0} \right) \exp \left( \frac{t}{t_c} \right) \right]
\]

(1)

\[
P_i = \frac{R_c \Delta p}{2L} \text{ (driving pressure)}
\]

(2)

\[
t_c = \frac{2\rho_{so} L}{C_\rho \Delta p} \text{ (characterising time of piping)}
\]

(3)

where \( t_c \) is the characteristic time of piping erosion, \( R_0 \) is the initial radius, \( \Delta p \) is the pressure drop in the hole, \( L \) is the hole length, \( \rho_{so} \) is the dry soil density, and \( C_\rho \) is the Fell coefficient of soil erosion. The later is similar to the Temple and Hanson [20] coefficient of erosion \( k_d \), as \( k_d = C_\rho / \rho_{so} \). The Fell erosion index is \( I_e = -\log C_\rho \) (\( C_\rho \) given in s/m).

The Hole Erosion Test

The hole erosion test was designed to simulate piping flow erosion in a hole. This test is not new [15]. An eroding fluid is driven through the soil sample to initiate erosion of the soil along a pre-formed hole. The results of the test are given in terms of the flow rate versus time curve with a constant pressure drop. Therefore, the flow rate is used as an indirect measurement of the erosion rate. For further details about this test, see [23]-[24]. The scaling law Equation (1) is compared with previously published data [23]-[24]. Analysis were performed in 18 tests, using 9 different soils (clay, sandy clay, clayey sand or siltysand). The initial radius and the length of the pipe were \( R_0 = 3 \) mm and \( L = 117 \) mm. Table 1 contains particle size distribution, and critical stress and Fell erosion index.

Figure 1 gives the effect of erosion process as the flow rate in relation to time, and shows that the use of \( t_c \) leads to efficient dimensionless scaling. Without this scaling, multiple graphs would be necessary to provide clarity of presentation. Scaled radius are plotted as a function of the scaling time in Figure 2. Nearly all the data can be seen to fall on a single curve. This graph confirms the validity of the scaling law Equation (1).

It is well known fact that different soils erode at different rate. Atempts were made to correlate erosion parameters - critical stress and coefficient of erosion - to common geotechnical or chemical soil properties in hope that simple equations could be developed for everyday use. All attempts failed to reach a reasonable correlation coefficient value [5]. It is strongly recommended carrying out hole erosion tests rather than using correlations, in order to evaluate the piping erosion parameters on any sample of cohesive soil from a site [23]-[24].

![Figure 1: Hole Erosion Tests (symbols) versus scaling law (continuous lines). Dimensionless flow rate is shown as a function of dimensionless time.](image1)

![Figure 2: Hole Erosion Tests (symbols) versus scaling law (continuous lines). Dimensionless radius is shown as a function of dimensionless scaling time.](image2)
Mechanically based relations for the time to failure and the peak flow

The rate of erosion has a significant influence on the time for progression of piping and development of a breach in earthdams, dykes or levees. This provides an indication of the amount of warning time available to evacuate the population at risk downstream of the dam, and hence has important implications for the management of dam safety. Given that erosion has initiated, and the filters are absent or unable to stop erosion, the hydraulics of flow in concentrated leaks are such that erosion will progress to form a continuous tunnel (the pipe). We consider the case of a straight and circular pipe, of current radius \( R(t) \), in an embankment of height \( H_{\text{dam}} \) and base width \( L_{\text{dam}} = c_{\text{t}} H_{\text{dam}} \) (Figure 3). The average quantities are defined as follows:

\[
L(t) = c_{\text{t}} \left[ H_{\text{dam}} - R(t) \right] \quad \text{(current pipe length)} \quad (4)
\]

\[
\Delta p_{\text{f}}(t) = \rho_{\text{w}} g \left[ \Delta H_{w} - R(t) \right] \quad \text{(average pressure drop)} \quad (5)
\]

![Figure 3: Sketch of the piping erosion in a water retaining structure.](image)

Although the head drop is likely to decrease with time, the situation for the case \( \Delta H_{w} = H_{\text{dam}} \) is more critical and, therefore, yields a conservative estimate of the time needed to initiate roof collapse. The rate of pipe enlargement is highly dependent on the erodibility of the soil as measured by the coefficient of erosion \( C_{\epsilon} \). The enlargement of the pipe causes roof collapse and creates a breach. The scaling law of the piping erosion process with a constant hydraulic gradient is given in Equation (1). We can now propose an expression for the remaining time to breaching. The piping process begins at time \( t_{0} \) with the initial radius \( R_{0} \), both unknown.

A sketch of our description is represented in Figure 4. A visual inspection defines the initial time \( t_{0} > t_{b} \) for detection, and can provide an estimation of the output flow rate, thus an estimation of the radius \( R_{b} > R_{0} \). We take \( R_{b} \) and \( t_{b} \) to denote the maximum radius of the pipe before roof collapse, and the collapse time, respectively. For \( t > t_{b} \), the piping failure continues to cause erosion in a similar way to an overtopping failure. The modeling of the overtopping breaching is fairly well documented [25].

The erosion onset radius can be neglected, as \( R_{e} \ll R_{b} \). The remaining time prior to breach \( \Delta t_{\epsilon} = t_{\epsilon} - t_{b} \) can therefore be estimated as follows

\[
\Delta t_{\epsilon} = t_{\epsilon} \ln \left( \frac{R_{b}}{R_{e}} \right) \quad (6)
\]

This important result establishes that the coefficient of erosion \( C_{\epsilon} \) can serve as an indicator of the remaining time to breaching, as \( \Delta t_{\epsilon} \propto C_{\epsilon}^{-1} \). The peak flow is assumed to correspond to the maximum radius of the pipe. Consequently, the time prior to breach \( \Delta t_{\epsilon} \) is also the time from detection (e.g. eyewitnesses observations) to peak discharge.

The average quantities are defined as follows:

\[
\Delta p_{\text{f}}(t) = \frac{1}{2} k \rho_{\text{w}} V^{2}(t) + \Delta p(t) \quad (7)
\]

\[
\tau_{\text{t}}(t) = \frac{R(t) \Delta p(t)}{2L(t)} \quad \text{(tangential stress)} \quad (8)
\]

\[
V(t) = \frac{\tau_{\text{t}}(t)}{\rho_{\text{w}}} \quad \text{(velocity)} \quad (9)
\]

\[
Q(t) = \pi R(t)^{2} V(t) \quad \text{(peak flow)} \quad (10)
\]

where \( k \) is the singular head loss coefficient (section sharpening of the pipe inlet), and \( f_{\text{t}} \) is the turbulent friction factor in the pipe.

<table>
<thead>
<tr>
<th>Soil</th>
<th>% Gravel</th>
<th>% Sand</th>
<th>% Fines</th>
<th>% &lt;2(\mu)m</th>
<th>(\tau_{\epsilon}) (Pa)</th>
<th>(I_{\epsilon})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lyell</td>
<td>1</td>
<td>70</td>
<td>29</td>
<td>13</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Fattorini</td>
<td>3</td>
<td>22</td>
<td>75</td>
<td>14</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>Pukaki</td>
<td>10</td>
<td>48</td>
<td>42</td>
<td>13</td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td>Jindabyne</td>
<td>0</td>
<td>66</td>
<td>34</td>
<td>15</td>
<td>6 - 72</td>
<td>3 - 4</td>
</tr>
<tr>
<td>Bradys</td>
<td>1</td>
<td>24</td>
<td>75</td>
<td>48</td>
<td>50 - 76</td>
<td>4</td>
</tr>
<tr>
<td>Shellharbour</td>
<td>1</td>
<td>11</td>
<td>88</td>
<td>77</td>
<td>99 - 106</td>
<td>4</td>
</tr>
<tr>
<td>Waranga</td>
<td>0</td>
<td>21</td>
<td>79</td>
<td>54</td>
<td>106</td>
<td>4</td>
</tr>
<tr>
<td>Matahina</td>
<td>7</td>
<td>43</td>
<td>50</td>
<td>25</td>
<td>128</td>
<td>4</td>
</tr>
<tr>
<td>Hume</td>
<td>0</td>
<td>19</td>
<td>81</td>
<td>51</td>
<td>66 - 92</td>
<td>4 - 5</td>
</tr>
</tbody>
</table>

Table 1: Hole Erosion Tests, Properties of Soils Samples, Critical Stress and Elk Erosion Index.
Inserting Equations (4)-(9) in Equation (10) yields:

$$Q_{\text{peak}} = \pi R_s^{1/2} \sqrt{\frac{2g(\Delta H_w - R_s)}{kr_s + 4f_s c_L(H_{\text{dam}} - R_s)}}$$  \hspace{1cm} (11)

It is emphasized that Equations (6) and (11) do not relate $\Delta t$, and $Q_{\text{peak}}$ to the reservoir storage, which is mechanically irrelevant, as it is usually proposed in the dam engineering literature. In other hand, $\Delta t$, and $Q_{\text{peak}}$ are directly related to $R_s$. The relations Equations (6) and (11) are therefore not useful as predictors if $R_s$ cannot be estimated with great certainty prior to a breach.

![Figure 4: Piping erosion in a water retaining structure, phases from initiation to breaching.](image)

**Order of magnitude of mechanical quantities on case studies**

Case study data provide only limited information. This is primarily due to the variations in interpretation of failure by the lay person who often is the only eyewitness to a dam failure. In the best case, the only information available are the time to breaching and the peak flow. The radius or the flow rate at detection is never reported. Inverting Equation (6) yields

$$C_s \approx \frac{2 \rho_{\text{water}} L}{\Delta t \Delta \rho} \ln \left( \frac{R_s}{R_d} \right)$$  \hspace{1cm} (12)

Few attempts have been made to propose a constitutive model to calculate the radius value prior to roof collapse. This estimate can be made on the basis of information derived from the peak flow value Equation (11). If the peak flow is unknown, an upper bound can however be obtained by taking $R_s = H_{\text{dam}} / 2$:

$$C_s \leq \frac{2 \rho_{\text{water}} L}{\Delta t \Delta \rho} \ln \left( \frac{H_{\text{dam}}}{2R_d} \right)$$  \hspace{1cm} (13)

The present statement is not intended to provide accurate values of the shear stress, the velocity or the flow rate. Rather, attention is focused explicitly on the more limited goal of giving numbers, and orders of magnitude.

Tables 1 and 2 contain data and results of this simplified analysis on 14 well documented piping failures cases. These cases were taken from the database presented in [22], where data on 108 case studies of actual embankment dam failures were collected from numerous sources in the literature. The dam height $H_{\text{dam}}$ ranged from 6 m to 93 m. The relative water level $\Delta H_w$ / $H_{\text{dam}}$ at failure ranged from 0.48 to 1. The coefficient $c_L = L_{\text{dam}} / H_{\text{dam}}$ ranged from 1.54 to 3. The failure time $\Delta t$ ranged from 0.5 h to 5.25 h. The peak flow $Q_{\text{peak}}$ ranged from 79 m$^3$/s to 65,120 m$^3$/s. The relative maximum radius $2R_s / H_{\text{dam}}$ estimated with Equation (11) ranged from 0.26 to 0.96.

The shear stress $\tau_s$, Equation (6) at failure (prior to roof collapse) ranged from 262 Pa to 8,051 Pa. The water velocity at failure $V$ Equation (7), estimated with $k=0.5$ and $f_s=0.005$, ranged from 7 m/s to 40 m/s. The choice of $k$, the singular head loss coefficient, corresponds to the section sharpening of the pipe inlet. The choice of $f_s$ is consistent with the Reynold number at failure, which ranged from $3 \times 10^2$ to $2 \times 10^4$. This estimate was obtained on the basis of investigations with several friction factor formula [1, 16, 19].

The radius at detection $R_d$ is unknown. As $R_d \ll \Delta H_w \leq H_{\text{dam}}$, the flow rate can roughly be estimated with $Q(R_d) = \pi R_d^{1/2} \sqrt{g / (2f_s c_L)}$. This gives $Q = 1$ m$^3$/s for $R_d = 4$ cm and $Q = 1$ m$^3$/s for $R_d = 20$ cm. These values can be considered as two extreme values for visual detection and emergency status.

The erosion index rate $I_e = -\log C_s$ was estimated as an average of four numbers, calculated with the RHS of Equations (12) and (13), with $R_s = 20$ cm and $R_d = 4$ cm. The average erosion index rate was found to range from 1.6 to 3.0. The standard deviation ranged from 0.08 to 0.22.

Wan and Fell [23, 24] found that the coefficient of erosion $C_v$ can differ by up to $10^4$ times across different soils from a series of hole erosion tests (13 soils). The coefficient of erosion was found to range from $10^3$ s.m$^{-1}$ to $10^7$ s.m$^{-1}$. In this study dealing with piping erosion, the coefficient of erosion $C_v$ inferred from data of case histories ranged from $10^3$ s.m$^{-1}$ to $10^7$ s.m$^{-1}$. Our results are consistent with previous finding.

For overtopping, Courivaud and Fry [6] reported values of breach widening rate inferred from data of 10 case histories, covering a range of dam height from 8 m to 60 m. These values ranged from 14 cm.min$^{-1}$ to 600 cm.min$^{-1}$. For piping flow erosion, the erosion rate prior to roof collapse can be estimated with $V_c = \tau_s C_s / \rho_{\text{water}}$ and $\rho_{\text{water}}=1,600$ kg/m$^3$. We found that $V_c$ ranged from 3 cm.min$^{-1}$ to 107 cm.min$^{-1}$. These comparisons confirm the validity of our statement: orders of magnitude can be inferred from field data with limited information.
Conclusion

Few attempts have been made so far to model the piping erosion process in soils. A simplified but mechanically based approach was used to establish new relationship for water retaining structures. The time to piping failure and the peak flow were related to the coefficient of erosion, and the maximum pipe diameter prior to roof collapse and breaching. Several mechanical quantities were inferred from 14 case studies: the shear stress, the velocity, the coefficient of erosion and the erosion rate. This is the first attempt to derive such numbers from field data for the piping erosion process. The comparisons with published data on erosion confirmed that the obtained orders of magnitude, inferred from case studies with limited information, are relevant. For water retaining structures, the coefficient of erosion is the first important parameter: it can serve as an indicator of the remaining time to breaching, but visual detection of the piping event and reporting is required. This coefficient can be obtained with the Hole Erosion Test. However, the issue of the way the change of scale (from the laboratory to the structure) could affect the coefficient of erosion remains to be addressed.

The second important parameter is the maximum pipe diameter prior to roof collapse and to breaching. It can serve as an indicator of the peak flow. However, constitutive relations to calculate this diameter as a function of mechanical quantities (such as the soil shear strength) are currently limited. To date, this information cannot be estimated with great certainty prior to a breach.

Acknowledgements

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References

### Table 2: Well documented piping failure cases. The first five columns are taken from [23]. The maximum radius is estimated by using Eq. (11)

<table>
<thead>
<tr>
<th>Dam name and location</th>
<th>$H_{\text{dam}}$ (m)</th>
<th>$\Delta H_{\text{w}}$ (m)</th>
<th>$c_L$</th>
<th>$\Delta t_u$ (h)</th>
<th>$Q_{\text{peak}}$ (m$^3$.s$^{-1}$)</th>
<th>$R_u$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland No. 5, Colo.</td>
<td>6.0</td>
<td>3.8</td>
<td>3.0</td>
<td>0.5</td>
<td>110</td>
<td>2.20</td>
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<tr>
<td>Lower Latham, Colo.</td>
<td>8.6</td>
<td>5.8</td>
<td>3.0</td>
<td>1.5</td>
<td>340</td>
<td>3.53</td>
</tr>
<tr>
<td>Frankfurt, Germany</td>
<td>9.8</td>
<td>8.2</td>
<td>3.0</td>
<td>2.5</td>
<td>79</td>
<td>1.42</td>
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<td>Kelly Barnes, Ga.</td>
<td>11.6</td>
<td>11.3</td>
<td>1.7</td>
<td>0.5</td>
<td>680</td>
<td>3.66</td>
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<td>French Landing, Mich.</td>
<td>12.2</td>
<td>8.5</td>
<td>2.8</td>
<td>1.16</td>
<td>929</td>
<td>5.30</td>
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<tr>
<td>Lake Latonka, Penn.</td>
<td>13.0</td>
<td>6.3</td>
<td>2.2</td>
<td>3</td>
<td>290</td>
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<td>13.7</td>
<td>2.9</td>
<td>2</td>
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<td>6.94</td>
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<td>Quail Creek, Utah</td>
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<td>16.7</td>
<td>3.0</td>
<td>1</td>
<td>3,110</td>
<td>7.53</td>
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<td>Hatchtown, Utah</td>
<td>19.2</td>
<td>16.8</td>
<td>2.3</td>
<td>4</td>
<td>3,080</td>
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<tr>
<td>Little Deer Creek, Utah</td>
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<td>22.9</td>
<td>2.4</td>
<td>0.66</td>
<td>1,330</td>
<td>4.37</td>
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<td>Bradfield, England</td>
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<td>29.0</td>
<td>1.7</td>
<td>0.5</td>
<td>1,150</td>
<td>3.75</td>
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<td>28.0</td>
<td>2.4</td>
<td>3.25</td>
<td>6,850</td>
<td>9.51</td>
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<tr>
<td>Hell Hole, Calif.</td>
<td>67.1</td>
<td>35.1</td>
<td>1.5</td>
<td>0.75</td>
<td>7,360</td>
<td>9.30</td>
</tr>
<tr>
<td>Teton, Idaho</td>
<td>93.0</td>
<td>77.4</td>
<td>2.7</td>
<td>5.25</td>
<td>65,120</td>
<td>22.73</td>
</tr>
</tbody>
</table>

### Table 3: Well documented piping failure cases. Coefficient of erosion and final erosion rate estimates. The erosion index rate is $I_{er} = -\log C_{er}$

<table>
<thead>
<tr>
<th>Dam name and location</th>
<th>$\tau_b$ (Pa)</th>
<th>$V_u$ (m.s$^{-1}$)</th>
<th>$I_{er}$ (mean ± std. dev.)</th>
<th>$C_{er}$ ($10^3$ s.m$^{-1}$)</th>
<th>$V_{er}$ (cm.mm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ireland No. 5, Colo.</td>
<td>262</td>
<td>7</td>
<td>1.6 ±0.13</td>
<td>24</td>
<td>24</td>
</tr>
<tr>
<td>Lower Latham, Colo.</td>
<td>379</td>
<td>9</td>
<td>2.0 ±0.11</td>
<td>10</td>
<td>14</td>
</tr>
<tr>
<td>Frankfurt, Germany</td>
<td>784</td>
<td>13</td>
<td>3.0 ±0.22</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Kelly Barnes, Ga.</td>
<td>1,309</td>
<td>16</td>
<td>2.0 ±0.12</td>
<td>10</td>
<td>47</td>
</tr>
<tr>
<td>French Landing, Mich.</td>
<td>552</td>
<td>11</td>
<td>1.8 ±0.09</td>
<td>15</td>
<td>31</td>
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<tr>
<td>Lake Latonka, Penn.</td>
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<td>10</td>
<td>2.5 ±0.14</td>
<td>4</td>
<td>6</td>
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<td>2.2 ±0.08</td>
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<td>25</td>
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<tr>
<td>Quail Creek, Utah</td>
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<td>2.0 ±0.09</td>
<td>10</td>
<td>56</td>
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<tr>
<td>Hatchtown, Utah</td>
<td>1,606</td>
<td>18</td>
<td>2.6 ±0.09</td>
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<td>Little Deer Creek, Utah</td>
<td>2,454</td>
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<td>Bradfield, England</td>
<td>3,378</td>
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<td>Apishapa, Colo.</td>
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<td>2.1 ±0.13</td>
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<td>Teton, Idaho</td>
<td>8,051</td>
<td>40</td>
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