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Fermi liquid behavior of the normal phase of a strongly interacting gas of cold atoms

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We measure the magnetic susceptibility of a Fermi gas with tunable interactions in the low temperature limit and compare it to quantum Monte Carlo calculations. Experiment and theory are in excellent agreement and fully compatible with the Landau theory of Fermi liquids. We show that these measurements shed new light on the nature of the excitations of the normal phase of a strongly interacting Fermi gas.

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In 1956 Landau developed an elegant description of interacting Fermi systems at low temperature relying on the existence of long-lived quasiparticles. While this Fermi liquid theory (FLT) describes well Helium 3 and many solid-state materials above the superfluid temperature, there exist notable exceptions such as underdoped cuprates [1], where despite tremendous theoretical and experimental efforts, the nature of the normal phase is not yet understood. Similarly to high critical temperature superconductors, the properties of the normal phase of strongly correlated atomic fermionic gases and the nature of its excitations are still debated. This issue was addressed recently for spin-balanced gases above the superfluid transition, through the measurement of equations of state [2, 5–7], the study of the single-particle excitation spectrum [8, 9] or of spin fluctuations [10]. On the one hand, recent photoemission spectroscopy experiments near the critical temperature were interpreted using a pseudogap model [9]. On the other hand, measurement of the temperature dependence of the specific heat displayed a linear behavior compatible with Fermi liquid’s prediction [2]. All these experimental probes give access to the properties of the normal phase of the unpolarized normal phase above the critical temperature $T_c$. This limitation can be overcome by stabilizing the normal state at $T < T_c$ by imposing a spin population imbalance in the trapped gas [11–13] and extrapolating its properties to zero imbalance. Previous works focused on the highly-polarized limit where minority atoms behave as impurities: $n_2 \ll n_1$, where $n_i$ is the density for species $i$ [2, 4, 14–21]. Here, we interpret the spin imbalance as the application of an effective magnetic field to the unpolarized normal gas at very low temperature and using a combination of Monte Carlo simulations and experimental results, we extract from the equation of state the magnetic spin response of the normal phase in the limit $T \ll T_c$. We show that our results are compatible with a Fermi Liquid description of the normal phase, and we extract the Fermi liquid parameters in the universal unitary limit where scattering length is infinite. The relationship between these parameters and the properties of low lying excitations of the system allow us to quantitatively interpret spectroscopic data from [8, 9].

The polarization dependence of the energy $E$ of the system directly reflects the presence of spin-singlet dimers in the sample. Indeed, the presence of a gap in the spin excitation spectrum implies a linear dependence of the energy $E$ with polarization $p = (N_1 - N_2)/(N_1 + N_2)$ at low temperature, and hence a zero spin susceptibility. We have performed quantum Monte Carlo simulations of the partially polarized Fermi gas at $T = 0$ in the BEC-BCS crossover. We make use of the fixed-node diffusion Monte Carlo method that was employed in earlier studies of polarized Fermi gases [15, 19]. The state of the system is forced to be in the normal phase by imposing the nodal surface of a many-body wave function incompatible with off-diagonal long-range order. A simple way to implement this requirement is by choosing the trial function of the Jastrow-Slater form

$$\psi_T(R) = \prod_{i,i'} f(r_{ii'}) D(N_1) D(N_2),$$

where $R = (r_1, ..., r_N)$ is the spatial configuration vector of the $N$ particles and $D$ denotes the Slater determinant of plane waves in a cubic box of size $L$ with periodic boundary conditions. The positive Jastrow correlation term $f(r)$ is determined as described in Ref. [15]: at short distances it corresponds to the lowest-energy solution of the two-body problem, while it satisfies the boundary condition on its derivative $f'(r = L/2) = 0$.

The results for the canonical equation of state $E(N_1, N_2)$ are shown in Fig.1. They are well fitted by

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FIG. 1: Canonical equation of state of a two-component Fermi gas calculated using quantum Monte Carlo simulation, for $1/k_F a = -1.5, -1, -0.6, -0.2, 0, 0.2, 0.4, 0.5$ (from top to bottom). The solid lines are fits of the low-polarization data with equation (2). Inset: Extracted values of the susceptibility $\tilde{\chi}$ as a function of $1/k_F a$. The dashed red line is the result of a perturbation expansion valid up to order $(k_F a)^2$.

the energy functional

$$E(p) = \frac{3}{5} N E_F \left( \xi_N + \frac{5}{9} \tilde{\chi}^{-1} p^2 + \ldots \right),$$

(2)

holding for a spin polarizable system at low temperature, where both $\xi_N$ and the dimensionless spin susceptibility $\tilde{\chi}$ (in units of the susceptibility of an ideal Fermi gas $3\hbar^2/2E_F$) depend on $1/k_F a$, where $k_F = (3\pi^2 n)^2/3$. The Monte Carlo method indicates the absence of spin gap, and thus of preformed molecules in the normal phase for $1/k_F a \lesssim 0.5$. Note that the extracted values of $\tilde{\chi}$ reported in the inset of Fig.1 show a rapid drop for positive values of $a$ when entering the BEC side of the Feshbach resonance. A likely explanation is the binding of fermions into spin-singlet pairs for some positive value of the interaction strength $1/k_F a$. Monte Carlo calculations for values of $1/k_F a \geq 0.7$ show that $E(p)$ is indeed linear rather than quadratic in $p$, indicating the emergence of a gap. However, pairing fluctuations play a major role for such values of the coupling and the nodal surface of the Jastrow-Slater state (see Auxiliary Materials) is no longer sufficient to enforce the normal phase. This behavior is reminiscent of the pairing transition investigated in the framework of BCS theory [23], as well as in the normal phase of the attractive Hubbard model, extrapolated to a temperature range below the superfluid transition [24, 25], while in our work the extrapolation is made towards a small spin imbalance.

We now compare these simulations with the grand-canonical equation of state (EoS) of a homogeneous system obtained experimentally in ref. [2, 4]. We prepare a deeply degenerate mixture of the two lowest internal states of $^4$Li, held in a cylindrically symmetric hybrid optical/magnetic trap, of radial (axial) frequency $\omega_r$ ($\omega_z$, respectively). The bias magnetic field $B_0$ is chosen between 822 G and 981 G, allowing to tune the strength of interactions $-1 < 1/k_F a < 0.2$. The final atom number is $2 \times 10^4$ atoms per spin state, and the gas temperature is smaller than $0.06 T_F$, as measured from the fully-polarized wings of a trapped gas [5]. From dimensional analysis, the EoS of a spin-imbalanced Fermi gas can be written as

$$P(\mu_1, \mu_2, a) = P_0(\mu) h \left( \delta = \frac{\hbar}{\sqrt{2m \mu a}}, b = \frac{\mu_1 - \mu_2}{\mu_1 + \mu_2} \right),$$

where $\mu = (\mu_1 + \mu_2)/2$ is the mean chemical potential and $P_0(\mu)$ is the pressure of a non-interacting unpolarized Fermi gas. $\delta$ is a grand-canonical analog of the interaction parameter $1/k_F a$, and $b$ is a dimensionless number proportional to the ‘spin-polarizing field’ $\mu_1 - \mu_2$.

At all values of the scattering length addressed in this work, the equation of state exhibits a clear discontinuity of its derivative at the critical field $b_c(\delta)$ (See Fig. 2), indicating a first-order phase transition from a superfluid state for $b < b_c$ to a normal state for $b > b_c$ where $h$ is linear in $b^2$. [4, 12]. The equation of state of the superfluid phase has been discussed in a previous work [4] and we focus here on the properties of the normal phase. We write:

$$h(\delta, b) = h_N(\delta) \left( 1 + \frac{15}{8} \chi^{GC}(\delta) b^2 + O(b^4) \right).$$

(3)

$h_N(\delta)$ is the grand-canonical equation of state in the normal phase, extrapolated to a spin-symmetric configuration. $\chi^{GC}(\delta)$ is a grand-canonical magnetic susceptibility. For an ideal two-component Fermi gas, the functions $h_N$ and $\chi^{GC}$ are equal to 1. Fitting our data in the normal phase with (3), we obtain the parameters $h_N(\delta)$ and $\chi^{GC}(\delta)$ in the BEC-BCS crossover shown in Fig.3 where we compare their values to the predictions of the Monte Carlo simulations. To this end, we fit the dependence with $1/k_F a$ of the parameters $\xi_N$ and $\tilde{\chi}$ determined by Monte Carlo simulations, and perform a Legendre transform to obtain the grand-canonical EoS $h_N(\delta)$ of the normal phase and magnetic susceptibility $\chi^{GC}(\delta)$ measured.

FIG. 2: Thermodynamic function $h(b)$ measured at different magnetic fields $B_0 = 871, 834, 822$ G. The blue lines correspond to the superfluid equation of state $h_S(\delta)$ measured in [4]. The red line is a linear fit of the data in the normal phase, $b > b_c$. The dashed line indicates the superfluid/normal phase transition ($b = b_c$).
thus suggests that the dip is either extremely narrow or
≃ in Fig. 1, 2, and in the whole range
above the Fermi surface. Combining the measurement of
the Quantum Monte Carlo results of Fig.1.
served neither in the experimental data of Fig.2 nor in
smaller on the BCS side of the resonance. This is ob-
served neither in the experimental data of Fig.2 nor in

The spin susceptibility in the superfluid phase .
spin excitations of the system are not gapped in the
symmetric configuration
FIG. 3: Fermi liquid equation of state extrapolated to a spin-
symmetric configuration $h_{\nu}(\delta)$. The black dots are the ex-
erimental data, and the red line is calculated from the Monte
Carlo data. Inset: Grand-canonical susceptibility $\chi^{GC}(\delta)$ of
a Fermi gas in the BEC-BCS crossover.

experimentally. In the investigated parameter range, the
agreement between theory and experiment is excellent.
We also remark that our value for the susceptibility of
the normal phase at unitarity is about twice larger than
the value measured in [10] on a gas with a 35% conden-
sate fraction, confirming a significant suppression of the
spin susceptibility in the superfluid phase .

Our findings demonstrate that for $1/k_{F}a \lesssim 0.5$, the
spin excitations of the system are not gapped in the
normal phase which therefore does not support “true”
molecules. However, a certain class of theories predicts
a reminiscence of this gap in the form of a dip in the
density of states over a range $\Delta^*$ around the Fermi level
[26]. $\Delta^*$ is often called the pseudogap, in relationship to
some features of high-critical temperature superconduc-
tors. These theories predict a departure of $E(p)$ from its
quadratic behavior when the Fermi levels of the two spin
species reach the edges of the dip, $\mu_2 - \mu_1 \simeq \Delta^*$, (see
Auxiliary Materials).
The absence of such an anomaly in Fig. 1, 2, and in the whole range $-1 < 1/k_{F}a < 0.5$
thus suggests that the dip is either extremely narrow or
very broad: the density of state remains flat over the
range of polarizations and interaction strength studied
in our work. For instance, at unitarity this range covers
$0 < b^2 < 3$. If a sizeable dip existed, then its width cannot
be smaller than $\simeq (\mu_1 + \mu_2)\sqrt{3} \simeq 1.4E_F$ where we
have used the unitary equation of state, $\mu = 0.41E_F$
[4]. Such a large pseudogap is not compatible with the
photoemission data of [9] (See below). Furthermore,
we would expect on physical grounds that $\Delta^*$ becomes
smaller on the BCS side of the resonance. This is ob-
erved neither in the experimental data of Fig.2 nor in
the Quantum Monte Carlo results of Fig.1.

On the contrary, Landau’s theory of Fermi liquids is
fully compatible with our observations. This theory as-
sumes the existence of long-lived fermionic excitations
above the Fermi surface. Combining the measurement of
the low-temperature compressibility $\kappa$ and specific heat
$C_v$ of [2] with the data presented here, we can fully
characterize the parameters of the theory at the unit-
ary limit. From the magnetic response of the $T = 0$
gas, we obtain here its magnetic susceptibility and an-
other determination of $\kappa$. The two determinations of $\kappa$ coincide within 5%, showing that the two approaches
indeed probe the same Fermi liquid. From this set of
thermodynamic quantities we derive, according to Lan-
dau’s Fermi liquid theory, a complete characterization
of the low-lying excitations of the unitary gas: besides
their effective mass $m^* = 1.13m$ and Landau param-
eters $F_0 = -0.42$, $F_1 = 0.39$ found in [2], we re-
cover here $F_0^* = -0.40$ and obtain the new parameter
$F_0^* = m^*/m\bar{\chi}(0) - 1 = 1.1(1)$. Note that $F_0^* > 0$
corresponds to magnetic correlations which do favor the
singlet configuration.

We can finally test FLT on the single-particle photo-
emission spectrum obtained at the unitary limit and at
the onset of superfluidity from ref. [9]. The experimen-
tal signal $\chi(k, \omega)$ is directly proportional to the spectral
function $A(k, \omega - \mu)$ averaged over the trap that we es-
imate using the following procedure: In the vicinity of
the Fermi surface, the dispersion relation of the Fermi
liquid quasi-particles reads $\hbar\omega_k = \mu + \hbar^2(k^2 - k_F^2)/2m^*$
where $m^* = 1.1m$. Assuming long-lived quasiparticles,
we approximate $A(k, \omega)$ by $\delta(\omega - \omega_k)$ and perform the
integration over the trap to obtain $\chi(k, \omega)$ given by [9]:

$$\chi(k, \omega) = \frac{4\hbar k^2}{\pi^2} \int d^3r \frac{A(k, \omega - \mu(r)/\hbar)}{1 + \exp^{\hbar\omega_k - \mu(r)/k_BT}},$$

where $\mu(r)$ is the local chemical potential at position
$r$. In order to calculate the integrated spectral function
$\chi(k, \omega)$ of a Fermi liquid, we replace the spectral func-
tion by $\delta(\omega - \omega_k)$, and perform the integral in (4).
$k_F(r)$ is calculated from the equation of state of the unitary
gas determined in [2]. The temperature is chosen at the
onset of superfluidity $k_BT/\mu^0 = 0.32$ [2, 32]. In order
to make a direct comparison with the experimental data,
we finally convolve our result with the experimental reso-
lution in $\omega$ [9], equal to $0.25E_F/\hbar$ and results for various
values of $k$ are shown in Fig. 4.

With no free parameter in the theory, FLT well
reproduces the experimental spectra for $\chi(k, \omega)$ in the
region $k < k_F$, with an excellent agreement in the region
$0.3k_F \leq k \leq k_F$ close to the most probable Fermi
level in the trap ($\simeq 0.7k_F$) where FLT is expected to be
more accurate. Interestingly, we observe that the width
of the peak at $k/k_F = 0.6$ is well reproduced by our
model meaning that the broadening of the line is not
limited by the lifetime of the quasiparticles, but rather
by trap inhomogeneity and measurement resolution. Sig-
nificant deviations between experiment and FLT appear
for $k > 1.1k_F$, far from the most probable Fermi wave-
vector. However in this region the energy spectrum signal
is very broad and weak, corresponding to an incoherent background in the spectral function. Our Fermi liquid description thus accounts for the coherent part of the excitation spectrum from [9].

In conclusion we have shown that the magnetic and thermal responses of the unitary Fermi gas support a description of the normal phase in terms of Fermi liquid theory despite the fact that this system exhibits a high critical temperature for superfluidity. This behaviour is in contrast with underdoped cuprate high $T_c$ materials displaying anomalous magnetic susceptibility or pseudogap physics in the normal phase. Recent quantum oscillation experiments on cuprates in high magnetic fields, aiming at studying the incipient normal state (somewhat analogously to the present work) do suggest long-lived quasiparticles [30]. The drop of the susceptibility on the BEC side of the resonance for $1/k_F a \gtrsim 0.5$ indicates the appearance of a spin gap in this regime that deserves further investigations. Finally, the magnetic susceptibility could be a key observable for characterizing the onset of itinerant ferromagnetism in a repulsive Fermi gas [29, 31].

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AUXILIARY MATERIAL

Measurement of the equation of state We recall here the procedure used to measure the equation of state \( P(\mu_1, \mu_2, a) \), that was already employed in [1] after the method proposed by [4]. We prepare a deeply degenerate mixture of the two lowest internal states of \(^6\)Li, held in a cylindrically symmetric hybrid optical/magnetic trap, of radial (axial) frequency \( \omega_r \) (\( \omega_z \)), respectively. The bias magnetic field \( B_0 \) is chosen between 882 G and 981 G, allowing to tune the strength of interactions. The final atom number is \( 2 \times 10^9 \) atoms per spin state, and the gas temperature is 0.03(3)\(T_F\), as measured from its in situ image. In the framework of local density approximation, this provides the grand-canonical equation of state \( P(\mu_1, \mu_2, a) \) at the local chemical potentials \( \mu_{iz} = \mu_1 - \frac{1}{2} m \omega_z^2 z_i^2 \), where \( \mu_1 \) is the global chemical potential for species \( i \). The global chemical potential \( \mu_1^0 \) for the majority species is directly obtained from the Thomas-Fermi radius \( R_1 \) of the fully polarized phase, according to \( \mu_1^0 = \frac{1}{2} m \omega_z^2 R_1^2 \). Similarly to [1], we obtain the global chemical potential \( \mu_2^0 \) by imposing that, at the outer radius \( R_2 \) of the minority species, the chemical potential ratio \( \mu_2/\mu_1 \) is given by the resolution of the impurity problem [8–13].

Fixed-Node Monte Carlo simulation The Hamiltonian of the \( N = N_1 + N_2 \) atoms of the two species is given by

\[
H = -\frac{\hbar^2}{2m}\left(\sum_{i=1}^{N_1} \nabla_i^2 + \sum_{i'=1}^{N_2} \nabla_{i'}^2\right) + \sum_{i,i'} V(r_{ii'}) , \tag{5}
\]

where \( i, j, \ldots \) and \( i', j', \ldots \) label, respectively, majority and minority fermions. We model interspecies interatomic interactions using an attractive square well potential: \( V(r) = -V_0 \) if \( r < R_0 \) and zero otherwise (\( V_0 > 0 \)). The short range \( R_0 \) is fixed by the condition \( n R_0^3 = 10^{-6} \), where \( n = n_1 + n_2 \) is the total atom density. The depth \( V_0 \) is instead chosen as to give the proper value of the scattering length \( a \) along the BEC-BCS crossover. We consider a system with fixed total number of particles \( N = 66 \) in a fixed volume \( V = L^3 \).

Finite-size effects have been reduced using the technique described in [7].

Gap and spin susceptibility Let us consider a system containing \( N_1 \) spin up and \( N_2 \) spin down particles. We define \( M = N_1 - N_2 \) and \( N = N_1 + N_2 \) the polarization and the total atom number and we note \( E(N, M) \) the energy of the system. If one assumes that the energy can be expanded in \( M \) then by symmetry the linear term vanishes and one gets \( E(N, M) = E(N, 0) + M^2/2\chi + \ldots \). With this definition, \( \chi \) is then the spin susceptibility of the system. Indeed, adding a magnetic field \( h \) contributes to a \( -hM \) term to the energy and we immediately see that the energy minimum is shifted from \( M = 0 \) to \( M = \chi h \).

This argument is no longer true in the case of a gapped system. Indeed, polarizing a spin balanced system costs the binding energy of the broken pairs. This definition applies to any system composed of spin-singlet dimers, from a fermionic superfluid composed of Cooper pairs, or a pure gas of uncondensed molecules, and leads to the following leading order expansion

\[
E(N, M, \Delta) = E(N, 0) + |M|\Delta + \ldots
\]

To evaluate the spin susceptibility, we add as above a magnetic field \( h \) changing the energy into \( E = -hM \). We see that for \( h \neq 0 \), the potential is tilted but the energy minimum stays located at \( M = 0 \) (as long as \( |h| < \Delta \) corresponding to the Pauli limit pointed out by Clogston and Chandrasekhar in the case of superconductors [20, 21]).

\[\text{FIG. A1: Dependence of energy } E \text{ with spin population imbalance } M \text{ for gapless (top) and gapped (bottom) systems.}
\]

\[\text{Top: Full line: The dependence with spin imbalance is quadratic and the curvature is equal to the inverse of the spin susceptibility } \chi. \text{ Dashed line: In the presence of a spin polarizing field } h, \text{ the energy minimum is shifted to } M = \chi h. \]

\[\text{Bottom: gapped system. Full line: Energy in the absence of external spin polarizing field. The slope is equal to the gap } \Delta. \text{ Dashed line: in the presence of a spin polarizing field, the energy profile is tilted but the minimum remains located at } M = 0.\]

\[\text{Thermodynamic signature of the pseudogap} \]

The pseudogap phenomenon can be defined as a dip in the
density of state $\rho(\varepsilon)$ close to $E_F$ reminiscent of the true cancellation of $\rho$ inside the superfluid gap. We note $\Delta^*$ the width of the dip, and for the sake of simplicity we assume that $\Delta^* \ll k_B T_F$. In a simple model where one assumes that the excitations of the system are described by the Fermi-Dirac distribution, one can show that the spin susceptibility of the system is equal to the density of states of the two spin species inside the dip. This results in a depletion of spin excitations and a reduction of the spin susceptibility. 

In this case, the spin susceptibility is given by $\chi = \rho_0 > \rho_0^*$ (see Fig. A2).

![FIG. A2: Top: sketch of the density of state in a pseudogap model. A footprint of the molecular state appears as a dip of width $\Delta^*$ and depth $\rho_0 - \rho_0^*$ in the density of state. Bottom: Polarization $p$ as a function of the magnetic field $b$, with $\rho_0^* = 0.5\rho_0$ and $\Delta^* = 0.1\mu$. At low imbalance, the Fermi levels of the two spin species lie inside the dip. This results in a depletion of spin excitations and a reduction of the spin susceptibility.](image)

**Polarizability of the unitary Fermi gas** It is interesting to express our data for the unitary Fermi gas in the usual variables of condensed matter physics, namely the polarization $p = (n_1 - n_2)/(n_1 + n_2)$ as a function of $(\mu_1 - \mu_2)/2E_F$, where $E_F$ is the Fermi energy. These quantities are calculated from the thermodynamic function at unitarity $h_0(b) = h(\delta = 0, b)$ according to

$$p = \frac{\chi_0(b)}{2} - \frac{b}{2} h_0(b) \chi_0^2 = \frac{\chi_0(b_0) - b h_0(b_0)}{(h_0(b) - \frac{b}{2} h_0^2(b))^{2/3}}.$$ 

This requires to take the derivative of our experimental data, which decreases the signal-to-noise ratio. We obtain the data plotted in Fig. A3. In the superfluid phase the polarization remains equal to 0 and jump to $p \simeq 0.4$ at the superfluid/normal transition (for $\mu_1 - \mu_2 \simeq 0.4 \cdot 2E_F$). The polarization then increases linearly with the magnetic field over a large polarization range, according to $p = \frac{\chi}{2} (\mu_1 - \mu_2)/2E_F$, with $\chi = 0.54$.

![FIG. A3: Polarization $p$ of the unitary Fermi gas as a function of the magnetic field $\mu_1 - \mu_2$, normalized to the Fermi energy $E_F = \hbar^2/(2m(3\pi^2 n)^{2/3})$. The red line corresponds to the relation $p = \frac{\chi}{2} (\mu_1 - \mu_2)/2E_F$, with $\chi = 0.54$.](image)


