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A MICROSTRUCTURALLY-BASED EFFECTIVE STRESS FOR UNSATURATED SOILS

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ABSTRACT: Current alternative choices of stress state variables in unsaturated soils are described and compared with a special focus on the use of an effective stress. Experimental data about stiffness and shear strength evolution with suction suggests that the proportion of suction contributing to the effective stress is often much smaller than predicted by the term “suction times degree of saturation” generally used in effective stress expressions of Bishop’s type. It is suggested that effective stress in unsaturated soils should be related to soil microstructure. An effective degree of saturation is defined to describe the volume of water filling partially the soil macroporosity. This effective degree of saturation defines the proportion of prevailing suction which actually contributes to the effective stress. Two alternative expressions (piece-wise linear and nonlinear) are proposed for the effective degree of saturation. They offer a similar performance. Available data on stiffness and shear strength variation with suction of a few different soils ranging from a markedly granular material to high plasticity clay have been analyzed. The analysis made supports the proposed microstructural interpretation of the effective stress. Indeed, for granular soils the effective degree of saturation is almost equal to the total degree of saturation and therefore, Bishop’s type expression generally used as an effective stress is recovered. As the soil becomes more plastic the proportion of free water reduces and the contribution of suction to the effective stress reduces. At the limit, when the proportion of free water is negligible (this is the case of high plasticity clays at high values of suction) the proposed effective stress reduces to the net stress (excess of total stress over the air pressure). The proposed effective stress equation may be identified if information on the amount of immobile water is available for a given soil. Water retention or porosimetry data provides this information. This has been shown by comparing the present proposal with independently obtained information about immobile water in high plasticity clays.

KEYWORDS: Effective stress, unsaturated soil, suction, degree of saturation, stiffness, strength, constitutive model.

INTRODUCTION

The modern development of constitutive models for unsaturated soils is tightly linked to the discussion on effective stress. The Bishop (1959) proposal (adding a fraction of current suction to total stress in order to obtain a unique effective stress) found a major objection: the impossibility to explain collapse behaviour (volumetric compression under suction reduction). This difficulty was soon recognized in early contributions (Bishop & Blight, 1963; Burland, 1965; Blight, 1965 and Aitchison, 1967). With the purpose of getting a consistent representation of volumetric behaviour they choose to represent void ratio or volumetric strain in terms of two independent stress variables: total stress and suction. The implication was that a single effective stress could not be used to represent unsaturated soil behaviour. Later, Fredlund and Morgenstern (1977) performed null triaxial tests aimed at showing that the two independent stress variables: net stress (excess of total stress over air pressure) and matric suction (air pressure minus water pressure) could consistently be used to represent unsaturated soil behaviour. Early elastoplastic constitutive models (Alonso et al, 1990; Gens and Alonso 1992) were formulated on these two independent stress fields.

In the years to follow a number of authors recovered the Bishop expression in their constitutive models (Kogho *et al.*, 1993, Modaressi and Abou-Bekr, 1994, Bolzon *et al.*, 1996, Loret and Khalili, 2000, 2002, Gallipoli *et al.*, 2003, Wheeler *et al.*, 2003, Sheng *et al.*, 2004, Ehlers *et al.*, 2004 and Santagiuliana and Schrefler, 2006 among others). Their formulations are widely different but they essentially required two independent stress components. One, which could be viewed as an alternative to the net stress is a "Bishop type" of stress and a second one which is essentially described by suction. Examples of elastoplastic models which combine an "effective" stress and an independent suction-based stress term have been summarized in Gens (1995). Jommi (2000) presented also a clear discussion on the selection of stress variables. Some of the authors mentioned preferred to describe their models as based on an "effective" or "generalized" stress proposition. This has caused some confusion in the sense that it could be inferred that a single effective stress (the Bishop based or "generalized" stress) could be enough to reproduce consistently unsaturated soil behaviour. The fact is that the mentioned models, even if they use an "effective" stress in their formulation require also an independent consideration of suction. This view is now widely accepted. The current discussion is no longer about the need to formulate the models in terms of two independent stress components but on the most convenient choice. An interesting contribution in this regard was given by Houlsby (1997). Based on considerations of work input into a representative volume of unsaturated soil he identified proper sets of conjugate stress and strain variables.

An additional ingredient to this discussion comes from a different consideration. It has been found that particular aspects of unsaturated soil behaviour, namely shear strength and elastic stiffness, could be satisfactorily explained with a single effective stress (of Bishop type) (Khalili *et al.*, 2004). In other words, if a model exists to describe the shear strength law or the elastic parameters of a saturated soil in terms of effective stress, in the Terzaghi sense, the derivation of a model for shear strength or elastic stiffness for the same soil, under unsaturated conditions, only requires the formal substitution of the Terzaghi stress by the Bishop-like stress. If this is the case, the accumulated experience and information on saturated states can be transferred immediately to the unsaturated range in a simple manner.

It is then natural to think that introducing such an effective stress (of Bishop type) into the formulation of constitutive equations will imply advantages in terms of model capabilities. Of course, a second stress component, essentially the current suction, will be required for a comprehensive soil modelling. In the remaining of this paper the term "effective stress" will be reserved to this mechanically based component, which is one of the two independent stresses necessary for a comprehensive constitutive characterization of unsaturated soil behaviour. This paper concentrates in the search for a suitable effective stress using two sources of data: the results of suction controlled strength and stiffness tests on different soil types and the interpretation of microstructural features.

However, a theoretical approach is also possible. Homogeneization techniques or thermodynamic approaches may be followed to derive an effective stress formulation. Homogenization often considers idealized geometrical configurations of the porous network (see, for instance, de Buhan & Dormieux, 1996; Chateau & Dormieux, 2002). Thermodynamic considerations have been invoked by Gray & Schrefler (2001), Laloui *et al.* (2003) and Coussy (2004), among others, to propose an effective stress equation in the form:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_g \mathbf{1} + S_r\, s \mathbf{1} \qquad (1)$$

In equation (1) $\boldsymbol{\sigma}'$ and $\boldsymbol{\sigma}$ are the effective and total stress tensors, p_g is the gas pressure, S_r, the degree of saturation, s, the prevailing suction ($s = p_g - p_l$, where p_l is the liquid pressure) and $\mathbf{1}$ is the unit tensor. The stress $\boldsymbol{\sigma}'$ is also referred to in some of the works mentioned as generalized effective stress, constitutive stress or even Bishop's stress in reference to the original proposal by Bishop (1959). It should however be noted that Bishop's proposal was more general since it included a parameter, χ, correcting the suction term, which was not claimed to be equal to S_r.

In thermodynamic approaches, derivations are conducted at the scale of the representative volume element of the soil and the solid constituent is often assumed to be incompressible. More generally, if a wider range of porous media (rocks, concrete…) is considered, the compressibility of the solid phase has to be accounted for and Biot coefficients (Biot, 1941) would appear in the expression (1); see for instance Coussy (2007). But the most important hypothesis when dealing with unsaturated media is that both fluid phases (generally water and air) are continuous. It should also be pointed out that a unique status is assigned to water so that, for the particular case of clays, no difference between the intra-aggregate water and the water partially filling the macropores is introduced. Furthermore, only simplified physico-chemical or energetic interactions between constituents are assumed. Intrinsic constitutive laws for the saturating fluids are also generally simplified (assuming, for instance, perfect fluids).

The degree of saturation S_r that appears in Equation (1) should be based on an areal definition because stresses (the surface interactions between solid skeleton and fluid phases) act on areas. As a consequence, S_r should be equal to the areal degree of saturation (see for instance, Vanapalli *et al.*, 1996, Loret & Khalili, 2002 or Laloui *et al.*, 2003). However, since this quantity is not easily measurable, it is generally assumed to be equal to the volumetric degree of saturation.

With the same perspective, Coussy (2007) shows that the Bishop coefficient χ can be identified with the liquid degree of saturation only under the assumption that the part of porous network in contact with water undergoes the same deformation as the part of porous network in contact with air. This is a particularly restrictive assumption.

Discussion on this basic topic continues to be present in unsaturated soil mechanics. However, after years of experiments in the laboratory, in the interpretation of strength and deformation data, in the formulation of constitutive models, and in the performance of numerical analyses, some conclusions could tentatively be advanced:

- A comprehensive constitutive description of unsaturated soils requires a set of two independent stress variables, involving gas pressure, water pressure and total stress.
- The two approaches most generally favoured are:
  a) To work with net stress and suction
  b) To work with an effective stress (usually defined as a combination of total stress and suction, possibly involving the degree of saturation) and the suction component

  It is worth saying that both approaches are supported by thermodynamic considerations under the assumptions previously mentioned.
- Approach a) is simple to use because no material parameters enter into the definition of the stresses. Within this framework, laboratory tests are conveniently planned, executed and interpreted. When used in numerical analyses, difficulties may be found to follow the transition between saturated and unsaturated states.
- It is acknowledged that approach b) provides a smooth transition from unsaturated to saturated states. Partial information on unsaturated soil constitutive behaviour (i.e. shear strength data, elastic data, as mentioned before) has been shown in some cases to be consistent with the use of a single effective stress (that is, avoiding the use of a two-stress representation). However, the use of an unsaturated effective stress may not be convenient when laboratory tests are planned, executed and interpreted since total stress and fluid pressures are the controllable quantities. A conversion between desired stress paths in terms of effective stress to those expressed in terms of controllable quantities will be necessary. This conversion may not be straightforward in practice. Indeed, if the effective stress as defined in Eq. (1) is used, a previous and complete determination of the water retention properties (which in turn depend on the current void ratio and

therefore on stress state and soil history) will be required. It should be added that water retention properties exhibit hysteretic behaviour when considering wetting and drying cycles.

In addition, and this is perhaps the strongest criticism to approach b), the term $S_r \cdot s$ (suction related component of effective stress if the χ factor is assumed equal to the degree of saturation of the liquid phase) often leads to gross prediction errors. This is specially the case in unsaturated clayey soils (very often found in geotechnical applications and in compacted soil structures) where the product $S_r \cdot s$ becomes, as suction increases, an extremely large term, which would compress the soil in an unrealistic manner. When analytical expressions are used to fit the water retention curve, this concern is equivalent to studying the limiting value of the term $S_r \cdot s$ at high values of suction. It can be shown that this limit may be infinite and that it depends on mathematical details of the equation of the water retention curve, a point stressed later. Again, this particular point highlights the importance of the choice of effective stress parameter χ and of the appropriate equation for the water retention curve if the χ factor is identified with the degree of saturation.

These considerations were behind the research reported here. It was found that, if due consideration is given to the nature of the microstructure of the soil, an integrated approach, which combines features of the two most widely choices may rationally be found to propose a suitable expression for the effective stress

EQUIVALENT PORE PRESSURE AND THE WATER RETENTION CURVE

Equivalent pore pressure

Defining an effective stress for unsaturated soils implies the definition of a stress variable gathering in a single term total stress, suction and, sometimes, degree of saturation. By analogy with the saturated case, the proposed expressions for effective stress (reviews have been published by Pereira *et al.*, 2003 and more recently by Nuth & Laloui, 2007) can be written in the following general form:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \pi(p_g, p_l, S_r)\mathbf{1} \qquad (2)$$

where the function $\pi(p_g, p_l, S_r)$ can be seen as an equivalent pore pressure corresponding to the pressure of a fluid saturating the porous space and resulting in the same overall behaviour as the unsaturated soil. As a sign convention, π is taken as positive when the soil is unsaturated so that it actually corresponds to the opposite sign of the equivalent pore pressure, if interpreted in the classical (Terzaghi) way. If Bishop original equation is considered:

$$\pi = -p_g + \chi(p_g - p_l) = -(1-\chi)p_g - \chi p_l \qquad (3)$$

where χ is a weighting factor, which varies between 1, under saturated conditions, and 0, when the material is dry.

Since the capillary pressure, $p_g - p_l$, is equal to the matric suction, s, equation (2) can be rewritten as:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_g\mathbf{1} + \chi s\mathbf{1} = \overline{\boldsymbol{\sigma}} + \chi s\mathbf{1} \qquad (4)$$

where $\overline{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - p_g\mathbf{1}$ is the net stress.

In his early work, Bishop suggested that "in the range of partial saturation the value of χ would depend mainly on the degree of saturation S_r, but also on the soil structure and the cycle of wetting, drying or stress change leading to a particular value of S_r" (Bishop, 1959). Because of lack of available data, it was often suggested to use S_r as a tentative candidate for χ. Jennings & Burland (1962) also reported that χ was a function of the degree of saturation, S_r.

The constitutive parameter χ weights the contributions of each of the fluid pressures entering in the definition of the effective stress (equation 3). This concept has been followed by a number of authors (e.g. Lewis & Schrefler, 1998; Jommi, 2000; Gallipoli *et al.*, 2003; Sheng *et al.*, 2003a,b; Wheeler *et al.*, 2003; Tamagnini, 2004; Pereira *et al.,* 2005) in their proposals to identify χ. Khalili & Khabbaz (1998) proposed a direct relationship between χ and suction. Coussy & Dangla (2002) proposed an equivalent pore pressure definition based on an energetic approach which accounts for the energy of interfaces between phases. Modaressi & Abou-Bekr (1994) and Kohgo *et al.* (1993) also proposed elastoplastic models for unsaturated soils based on particular forms of effective stress.

The role of the water retention curve

The basic information required to find the equivalent fluid pressure in Bishop-type formulations or in the Coussy & Dangla proposal is the water retention curve (WRC). Consider three analytical expressions, which are often used:

- Brooks & Corey (1964) –BC:

$$S_r(s) = \left(\frac{s_e}{s}\right)^{1/\alpha_b} \qquad (5)$$

where s_e and α_b are model parameters

- van Genuchten (1980) –VG:

$$S_r(s) = \left(1 + (\alpha_I s)^n\right)^{-m} \qquad (6)$$

where α_I, n, and m are model parameters. n and m are often assumed to be linked through the following relationship:

$$n = \frac{1}{1-m} \qquad (7)$$

- modified van Genuchten (Romero and Vaunat, 2000) –MVG:

$$S_r(s) = \left(1 + (\alpha_I s)^n\right)^{-m} \left[1 - \frac{\ln\left(1 + \frac{s}{s_{res}}\right)}{\ln\left(1 + \frac{a}{s_{res}}\right)}\right] \qquad (8)$$

where a is a limiting suction value, which effectively desaturates the soil ($S_r = 0$ for $s = a$). s_{res} is a parameter, which may adopt values in the range 0.1 to a. For simplicity, it may be taken equal to a.

The main concern, when calculating equivalent fluid pressures when assuming $\chi = S_r$ in the effective stress definition and when using mathematical functions for the water retention curve, is the evolution of the term $s{\cdot}S_r$ as suction increases. When using Brooks & Corey's model it can easily be shown that the term $s{\cdot}S_r(s)$ diverges when s tends to infinity whatever is the value of $\alpha_b > 1$. Concerning van Genuchten's function, $s.S_r(s)$ tends to 0 when $m > 0.5$ and to infinity otherwise (assuming that m and n are linked according to (7)). The modified version of van Genuchten excludes the possibility of such a divergence since the correction term introduces a suction, a, beyond which the degree of saturation vanishes. Figure 1 illustrates these remarks by comparing theoretical equivalent pore pressures π using different water retention curves: Brooks & Corey's ($s_e = 0.1$, $\alpha_b = 3$) model and van Genuchten's model for two different values of m ($m = 0.4$ and 0.6, n calculated according to (7) and $\alpha_I = 3.33$).

The fact that the equivalent pore pressure could tend to infinite values when drying paths are imposed is clearly impossible, because it would imply an indefinite compression of the soil.

Even if this remark mainly lies in the choice of the mathematical model used to fit the water retention curve, experimental cases may be found where the WRC is reasonably well fitted but computed values of the effective stress are unrealistic. This is typically the case for clays as shown later. The common experience is that, as the degree of saturation decreases, the overall influence of suction on the effective stress and on mechanical properties such as the soil strength reduces. This observation implies that χ values must tend to zero faster than the degree of saturation does when suction increases.

Experimental data on WRC is abundant. An interesting exercise is to calculate the product ($s.S_r$) when S_r decreases. A few examples are given in Figure 2. Figure 2a provides data on a compacted silty sand used in the core of Vallfornés dam (SM, non-plastic; %<2μm = 6; %< 74μm = 45.6; Normal Proctor w_{opt} = 13%). The "Sadurní" soil, also represented in Figure 2a, is a low plasticity clay (CL, w_L = 42.5 %; PI = 21.2%; %<2μm = 30.7; %<74μm = 91.3; Normal Proctor w_{opt} = 18%). Figure 2b provides the same information for high plasticity Boom clay (Belgium). Note the very high values of the product ($s.S_r$) when the degree of saturation decreases. Equivalent stresses in the order of a few MPa will certainly compress the soil substantially, against observations. In Figure 2c, data on fine sand is presented. The product ($s.S_r$) now reaches a maximum for S_r around 0.85 and decreases at lower suctions to increase again below S_r=0.2. Looking at these examples it is concluded that the product ($s.S_r$) does not provide a good measure of an equivalent fluid pressure for a wide range of suction changes.

Further evidence in this regard was provided by the shear moduli measured in compacted samples of Vallfornés soil tested in a resonant column apparatus. Samples were compacted statically at dry densities close to 17.9 kN/m^3, slightly above optimum Normal Proctor, and varying moisture contents (3% to 16%). Water content was measured at the end of the tests and suction was determined by filter paper techniques (Chandler and Gutierrez, 1986). The measured value for G_{max} (for small shear deformations, close to 10^{-6}) is represented in Figure 3 in terms of suction and total mean confining stress. After a phase of fast increase of the modulus with suction, soil stiffness remains nearly constant for increasing suctions. G_{max} for saturated conditions (G_{m0}) was found to increase linearly with increasing effective mean confining stress p' (Alonso, 1998):

$$G_{m0} = Ap' \qquad (9)$$

where A is a parameter.

The validity of Bishop's expression has been analyzed in this case taking into account the retention curve in Figure 2a. Moduli for unsaturated conditions are calculated from expression (9) by extending p' to its unsaturated definition (Eq. 4 with $\chi = S_r$):

$$G_{\max} = A(\overline{p} + S_r s) \qquad (10)$$

Results are indicated also in Figure 3. It is obvious that the effective stress component associated with suction is a much smaller term than $s \cdot S_r$.

MICROSTRUCTURAL INTERPRETATION OF WATER RETENTION CURVE AND EFFECTIVE STRESS

The previous discussion suggests that the main problems in the application of the Bishop expression (or the equivalent fluid pressure concept) arise in fine soils that maintain a significant water content (or degree of saturation) under high suctions. In these cases, the effective stress term related to suction ($s.S_r$) increases unrealistically as s increases.

On the other hand, current knowledge on the microstructure of clayey soils indicates that pore sizes may be grouped into two categories: relatively large open pores, where capillary effects are the governing phenomena, and small pores, associated with inter-particle distances, or even with inter-crystalline spacing in the case of highly swelling clays (montmorillonite) (Romero *et al*., 1999). Water held in this part of the total pore space is attached to the solid by physico-chemical bonds.

The previous two points indicate that the degree of saturation, S_r, could be separated into two contributions:

- A macroscopic degree of saturation S_r^M, which describes the occupation of macro-pores by water and is linked mainly to capillary effects.
- A microscopic degree of saturation S_r^m, which concerns the water within micropores.

As a first approximation, this second contribution can be considered constant irrespective of applied suction or mechanical load.

Then, the degree of saturation can be split as follows:

$$S_r = S_r^m + S_r^M \tag{11}$$

Let us now define an effective degree of saturation such that:

$$S_r^e = \left\langle \frac{S_r - S_r^m}{1 - S_r^m} \right\rangle \tag{12}$$

where <x> = 0.5 (x+|x|) defines the Macauley brackets. With this definition, the effective degree of saturation S_r^e is thus a measure of the freely available water filling the macroporosity, in a scale that extends from 0, when all the water is stored in micropores, to 1, when the soil is fully saturated. Note that for $S_r \leq S_r^m$, $S_r^e = 0$.

The proposal is now that the equivalent fluid pressure in the definition of effective stress given in (2) is defined by $\pi(p_g, p_l, S_r) = -p_g + S_r^e(p_g - p_l)$ and then, the effective stress becomes:

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_g \mathbf{1} + S_r^e s \mathbf{1} \tag{13}$$

The weighting factor χ is thus assumed to satisfy the identy $\chi = S_r^e$ in Bishop stress expression.

It is expected that in granular soils $S_r^m \rightarrow 0$ and (13) reduces to Bishop expression with $\chi = S_r$ assumption. However, as the clay content increases S_r^m will increase. In a limiting case, when $S_r^m \rightarrow S_r$ the effective stress becomes equal to the net stress if (13) is applied.

This effective degree of saturation has the character of a soil parameter which enters the proposed form for the effective stress. The purpose of the remaining of the paper is to explore the validity of Equations (11)-(13) by analyzing published experimental data on a range of soils varying from granular soils to high plasticity clays. The analyzed data refers to elastic behaviour and to shear strength data. In all cases information on the water retention curve is required.

The transition from macro to micropores is not clear-cut in soils. It is more precise to think in terms of a continuous pore size distribution with two dominant sizes. Therefore, the second order discontinuity (slope change) of equation (12) at $S_r = S_r^m$ should be, rather, a smooth transition. A simple continuous equation, which approximates Equation (12) and which will be shown to be as suitable as the preceding one, is:

$$S_r^e = (S_r)^\alpha \tag{14}$$

where α ($\alpha \geq 1$) is a material parameter. Therefore:

$$S_r^e \leq S_r \text{ for } S_r \in [0,1] \tag{15}$$

Equations (12) and (14) have been plotted in Figure 4a and b for a few S_r^m and α values, respectively.

The main difference between the piecewise description (equation 12) and the continuous version (14) is that the latter maintains non zero values of the effective saturation for low degrees of saturation. It is expected, therefore, that the piecewise approximation will be more effective in eliminating suction effects for large suction values. However, as it will be seen in the following section, the available test results do not offer enough discriminating possibilities to pinpoint one of the two proposals for S_r^e (Equations 12 or 14) as clearly superior in terms of predictive capabilities over the other. This situation may change if more precise experimental data becomes available.

IDENTIFICATION OF EFFECTIVE DEGREE OF SATURATION

From shear strength data

Substitution of equation (13) into the classical Mohr-Coulomb equation in terms of effective stress gives:

$$\tau = c' + \sigma' \tan\varphi' = c' + (\sigma - p_g + S_r^e s)\tan\varphi' = c' + (\sigma - p_g)\tan\varphi' + S_r^e s \tan\varphi' \quad (17)$$

where S_r^e is given by equations (12) or (14). Therefore, for a given soil, if shear strength data is available for varying suctions and a given confining net stress, parameters of the equations defining the effective degree of saturation could be determined by fitting equation (17) to experimental data. A few examples have been collected and analyzed from this perspective hereafter.

Vanapalli *et al.* (1996) presented shear strength data on a Canadian glacial till (Sand/Silt/Clay fractions: 28%, 42%, 30%; w_L = 35.5%; w_P = 16.8%). An effective friction angle φ' = 23º and a zero effective cohesion were reported. The data plotted in Figure 5b were measured for a normal confining stress of 25 kPa. The water retention curve (drying branch of preconsolidated samples using pressure plate and desiccators) was also available (Figure 5a). A modified van Genuchten law with parameters α_l = 0.72 MPa^{-1}, n = 0.59, m = 0.67 and a = 10^3 MPa fits well the data in Figure 5a. Figures 5b shows the measured shear strength data and the predictions based on the Bishop relationship (using $\chi = S_r$) and on the effective degree of saturation (equations 12 and 14). A good fit of the predicting curves was found for the following parameters of the effective degree of saturation: S_r^m = 0.64 (if the piece-wise approximation (Eq. 12) is selected) and α = 4.2 (if the power law (Eq. 14) is selected) (see Figure 5b).

A similar analysis was made using the direct shear strength data reported by (Fredlund *et al.*, 1996) on a decomposed tuff from Hong-Kong a fine to medium-grained saprolitic soil (Sand/Silt/Clay fractions: 60%, 35%, 5%; non-plastic) (Gan & Fredlund, 1996). An effective friction angle φ' = 38.7º and a zero effective cohesion were reported. Plotted strength data in Figure 6b corresponds to a confining stress of 100 kPa. Parameters of the modified van Genuchten equation which fit the experimental water retention curve (Figure 6a) are α_l = 36.1 MPa^{-1}, n = 3.63, m = 0.144 and a = 10^3 MPa. A good approximation was found for the following parameters of the effective degree of saturation: S_r^m = 0.02 and, alternatively, α = 1.03 (Figure 6b). This shows that in this case the common assumption $\chi = S_r$ in Bishop's effective stress relation explains convincingly the strength data in the studied suction range. The soil is non-plastic and behaves as a "pure" granular material. Micropores amount to a very small proportion of the total porosity.

A final example is provided in Figure 7. Geiser *et al.* (2006) tested a sandy silt from Sion, Switzerland (Sand/Silt/Clay fractions: 20%, 72%, 8%; w_L = 25.4%; w_P = 16.7%). An effective friction angle φ' = 32.3º and a zero effective cohesion were reported. The water retention data was approximated by a modified van Genuchten equation (α_l = 19.08 MPa^{-1}, n = 3.25, m = 0.24 and a = 10^3 MPa) (see Figure 7a). Figure 7b shows the peak strength data (p_{ini} refers to the confining pressure, expressed in terms of total stress) and the predictions of the three models used in all previous cases. A good fit is provided by S_r^m = 0.4 and, alternatively, by α = 2.5. It should be noted that the prediction was made through parameters derived from the back analysis of a single set of experimental data, namely those corresponding to an initial confining pressure p_{ini} equal to 300 kPa. Predicted strength values were then found for the two other confining pressures p_{ini} = 200 and 1000 kPa. The agreement is satisfactory.

From stiffness data

Under isotropic or oedometric loading, elastic soil stiffness for saturated states may be defined by a compressibility coefficient, κ,

$$d\varepsilon_v = \kappa \frac{dp'}{p'} \tag{18}$$

If this relationship is accepted also for unsaturated conditions, the mean effective confining stress, p' should be replaced by expression (13). Then, in loading stress paths at constant suction, equation (18) becomes,

$$d\varepsilon_v = \kappa \frac{d\overline{p}}{(\overline{p} + S_r^e s)} \tag{19}$$

If an elastic compressibility coefficient, $\overline{\kappa}$, is defined in terms of changes in net mean stress, (or total mean stress) for the same loading stress paths,

$$d\varepsilon_v = \overline{\kappa} \frac{dp}{p} \tag{20}$$

Therefore,

$$\overline{\kappa} = \kappa \frac{\overline{p}}{(\overline{p} + S_r^e s)} \tag{21}$$

Since κ is assumed to be a constant in the effective stress model, its counterpart $\overline{\kappa}$ in terms of net stress is thus dependent on suction. This point is corroborated by experimental evidences. For instance, Romero (1999) reported values of $\overline{\kappa}$ (that is the suction dependent elastic parameter) for specimens of compacted Boom clay at two different dry densities tested in a suction controlled oedometer cell. Boom clay is a high plasticity clay (Sand/Silt/Clay fractions: 18%, 30%, 52%; w_L = 55.7%; w_P = 26.9%). Effective friction angles in the range 18º-24º have been reported by Coll (2005) based on triaxial tests. The water retention curves (wetting branch) was approximated by a modified van Genuchten equation for both densities (α_l = 1.14 MPa^{-1}, n = 0.196, m = 21.29 and a = 274 MPa for γ_d = 13.7 kN/m^3 and α_l = 0.748 MPa^{-1}, n = 0.354, m = 1.55 and a = 274 MPa for γ_d = 16.7 kN/m^3)(Figure 8a). Figures 8b and 8c show the measured stiffness coefficients and the predictions based on the Bishop relationship and on the effective degree of saturation (equations 12 and 14). The best fit is achieved for (S_r^m = 0.418; S_r^m = 0.628) and (α = 4.4; α = 6.4) for the two dry densities (γ_d =13.7 kN/m^3; γ_d =16.7 kN/m^3). It can be noted that it is possible to reproduce the maximum stiffness reached for an intermediate suction value for the density of 16.7 kN/m^3. The plastic Boom clay is thus characterized by relatively high values of the microstructural degree of saturation S_r^m . This situation corresponds to high values of the power coefficient α.

Similar data on elastic parameter κ on Jossigny silt are reported by Vicol (1990). The water retention curve was approximated by a modified van Genuchten equation (α_l = 35.54 MPa^{-1},

$n = 4.56$, $m = 0.026$ and $a = 10^3$ MPa. Figure 9 shows the measured stiffness coefficients and the prediction based on the Bishop relationship and on the effective degree of saturation (equations (12) and (14)). The best fit is achieved for $S_r^m = 0.56$ and $\alpha = 3.5$. Again, such a fit is able to reproduce the evolution of stiffness with suction.

The resonant column tests reported previously on compacted specimens of the core of Vallfornés dam also provides suitable data to be interpreted with the help of the proposed effective stress framework. Data on this soil was given before (see Figures 2 and 3). If equation (9) is now modified as follows:

$$G_{\max} = A(\bar{p} + S_r^e s) \tag{22}$$

the measured values of G_{max} can be used to identify the effective degree of saturation. A relatively good fit (Figure 10) is achieved for $S_r^m = 0.25$ (piece-wise approximation) and $\alpha = 2.0$ (power law).

DISCUSSION

Parameters of the water retention curve of the soils analyzed have been collected in Table 1. Parameter $(1/\alpha_I)$ provides information on the air entry value, a concept related to the characteristic size of the macro pores of the soil. The parameters of the effective degree of saturation, either the microscopic degree of saturation, S_r^m, (equation 12) or the power α (equation 14), are given in Table 2.

High plasticity clays, such as Boom clay, are expected to have high values of S_r^m. The density of the specimen also controls the relative values of the micro and macro degrees of saturation. Table 2 shows that the densest specimen has the highest S_r^m value. Romero (1999) also provided an estimation of a similar parameter: the "quasi-immobile intra-aggregate water fraction" which was expressed as a percentage of the total pore volume. This water fraction is therefore identical to the S_r^m parameter. He reported values of this "water fraction" for Boom clay, interpreting different types of tests (relative permeability data; water retention characteristics of compacted Boom clay at different densities and mercury intrusion porosimetry; see Romero *et al.*, 1999). Romero found that alternative procedures led to very similar values for the immobile water fraction. Water fraction values are plotted in Figure 11 as a function of dry density. Also plotted in this figure are the two S_r^m values determined here. They fit nicely into the set of independent estimations of the immobile water fraction given in Figure 11 for Boom clay. The interesting point is that the estimation made in this paper is based in a completely different procedure, which uses in a joint manner soil compressibility results and water retention data.

α values are directly correlated with S_r^m. High plasticity clays are expected to have high α powers and this is also shown in Table 2. Since Figure 11 provides data on the proportion of immobile water of two types of smectite compacted at different densities, this information could be used directly to derive expressions for the effective degree of saturation for them. It is clear that these materials have a very low value of effective degree of saturation, S_r^e, for all densities.

Results in Table 2 are sorted in the order of increasing S_r^m and α. Low values (S_r^m close to 0 and α close to 1) were found in fairly granular materials (decomposed tuff). The glacial till and the Sion and Jossigny silts exhibit intermediate microstructures between granular materials and plastic clays. This is reflected by the obtained values of S_r^m and α. The clays are characterized by relatively high values of S_r^m and high values of the power α. This is consistent with the microstructural interpretation proposed here.

The effective degree of saturation for all the soils investigated has been plotted in Figure 12. It is interesting to compare the S_r^e function, given by the power law, (14), with the Bishop χ value reported in the classic contribution of Jennings & Burland (1962) (Figure 13a). Soils 2 (compacted shale) and 5 (silty clay) are the finer soils of all the materials included in the plot (S_r,χ). The original (S_r,χ) results have been plotted in Figure 13b together with the effective S_r^e function based on the power law. It may be interpreted that the α parameter for the compacted shale and the silty clay was approximately 4 and 6 respectively. It is worth noting that these values are similar to those obtained here for Boom clay. For the silty soils numbered 3 and 4, α parameter was closer to 1. Within the interpretation and results obtained here, they would be characterized by an effective degree of saturation S_r^e close to S_r.

CONCLUSIONS

The paper discusses the relative merits of two alternative sets of effective stress proposals: the pairs (net stress; suction) and (Bishop effective stress, using parameter S_r; suction). Both have received much audience and have been proved to be thermodynamically consistent. The first approach is a convenient choice to interpret and analyze suction-controlled laboratory tests and leads to simpler formulations of constitutive models. However, it does not facilitate the transition from unsaturated to saturated states and the interpretation of some aspects of unsaturated soil behaviour (shear strength or stiffness dependence upon suction) requires the joint consideration of the two stress variables. The second approach offers the possibility of describing properly some aspects of unsaturated soil behaviour (strength and elastic stiffness, in particular) in terms of a single effective stress (the Bishop expression having a χ factor identified with the degree of saturation). This is particularly the case of granular soils (sands) and, to some extent, of fine granular soils such as silts, whose unsaturated behaviour is mainly controlled by capillary effects. This advantage is progressively lost when the clay fraction of the soil increases. For high plasticity clayey soils, Bishop effective stress (always within the interpretation $\chi = S_r$) becomes exceedingly large as suction increases. It has been shown that details of the analytical expression describing the water retention curve may lead to unrealistic results as suction increases. Bishop's expression may also be inconvenient to use in practice, especially in the interpretation of laboratory tests. These are severe limitations in practice.

The main proposition of the paper is that the concept of effective stress in unsaturated soils cannot be dissociated from the soil microstructure. In order to introduce this relationship the volume of water existing in the soil for a given suction is conceptually divided into two parts: the free water filling partially the macropores and the "immobile" water closely attached to the clay minerals. An effective degree of saturation, in a scale ranging from zero to one, is then defined to characterize the amount of free water. The proposal made is that this effective degree of saturation weights suction effects into the effective stress definition. Even if two stress variables are needed to describe in a comprehensive way the behaviour of unsaturated soils, particular aspects (strength and elastic behaviour have been selected here) may be properly interpreted in terms of the single effective stress proposed here. It is therefore suggested that the effective stress defined in this paper may conveniently be used as one of the two independent stress states required for a full description of the unsaturated soil.

An interpretation of data on strength and stiffness changes with suction for a variety of soils has shown that the proposed effective stress definition is consistent with experimental results. The analysis performed led also to the identification of the material parameter involved in the definition of the effective degree of saturation. This parameter characterizes the immobile water fraction and, in one of the cases analyzed (Boom clay), it has been shown to be very close to independent determinations using other techniques (porosimetry, WRC data, relative permeability determinations). These are relatively simple tests and therefore the effective stress equation proposed here (in particular, parameter S_r^m , in equation 12, or α in equation 14) may be conveniently identified in practice.

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FIGURE CAPTIONS

Fig. 1. Comparison between theoretical equivalent pore pressures π using different water retention curves: Brooks & Corey's ($s_e = 0.1$, $\alpha_b = 3$) model and van Genuchten's model for two different values of m ($m = 0.4$ and 0.6, n calculated according to (11) and $\alpha_I = 3.33$).

Fig. 2. Water retention data and calculated values of $S_r \cdot s$ for: a) Vallfornés and Sant Sadurní soils; b) Compacted Boom clay at a dry density of 13.7 kN/m^3; c) Fine sand, reported in Brooks & Corey (1964).

Fig. 3. Shear moduli of compacted silty sand used in the core of Vallfornés dam: measured data in resonant column tests (symbols) and theoretically predicted values using $\chi = S_r$ (dashed lines).

Fig. 4. Effective degree of saturation: (a) Equation (12); (b) Equation (14).

Fig. 5. (a):WRC of glacial till. (b): Variation of shear strength with suction: experimental data and simulations using the effective degree o.f saturation. Test results reported by Vanapalli *et al.* (1996)

Fig. 6. (a): WRC of decomposed tuff from Hong Kong. (b): Variation of shear strength with suction: experimental data and simulations using the effective degree of saturation. Test results reported by Fredlund *et al.* (1996).

Fig. 7. (a): WRC of a remoulded silt. (b), (c) and (d): Experimental and simulated shear strength (q_{peak}) as a function of suction for total confining pressures p_{ini} = 200, 300 and 1000 kPa respectively. Experimental results reported by Geiser et al. (2006).

Fig. 8. (a): WRC of compacted Boom clay at two dry densities. (b): Variation of elastic compressibility parameter, $\bar{\kappa}$, for increasing suction: experimental data and simulations using the effective degree of saturation for the dry density of 1.37. (c): Same as (b) but for dry density of 1.67. Experimental data reported by Romero (1999).

Fig. 9. Variation of elastic compressibility parameter, $\bar{\kappa}$, for increasing suction: experimental data and simulations using the effective degree of saturation (experimental data reported by Vicol, 1990).

Fig. 10. (a): WRC of compacted specimens of Vallfornés dam core. (b): Variation of G_{max} with suction and applied confining stress. Model predictions for an effective stress based on the effective degree of saturation. Experimantal data has been obtained using the resonant column method.

Fig. 11. Quasi-immobile intra-aggregate water fraction (% of total porous volume). Experimental data after Romero (1999).

Fig. 12. Identified effective degrees of saturation for all soils investigated. (a): Model using the microstructural degree of saturation, eq. (12). (b): Model using a power law, eq. (14).

Fig. 13. (a): Variation of χ coefficients with suction from Jennings & Burland (1962) and (b): comparison with the effective degree of saturation proposed in this study (right)

LIST OF Tables

Table 1. Parameters of the water retention curves identified using modified van Genuchten's model

	n	**m**	α_I *(MPa^{-1})*	1/α_I *(MPa)*	**s_r** *(MPa)*	**a** *(MPa)*
Glacial till, Vanapalli **et al.** ***(1996)***	***0.59***	***0.67***	***0.72***	***1.4***	***1000***	***1000***
Decomposed tuff, Fredlund **et al.** ***(1996)***	***3.63***	***0.14***	***36.14***	***0.027***	***1000***	***1000***
Sion silt, Geiser **et al.** ***(2006)***	***3.25***	***0.24***	***19.08***	***0.05***	***1000***	***1000***
Vallfornés dam core (Alonso, 1998)	***1.11***	***0.67***	***0.38***	***2.63***	***1000***	***1000***
Jossigny silt, Cui & Delage (1996)	***4.56***	***0.026***	***35.54***	***0.028***	***1000***	***1000***
Boom clay (γ_d=13.7 kN/m^3), Romero (1999)	***1.14***	***0.196***	***21.29***	***0.046***	***274***	***274***
Boom clay (γ_d=16.7 kN/m^3), Romero (1999)	***0.75***	***0.354***	***1.55***	***0.64***	***274***	***274***

Table 2. Parameters of the effective degree of saturation

	α *(-)*	**S_r^m** *(-)*
Decomposed tuff, Fredlund **et al.** ***(1996)***	***1.03***	***0.02***
Vallfornés dam core (Alonso, 1998)	***2.0***	***0.25***
Sion silt, Geiser **et al.** ***(2006)***	***2.5***	***0.4***
Jossigny silt, Vicol (1990)	***3.5***	***0.56***
Glacial till, Vanapalli **et al.** ***(1996)***	***4.2***	***0.64***
Boom clay (γ_d=13.7 kN/m^3), Romero (1999)	***4.4***	***0.42***
Boom clay (γ_d=16.7 kN/m^3), Romero (1999)	***6.4***	***0.63***

LIST OF FIGURES

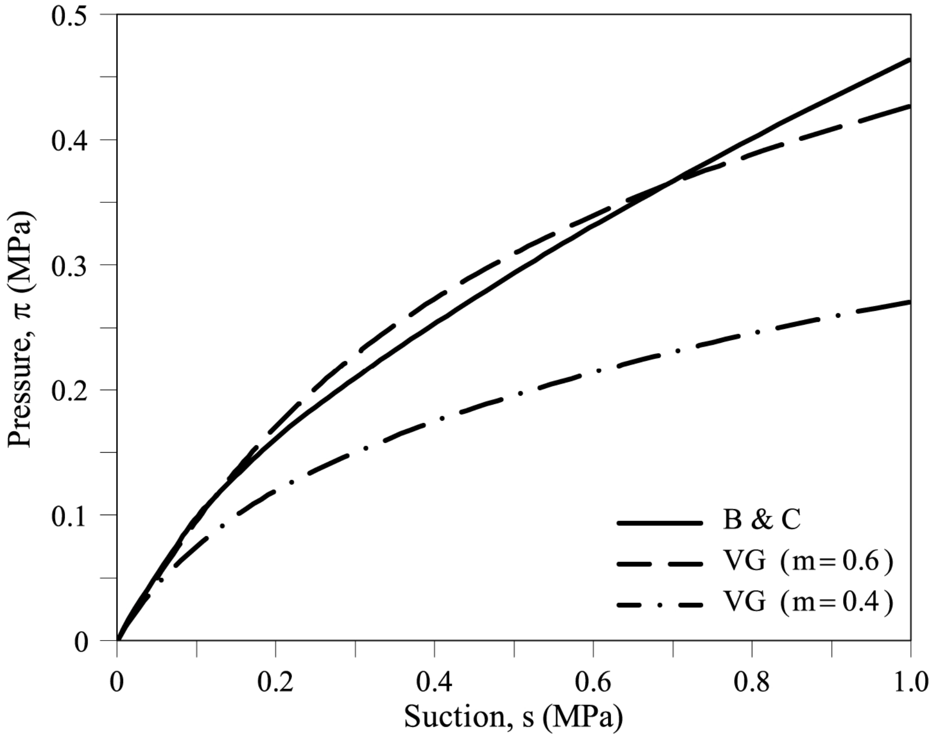


Figure 1

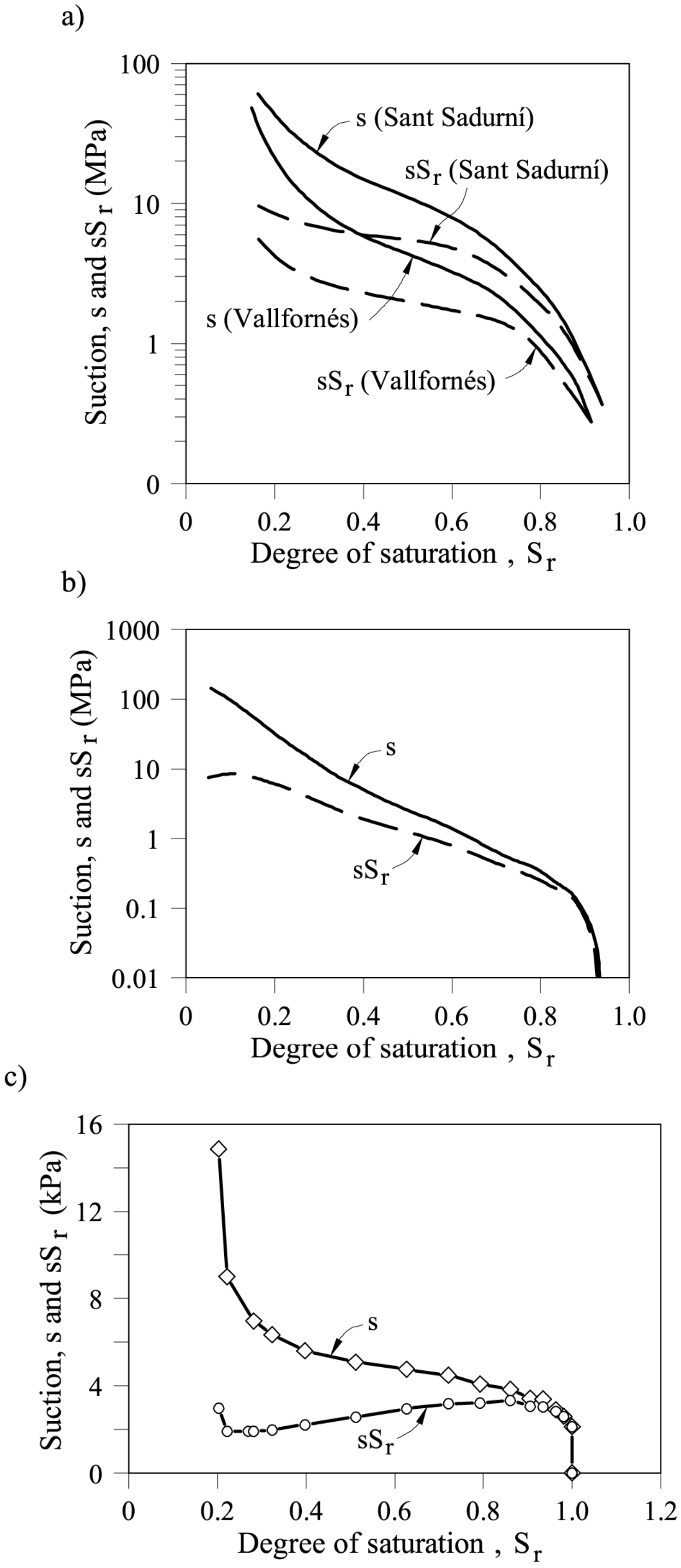

a)
Suction, s and sS r (MPa)
100
10
1
0
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation , S r
s (Sant Sadurní)
sS r (Sant Sadurní)
s (Vallfornés)
sS r (Vallfornés)
b)
Suction, s and sS r (MPa)
1000
100
10
1
0.1
0.01
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation , S r
s
sS r
c)
Suction, s and sS r (kPa)
16
12
8
4
0
0
0.2
0.4
0.6
0.8
1.0
1.2
Degree of saturation , S r
s
sS r

Figure 2

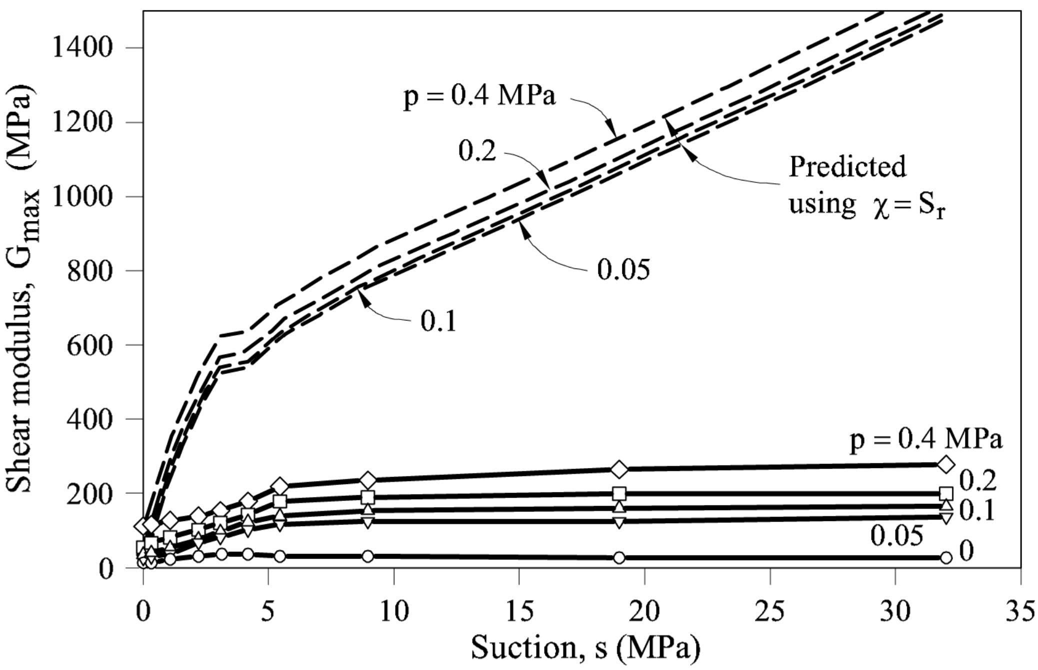

Shear modulus, G_{max} (MPa)
0
200
400
600
800
1000
1200
1400
Suction, s (MPa)
0
5
10
15
20
25
30
35
p = 0.4 MPa
0.2
0.1
0.05
Predicted using $\chi = S_r$
p = 0.4 MPa
0.2
0.1
0.05
0

Figure 3

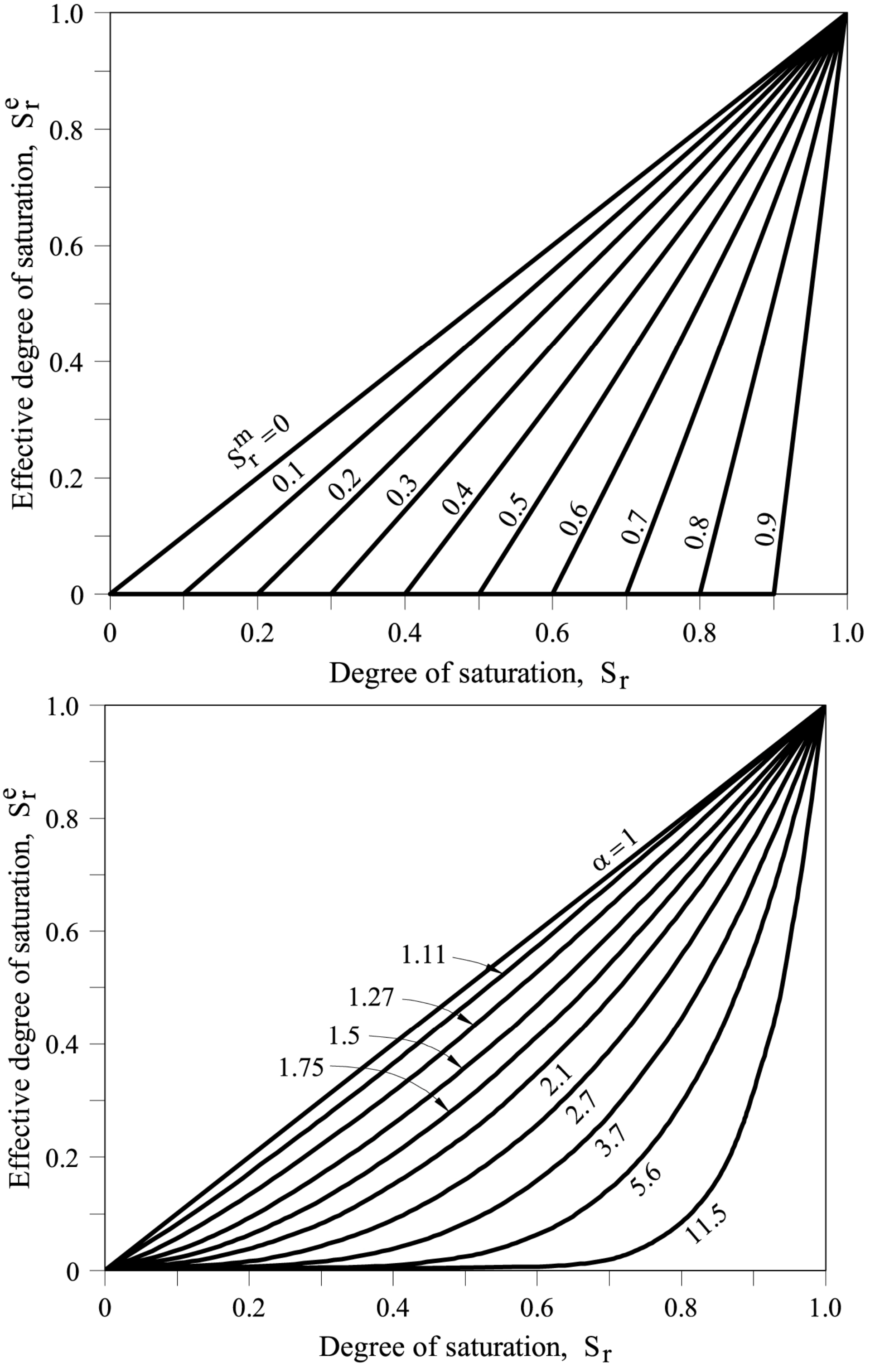

1.0
0.8
0.6
0.4
0.2
0
Effective degree of saturation, S_r^e
$S_r^m = 0$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation, S_r
1.0
0.8
0.6
0.4
0.2
0
Effective degree of saturation, S_r^e
$\alpha = 1$
1.11
1.27
1.5
1.75
2.1
2.7
3.7
5.6
11.5
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation, S_r

Figure 4

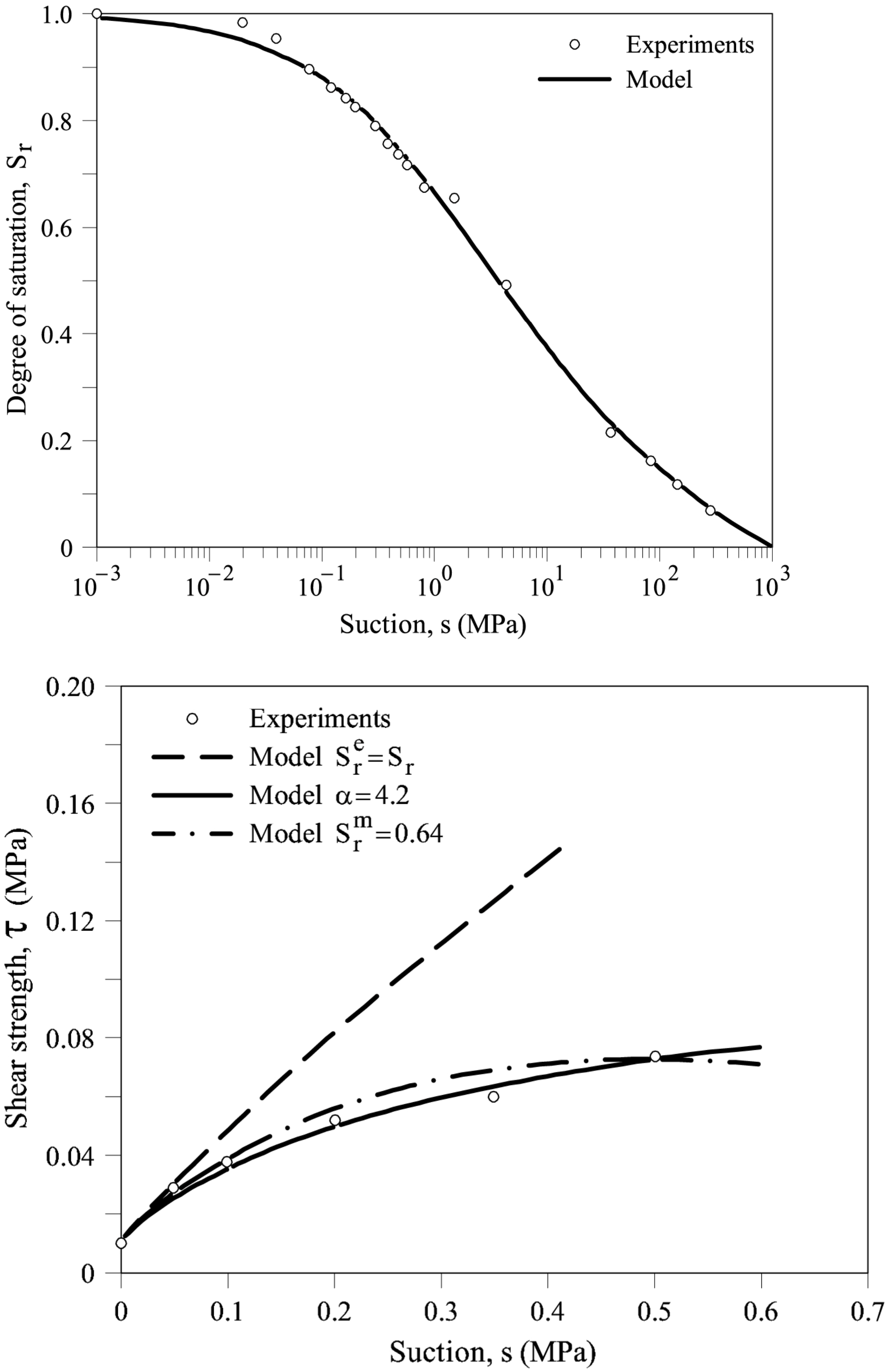

1.0
0.8
0.6
0.4
0.2
0
Degree of saturation, S_r
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Suction, s (MPa)
Experiments
Model
0.20
0.16
0.12
0.08
0.04
0
Shear strength, τ (MPa)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
Suction, s (MPa)
Experiments
Model $S_r^e = S_r$
Model $\alpha = 4.2$
Model $S_r^m = 0.64$

Figure 5

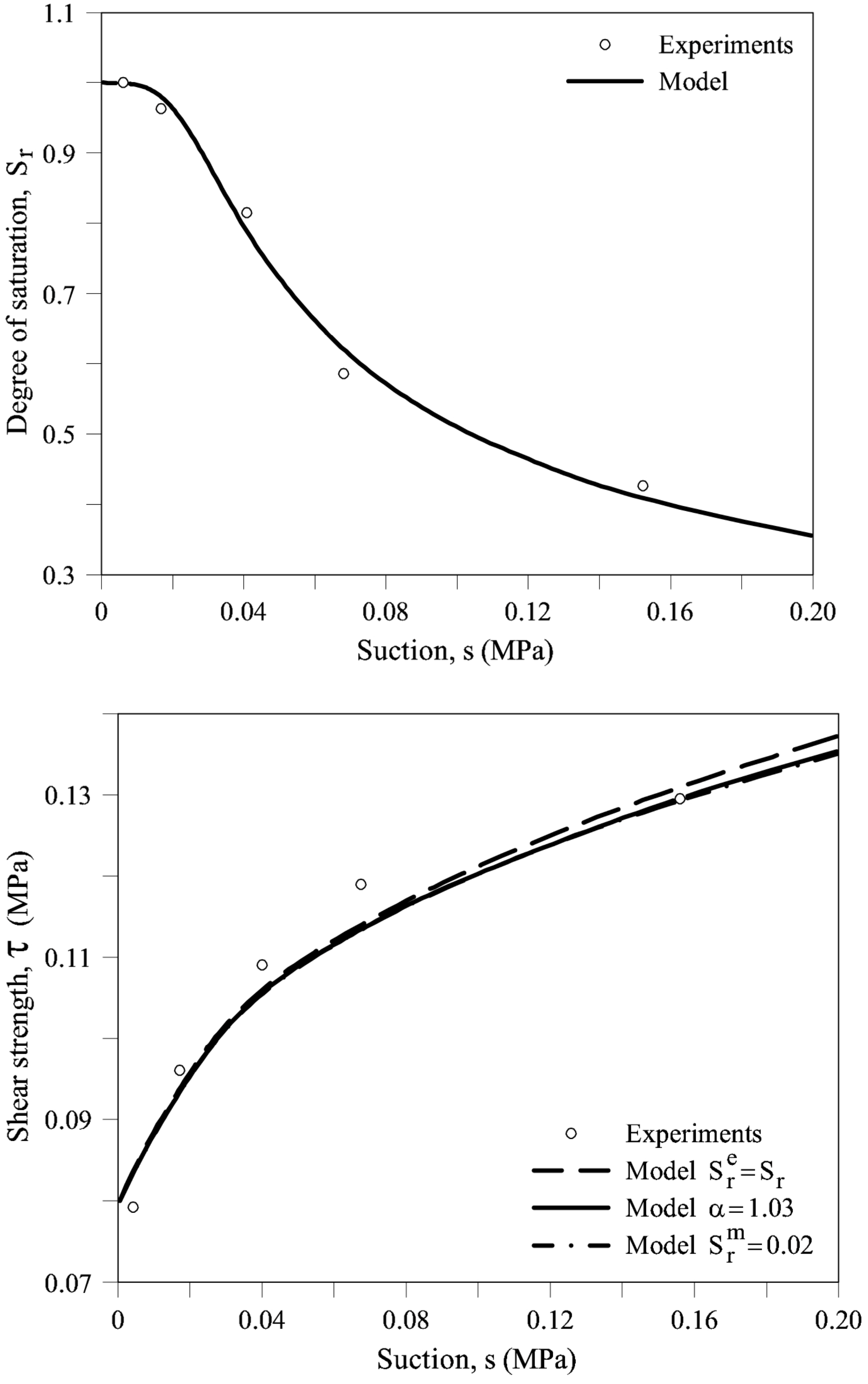
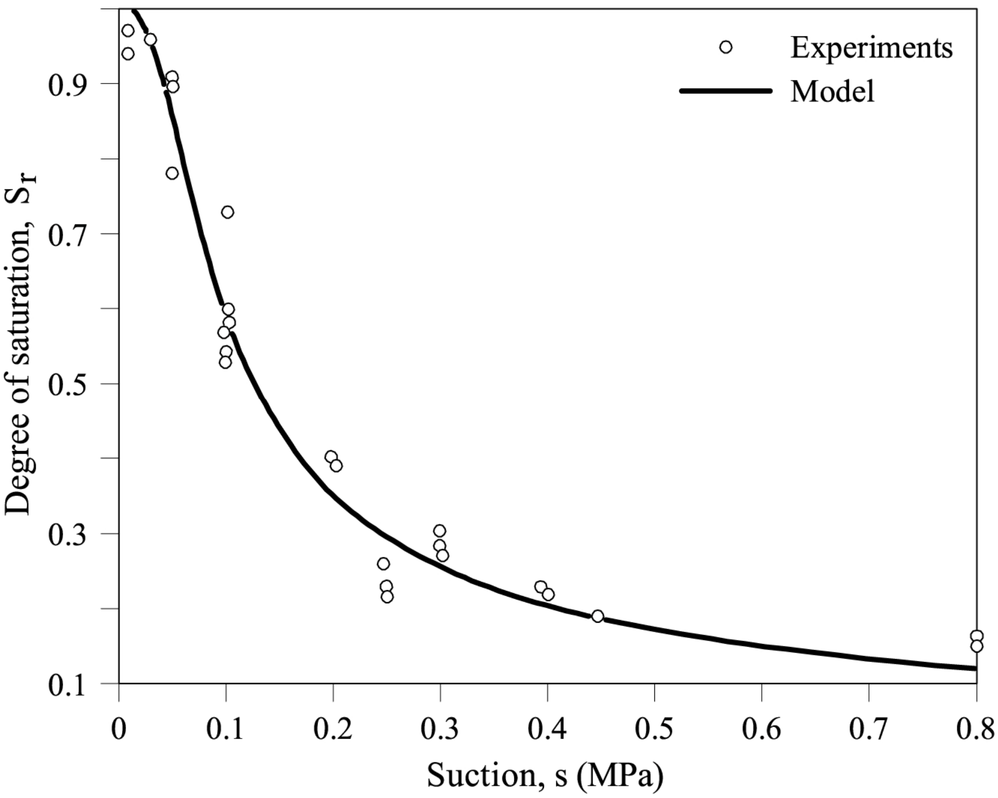
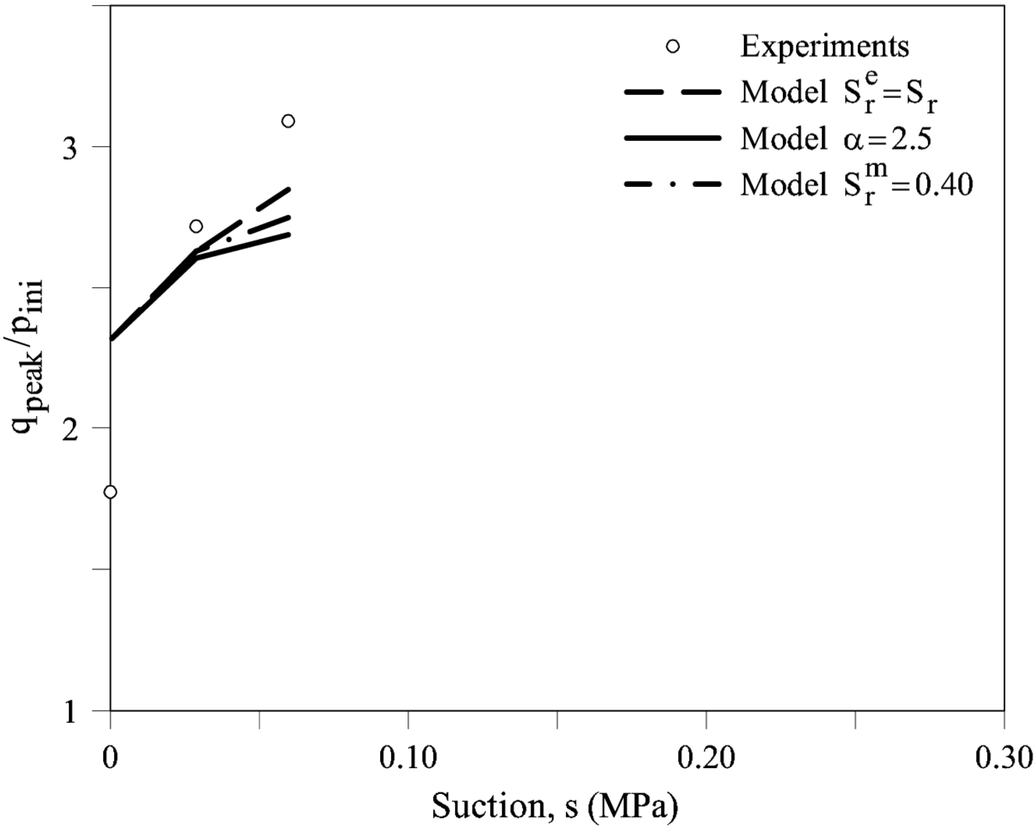
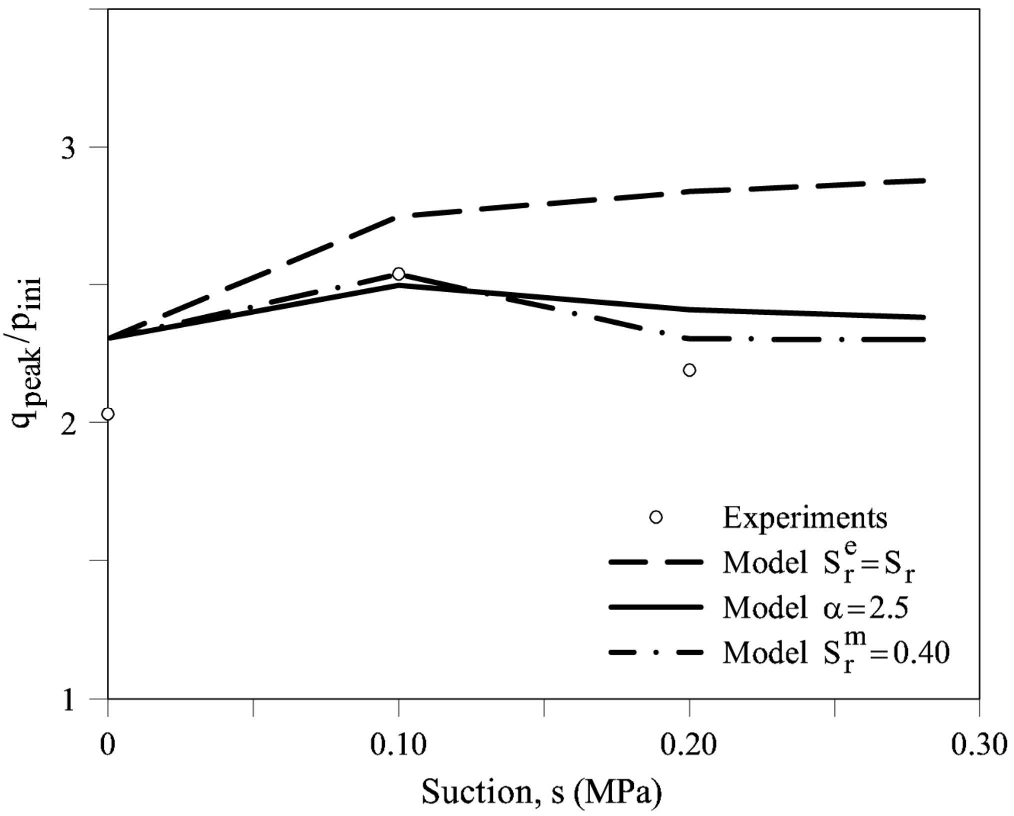

1.1
0.9
0.7
0.5
0.3
Degree of saturation, S_r
Experiments
Model
0
0.04
0.08
0.12
0.16
0.20
Suction, s (MPa)
0.13
0.11
0.09
0.07
Shear strength, τ (MPa)
Experiments
Model $S_r^e = S_r$
Model $\alpha = 1.03$
Model $S_r^m = 0.02$
0
0.04
0.08
0.12
0.16
0.20
Suction, s (MPa)

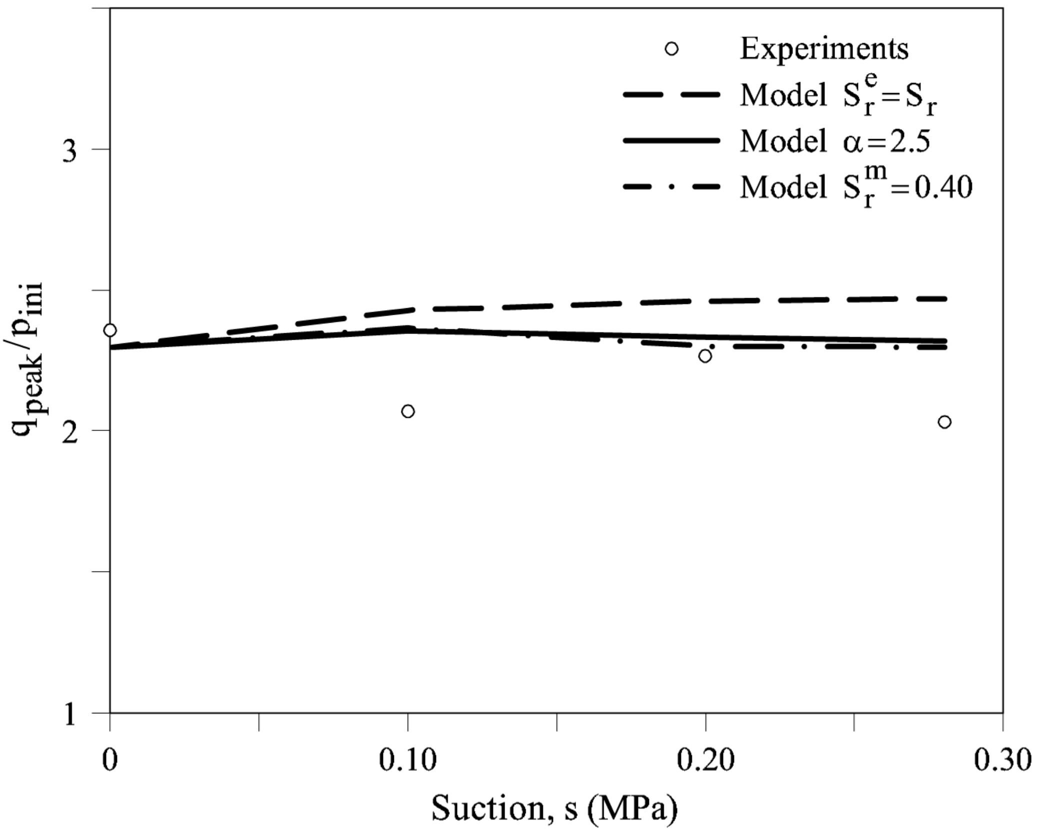
Figure 6


Experiments
Model
Degree of saturation, S_r
0.1
0.3
0.5
0.7
0.9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
Suction, s (MPa)


Experiments
Model $S_r^e = S_r$
Model $\alpha = 2.5$
Model $S_r^m = 0.40$
q_{peak}/p_{ini}
1
2
3
0
0.10
0.20
0.30
Suction, s (MPa)

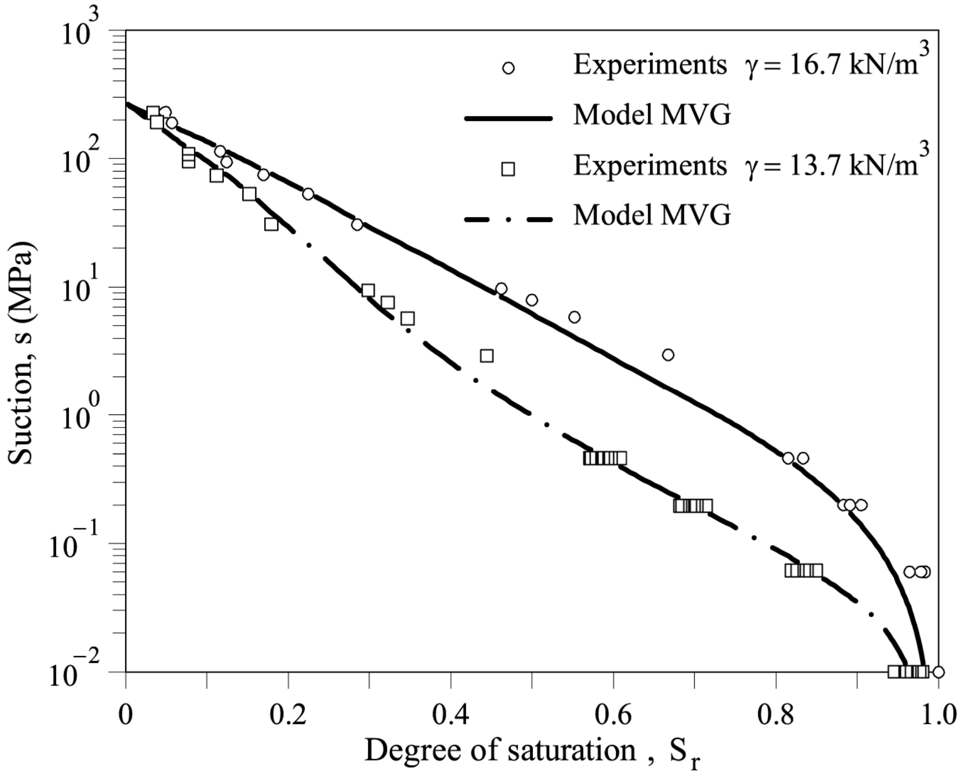


q_{peak}/p_{ini}
1
2
3
0
0.10
0.20
0.30
Suction, s (MPa)
Experiments
Model $S_r^e = S_r$
Model $\alpha = 2.5$
Model $S_r^m = 0.40$

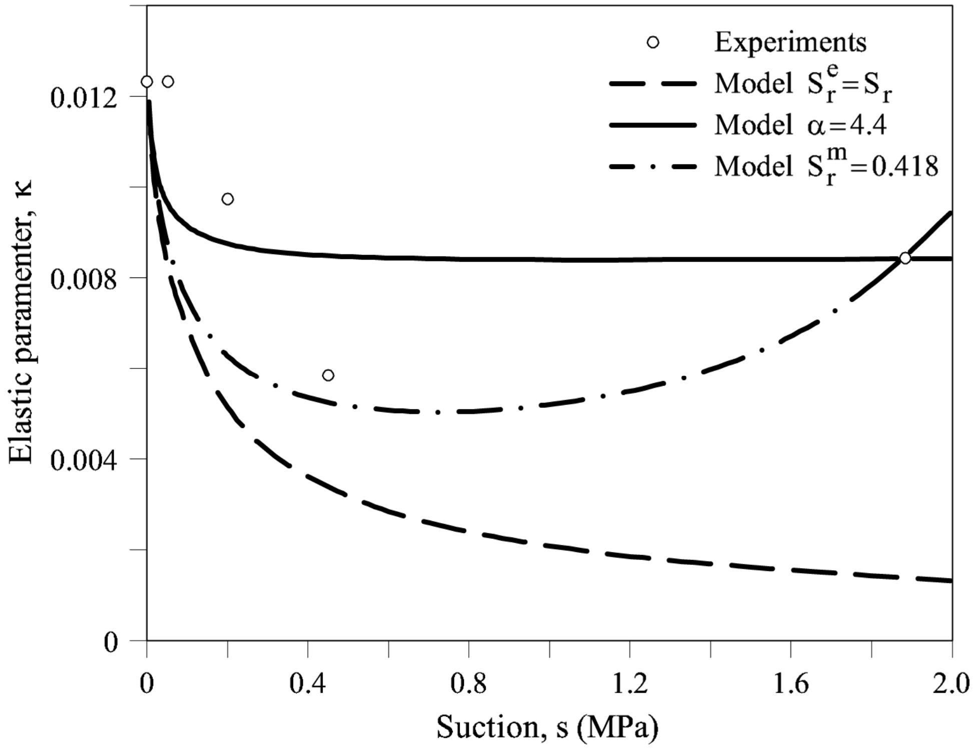


q_{peak}/p_{ini}
1
2
3
0
0.10
0.20
0.30
Suction, s (MPa)
Experiments
Model $S_r^e = S_r$
Model $\alpha = 2.5$
Model $S_r^m = 0.40$

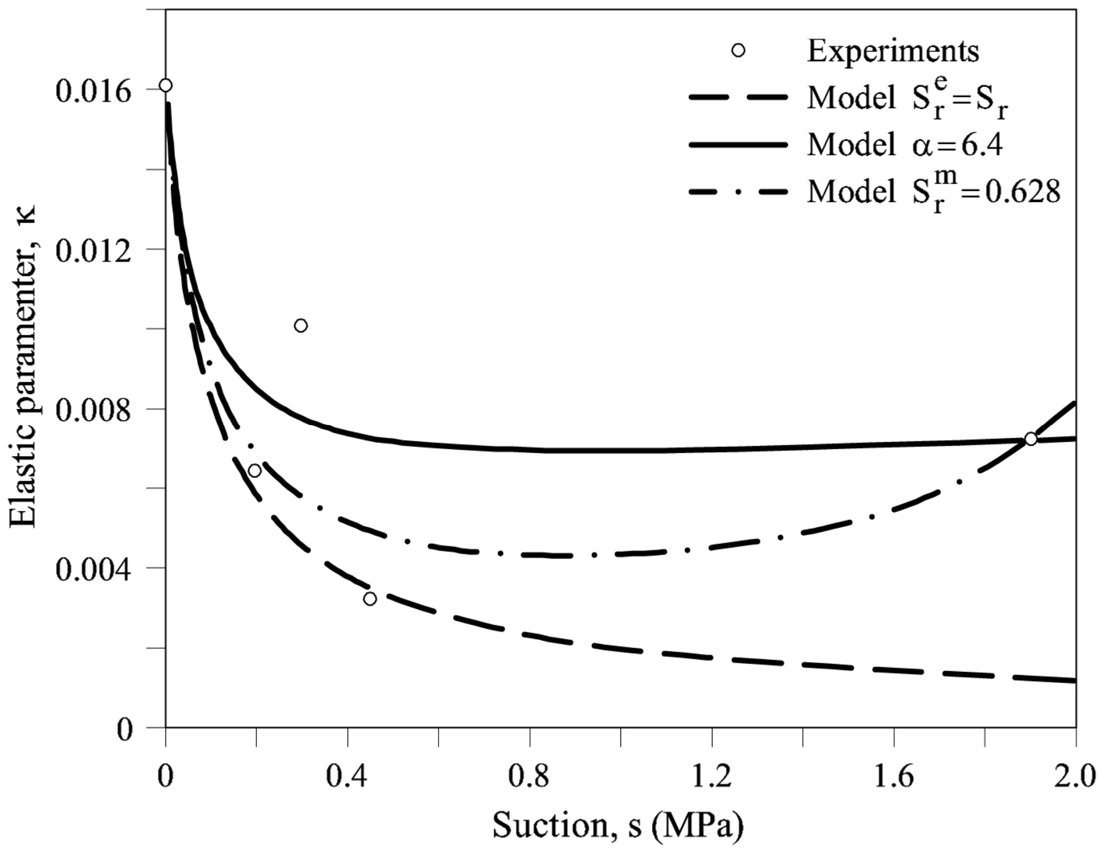
Figure 7



Experiments γ = 16.7 kN/m3
Model MVG
Experiments γ = 13.7 kN/m3
Model MVG
Suction, s (MPa)
Degree of saturation , Sr
10^3
10^2
10^1
10^0
10^−1
10^−2
0
0.2
0.4
0.6
0.8
1.0



Experiments
Model Sre=Sr
Model α=4.4
Model Srm=0.418
Elastic paramenter, κ
Suction, s (MPa)
0.012
0.008
0.004
0
0
0.4
0.8
1.2
1.6
2.0



Experiments
Model $S_r^e = S_r$
Model $\alpha = 6.4$
Model $S_r^m = 0.628$
Elastic parameter, κ
0
0.004
0.008
0.012
0.016
0
0.4
0.8
1.2
1.6
2.0
Suction, s (MPa)

Figure 8

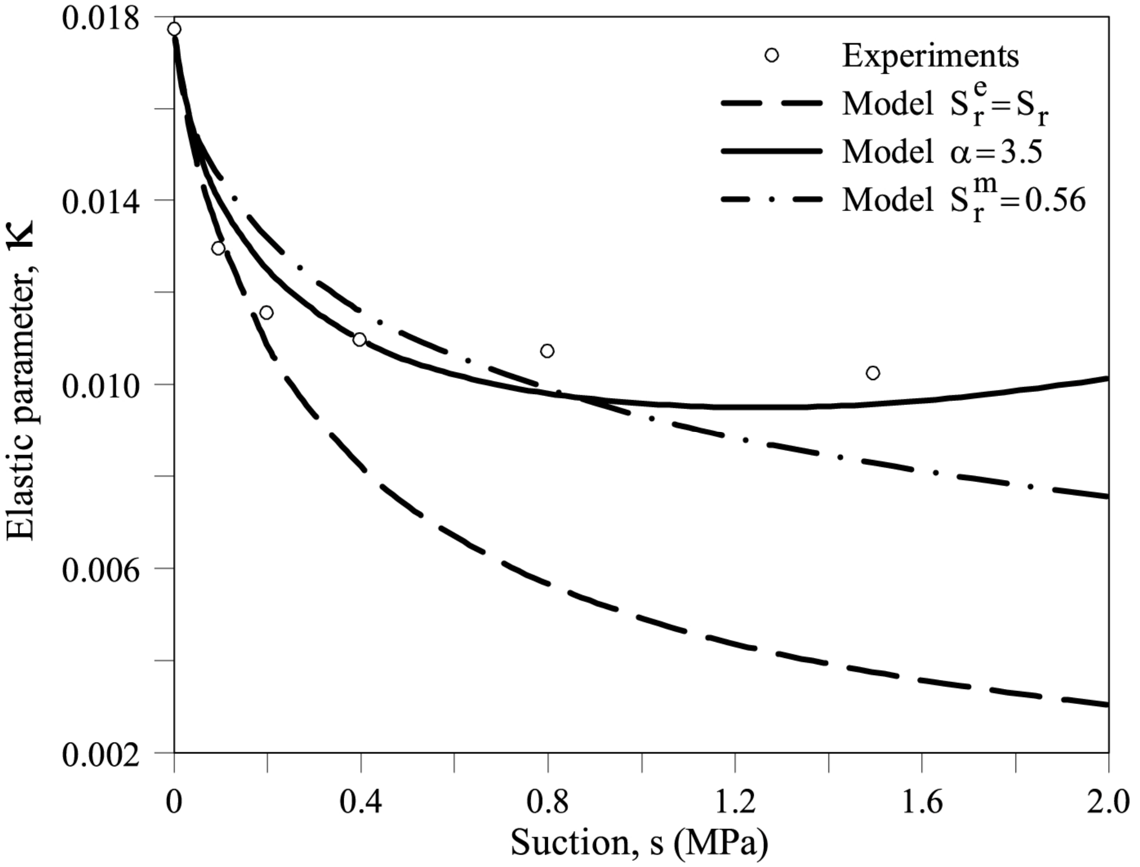

0.018
0.014
0.010
0.006
0.002
Elastic parameter, κ
0
0.4
0.8
1.2
1.6
2.0
Suction, s (MPa)
Experiments
Model $S_r^e = S_r$
Model $\alpha = 3.5$
Model $S_r^m = 0.56$

Figure 9

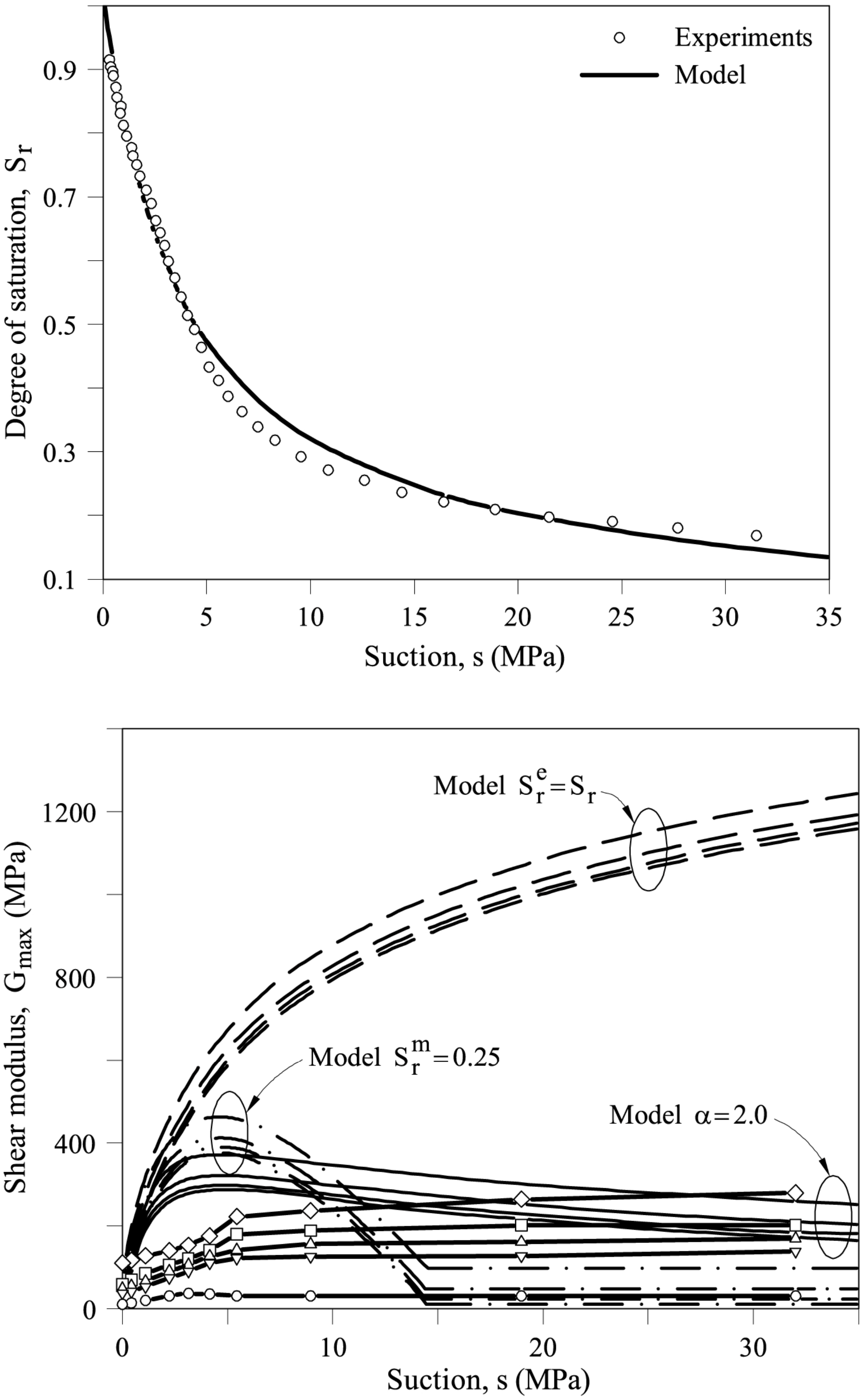

Experiments
Model
Degree of saturation, S_r
0.1
0.3
0.5
0.7
0.9
0
5
10
15
20
25
30
35
Suction, s (MPa)
Model $S_r^e = S_r$
Model $S_r^m = 0.25$
Model $\alpha = 2.0$
Shear modulus, G_{max} (MPa)
0
400
800
1200
0
10
20
30
Suction, s (MPa)

Figure 10

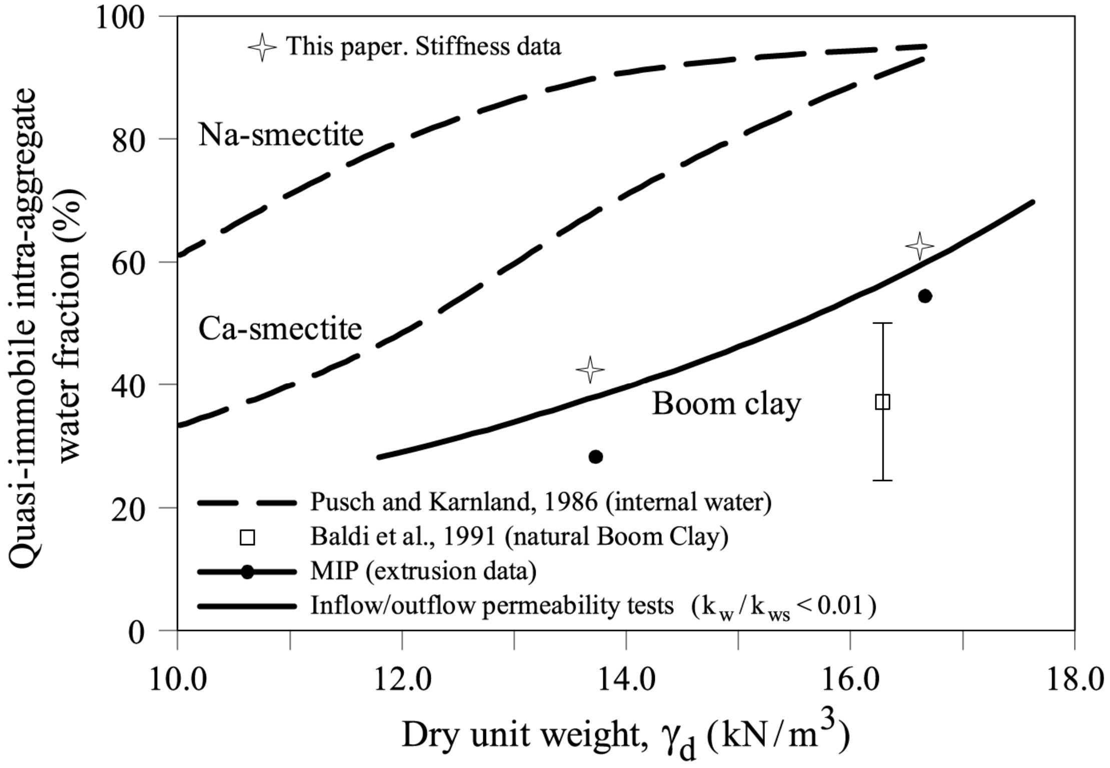

This paper. Stiffness data
Na-smectite
Ca-smectite
Boom clay
Pusch and Karnland, 1986 (internal water)
Baldi et al., 1991 (natural Boom Clay)
MIP (extrusion data)
Inflow/outflow permeability tests ($k_w/k_{ws} < 0.01$)
Quasi-immobile intra-aggregate water fraction (%)
0
20
40
60
80
100
10.0
12.0
14.0
16.0
18.0
Dry unit weight, γ_d (kN/m^3)

Figure 11

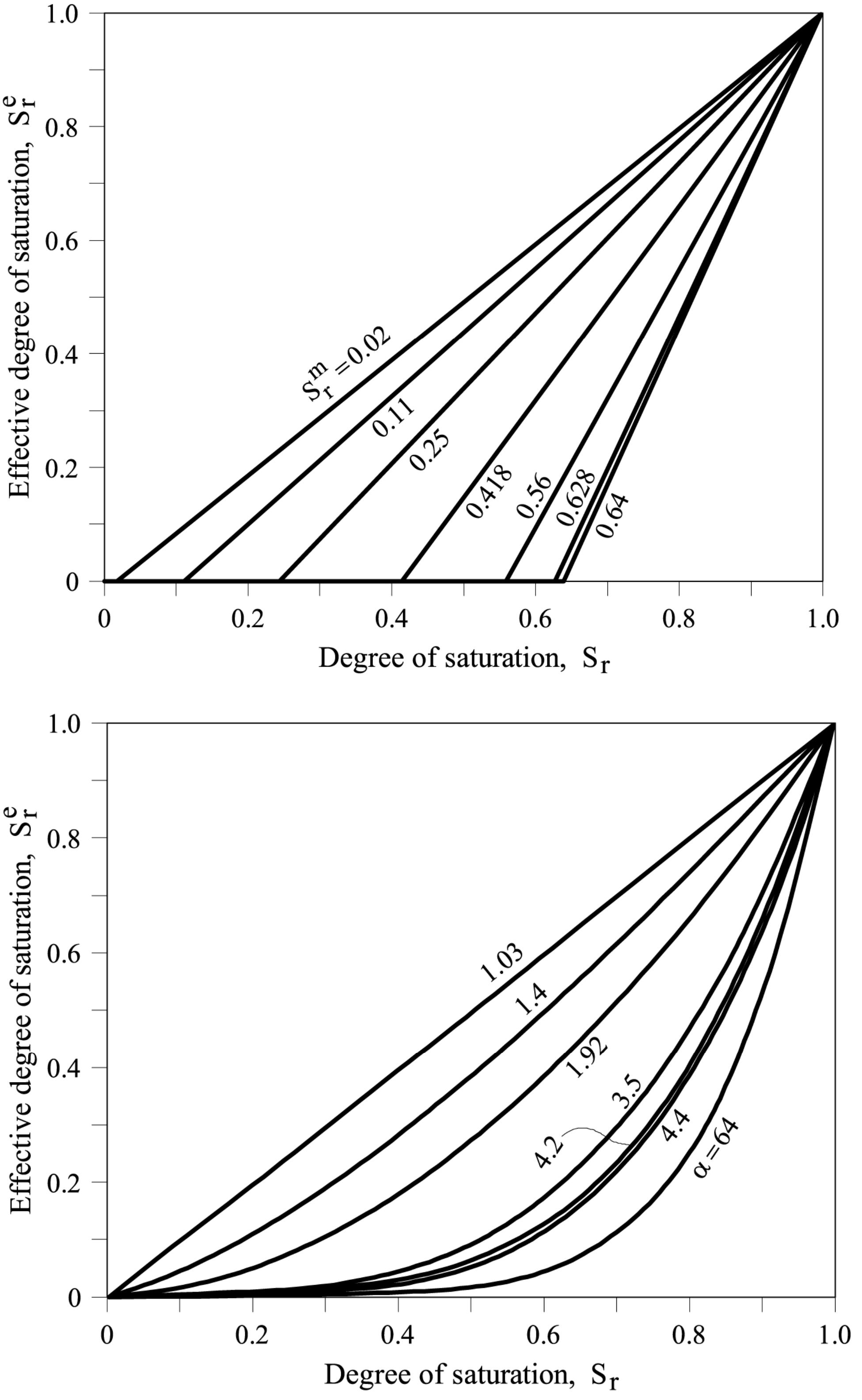

1.0
0.8
0.6
0.4
0.2
0
Effective degree of saturation, S_r^e
S_r^m =0.02
0.11
0.25
0.418
0.56
0.628
0.64
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation, S_r
1.0
0.8
0.6
0.4
0.2
0
Effective degree of saturation, S_r^e
1.03
1.4
1.92
3.5
4.2
4.4
α=64
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation, S_r

Figure 12

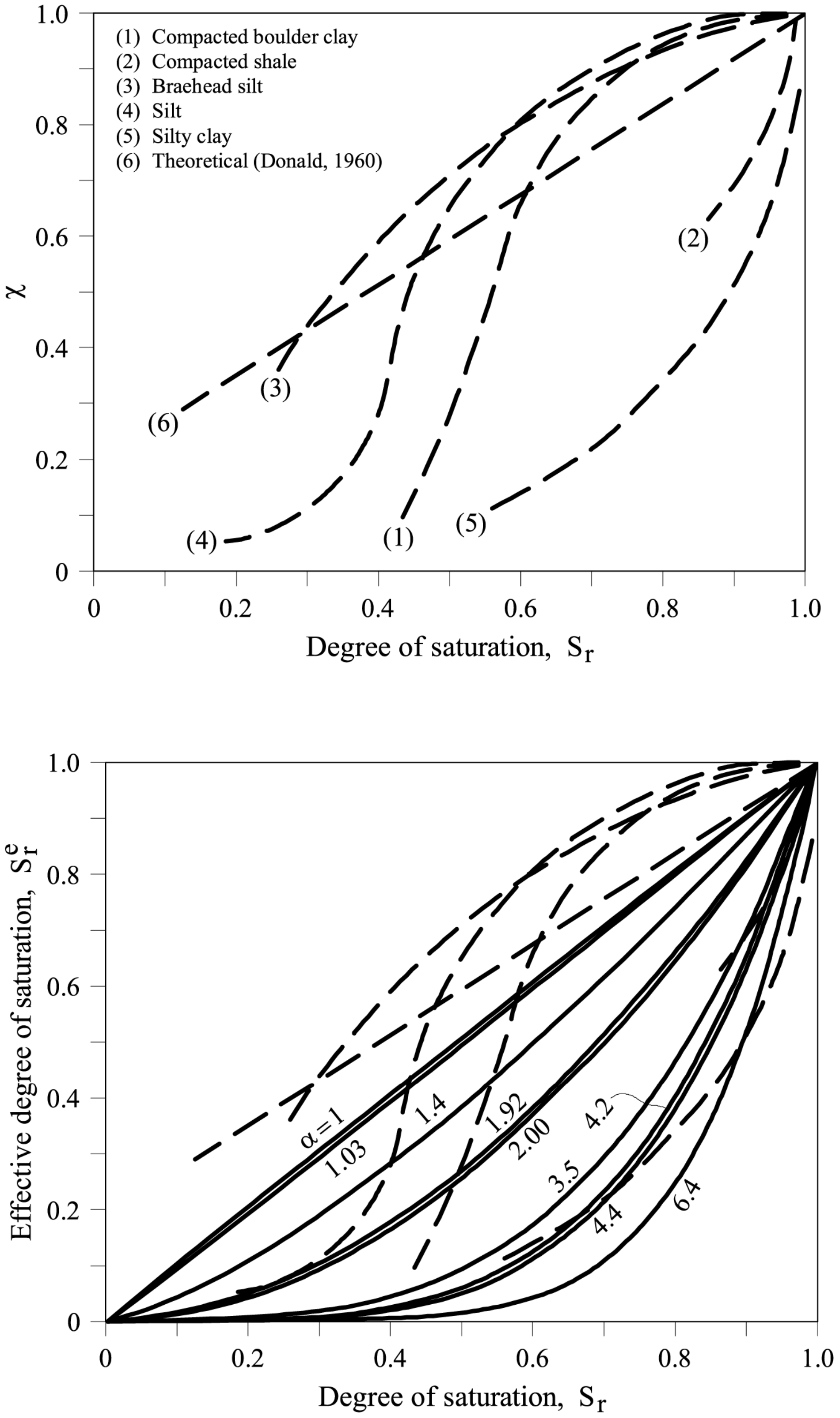

1.0
0.8
0.6
0.4
0.2
0
χ
(1) Compacted boulder clay
(2) Compacted shale
(3) Braehead silt
(4) Silt
(5) Silty clay
(6) Theoretical (Donald, 1960)
(1)
(2)
(3)
(4)
(5)
(6)
0
0.2
0.4
0.6
0.8
1.0
Degree of saturation, S_r
Effective degree of saturation, S_r^e
α=1
1.03
1.4
1.92
2.00
3.5
4.2
4.4
6.4
Degree of saturation, S_r

Figure 13