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Channel Estimation for MIMO-OFDM Systems in Fast Time-Varying Environments

Hussein Hijazi¹, Eric Pierre Simon¹, Martine Liénard¹ and Laurent Ros²

¹Laboratoire TELICE, U.S.T.L. Lille 1 - IEMN/UMR 8520 Bâtiment P3 59655 Villeneuve d’Ascq - FRANCE
²GIPSA-lab, Department Image Signal, BP 46 - 38402 Saint Martin d’Hères - FRANCE
hussein.hijazi@hotmail.fr, eric.simon@univ-lille1.fr, martine.lienard@univ-lille1.fr, laurent.ros@gipsa-lab.inpg.fr

Abstract—A channel estimation algorithm for MIMO-OFDM systems in Fast Time-Varying Environments is proposed. The channel estimation function is based on the equivalent discrete-time channel taps or on the physical propagation channel parameters. To handle rapid variations of channels within a transmission block, we approximate the channel by a basis expansion model (BEM). Based on the Jakes process, an auto-regressive (AR) model of the BEM coefficients dynamics is built, making it possible to estimate and track the BEM coefficients using Kalman filter. Hence, the channel matrix is easily computed, and the data symbol is detected with free ICI. Our claims are supported by theoretical analysis and simulation results, which are obtained considering Jakes’ channels with high Doppler spreads.

Index Terms—MIMO systems, OFDM, time-varying Rayleigh channels, channel estimation, Kalman filter.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) antennas with Orthogonal Frequency Division Multiplexing (OFDM) provide high data rates and are robust to multi-path delay in wireless communications. Channel parameters are required for diversity combining, coherent detection and decoding. Therefore, channel estimation is essential in MIMO-OFDM system design. Various OFDM channel estimation schemes have been proposed in the literature, mostly for single antenna systems [8] [12] [14] [9] [10]. For MIMO-OFDM systems, most of the channel estimation schemes have focused on pilot-assisted approaches [1] [3] [4], based on a quasi-static fading model that allows the channel to be invariant within a MIMO-OFDM block. However, in fast fading channels, the time-variation of the channel within a MIMO-OFDM block results in a loss of subcarrier orthogonality which leads to intercarrier interference (ICI) [8] [2]. To support high speed mobile channels, the channel time-variation within a block must be considered. Channel estimation can be summarized by two approaches. The first one is to estimate the equivalent discrete-time channel taps [2] [12] whereas the second approach is to directly estimate the physical propagation channel parameters such as multi-path delays and multi-path complex gains [14] [8]. In Radio-Frequencies transmission, the delays are quasi invariant over several MIMO-OFDM blocks [11] [9] whereas the complex gains may change significantly, even within one MIMO-OFDM block. Exploiting the channel nature and assuming the availability of delay information, a lot of methods estimate the multi-path complex gains [8] [10] [9]. There are more channel parameters in fast fading channels than in quasi-static fading channels. Therefore, in single antenna systems, many existing works resort to the basis expansion model (BEM) in order to estimate the time-variation of the channel taps or the multi-path complex gains within a block [14] [8]. The BEM methods are discrete Karhunen-Loève BEM (DKL-BEM), discrete prolate spheroidal BEM (PS-BEM), complex-exponential BEM (CE-BEM), generalized CE-BEM (GCE-BEM) and polynomial BEM (P-BEM).

In this paper, we present a new iterative algorithm for joint Rayleigh channel (taps or complex gains) estimation and data recovery in fast time-varying environments. After approximated the time-variation of the channel by a BEM, an auto-regressive (AR) model of the BEM coefficients dynamics is built based on the Jakes process. Hence, the BEM coefficients are estimated and tracked by using the Kalman filter. In order to perform BEM coefficients estimation, we use the estimate along with the channel matrix output to recover the transmitted data. One can, in turn, use the detected data along with pilots to enhance the BEM coefficients estimate giving rise to an iterative technique for channel estimation and data recovery. The data detection is performed by using the MMSE-equalizer [13] or the QR-equalizer [8] [7].

This paper is organized as follows: Section II introduces the MIMO-OFDM system and the BEM modeling. Section III describes the AR model for the BEM coefficients and the Kalman filter. Section IV covers the algorithm for joint channel estimation and data recovery. Section V presents the simulations results which validate our technique. Finally, our conclusions are presented in Section VI.

The notations adopted are as follows: Upper (lower) bold face letters denote matrices (column vectors). [x]k denotes the kth element of the vector x, and [X]k,m denotes the [k, m]th element of the matrix X. We will use the matlab notation X[k1:k2, m1:m2] to extract a submatrix within X from row k1 to row k2 and from column m1 to column m2. I_N is a N × N identity matrix and 0_N is a N × N matrix of zeros. diag[x] is a diagonal matrix with x on its main diagonal and blkdiag{X, Y} is a block diagonal matrix with the matrices X and Y on its main diagonal. The superscripts (·)T, (·)∗ and (·)H stand respectively for transpose, conjugate and Hermitian operators. Tr(·) and E[·] are respectively the determinant and expectation operations. J0(·) is the zeroth-order Bessel function of the first kind.
II. MIMO-OFDM SYSTEM AND CHANNEL MODELS

A. MIMO-OFDM System Model

Consider a MIMO-OFDM system with $N_T$ transmit antennas, $N_R$ receive antennas, $N$ sub-carriers, and a cyclic prefix length $N_p$. The duration of a MIMO-OFDM block is $T = vT_s$, where $T_s$ is the sampling time and $v = N + N_p$. Let $\mathbf{x}_{(n)} = [x_{(n),1}, x_{(n),2}, \ldots, x_{(N_T,n)}]^T$ be the $n$th transmitted MIMO-OFDM block, where $\mathbf{x}_{(n)} = [x_{(n),1}, x_{(n),2}, \ldots, x_{(n),N_T}]^T$ is the $n$th transmitted OFDM symbol by the $t$th transmit antenna and the $r$th receive antenna, and not necessarily integers. In the sequel, we will make the calculus based on the second approach (physical channel) and can deduce the results of the first approach (channel taps) by replacing $L_{(r,t)}$ by $L'_{(r,t)}$ and the set of delays $\{\tau_{(r,t)}\}$ by $\{l - 1, l = 1 : L'_{(r,t)}\}$.

B. BEM Channel Model

Let $L = \sum_{r=1}^{N_R} L_{(r)} = \sum_{r=1}^{N_R} L_{(r)}$ be the total number of complex gains for the MIMO channel and designate by $(t,r)$ the branch between the $t$th transmit antenna and the $r$th receive antenna. Since the number of samples to estimate $Lv$ is greater than the number of observation equations $NcR$, it is not efficient to estimate the time-variation of the complex gains, using directly the observation model in (1). Thus, we need to reduce the number of parameters to estimate. In this section, our aim is to accurately model the time-variation of $\alpha^{(n)}_{(t,r)}(qT_s)$ by using a BEM. Collecting the samples of the $t$th path of the branch $(r,t)$ within the $n$th MIMO-OFDM block in a $v \times 1$ vector $\alpha^{(n)}_{(t,r)} = [\alpha^{(n)}_{(t,r)}(-N_p T_s), \ldots, \alpha^{(n)}_{(t,r)}((N - 1)T_s)]^T$, we can express $\alpha^{(n)}_{(t,r)}$ as:

$$\alpha^{(n)}_{(t,r)} = \alpha^{(n)}_{BEM,(t,r)} + \xi^{(n)}_{(t,r)} = Q^{(n)} c^{(n)}_{(t,r)} + \xi^{(n)}_{(t,r)}$$

where $Q^{(n)}$ is a $v \times N_c$ matrix that collects $N_c$ orthonormal basis function $q_k$ as columns, $c^{(n)}_{(t,r)} = [c^{(n)}_{(1,t,r)}, \ldots, c^{(n)}_{(L_{(r,t)},t,r)}]$ represents the $N_c$ BEM coefficients for the complex gain of the branch $(r,t)$ of the $n$th MIMO-OFDM block, and $\xi^{(n)}_{(t,r)}$ represents the corresponding BEM modeling error, which is assumed to be minimized in the MSE sense [16] [5]. Under this criterion, the optimal BEM coefficients and the corresponding model error are given by:

$$\alpha^{(n)}_{BEM,(t,r)} = (Q^H Q)^{-1} Q^H \alpha^{(n)}_{(t,r)}$$

$$\xi^{(n)}_{(t,r)} = (I_v - S)\alpha^{(n)}_{(t,r)}$$

where $Q = [q_1, \ldots, q_{N_c}]$ is a $v \times N_c$ matrix, $c^{(n)}_{(t,r)}$ and $\xi^{(n)}_{(t,r)}$ are respectively the number of channel taps and the number of paths for the branch between the $t$th transmit antenna and the $r$th receive antenna. The delays are normalized by $T_s$ and not necessarily integers $(\tau_{(r,t)} < N_p)$. The $L_{(r,t)}$ elements of $\{\alpha^{(n)}_{(t,r)}(qT_s)\}$ are uncorrelated, unless that the delays are multiple of $T_s$ as mostly assumed in the literature. They are wide-sense stationary (WSS), narrow-band zero-mean complex Gaussian processes of variances $\sigma_d^2$ and $\sigma_{\alpha}^2$, with the so-called Jakes’ power spectrum of maximum Doppler frequency $f_d$ [18]. The average energy of each channel approach is normalized to one, i.e., $\sum_{l=1}^{L_{(r,t)}} \sigma_d^2 = 1$ and $\sum_{l=1}^{L_{(r,t)}} \sigma_{\alpha}^2 = 1$.

$$\text{MMSE}_{(t,r)} = \frac{1}{v} \mathbf{E} \left[ \xi^{(n)}_{(t,r)} \xi^{(n)}_{(t,r)}^H \right] = \frac{1}{v} \text{Tr} \left( (I_v - S)\mathbf{R}^{(0)}_{\alpha^{(n)}_{(t,r)}} (I_v - S)^H \right)$$

where $\mathbf{R}^{(0)}_{\alpha^{(n)}_{(t,r)}} = \mathbf{E} \left[ \alpha^{(n)}_{(t,r)} \alpha^{(n-s)H}_{(t,r)} \right]$ is the $v \times v$ correlation matrix of $\alpha^{(n)}_{(t,r)}$ with elements given by:

$$\mathbf{R}^{(0)}_{\alpha^{(n)}_{(t,r)}}_{k,m} = \sigma_{\alpha}^2 \delta_{m,k} J_0 \left( 2\pi f_d T_s (k - m + sv) \right)$$

Various traditional BEM designs have been reported to model the channels time-variations, e.g., the CE-BEM $\mathbf{Q}_{k,m} = e^{j2\pi (\frac{m-k}{N_T - 1}) / (m-1)}$ [15], the GCE-BEM $\mathbf{Q}_{k,m} = \mathbf{R}^{(0)}_{\alpha^{(n)}_{(t,r)}}_{k,m}$.
where $\mathbf{L}_n = \mathbf{L}_n \times \mathbf{L}_n$ is a $N_c \times L_{N_c}$ matrix of size $\mathbf{L}_n \times \mathbf{L}_n$ respectively. We initialize the
state and the observation models (21) (22) allow us to use
the Kalman Filter.

Based on the AR model of $\mathbf{c}(n)$ in (20), we define the
state space model for the MIMO-OFDM system as $\mathbf{g}(n) = \{ \mathbf{c}(n), \mathbf{c}(n-1), \mathbf{c}(n-2), \mathbf{c}(n-3), \mathbf{c}(n-4) \}$. Thus, using (20) and (11), we obtain:

$$\mathbf{g}(n) = \mathbf{S}_1 \mathbf{g}(n-1) + \mathbf{S}_2 \mathbf{u}(n)$$

Using (16), we obtain the AR model of order $p$ for $\mathbf{c}(n)$:

$$\mathbf{c}(n) = -\sum_{i=1}^{p} \mathbf{A}_i \mathbf{c}(n-i) + \mathbf{u}(n)$$

The optimal BEM coefficients $\mathbf{c}(n)$ are correlated complex Gaussian variables with zero-means and correlation matrix given by:

$$\mathbf{R}_c(n) = \mathbf{E} \left[ \mathbf{c}(n) \mathbf{c}^H(n) \right]$$

$$\mathbf{R}_c(n) = \mathbf{Q}^H \mathbf{R}_c(n) \mathbf{Q} \mathbf{Q}^H \mathbf{R}_c(n) \mathbf{Q}^H \mathbf{Q}^{-1}$$

Therefore, the dynamics of $\mathbf{c}(n)$ can be well modeled by an auto-regressive (AR) process [19] [20] [8]. A complex AR process of order $p$ can be generated as:

$$\mathbf{c}(n) = -\sum_{i=1}^{p} \mathbf{A}_i \mathbf{c}(n-i) + \mathbf{u}(n)$$

where $\mathbf{A}(1), \ldots, \mathbf{A}(p)$ are $N_c \times N_c$ matrices and $\mathbf{u}(n)$ is a $N_c \times 1$ complex Gaussian vector with covariance matrix $\mathbf{U}(n)$.
where \(\mathbf{w}(s) = 1 + LN_c : (s + 1)LN_c\) with \(s \in [0, p - 1]\) and \(\mathbf{R}^{(s)}_{(r,t)}\) is the correlation matrix of \(c^{(s)}_{(l,t)}\) defined in (15). The Kalman filter is a recursive algorithm composed of two stages: Time Update Equations and Measurement Update Equations. These two stages are defined as:

**Time Update Equations:**

\[
\begin{align*}
\mathbf{\hat{g}}_{(n)} & = \mathbf{S}_{(n)} \mathbf{\hat{g}}_{(n-1)(n-1)} \\
\mathbf{P}_{(n)} & = \mathbf{S}_{(n)} \mathbf{P}_{(n-1)(n-1)} \mathbf{S}_{(n)}^H + \mathbf{S}_{(n)} \mathbf{U} \mathbf{S}_{(n)}^H
\end{align*}
\]

(25)

**Measurement Update Equations:**

\[
\begin{align*}
\mathbf{K}_{(n)} & = \mathbf{P}_{(n)} \mathbf{S}_{(n)}^H \left( \mathbf{S}_{(n)} \mathbf{P}_{(n)} \mathbf{S}_{(n)}^H + N T \sigma^2 \mathbf{I}_{N_r N_c} \right)^{-1} \\
\mathbf{\hat{g}}_{(n)} & = \mathbf{\hat{g}}_{(n)} + \mathbf{K}_{(n)} \left( \mathbf{\hat{y}}_{(n)} - \mathbf{S}_{(n)} \mathbf{\hat{g}}_{(n)} \right) \\
\mathbf{P}_{(n)} & = \mathbf{P}_{(n)} - \mathbf{K}_{(n)} \mathbf{S}_{(n)} \mathbf{P}_{(n)}
\end{align*}
\]

(26)

where \(\mathbf{K}_{(n)}\) is the Kalman gain. The Time Update Equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The Measurement Update Equations are responsible for the feedback, i.e., for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The Time Update Equations can also be thought of a predictor equations, while the Measurement Update Equations can be thought of a corrector equations.

### IV. Joint Data Detection and Kalman Estimation

#### A. Iterative Algorithm

In the iterative algorithm for joint data detection and channel Kalman estimation, the \(N_p\) pilots subcarriers are evently inserted into the \(N\) subcarriers at the positions \(\mathcal{P} = \{ p_1, \ldots, p_r \} = (r - 1)L_f + 1, \quad r = 1, \ldots, N_p\), where \(L_f\) is the distance between two adjacent pilots. We test two channel equalizers for the data detection: the MMSE equalizer [13] and the QR-equalizer [8] [7]. The first one is based on the noise variance \(NT \sigma^2\) whereas the second equalizer allows us to estimate the data symbol with free ICI by performing a so-called QR-decomposition. The algorithm proceeds as follows:

**initialization:**

- \(\mathbf{\hat{g}}_{(000)} = \mathbf{0}_{p L N_c}, 1\)
- compute \(\mathbf{P}_{(000)}\) using (24)
- \(n \leftarrow n + 1\)
- execute the Time Update Equations of Kalman filter (25)
- compute the channel matrix using (14)
- \(i \leftarrow i + 1\)

**recursion:**

1) remove the pilot ICI from the received data subcarriers
2) Detection of data symbols
3) execute the Measurement Update Equations of Kalman filter (26)
4) compute the channel matrix using (14)
5) \(i \leftarrow i + 1\)

where \(i\) represents the iteration number.

### B. Mean Square Error (MSE) Analysis

The error between the \(l\)th exact complex gain and the \(l\)th estimated polynomial \(c^{(n)}_{(l,t)}\) is given by:

\[
\mathbf{c}^{(n)}_{(l,t)} = \mathbf{\alpha}_{(l,t)}^{(n)} - \mathbf{\alpha}_{\text{BM}}^{(n)} = \mathbf{\xi}^{(n)}_{(l,t)} + \mathbf{Q} \mathbf{e}^{(n)}_{(l,t)}
\]

(27)

where \(\mathbf{e}^{(n)}_{(l,t)} = \mathbf{c}^{(n)}_{(l,t)} - \mathbf{\alpha}^{(n)}_{(l,t)}\) and \(\mathbf{\alpha}^{(n)}_{(l,t)}\) is the BEM model error defined in (7). Neglecting the cross-covariance terms between \(\mathbf{\xi}_{(l,t)}^{(n)}\) and \(\mathbf{e}^{(n)}_{(l,t)}\), the mean square error (MSE) between \(\mathbf{\alpha}_{(l,t)}^{(n)}\) and \(\mathbf{\alpha}_{\text{BM}}^{(n)}\) is given by:

\[
\text{MSE}_{(l,t)} = \frac{1}{v} \text{Tr} \left( \mathbf{Q} \mathbf{MSE}_{(l,t)} \mathbf{Q}^H \right)
\]

(28)

provided that the data symbols are perfectly estimated (i.e., data-aided context), where the sequence of indices \(f(l, r, t) = 1 + (l - 1 + a_{r,t})N_c : (l + a_{r,t})N_c\) with \(a_{r,t} = \sum_{r'=1}^{r-1} L(r', t')\).

The on-line Bayesian Cramer-Rao Bound (BCRB) is an important criterion for evaluating the quality of the complex gains Kalman estimation. The on-line BCRB for the estimation of \(\mathbf{\alpha}_{(l,t)}^{(n)}\) in data-aided (DA) context, is given by:

\[
\text{BCRB}(\mathbf{\alpha}_{(l,t)}^{(\infty)}) = \text{MMSE}_{(l,t)} + \frac{1}{v} \text{Tr} \left( \mathbf{Q} \text{BCRB}(\mathbf{\alpha}_{(l,t)}^{(\infty)}) \mathbf{Q}^H \right)
\]

(30)

where \(\text{BCRB}(\mathbf{\alpha}_{(l,t)}^{(K)})\) is the on-line BCRB associated to the estimation of \(\mathbf{\alpha}_{(l,t)}^{(K)}\) which is given by:

\[
\text{BCRB}(\mathbf{\alpha}_{(l,t)}^{(K)}) = \text{BCRB}(\mathbf{\alpha}_{f(l,t),f(l,t)}^{(K)})
\]

(31)

\(\text{BCRB}(\mathbf{\alpha}_{(l,t)})\) is the on-line BCRB for the estimation of \(\mathbf{\alpha} = [\mathbf{c}_{(l,t)}^T, \ldots, \mathbf{c}_{(l,t)}^T]^T\) in DA context which is given by:

\[
\text{BCRB}(\mathbf{\alpha}_{(l,t)}) = \left( \text{blkdiag} \left\{ \mathbf{J}_{(1)}, \ldots, \mathbf{J}_{(2)}' \right\} + \mathbf{R}_{s'} \right)^{-1}
\]

(32)

where \(\mathbf{R}_{s'}\) is calculated in the same way as \(\mathbf{P}_{(000)}\) with \(s, s' \in [0, K - 1]\), and \(\mathbf{J}_{(n)}\) is a \(L N_c \times L N_c\) matrix given by [5]:

\[
\mathbf{J}_{(n)} = \text{blkdiag} \left\{ \mathbf{J}_{(1,n)}, \ldots, \mathbf{J}_{(N_r, n)} \right\}
\]

(33)

\[
\mathbf{J}_{(r,n)} = \begin{bmatrix}
\mathbf{J}_{(r,1,n)} & \cdots & \mathbf{J}_{(r,1,N_r, n)} \\
\vdots & \ddots & \vdots \\
\mathbf{J}_{(r,N_r,1,n)} & \cdots & \mathbf{J}_{(r,N_r, N_r, n)}
\end{bmatrix}
\]

(34)

\[
\mathbf{J}_{(r,t,t',n)} = \frac{1}{N^2 \sigma^2 \mathcal{F}(r, t, t', n)} \mathcal{M} \mathcal{F}(r, t, t', n)
\]

(35)
\[ \mathcal{M} = \begin{bmatrix} M_{11} & \cdots & M_{1N_c} \\ \vdots & \ddots & \vdots \\ M_{N_c1} & \cdots & M_{N_cN_c} \end{bmatrix} \]

\[ \mathcal{F}_{(r,t,n)} = \begin{bmatrix} \mathcal{F}_{(1,r,t,n)} & \cdots & \mathcal{F}_{(N_c,r,t,n)} \end{bmatrix} \]

where the \( N N_c \times N_c \) matrix \( \mathcal{F}_{(r,t,n)} \) is given by:

\[ \mathcal{F}_{(r,t,n)} = \text{blkdiag} \{ \text{diag} \{ x_{(t,n)} \} f_{(r,t)}, \ldots, \text{diag} \{ x_{(t,n)} \} f_{(r,t)} \} \]

V. SIMULATION

In this section, we verify the theory by simulation and test the performance of the iterative algorithm based on the second approach (i.e., estimation of \( \alpha \) assuming the availability of delay information). We assume that the channels from different transmitters to the different receivers have the same delay and fading property (i.e., same number of paths, \( \sigma^2_{p_{0,c,t}} \) and \( t_{(r,t)} \)) and we use the Rayleigh channel model given in [8] [5]. It is reasonable because the transmitters and the receivers are very close to each other in practice. A normalized 4QAM MIMO-OFDM system, with two transmit and two receive antennas, \( N = 256 \) subcarriers, \( N_p = \frac{N}{4} \) pilots (i.e., \( L_f = 4 \)) and \( \frac{1}{T} = 2 MHz \) is used (note that SNR = \( \frac{1}{P} \)). The MSE and the BER are evaluated under a fast time-varying environment. The rapid time-variation of the channel within one MIMO-OFDM block are approximated by a BEM model. The BEM coefficients are tracked and estimated using the Kalman filter. The data symbols are estimated by performing a QR-decomposition of the channel matrix or by using a MMSE equalizer. Theoretical analysis and simulation results show that our algorithm has a good performance for high Doppler spread.

VI. CONCLUSION

In this paper, we have presented a new iterative algorithm for joint multi-path Rayleigh channel estimation and data recovery in fast time-varying environments. The rapid time-variation of the channel within one MIMO-OFDM block are approximated by a BEM model. The BEM coefficients are tracked and estimated using the Kalman filter. The data symbols are estimated by performing a QR-decomposition of the channel matrix or by using a MMSE equalizer. Theoretical analysis and simulation results show that our algorithm has a good performance for high Doppler spread.

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