Channel Estimation for MIMO-OFDM Systems in Fast Time-Varying Environments
Hussein Hijazi, Eric Simon, Martine Liénard, Laurent Ros

To cite this version:
Hussein Hijazi, Eric Simon, Martine Liénard, Laurent Ros. Channel Estimation for MIMO-OFDM Systems in Fast Time-Varying Environments. 4th International Symposium on Communications, Control and Signal Processing (ISCCSP 2010), Mar 2010, Limassol, Cyprus. 5 p., 2010. <hal-00539137>

HAL Id: hal-00539137
https://hal.archives-ouvertes.fr/hal-00539137
Submitted on 24 Nov 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Channel Estimation for MIMO-OFDM Systems in Fast Time-Varying Environments

Hussein Hijazi\textsuperscript{1}, Eric Pierre Simon\textsuperscript{1}, Martine Liénard\textsuperscript{1} and Laurent Ros\textsuperscript{2}
\textsuperscript{1}Laboratoire TELICE, U.S.T.L. Lille 1 - IEMN/UMR 8520 Bâtiment P3 59655 Villeneuve d’Ascq - FRANCE
\textsuperscript{2}GIPSA-lab, Department Image Signal, BP 46 - 38402 Saint Martin d’Hères - FRANCE
hussein.hijazi@hotmail.fr, eric.simon@univ-lille1.fr, martine.lienard@univ-lille1.fr, laurent.ros@gipsa-lab.inpg.fr

Abstract—A channel estimation algorithm for MIMO-OFDM systems in Fast Time-Varying Environments is proposed. The channel estimation function is based on the equivalent discrete-time channel taps or on the physical propagation channel parameters. To handle rapid variations of channels within a transmission block, we approximate the channel by a basis expansion model (BEM). Based on the Jakes process, an auto-regressive (AR) model of the BEM coefficients dynamics is built, making it possible to estimate and track the BEM coefficients using Kalman filter. Hence, the channel matrix is easily computed, and the data symbol is detected with free ICI. Our claims are supported by theoretical analysis and simulation results, which are obtained considering Jakes’ channels with high Doppler spreads.

Index Terms—MIMO systems, OFDM, time-varying Rayleigh channels, channel estimation, Kalman filter.

I. INTRODUCTION

Multiple-Input-Multiple-Output (MIMO) antennas with Orthogonal Frequency Division Multiplexing (OFDM) provide high data rates and are robust to multi-path delay in wireless communications. Channel parameters are required for diversity combining, coherent detection and decoding. Therefore, channel estimation is essential in MIMO-OFDM system design. Various OFDM channel estimation schemes have been proposed in the literature, mostly for single antenna systems [8] [12] [14] [9] [10]. For MIMO-OFDM systems, most of the channel estimation schemes have focused on pilot-assisted approaches [1] [3] [4], based on a quasi-static fading model that allows the channel to be invariant within a MIMO-OFDM block. However, in fast fading channels, the time-variation of the channel within a MIMO-OFDM block results in a loss of subcarrier orthogonality which leads to intercarrier interference (ICI) [8] [2]. To support high speed mobile channels, the channel time-variation within a block must be considered. Channel estimation can be summarized by two approaches. The first one is to estimate the equivalent discrete-time channel taps [2] [12] whereas the second approach is to directly estimate the physical propagation channel parameters such as multi-path delays and multi-path complex gains [14] [8]. In Radio-Frequencies transmission, the delays are quasi invariant over several MIMO-OFDM blocks [11] [9] whereas the complex gains may change significantly, even within one MIMO-OFDM block. Exploiting the channel nature and assuming the availability of delay information, a lot of methods estimate the multi-path complex gains [8] [10] [9]. There are more channel parameters in fast fading channels than in quasi-static fading channels. Therefore, in single antenna systems, many existing works resort to the basis expansion model (BEM) in order to estimate the time-variation of the channel taps or the multi-path complex gains within a block [14] [8]. The BEM methods are discrete Karhunen-Loeve BEM (DKL-BEM), discrete prolate spheroidal BEM (PS-BEM), complex-exponential BEM (CE-BEM), generalized CE-BEM (GCE-BEM) and polynomial BEM (P-BEM).

In this paper, we present a new iterative algorithm for joint Rayleigh channel (taps or complex gains) estimation and data recovery in fast time-varying environments. After approximated the time-variation of the channel by a BEM, an auto-regressive (AR) model of the BEM coefficients dynamics is built based on the Jakes process. Hence, the BEM coefficients are estimated and tracked by using the Kalman filter. In order to perform BEM coefficients estimation, we use the estimate along with the channel matrix output to recover the transmitted data. One can, in turn, use the detected data along with pilots to enhance the BEM coefficients estimate giving rise to an iterative technique for channel estimation and data recovery. The data detection is performed by using the MMSE-equalizer [13] or the QR-equalizer [8] [7].

This paper is organized as follows: Section II introduces the MIMO-OFDM system and the BEM modeling. Section III describes the AR model for the BEM coefficients and the Kalman filter. Section IV covers the algorithm for joint channel estimation and data recovery. Section V presents the simulations results which validate our technique. Finally, our conclusions are presented in Section VI.

The notations adopted are as follows: Upper (lower) bold face letters denote matrices (column vectors). \( \mathbf{x}_k \) denotes the \( k \)th element of the vector \( \mathbf{x} \), and \( \mathbf{X}_{k,m} \) denotes the \( [k,m] \)th element of the matrix \( \mathbf{X} \). We will use the matlab notation \( \mathbf{X}_{[k_1:k_2,m_1:m_2]} \) to extract a submatrix within \( \mathbf{X} \) from row \( k_1 \) to row \( k_2 \) and from column \( m_1 \) to column \( m_2 \). \( \mathbf{I}_N \) is a \( N \times N \) identity matrix and \( \mathbf{0}_N \) is a \( N \times N \) matrix of zeros. \( \text{diag}(\mathbf{x}) \) is a diagonal matrix with \( \mathbf{x} \) on its main diagonal and \( \text{blkdiag}(\mathbf{X}, \mathbf{Y}) \) is a block diagonal matrix with the matrices \( \mathbf{X} \) and \( \mathbf{Y} \) on its main diagonal. The superscripts \( (\cdot)^T, (\cdot)^* \) and \( (\cdot)^H \) stand respectively for transpose, conjugate and Hermitian operators. Tr(\( \cdot \)) and E[\( \cdot \)] are respectively the determinant and expectation operations. \( J_0(\cdot) \) is the zeroth-order Bessel function of the first kind.
II. MIMO-OFDM System and Channel Models

A. MIMO-OFDM System Model

Consider a MIMO-OFDM system with $N_T$ transmit antennas, $N_R$ receive antennas, $N$ sub-carriers, and a cyclic prefix length $N_g$. The duration of a MIMO-OFDM block is $T = v T_s$, where $T_s$ is the sampling time and $v = N + N_g$. Let $x_{(n)} = [x_{(1,n)}, x_{(2,n)}, ..., x_{(N_R,n)}]^{T}$ be the $n$th transmitted MIMO-OFDM block, where $x_{(t,n)} = [x_1(t,n), x_2(t,n), ..., x_{N_T}(t,n)]^{T}$ is the $n$th transmitted OFDM symbol by the $t$th transmit antenna and $[x_1(t,n)]$ are normalized symbols (i.e., $E[x_1(t,n)x_1^{*}(t,n)] = 1$). After transmission over a multi-path Rayleigh channel, the $n$th received MIMO-OFDM block $y_{(n)} = [y_1(n), y_2(n), ..., y_{N_R}(n)]^{T}$, where $y_{(r,n)} = [y(r,n) - \frac{N}{2}, y(r,n) - \frac{N}{2} + 1, ..., y(r,n) - 1]^{T}$ is the $n$th received OFDM symbol by the $r$th receive antenna, is given by [9] [12]:

$$y_{(n)} = H_{(n)} x_{(n)} + w_{(n)}$$

(1)

where $w_{(n)} = [w_1(n), w_2(n), ..., w_{N_R}(n)]^{T}$, with $w_{(r,n)} = [w_1(n) - \frac{N}{2}, w_1(n) - \frac{N}{2} + 1, ..., w_1(n) - 1]^{T}$ is a white complex Gaussian noise vector of covariance matrix $N_0 \Sigma^2$ and $H_{(n)}$ is a $N_R \times N_T$ MIMO channel matrix given by:

$$H_{(n)} = \begin{bmatrix}
H_{(1,1,n)} & \cdots & H_{(1,N_T,n)} \\
\vdots & \ddots & \vdots \\
H_{(N_R,1,n)} & \cdots & H_{(N_R,N_T,n)}
\end{bmatrix}$$

(2)

where $H_{(r,t,n)}$ is the channel matrix between the $r$th transmit antenna and the $t$th receive antenna. The elements of channel matrix $H_{(r,t,n)}$ can be written in terms of equivalent channel taps [2] $\{g_{(r,t,n)}(q_{T_s}) = g_{(r,t,n)}(n T_s + q_{T_s})\}$ or in terms of physical channel parameters [8]: delays $\{\tau_{(r,t)}\}$ and complex gains $\{\alpha_{(r,t)}^{(n)}(q_{T_s}) = \alpha_{(r,t)}^{(n)}(n T_s + q_{T_s})\}$, as:

$$H_{(r,t,n)} = \sum_{l=1}^{L_r} \sum_{c=0}^{C_{(r,t)}} e^{-j2 \pi \\frac{c}{C_{(r,t)}} \cdot \frac{1}{2}(l-1)} N_{r,b} g_{(r,t,n)}^{(b)}(q_{T_s}) e^{j2 \pi \frac{m}{N_{r,b}} q_{T_s}}$$

(3)

$$= \sum_{l=1}^{L_r} \sum_{c=0}^{C_{(r,t)}} e^{-j2 \pi \frac{c}{C_{(r,t)}} \cdot \frac{1}{2}(l-1)} N_{r,b} g_{(r,t,n)}^{(b)}(q_{T_s}) e^{j2 \pi \frac{m}{N_{r,b}} q_{T_s}}$$

(4)

where $L_{r,c} = \sum_{l=1}^{L_r} \sum_{c=0}^{C_{(r,t)}} g_{(r,t,n)}^{(b)}(q_{T_s}) e^{j2 \pi \frac{m}{N_{r,b}} q_{T_s}}$ and $L_{c,r} = \sum_{l=1}^{L_r} \sum_{c=0}^{C_{(r,t)}} g_{(r,t,n)}^{(b)}(q_{T_s}) e^{j2 \pi \frac{m}{N_{r,b}} q_{T_s}}$ are the number of channel taps and the number of paths for the branch between the $r$th transmit antenna and the $r$th receive antenna. The delays are normalized by $T_s$ and not necessarily integers ($\tau_{(r,t)} < N_g$). The $L_{r,c}$ elements of $\{g_{(r,t,n)}^{(b)}(q_{T_s})\}$ are correlated, unless that the delays are multiple of $T_s$ as mostly assumed in the literature. They are wide-sense stationary (WSS), narrow-band zero-mean complex Gaussian processes of variances $\sigma_{g_{(r,t,n)}^{(b)}}^2$ and $\sigma_{g_{(r,t,n)}^{(b)}}^2$ with the so-called Jakes’ power spectrum of maximum Doppler frequency $f_d$ [18]. The average energy of each channel approach is normalized to one, i.e., $\sum_{l=1}^{L_{c,r}} \sigma_{g_{(r,t,n)}^{(b)}}^2 = 1$ and $\sum_{l=1}^{L_{r,c}} \sigma_{g_{(r,t,n)}^{(b)}}^2 = 1$.

In the sequel, we will make the calculus based on the second approach (physical channel) and can deduce the results of the first approach (channel taps) by replacing $L_{r,c}$ by $L'_{r,c}$ and the set of delays $\{\tau_{(r,t)}\}$ by $\{l-1, l = 1 : L'_{r,c}\}$.

B. BEM Channel Model

Let $L = \sum_{r=1}^{N_R} L_{r,c} = \sum_{r=1}^{N_R} \sum_{t=1}^{N_T} L_{r,c}$ be the total number of complex gains for the MIMO channel and designate by $(l, r)$ the branch between the $t$th transmit antenna and the $r$th receive antenna. Since the number of samples to estimate $L_v$ is greater than the number of observation equations $N_R N$, it is not efficient to estimate the time-variation of the complex gains, using directly the observation model in (1). Thus, we need to reduce the number of parameters to estimate. In this section, our aim is to accurately model the time-variation of $\alpha_{(r,t)}^{(n)}(q_{T_s})$ by using a BEM. Collecting the samples of the $l$th path of the branch $(r, t)$ within the $n$th MIMO-OFDM block in a $v \times 1$ vector $\alpha_{(r,t)}^{(n)} = [\alpha_{(r,t)}^{(n)}(-N_g T_s), ..., \alpha_{(r,t)}^{(n)}((N-1)T_s)]^{T}$, we can express $\alpha_{(r,t)}^{(n)}$ as:

$$\alpha_{(r,t)}^{(n)} = \alpha_{BEM_{(r,t)}}^{(n)} + \xi_{(r,t)}^{(n)} = Q c_{(r,t)}^{(n)} + \xi_{(r,t)}^{(n)}$$

(5)

where $Q = [q_1, ..., q_N]_n$ is a $v \times N_c$ matrix that collects $N_c$ orthonormal basis function $q_k$ as columns, $c_{(r,t)}^{(n)} = [c_{1(r,t)}^{(n)}, ..., c_{N_T(r,t)}^{(n)}]$ represents the $N_c$ BEM coefficients for the $l$th complex gain of the branch $(r, t)$ of the $n$th MIMO-OFDM block, and $\xi_{(r,t)}^{(n)}$ represents the corresponding BEM modeling error, which is assumed to be minimized in the MSE sense [16] [5]. Under this criterion, the optimal BEM coefficients and the corresponding model error are given by:

$$\alpha_{BEM_{(r,t)}}^{(n)} = (Q^{H}Q)^{-1}Q^{H}c_{(r,t)}^{(n)}$$

(6)

$$\xi_{(r,t)}^{(n)} = (I_v - S)\alpha_{BEM_{(r,t)}}^{(n)}$$

(7)

where $S = Q \left(Q^{H}Q\right)^{-1}Q^{H}$ is a $v \times v$ matrix. It provides the MMSE approximation for all BEM containing $N_c$ coefficients, given by:

$$\text{MMSE}_{(r,t)} = \frac{1}{v} E \left[\xi_{(r,t)}^{(n)}(\xi_{(r,t)}^{(n)})^{H}\right]$$

(8)

$$= \frac{1}{v} tr \left( (I_v - S) R_{\alpha_{BEM_{(r,t)}}}^{(n)} (I_v - S)^{H} \right)$$

(9)

where $R_{\alpha_{BEM_{(r,t)}}}^{(n)} = E \left[\alpha_{BEM_{(r,t)}}^{(n)}(\alpha_{BEM_{(r,t)}}^{(n)})^{H}\right]$ is the $v \times v$ correlation matrix of $\alpha_{BEM_{(r,t)}}^{(n)}$ with elements given by:

$$R_{\alpha_{BEM_{(r,t)}}}^{(n)} = \sigma_{\alpha_{BEM_{(r,t)}}}^2 J_0(2 \pi f_d T_s (k - m))$$

(10)

Various traditional BEM designs have been reported to model the channels time-variations, e.g., the CE-BEM $Q|_{k,m} = e^{j2 \pi ((k-m)/(N_T-1)) m}$, the GCE-BEM $Q|_{k,m} = e^{j2 \pi ((k-m)/(N_T-1)) m}$.
\[ e^{i2\pi \left(\frac{m-k}{N_{c}}\right)(m-1 - \frac{n-1}{N_{c}})} \] with \( 1 < a \leq \frac{N_{c}-1}{2} \) [16], the P-BEM \( [Q_{k,m} = (k - N g - 1)(m-1)] \) [5] and the DKL-BEM which employs basis sequences that corresponds to the most significant eigenvectors of the autocorrelation matrix \( R_{\alpha}(t,r,t) \) [17]. From now on, we can describe the MIMO-OFDM system model derived previously in terms of the BEM. Substituting (5) in (1) and neglecting the BEM model error, we obtain after some algebra:

\[
y_{(n)} = K_{c}(n) e_{(n)} + w_{(n)} \tag{11}
\]

where the \( L_{N_{c}} \times 1 \) vector \( e_{(n)} \) and the \( N_{R}N \times L_{N_{c}} \) matrix \( K_{c}(n) \) are given by:

\[
e_{(n)} = \left[ c_{(1,1,n)}^T, \ldots, c_{(1,N_{R},n)}^T, c_{(N_{R},N_{R},n)}^T \right]^T
\]

\[
c_{(r,t,n)} = \left[ c_{(r,t,n)}^T \right]^T
\]

\[
K_{c}(n) = \text{blkdiag} \left\{ K_{c}(1,1,n), \ldots, K_{c}(N_{R},N_{R},n) \right\}
\]

\[
K_{c}(r,t,n) = \frac{1}{N} \left[ \sum_{q=0}^{N_{c-1}} Z_{(n)}^{(1),r,t} \right] \tag{13}
\]

\[ F_{(r,t)} = e^{-\frac{j2\pi}{N_{R}} \sum_{q=0}^{N_{c-1}}} \]

\[ M_{(d)} = \sum_{q=0}^{N_{c-1}} \left[ Q_{(q+1),d} \right] e^{2\pi i \frac{n-1}{N_{c}}} \tag{14}
\]

Moreover, the channel matrix of the branch \((r,t)\) is easily computed by using the BEM coefficients [9]:

\[ H_{(r,t,n)} = \sum_{d=1}^{N_{c}} M_{(d)} \text{diag} \left\{ \chi_{(n)}^{(d)}(t,r,t) \right\} \tag{14}
\]

where \( \chi_{(n)}^{(d)} = \left[ c_{(1,1,d)}^{(n)}, \ldots, c_{(1,N_{R},d)}^{(n)} \right] \), the matrices \( M_{(d)} \) can be computed and stored.

### III. AR MODEL AND KALMAN FILTER

#### A. The AR Model for \( c(n) \)

The optimal BEM coefficients \( c_{(r,t)} \) are correlated complex Gaussian variables with zero-means and correlation matrix given by:

\[
R_{c_{(r,t)}}^{(a)} = E[c_{(r,t)}(a) c_{(r,t)}^{H}(a-n)] = (Q^{H}Q)^{-1} Q^{H} R_{\alpha_{(r,t)}}^{(a)} Q (Q^{H})^{-1} \tag{15}
\]

Hence, the dynamics of \( c_{(r,t)}^{(n)} \) can be well modeled by an auto-regressive (AR) process [19] [20] [8]. A complex AR process of order \( p \) can be generated as:

\[
c_{(r,t)}^{(n)} = -\sum_{i=1}^{p} A_{i}^{(p)} c_{(r,t)}^{(n-i)} + u_{(r,t)}^{(n)} \tag{16}
\]

where \( A_{1}^{(1)}, \ldots, A_{p}^{(p)} \) are \( N_{c} \times N_{c} \) matrices and \( u_{(r,t)}^{(n)} \) is a \( N_{c} \times 1 \) complex Gaussian vector with covariance matrix \( U_{(r,t)}^{(n)} \). The matrices \( A_{1}^{(1)}, \ldots, A_{p}^{(p)} \) and \( U_{(r,t)}^{(n)} \) are the AR model parameters obtained by solving the set of Yule-Walker equations defined as:

\[ T A = -V \quad \text{and} \quad U_{r,t} = R_{(r,t)}^{(0)} + \sum_{i=1}^{p} A_{i}^{(p)} R_{(r,t)}^{(i)} \tag{17} \]

where \( A = [A_{1}^{(1)}, \ldots, A_{p}^{(p)}]^{T} \) is a \( p \times N_{c} \) matrix, \( V \) and \( T \) are \( p \times N_{c} \) and \( p \times N_{c} \) matrices defined by:

\[
V = \frac{1}{\sigma_{A}^{2}} \left[ R_{c_{(r,t)}}^{(0)}, \ldots, R_{c_{(r,t)}}^{(p)} \right]^{T} \tag{18}
\]

\[
T = \frac{1}{\sigma_{A}^{2}} \left[ R_{c_{(r,t)}}^{(0)}, \ldots, R_{c_{(r,t)}}^{(p)} \right] \tag{19}
\]

Using (16), we obtain the AR model of order \( p \) for \( e_{(n)} \):

\[
e_{(n)} = -\sum_{i=1}^{p} A_{i}^{(p)} e_{(n-i)} + u_{(n)} \tag{20}
\]

where \( A_{i} = \text{blkdiag} \{ A_{i}^{(1)}, \ldots, A_{i}^{(p)} \} \) is a \( L_{N_{c}} \times L_{N_{c}} \) matrix and \( u_{(n)} = \left[ u_{(n)}^{(1)}, \ldots, u_{(n)}^{(p)} \right]^{T} \) is a \( L_{N_{c}} \times 1 \) complex Gaussian vector with covariance matrix \( U = \text{blkdiag} \{ U_{(1,1)}, \ldots, U_{(L_{N_{c}},L_{N_{c}})} \} \).

#### B. The Kalman Filter

Based on the AR model of \( e_{(n)} \) in (20), we define the state space model for the MIMO-OFDM system as \( g_{(n)} = [c_{(n)}^{T}, \ldots, c_{(n-p+1)}^{T}]^{T} \). Thus, using (20) and (11), we obtain:

\[
g_{(n)} = S_{1} g_{(n-1)} + S_{2} u_{(n)} \tag{21}
\]

\[
\dot{y}_{(n)} = S_{1} \dot{g}_{(n)} + w_{(n)} \tag{21}
\]

where \( S_{2} = [I_{L_{N_{c}}}, 0_{L_{N_{c}},(p-1)L_{N_{c}}}^{T}] \) is a \( pL_{N_{c}} 	imes L_{N_{c}} \) matrix, \( S_{1} = [K_{c}(1), 0_{N_{R},(p-1)L_{N_{c}}}^{T}] \) is a \( N \times pL_{N_{c}} \) matrix and \( S_{1} \) is a \( pL_{N_{c}} \times pL_{N_{c}} \) matrix defined as:

\[
S_{1} = \left[ \begin{array}{cccc}
-A_{1}^{(1)} & -A_{2}^{(1)} & \cdots & -A_{p}^{(1)} \\
0_{L_{N_{c}}} & 0_{L_{N_{c}}} & \cdots & 0_{L_{N_{c}}} \\
I_{L_{N_{c}}} & 0_{L_{N_{c}}} & \cdots & 0_{L_{N_{c}}} \\
0_{L_{N_{c}}} & I_{L_{N_{c}}} & \cdots & 0_{L_{N_{c}}} \\
\vdots & \vdots & \ddots & \vdots \\
0_{L_{N_{c}}} & 0_{L_{N_{c}}} & \cdots & I_{L_{N_{c}}}
\end{array} \right] \tag{23}
\]

The state and the observation models (21) (22) allow us to use Kalman filter to adaptively track the BEM coefficients \( c_{(n)} \). Let \( \hat{g}_{(n)} \) be our priori state estimate at step \( n \) given knowledge of the process prior to step \( n \) \( \hat{g}_{(n|n)} \) be our a posteriori state estimate at step \( n \) given measurement \( y_{(n)} \). \( P_{0}(n) \) and \( P_{p}(n) \) are the a priori and the a posteriori error covariance matrices of size \( pL_{N_{c}} \times pL_{N_{c}} \) respectively. We initialize the Kalman filter with \( g_{(0|0)} = 0_{pL_{N_{c}},1} \) and \( P_{0}(0|0) \) given by:

\[
P_{0}(0|0)_{u_{(r,t)},u_{(r,t)}} = R_{u_{(r,t)}}^{(0)} \tag{24}
\]

\[
R_{u_{(r,t)}}^{(0)} = \text{blkdiag} \{ R_{u_{(r,t),r,t}}^{(1)}, \ldots, R_{u_{(r,t),r,t}}^{(p)} \}
\]

\[
R_{u_{(r,t)}}^{(0)} = \text{blkdiag} \{ R_{u_{(r,t),r,t}}^{(1)}, \ldots, R_{u_{(r,t),r,t}}^{(p)} \}
\]
where $\mathbf{u}(s) = 1 + LN_c : (s + 1)LN_c$ with $s \in [0, p - 1]$ and
$\mathbf{R}_e^{(s)}_{(r,t)}$ is the correlation matrix of $\mathbf{e}_{(l,r,t)}^{(n)}$ defined in (15). The
Kalman filter is a recursive algorithm composed of two stages:
Time Update Equations and Measurement Update Equations. These two stages are defined as:

**Time Update Equations:**

\[ \mathbf{K}_{(n)} = \mathbf{P}_{(n)} S_{1}^{H} \left( \mathbf{S}_{1} \mathbf{P}_{(n-1)} S_{1}^{H} + N_{T} \cdot \sigma^{2} \mathbf{I}_{N_{c}N_{c}} \right)^{-1} \]
\[ \hat{\mathbf{g}}_{(n)} = \mathbf{P}_{(n)} \left( \mathbf{S}_{1} \mathbf{S}_{1}^{H} \mathbf{P}_{(n)}^{-1} \right) \]
\[ \mathbf{P}_{(n)} = \mathbf{P}_{(n)} - \mathbf{K}_{(n)} \mathbf{S}_{1} \mathbf{P}_{(n)} \] (25)

**Measurement Update Equations:**

\[ \mathbf{K}_{(n)} = \mathbf{P}_{(n)} S_{1}^{H} \left( \mathbf{S}_{1} \mathbf{P}_{(n-1)} S_{1}^{H} + N_{T} \cdot \sigma^{2} \mathbf{I}_{N_{c}N_{c}} \right)^{-1} \]
\[ \hat{\mathbf{g}}_{(n)} = \mathbf{P}_{(n)} \left( \mathbf{S}_{1} \mathbf{S}_{1}^{H} \mathbf{P}_{(n)}^{-1} \right) \]
\[ \mathbf{P}_{(n)} = \mathbf{P}_{(n)} - \mathbf{K}_{(n)} \mathbf{S}_{1} \mathbf{P}_{(n)} \] (26)

where $\mathbf{K}_{(n)}$ is the Kalman gain. The Time Update Equations are responsible for projecting forward (in time) the current estimates for the next time step. The Measurement Update Equations are responsible for updating the error covariance estimates to obtain the a posteriori estimates. The Time Update Equations can also be thought of a predictor equations, while the Measurement Update Equations can be thought of a corrector equations.

**B. Mean Square Error (MSE) Analysis**

The error between the $l$th estimated polynomial $\hat{\alpha}_{(l,r,t)}^{(n)}$ and the $l$th exact complex gain and the $l$th estimated polynomial $\hat{\alpha}_{(l,r,t)}^{(n)}$ is given by:

\[ \mathbf{e}_{(l,r,t)}^{(n)} = \alpha_{(l,r,t)}^{(n)} - \hat{\alpha}_{(l,r,t)}^{(n)} = \xi_{(l,r,t)}^{(n)} + \mathbf{Q} \mathbf{e}_{(l,r,t)}^{(n)} \] (27)

where $\mathbf{e}_{(l,r,t)}^{(n)} = \alpha_{(l,r,t)}^{(n)} - \hat{\alpha}_{(l,r,t)}^{(n)}$ and $\mathbf{Q}$ is the BEM model error defined in (7). Neglecting the cross-covariance terms between $\xi_{(l,r,t)}^{(n)}$ and $\mathbf{e}_{(l,r,t)}^{(n)}$, the mean square error (MSE) between $\alpha_{(l,r,t)}^{(n)}$ and $\hat{\alpha}_{(l,r,t)}^{(n)}$ is given by:

\[ \text{MSE}_{\xi_{(l,r,t)}} = \frac{1}{V} \mathbf{E} \left[ \mathbf{e}_{(l,r,t)}^{(n)^{H}} \mathbf{e}_{(l,r,t)}^{(n)} \right] \]
\[ = \text{MMSE}_{\xi_{(l,r,t)}} + \frac{1}{V} \text{Tr} \left( \mathbf{Q} \text{MSE}_{\xi_{(l,r,t)}} \mathbf{Q}^{H} \right) \] (28)

provided that the data symbols are perfectly estimated (i.e., data-aided context), where the sequence of indices $f(l, r, t) = 1 + (l - 1 + a_{(r,t)} N_c : (l + a_{(r,t)}) N_c$ with $a_{(r,t)} = \sum_{r'=1}^{N_{r}} \sum_{t'=1}^{N_{t}} L(r', t')$.

The on-line Bayesian Cramer-Rao Bound (BCRB) is an important criterion for evaluating the quality of the complex gains Kalman estimation. The on-line BCRB for the estimation of $\alpha_{(l,r,t)}^{(n)}$ in data-aided (DA) context, is given by:

\[ \text{BCRB}^{(c)}_{(l,r,t)} = \text{MMSE}_{\xi_{(l,r,t)}} + \frac{1}{V} \text{Tr} \left( \text{QBCRB}_{(l,r,t)}^{(c)} \mathbf{Q}^{H} \right) \] (30)

where $\text{BCRB}^{(c)}_{(l,r,t)}$ is the on-line BCRB associated to the estimation of $\mathbf{c}_{(l,r,t)}^{(K)}$ which is given by:

\[ \text{BCRB}^{(c)}_{(l,r,t)} = \text{BCRB}^{(c)}_{f(l,r,t), f(l,r,t)} \] (31)

\[ \text{BCRB}^{(c)}_{(l,r,t)} = \left\{ \text{blmdiag} \left( \mathbf{J}_{(1)}^{T}, \mathbf{J}_{(2)}^{T}, \mathbf{J}_{(3)}^{T}, \ldots \right) \right\}^{-1} \] (32)

where $\mathbf{R}_{e}$ is calculated in the same way as $\mathbf{P}_{(000)}$ with $s, s' \in [0, K - 1]$, and $\mathbf{J}_{(n)}$ is a $LN_{c} \times LN_{c}$ matrix given by [5]:

\[ \mathbf{J}_{(n)} = \text{blmdiag} \left( \mathbf{J}_{(1,n)}^{T}, \mathbf{J}_{(2,n)}^{T}, \mathbf{J}_{(3,n)}^{T}, \ldots \right) \] (33)

\[ \mathbf{J}_{(r,n)} = \left[ \mathbf{J}_{(r,1,n)} \ldots \mathbf{J}_{(r,N_{r},n)} \right] \] (34)

\[ \mathbf{J}_{(r,t,n)} = \left[ \mathbf{J}_{(r,1,t,n)} \ldots \mathbf{J}_{(r,N_{t},t,n)} \right] \] (35)

IV. JOINT DATA DETECTION AND KALMAN ESTIMATION

**A. Iterative Algorithm**

In the iterative algorithm for joint data detection and channel Kalman estimation, the $N_{p}$ pilots subcarriers are evently inserted into the $N$ subcarriers at the positions $\mathbf{P} = \{ p_{1}, \ldots, p_{N_{p}} \} = \{r - 1 \} L_{f} + 1$, $r = 1, \ldots, N_{p}$, where $L_{f}$ is the distance between two adjacent pilots. We test two channel equalizers for the data detection: the MMSE equalizer [13] and the QR-equalizer [8] [7]. The first one is based on the noise variance $N_{T} \cdot \sigma^{2}$ whereas the second equalizer allows us to estimate the data symbol with free ICI by performing a so-called QR-decomposition. The algorithm proceeds as follows:

**initialization:**

- $\mathbf{g}_{(0)} = 0_{p \cdot LN_{c}, 1}$
- compute $\mathbf{P}_{(000)}$ as (24)
- $n \leftarrow n + 1$
- execute the Time Update Equations of Kalman filter (25)
- compute the channel matrix using (14)
- $i \leftarrow 1$

**recursion:**

1) remove the pilot ICI from the received data subcarriers
2) Detection of data symbols
3) execute the Measurement Update Equations of Kalman filter (26)
4) compute the channel matrix using (14)
5) $i \leftarrow i + 1$

where $i$ represents the iteration number.
normalized 4QAM MIMO-OFDM system, with two transmit
and the receivers are very close to each other in practice. A
given in [8] [5]. It is reasonable because the transmitters
have the same delay and fading property (i.e.,
from different transmitters to the different receivers have the
availability of delay information). We assume that the chan-
nels are approximated by a BEM model. The BEM coefficients
are tracked and estimated using the Kalman filter. The data
variation of the channel within one MIMO-OFDM block
are very close, particularly in regions of
low and moderate SNR. At very high
SNR, it is normal to not reach the reference because we have a small error floor due
to the data symbol detection error. It would be noted that the
curve floor with QR-equalizer is larger than that of the MMSE
equalizer.

In this section, we verify the theory by simulation and
test the performance of the iterative algorithm based on the
second approach (i.e., estimation of α assuming the
availability of delay information). We assume that the
channels from different transmitters to the different receivers have the
same delay and fading property (i.e., same number of paths,
\(σ^2_{d(0,t,c)}\) and \(τ(t,r,t)\)) and we use the Rayleigh channel model
given in [8] [5]. It is reasonable because the transmitters
and the receivers are very close to each other in practice. A
normalized 4QAM MIMO-OFDM system, with two transmit
and two receive antennas, \(N = 256\) subcarriers, \(N_p = \frac{N}{4}\),
\(N_p = \frac{N}{4}\) pilots (i.e., \(L_f = 4\)) and \(\frac{1}{T} = 2\) MHz is used (note
that SNR = \(\frac{1}{T}\)). The MSE and the BER are evaluated under a
rapid time-varying channel such as \(f_dT = 0.5\) corresponding to
a vehicle speed \(V_m = 400 km/h\) for \(f_c = 10\) GHz. In order
to decrease the complexity of the Kalman filter, we choose an
AR model of order \(p = 1\).

Fig. 1 and Fig. 2 show respectively the evolution of MSE and BER versus SNR, with the iterations, for \(f_dT = 0.5\) and GCE-BEM. We select \(N_c = 4\) in order to reduce the impact of the BEM modeling error. The equalizers used for data detection are: the MMSE equalizer [13] and the QR-equalizer [8] [7]. From Fig. 1, it is shown that, with DA, the MSE obtained by simulation agrees with the theoretical value of MSE given by (29). We also observe that MSE with DA is very close to the on-line BCRB. This means that the Kalman filter works very well. After three and nine iterations, a great improvement is realized and the MSE is close to the

\[ M = \begin{bmatrix} M^{H}_{(1)}M_{(1)} & \cdots & M^{H}_{(1)}M_{(N_c)} \\ \vdots & \ddots & \vdots \\ M^{H}_{(N_c)}M_{(1)} & \cdots & M^{H}_{(N_c)}M_{(N_c)} \end{bmatrix} \]

where the \(N N_c \times N_c\) matrix \(F_{(t,r,t,n)}\) is given by:

\[ F_{(t,r,t,n)} = \text{blkdiag} \{ \text{diag}(x_{(t,n)})f_{t(r,t)}, \ldots, \text{diag}(x_{(t,n)})f_{t(r,t)} \} \]

where \(M_{(1)} = N N_c \times N_c\) and \(GCE-BEM\) matrices, respectively, defined as:

\[ M = \begin{bmatrix} M^{H}_{(1)}M_{(1)} & \cdots & M^{H}_{(1)}M_{(N_c)} \\ \vdots & \ddots & \vdots \\ M^{H}_{(N_c)}M_{(1)} & \cdots & M^{H}_{(N_c)}M_{(N_c)} \end{bmatrix} \]

The MSE with DA, particularly in regions of moderate and high
SNR. In Fig. 2, we plotted, as reference, the performance
obtained with perfect channel knowledge. After three and nine
iterations, a significant improvement occurs; the performa-
nce of our algorithm and the performance obtained with perfect
channel knowledge are very close, particularly in regions of
low and moderate SNR. At very high SNR, it is normal to not
reach the reference because we have a small error floor due
to the data symbol detection error. It would be noted that the
error floor with QR-equalizer is larger than that of the MMSE
equalizer.

VI. CONCLUSION

In this paper, we have presented a new iterative algorithm for
joint multi-path Rayleigh channel estimation and data recovery in fast time-varying environments. The rapid time-
variation of the channel within one MIMO-OFDM block are
approximated by a BEM model. The BEM coefficients
are tracked and estimated using the Kalman filter. The data
symbols are estimated by performing a QR-decomposition
of the channel matrix or by using a MMSE equalizer. Theoretical
analysis and simulation results show that our algorithm has a
good performance for high Doppler spread.

VII. ACKNOWLEDGEMENT

This work has been carried out in the framework of the
CISIS (Campus International sur la Sécurité et Intermodalité
des Transports) project and funded by the French Ministry of
Research, the Region Nord Pas de Calais and the European
Commission (FEDER funds)

REFERENCES

2005.
2002.
Estimation via Probabilistic Data Association Based TOAs ” in IEEE


