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Optimal Highway Maintenance Policies under Uncertainty

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Key Words: maintenance optimization, gamma process, Markov semi-renewal

SUMMARY & CONCLUSIONS

We develop an inspection and maintenance policy to minimize the cost of maintaining a given section of road or highway when there is a great deal of uncertainty in the degradation process. We propose to model the degradation of a section of road based on the proliferation and growth of cracks. We utilize a combination of a Poisson and gamma process to account for the tremendous amount of uncertainty and difficulty in predicting the proliferation of cracks. Our policy defines the optimal inspection interval as well as the minimum threshold at which to perform crack repairs. Furthermore, our policy contains a safety constraint to prevent the probability of a “catastrophic” failure from exceeding a pre-determined reliability value. Numerical calculations have shown that our model will extend the lifecycle of the road by performing preventive, conditioned-based maintenance to slow down the growth of cracks. Classical preventive maintenance policies usually shorten the lifecycle by forcing earlier renewals.

1 INTRODUCTION

In the year 2000, the United States had almost US$31 billion in highway maintenance expenditures [1]. In order to devise an effective maintenance/rehabilitation strategy, an accurate model for the degradation of roads is needed. Cracks, specifically longitudinal wheel-path cracks in the pavement, are the primary cause for road maintenance. Myers et al. [2] reported that over 90% of road sections in Florida that are in need of repair have such cracks. The proliferation of a crack in asphalt concrete operates as follows. The traffic load applied to the pavement surface generates tensile stresses at the bottom of the asphalt layer. For new pavement, the tensile strength of the asphalt concrete initially exceeds the stress applied by the traffic load, and thus will remain free of cracks for a period of time. However, as the tensile strength of the road begins to degrade, fatigue cracks will initiate at weak spots at the bottom of the asphalt layer and over time these cracks will propagate through the asphalt layer to the surface. The size of the crack will eventually reach some unacceptable level at which maintenance and/or rehabilitation becomes necessary. As cracks increase in size, contaminants are able to more easily infiltrate the road and disrupt the bond of pavement materials if they are not properly sealed, thus leading to further degradation.

We propose to develop an accurate model of road degradation using longitudinal surface cracks. We decompose the cracking process into two phases: 1) the crack initiation and 2) the crack propagation. Based on this degradation model we will formulate an optimal policy for highway inspection in conjunction with maintenance and rehabilitation actions.

2 MOTIVATION

The development of mathematical models for road maintenance became prevalent with the Pavement Management System (PMS) developed by Golbai et al. [3] for the Arizona Department of Transportation (ADODT) and Woodward-Clyde Consultants. The PMS contains a Network Optimization System (NOS) which consists of two Markov decision models. Wang and Zaniewski [4] report that the PMS achieved dramatic cost savings at the ADODT and has gone on to be implemented in various forms by other highway agencies.

Markov processes make up the majority of stochastic modeling approaches for pavement maintenance/rehabilitation [5]. The underlying degradation model of the Markov process is obtained either through statistical analysis of pavement data or some representative model. The greatest challenge in this approach is that an accurate degradation model is needed to construct the transition probability matrix of the Markov chain. Specifically a probability associated with each state transition.

Many degradation models also deal with the propagation of cracks in a wide variety of settings and are largely based on the Paris and Erdogan [6] equation. The transition probability matrix approach may be utilized with either the general or crack-specific approach, while the Paris and Erdogan (1963) equation applies specifically to propagation of cracks as a function of applied stress.

Wu and Ni [7] found that a stationary Markov chain model with a transition probability matrix based on statistical data does not accurately describe a crack growth process. They also note that while the Markov chain approach is often criticized for its accuracy, Yang’s model [5] and the polynomial model have parameters that are difficult to determine and must be obtained from a large sample of crack growth measurements and observations.

There is a tremendous amount of uncertainty in degradation models due to the variability in material properties, environmental conditions, and traffic patterns [5].
Due to the difficulty in estimating the underlying degradation of the Markov chain approach and parameters for the Paris and Erdogan equation we propose the use of gamma processes to model the crack growth. Gamma processes have recently received a tremendous amount of attention in the reliability and maintainability literature as means to the model degradation of many civil engineering structures under uncertainty [8].

The use of a gamma process to model crack growth in conjunction with a Poisson for the appearance of a crack was first proposed by van Noortwijk et al. [9] for the modeling of scour holes of the Eastern-Scheldt barrier in the Netherlands. This work was then extended by van Noortwijk and Klatter [10] to allow the scour erosion to be decreasing rather than constant. We propose to further extend this model by applying it to road maintenance. We model the appearance of a crack in the road as a Poisson process and the propagation of the crack as a gamma process. The two previous models assumed that only one action was possible upon detection of a scour hole. No decision had to be made; a repair was automatically initiated. We allow for three possible actions: 1) do nothing, 2) maintenance: repair the crack, or 3) rehabilitation: completely resurface the section of road. Furthermore, we implement a safety constraint to ensure that the state of the road does not fall below a pre-determined threshold with a given reliability.

The evaluation of the size of a crack is a valuable metric for determining the optimal maintenance and rehabilitation strategy for roads [11]. The primary method used by most state departments of transportation in the United States to evaluate crack depths in pavement is core sampling. This method is time-consuming, costly, and destructive. Nondestructive techniques such as impact-echo or ultrasound do exist [12, 13], however they are designed for use on concrete and not suitable for the asphalt pavement of roads. However, longitudinal surface cracks may be inspected visually and accurately without causing further damage to the road itself. It can also be assumed that a strong correlation exists between the length and the depth of the crack. The objective of this paper is to formulate an optimal inspection and maintenance/rehabilitation policy based on cracks in the road.

3 MODEL ASSUMPTIONS

Surface cracks can be detected via visual inspection. As previously mentioned, this has the advantage of being both non-destructive and extremely accurate. For our model, inspection is assumed to occur in a constant, periodic manner where the detection and measurement of a crack are assumed to be perfect. We denote the length of equivalent inspection intervals as \( \lambda \). A periodic inspection policy is chosen for practical purposes so transportation departments can effectively budget and schedule the funds and manpower necessary. Inspection is also assumed to occur instantaneously. In practice, a special vehicle outfitted with a video camera films the road so that the actual human visual assessment of the road can occur in a lab. This prevents road closures and allows for more accurate measurements than in the field. For our cost model formulation we let \( c_i \) represent the fixed cost for inspecting a given length of road.

Let \( N \) be the number of cracks that appear at the end of the \( t_i \) period since the last inspection. Further, consider some crack \( j \in \{1, 2, \ldots, N_i \} \) where the length of the crack \( j \) at time \( t \). If \( 0 \) is denoted as \( L(t) \) where \( l(0) = 0 \) \( \ell \) denotes a specific instance of the random variable \( L_j \). The arrival times \( (T_1, T_2, \ldots) \) of initiated cracks are assumed to follow a Poisson process with intensity parameter \( \lambda \) and thus be exchangeable and memoryless. While we recognize that the initiation of cracks overtime is most likely a non-stationary process, the stationary assumption is made to facilitate parameter estimation in practice. Once a crack has been initiated, its length propagation is modeled by a gamma process with shape function \( \beta > 0 \) and scale parameter \( \alpha > 0 \). This process denotes a constant crack growth with a linear trend. The initiation and growth process of a crack is shown in Figure 1.

![Figure 1 – Crack Initiation and Growth Process](image)

4 PROBLEM STATEMENT

As mentioned above, there are three possible actions after inspection occurs. It must first be decided whether to “do nothing” or take some action (i.e., maintenance or rehabilitation). No action will be taken during a given inspection period if the total length of all observed cracks is less than the decision variable \( \gamma \). We must optimize \( \gamma \) in order to determine the maximum threshold in which it is optimal to take no action with the road. Therefore, the “do nothing” action is taken if:

\[
\sum_{j=1}^{N_i} L_j(t) < \gamma.
\]

If the decision is made that some action must be taken, then an evaluation must be made as to whether to maintain or rehabilitate the road. A rehabilitation action is defined as the complete resurfacing of the road to an as-good-as-new condition. Thus, rehabilitation is a complete renewal point. A maintenance action is defined as repairing each crack on an individual basis to an as-good-as-new condition. For our model, this is a semi-renewal point as we will be tracking the total number of repaired cracks as an indicator for when a rehabilitation action is needed.

Repairs, if carried out, are assumed to occur instantaneously and immediately after their detection and be perfect in nature. Let the cost of maintenance at the end of the \( t_i \) time period be \( c_{m}(t) \) with both a fixed and a linear variable cost component denoted \( c_i \) and \( c_{v} \) respectively. Their
relationship is defined as follows:
\[ C_m(t) = c_f + c_r \sum_{j=1}^{N_i} L_j(t). \] (2)

The cost of rehabilitation, denoted \( c_r \), is a single fixed cost per section of road under consideration and leads to a complete renewal of the process. Furthermore, it is assumed that \( c_r \gg c_m \), such that \( c_r + c_m \approx c_r \). Stipulating that the cost of rehabilitation is much larger than the cost to repair individual cracks ensures our model will not choose to preemptively rehabilitate and also reflects the real-world cost, man-power, and political problems associated with shutting down and resurfacing a section of road or highway. A rehabilitation action is considered to occur immediately upon recognition of the need and be instantaneous. The immediate and instantaneous assumption of the maintenance and rehabilitation actions is justified due to the relatively small amount of time required to carry out these actions in comparison to the total life of the road and the length of the inspection intervals. For quality and safety reasons, a rehabilitation action is deemed necessary when the total length of all cracks previously maintained, \( L_m \), plus the total length of cracks in need of maintenance exceeds a threshold \( l_r \). \( l_r \) is a given parameter that will vary depending on the length of the section of road under consideration and the quality standards of a particular area. This is necessary to prevent the road from becoming a complete patchwork of repaired cracks. It is a value set by considering a number of different points of view such as engineering, management, cost control, local government priorities, and how much past repair is deemed tolerable on a given section of road. Thus, rehabilitation will occur if the following is true
\[ L_m + \sum_{j=1}^{N_i} L_j(t) > l_r. \] (3)

Finally, we allow for a “safety rule” to ensure that our optimization model does not allow the probability of a given total length of all cracks in a section of road to exceed a specified safety threshold within an inspection interval. This requires two parameters: 1) the maximum total length of all cracks allowed at which point the road would be unsafe or unusable, denoted \( l_s \), and 2) the maximum allowable probability that the total length of all cracks will exceed the safety threshold \( l_s \). This maximum allowable probability is denoted \( p_s \). This safety rule is given by
\[ \Pr \left[ \sum_{j=1}^{N_i} L_j(t) > l_s \right] < p_s. \] (4)

The safety constraint also ensures that in our cost optimization model, the “do nothing” action is not taken at every inspection interval.

The possible actions in our model are summarized in Figure 2 based on a hypothetical scenario that begins with a brand new road. The first inspection occurs at time \( \tau \). Since the road is new, there are no previously repaired cracks and thus \( L_r(\tau) = 0 \) and no rehabilitation action is taken. At end of \( \tau \), two cracks have formed, however, they are less than the repair threshold \( \tau \), so no maintenance will occur either, leaving managers to “do nothing.” At the end of \( 2\tau \), the two initial cracks have continued to grow and a third has formed. There are still no previously repaired cracks and the total length of the three cracks does not exceed \( l_r \), so no rehabilitation action is performed. However, the total length of all three cracks now exceeds \( \tau \) and so a maintenance action is taken and the cracks are repaired. At the end of \( 3\tau \), two new cracks have formed. The previous cracks do not appear because they have been repaired. The total length of the two new cracks plus the total length of all previously repaired cracks do not yet exceed \( l_r \), so no rehabilitation action is performed. The two new cracks do exceed \( \tau \) so a repair action is taken. Finally at the end of the 4th inspection cycle, three more cracks have formed pushing the total length of all existing cracks plus the total length of all previous repaired cracks over \( l_r \), and the rehabilitation action is taken to completely renew the road.

\[\text{Figure 2 – Summary of Actions}\]

The goal of the maintenance decision model is to determine the optimal inspection interval \( \tau \), and the optimum maintenance threshold \( \tau \) that minimizes the expected average cost per time period given by
\[ \lim_{t \to \infty} \frac{C(t)}{t} = \frac{E_{\tau}(C(\tau))}{\tau} = c_i + c_f P_m + c_r E(X(\tau)) + c_e P_R \] (5)

where \( E(\cdot) \) is the expectation defined by a Markov renewal cycle, \( P_m \) and \( P_R \) are the long-run probabilities associated with performing maintenance and rehabilitation, respectively, and \( E(X(\cdot)) \) is the expected value of the cumulative length of all cracks at time \( \tau \). The derivation of these values is discussed in the next section.

5 DETERMINING THE STATIONARY LAW

We propose to solve our model by making use of Markov semi-renewal theory which allows us to only consider the semi-renewal period in evaluating the optimal infinite horizon solution. We define an inspection point (\( \tau, 2\tau, \ldots \)) as a semi-renewal point of the Markov process. It is not a complete renewal as we are tracking the amount of crack repairs. We must determine the stationary law of the semi-renewal interval \( \tau \). The stationary law dictates that the long-run probability distribution that governs the process for any time \( \tau \) in one inspection interval is equivalent for all other inspection intervals. We begin by defining the state of the system at time \( t, t \in (0, \tau) \):
The initial state of the system in Markov semi-renewal theory can be specified based upon our knowledge of the system at a defined time \( t = 0 \). The stochastic processes being applied in our model allow for estimating the probabilities of system states before and after inspection and subsequent to any repairs or rehabilitation actions at any time \( t \), based upon our specification of the initial state. The evolution of the system between 0 and \( t \) is referred to as \( f(x,n,x_0,n_0) \) where \( x_0 \) is the initial cumulative length of all unrepaired cracks at the beginning of the inspection cycle and \( n_0 \) is the initial number of cracks.

We find that the expected length of crack growth in a given inspection period \( \bullet \) given an initial number of cracks \( n_0 \), and initial cumulative growth length \( x_0 \) is given by

\[
E(X(t)) = x_0 + \frac{n_0 \alpha}{\beta} \left( \frac{\alpha}{\beta} \right) t + \frac{\alpha \beta^2}{2} t^2.
\]  

It should be noted that \( x_0 \in [0, y) \), \( n_0 \in N \), and \( y_0 < l \).

To define the density function for multiple cracks, we introduce the function \( f^n(x) \) that represents the nth convolution of the function \( f(x) \). Thus, \( \forall n \in N \), the density function is given by

\[
f_f(x,n) = \left[ \text{Ga}(\bullet | n_0 \alpha, \beta) \right] \left[ \sum_{n_0=0}^{n} \binom{n}{n_0} \left( \frac{\alpha}{\beta} \right)^{n-n_0} \left( \frac{\beta}{\beta} \right)^{n-n_0} e^{-\beta t} \right]
\]

where \( \text{Ga}(\bullet | \alpha, \beta) \) denotes the density function of the gamma distribution and

\[
f_f(x)(t) = \frac{e^{-\beta x}}{x^\beta \Gamma(\alpha)}
\]

Finally, we define the stationary law in two parts. Let \( I \) denote a binary variable indicating whether a repair has been made since the last rehabilitation: \( I_{1} = 0 \) (no repairs), \( I_{1, \ldots, k} \) (repairs have been made). The stationary law is denoted as \( \pi(x, y, z) \) which represents the long-run probability of being in state \( (x, y, z) \). It is given by

\[
\pi(x,y,z) = I_{\{y=0\}} \left[ \sum_{n_0=0}^{\min(y)} \left( \sum_{n_0=0}^{\min(y)} \binom{n}{n_0} \pi(x_0,n_0) f_{x,n,x_0,n_0} dx_0 \right) \right] + I_{\{y<z\lim\}} \left[ \sum_{n_0=1}^{\min(y,z)} \left( \sum_{n_0=1}^{\min(y,z)} \pi(x_0,n_0,y) f_{x,n,x_0,n_0} dx_0 \right) \right]
\]

where \( P_s \) is given by

\[
P_s = \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \pi(x,n,0) dx + \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \pi(x,n,y) dx dy
\]

Recall that \( P_s \) is the long-run probability of performing a rehabilitation action. Thus, using the stationary law, we can compute the probability of performing a maintenance action as

\[
P_s = \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \pi(x,n,0) dx + \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \pi(x,n,y) dx dy
\]

We implement a 4-step algorithm for computing the stationary law beginning with an arbitrary value of \( P_s \). The algorithm operates as follows:

1. Determine \( \hat{\pi}(0,0,0) \).
2. Iteratively evaluate \( \hat{\pi}(x,n,0) \) for all \( x > 0, n \).
3. Evaluate \( \hat{\pi}(x,n,y) \) for all \( x > 0, n, y > 0 \).
4. Solve for \( \pi(x,n,y) \) by normalizing \( \hat{\pi}(x,n,y) \) i.e.,

\[
\pi(x,n,y) = \frac{\hat{\pi}(x,n,y)}{P_{\text{norm}}}
\]

where

\[
P_{\text{norm}} = \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \hat{\pi}(x,n,0) dx
\]

\[+ \int_{y=0}^{\infty} \sum_{n=1}^{\infty} \hat{\pi}(x,n,y) dx dy + \int_{y=0}^{\infty} \hat{\pi}(0,0,0) dy\]

\( \hat{\pi}(0,0,0) \) is an estimate of \( \pi(x,n,y) \) based on an arbitrary value of \( P_s \) and is thus not a true density function as the probabilities do not sum to 1. It is normalized by \( P_{\text{norm}} \) which will cancel out \( P_s \) and leave the true value of \( \pi(x,n,y) \). A three-dimensional plot of the stationary law for a given set of parameters is shown in Figure 3. The \( x \) and \( y \) axes represent the \( x \) and \( y \) variables, respectively. The \( z \)-axis represents \( \sum_{n=1}^{\infty} \pi(x,n,y) \).

In Figure 3 the semi-renewals are clearly seen as the value of \( y \) cycles through multiples of \( y \), in this instance \( y = 10 \). The initial cycle after a rehabilitation has the highest probability of occurrence and each cycle after has a decreasing probability of occurrence due to the increase variability in the combinations of \( x \) and \( y \) values.

From the stationary law, the optimal policy is found by searching for the minimum cost over an iteration of potential \( \bullet \) and \( \bullet \) values. The cost function (5) is a convex function of these parameters and the algorithm may be terminated once the cost begins to increase (i.e., if the iteration starts from \( \bullet = 0 \) and \( \bullet = 0 \), the cost function will decrease until the optimum value is reached, after which the function will increase).

### 6 Numerical Example

Consider the following parameters for road degradation, maintenance costs and the limit on the maximum allowable repaired cracks:

\[\bullet = 0.1 \quad \bullet = 3 \quad \bullet = 10 \quad l = 60 \quad c_i = 10 \quad c_i = 10 \quad c_i = 5 \quad c_i = 1000\]
Figure 3 – Density Function of the Stationary Law

The plot in Figure 4 demonstrates the convexity of the cost function as a function of $\gamma$ for a fixed value of $\gamma = 10$.

Due to the convexity one knows they have reach the minimum cost as the cost curve will continue to decrease as a function of increasing $\gamma$ values until the minimum is obtained and subsequent values of $\gamma$ will cause the cost function to increase.

7 DISCUSSION

In this paper we have formulated an optimal inspection and maintenance policy for highways based on the state of the road. We are able to formulate this policy under conditions of great uncertainty as only 3 parameters are required for the degradation model, namely $\lambda$, $\alpha$ and $\beta$. This is a drastic reduction compared to other statistical and Markov chain approaches in the literature which require a tremendous amount of data for the statistical models. In practice one can easily estimate values of $\lambda$ based on rate of appearance of cracks. $\alpha$ and $\beta$ can also be easily be found and adjusted for various highway material compositions, traffic patterns, and environmental conditions without having to collect a large amount of data. Our policy is very easy to implement in practical settings as it gives maintenance managers the periodic inspection interval $\tau$ and clear guidelines on whether to "do nothing", repair, or rehabilitate after each inspection. Solving for the optimal policy can be done quickly as the equations and algorithm have been coded to run on a computer.

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BIOGRAPHIES

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