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Decision-Directed Channel Estimation and High I/Q Imbalance Compensation in OFDM Receivers

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Abstract—Direct-conversion architectures suffer from the mismatch between the In-phase (I) and the Quadrature-phase (Q) branches, commonly called I/Q imbalance. Even low I/Q imbalances imply poor performance of Orthogonal Frequency Division Multiplexing (OFDM) systems. In this paper, we propose a new algorithm that uses both training and data symbols in a decision-directed fashion to jointly estimate the channel and compensate for high receiver I/Q imbalance. Simulation results show that our method can compensate for high I/Q imbalance values and also estimate a frequency selective channel.

Index Terms—I/Q imbalance, Orthogonal Frequency Division Multiplexing (OFDM), zero-IF, equalization.

I. INTRODUCTION

DIRECT-CONVERSION architectures, also called zero-IF architectures, are good candidates for producing low-complexity OFDM receivers. The receiver I/Q imbalance resulting from imperfect matching of the analog components in the In-phase (I) and Quadrature-phase (Q) branches drastically degrades the performance of OFDM systems [1]. In order to design a high performance receiver, it is necessary to estimate and compensate for the I/Q imbalance. A very promising approach is digital compensation. The I/Q imbalance estimation problem has been addressed in the literature, and there are two main approaches. The first one is based on the received training symbols. Tubbax et al. [1] and Yu et al. [2] use the fact that the channel frequency response estimation has several sharp transitions in the presence of I/Q imbalance. Their technique estimates the I/Q imbalance when the frequency channel response on adjacent tones is highly correlated and the I/Q imbalance is low. Tarighat et al. [3] and Schuchert et al. [4] propose an estimation technique based on a least-mean-square filtering that converges slowly and requires a high number of training symbols. Tarighat et al. [3] and Windisch et al. [5] propose a technique that can estimate the I/Q imbalance by inserting null tones in the training symbols. Thus, the received training symbols are not affected by mutual interference between pairs of symmetric subcarriers. This technique gives an estimate of the I/Q imbalance using a single OFDM training symbol, but it requires the modification of the training symbols. The second approach is based on data symbols only. A blind estimation is proposed by Windisch et al. [6]: it uses the fact that the transmitted subcarriers are uncorrelated. This method needs a high number of received data symbols for a good statistical estimation of the I/Q imbalance. If the number of data symbols is not sufficient, an error floor occurs even for high signal-to-noise ratios.

In this paper we are interested in the case where: (i) the adjacent subcarriers’ correlation is low corresponding to a dense multipath environment; (ii) there are few training and data symbols; (iii) the I/Q imbalance can be strong; (iv) the data subcarriers carry high-order modulations and are very sensitive to I/Q imbalance. In these conditions the estimation methods presented above cannot be used. We propose to jointly estimate and compensate for high I/Q imbalance and to estimate and equalize the channel frequency response. Our method can be used for any OFDM system, and converges very quickly as it needs only few training and data symbols. It is original in the sense that it uses both the training and data received symbols in a decision-directed fashion. The remainder of the paper is organized as follows: Section II introduces the I/Q imbalance modeling in OFDM systems. Section III presents the proposed method in the case of the reception of two different training symbols. Section IV deals with the more general case of the reception of a single training symbol, and describes an iterative algorithm using both the training symbol and the data symbols. Finally Section V presents the choice of the parameters setting of our algorithm and the experimental results.

II. I/Q IMBALANCE MODELING IN OFDM SYSTEMS

The I/Q imbalance is caused by a gain mismatch $g$ and a phase mismatch $\phi$. The gain mismatch is due to a difference of the overall gain between the I and Q branches. The phase mismatch is due to a non-ideal layout, i.e., the fact that the lines between the mixer and the local oscillator of the I and Q branches are not exactly of the same length. Let $K_1 \triangleq (1 + g e^{-j\phi})/2$ and $K_2 \triangleq (1 - g e^{+j\phi})/2$ be the I/Q imbalance parameters as defined in [6]. For an OFDM system impaired by the receiver I/Q imbalance, the received signal on the $k^{th}$ subcarrier during the $n^{th}$ symbol $R_k(n)$ can be written as [6]:

$$R_k(n) = T_k(n)H_kK_1 + T^*_k(n)H^*_kK_2 = T_k(n)\alpha_k + T^*_k(n)\beta_k,$$

where $\alpha_k \triangleq H_kK_1$ and $\beta_k \triangleq H^*_kK_2$. $T_k(n)$, $H_k$ and $N_{DFT}$ denote respectively the transmitted symbols, the channel fre-
frequency response not affected by I/Q imbalance, and the block size of the Discrete Fourier Transform. The matrix form of the received symbols defined by equation (1) is:

\[
\begin{bmatrix}
R_k(n) \\
R^*_{-k}(n)
\end{bmatrix} =
\begin{bmatrix}
K_1 & K_2 \\
K_2^* & K_1^*
\end{bmatrix}
\begin{bmatrix}
H_k & 0 \\
0 & H^*_{-k}
\end{bmatrix}
\begin{bmatrix}
T_k(n) \\
T^*_{-k}(n)
\end{bmatrix},
\]

(2)

Note that our model does not take into account the unavoidable frequency offset between the transmitter/receiver local oscillators. The estimation of the frequency offset in the presence of I/Q imbalance can be performed using the method proposed in [7] for example. Thus we assume a pre-compensation at the transmitter or an analog pre-compensation of this effect at the receiver.

### III. TWO DIFFERENT LONG TRAINING SYMBOLS

In this section, we show that when two different Long Training Symbols (LTS) are transmitted at the beginning of each packet, it is possible to estimate the I/Q imbalance and thus to jointly estimate the channel and compensate for the I/Q imbalance. The channel estimate \( C_k \) is obtained by dividing the received LTS \( (P_{R_k}) \) by the transmitted LTS \( (P_{T_k}) \) that are known to the receiver. Note that \( P_{R_k} \) and \( P_{T_k} \) correspond respectively to \( R_k \) and \( T_k \) in the system model given by equation (1). When two different LTS are transmitted consecutively, the two channel estimations \( C_k(1) \) and \( C_k(2) \) affected by I/Q imbalance form a system of linear equations in two variables:

\[
\begin{align*}
C_k(1) &= \frac{P_{R_k}(1)}{P_{T_k}(1)} = \alpha_k + L_k(1)\beta_k, \\
C_k(2) &= \frac{P_{R_k}(2)}{P_{T_k}(2)} = \alpha_k + L_k(2)\beta_k,
\end{align*}
\]

(3)

where \( L_k(1) = \frac{P_{T_{-k}}(1)}{P_{T_k}(1)} \) and \( L_k(2) = \frac{P_{T_{-k}}(2)}{P_{T_k}(2)} \) are known to the receiver. If the condition \( C \) defined by \( L_k(1) = L_k(2) \) is satisfied, the system (3) can be solved. By definition \( K_1 + K_2^* = 1 \), and knowing \( \alpha_k \) and \( \beta_k \) (or their estimate), the channel frequency response is given by:

\[
H_k = \alpha_k + \beta^*_k.
\]

(4)

Since \( \alpha_k = K_1 H_k \), equation (4) yields to the following I/Q imbalance parameters:

\[
\begin{align*}
K_1 &= \frac{\alpha_k}{\alpha_k + \beta^*_k}, \\
K_2 &= 1 - K_1^*.
\end{align*}
\]

(5)

Once the I/Q parameters and the channel frequency response have been estimated, the data symbols are compensated for I/Q imbalance inverting matrix \( A_1 \) in equation (2), and the channel equalization can be performed.

### IV. A SINGLE LONG TRAINING SYMBOL

In many OFDM standards the same LTS is sent twice [8]. In that case the condition \( C \) is not satisfied, and it is not possible to apply the method described in Section III. In this Section we propose a new method for joint compensation of the I/Q imbalance and estimation of the channel frequency response using only a single LTS and data symbols. Note that if known pilots subcarriers are inserted in the data symbols satisfying the criteria \( C \), they can be used as a second LTS as defined in Section III. Since pilot subcarriers do not always satisfy the condition \( C \) and with the aim of reducing the number of pilots, in the sequel we only consider data subcarriers. The proposed algorithm is iterative. In first subsection we present the algorithm with a single iteration, and in the second sub-section we give the general case of \( i, i > 1 \), iterations.

#### A. Algorithm with a single iteration

We now present the 4 steps of our algorithm.

1) **Hard decisions on \( N \) received data symbols:** Even if the I/Q imbalance is pronounced, the \( N \) received symbols \( R_k(n) \) equalized by the channel estimation \( C_k \) are good approximations of \( T_k(n) \). We denote the hard decisions of these equalized symbols as \( D[R_k(n)/C_k] \).

2) **Estimation of the I/Q imbalance parameters \( K_1 \) and \( K_2 \):** If \( T_k(n) \) and \( T^*_{-k}(n) \) are replaced by the previous hard decisions in equation (1), we obtain an equation equivalent to the second received LTS, as described in Section III. Let \( \zeta \) be a set of all subcarriers satisfying \( D[R_k(n)/C_k] \neq D[R^*_{-k}(n)/C^*_{-k}] \). For each subcarrier \( k \) in \( \zeta \), the estimation of \( \alpha_k \) and \( \beta_k \) is possible. Since \( K_1 \) is the same for all the subcarriers, and since we use \( N \) data symbols, it is possible to average the estimates over time and frequency:

\[
\hat{K}_1 = \frac{1}{\text{card}(\zeta)} \sum_{k \in \zeta} \hat{\alpha}_k + \beta^*_k.
\]

(7)

\( \hat{K}_2 \) is obtained by \( \hat{K}_2 = 1 - \hat{K}_1^* \).

3) **I/Q imbalance compensation of data symbols:** Once \( K_1 \) and \( K_2 \) have been estimated, the data symbols are compensated for the I/Q imbalance by inverting the matrix \( A_1 \) given in equation (2). The data symbols compensated for the I/Q imbalance are:

\[
\hat{D}_k(n) = \frac{\hat{K}_1^* R_k(n) - \hat{K}_2 R^*_{-k}(n)}{|\hat{K}_1|^2 - |\hat{K}_2|^2}.
\]

(8)

4) **I/Q imbalance compensation of the channel estimation:** The estimates \( \hat{H}_k \) deduced from \( \hat{H}_k = \hat{\alpha}_k + \beta^*_k \) are not reliable for two reasons. The first one is that for each received data symbol, \( \hat{H}_k \) is not available when \( k \) does not belong to \( \zeta \). Secondly, \( \hat{H}_k \) is severely corrupted by hard decision errors because there is no averaging over the frequency range. Since the estimations of the I/Q imbalance parameters are reliable, the I/Q imbalance compensation of the rough channel estimation \( C_k \) gives a reliable estimation of \( H_k \) in a similar way as in the step 3:

\[
\hat{H}_k = \frac{\hat{K}_1^* C_k - L_k \hat{K}_2^* C^*_{-k}}{|\hat{K}_1|^2 - |\hat{K}_2|^2}.
\]

(9)

#### B. The algorithm with \( i \) iterations

The matrix form of the data symbols compensated for I/Q imbalance after the first iteration can be written as follows:

\[
\begin{bmatrix}
\hat{D}_{1k}(n) \\
\hat{D}^*_{1-k}(n)
\end{bmatrix} = \begin{bmatrix} A_1^{-1} A_1 \end{bmatrix} \begin{bmatrix}
H_k & 0 \\
0 & H^*_{-k}
\end{bmatrix} \begin{bmatrix}
T_k(n) \\
T^*_{-k}(n)
\end{bmatrix}.
\]

(10)
where \( \hat{A}_1 \) and \( A_2 \) correspond respectively to the estimation-compensation matrix of the I/Q imbalance, and to the remaining I/Q imbalance matrix after the first iteration. Simple algebra leads to \( A_2 \) in the following form:

\[
A_2 = \begin{bmatrix} K_{11} & K_{21} \\ K_{21}^* & K_{11}^* \end{bmatrix},
\]

\[
K_{11} = \frac{K_1^2 - K_2^2}{|K_1|^2 - |K_2|^2}, \quad K_{21} = \frac{K_1^* K_2 - K_2 K_1^*}{|K_1|^2 - |K_2|^2}.
\]

If the estimation of the I/Q imbalance parameters obtained after the first iteration was perfect, then \( A_2 \) would be the identity matrix. In this case \( \hat{D}_{ik}(n) = H_k T_k(n) \), and the I/Q imbalance would be perfectly compensated. However, in the presence of hard decision errors, the algorithm estimates imperfect I/Q imbalance parameters. From \( K_1 + K_2 = 1 \) and \( \hat{K}_1 + \hat{K}_2 = 1 \), it follows after simple algebra that \( K_{11} + K_{21} = 1 \), regardless of the values of \( K_1, K_2 \) and their estimates. Therefore equation (10) is similar to equation (2). It is possible to reiterate the estimation of the remaining I/Q imbalance parameters returning to the step 1 of the core algorithm. The data symbols and the channel estimation necessary in step 1 at the iteration \( i \) are those obtained in the iteration \( i - 1 \). Thus, the matrix \( A_{i+1} \) after the \( i^{th} \) iteration is:

\[
A_{i+1} = \hat{A}_1 \hat{A}_{i-1}^{-1} \cdots \hat{A}_2 \hat{A}_1^{-1} A_1. \tag{11}
\]

The experimental results evidence that \( A_{i+1} \) tends to the identity matrix when there is no noise and \( i \) goes to infinity. Due to the space limitations, details of the iterative compensation of the channel estimation are not given, but it can be easily checked that it leads to a similar matrix as equation (11).

V. Simulation Results

In this section, we demonstrate the capabilities of the proposed I/Q imbalance compensation and channel estimation algorithm in the context of the 802.11a Standard [8]. The data subcarriers are modulated by a 64-QAM (Quadrature Amplitude Modulation). An independent channel realization of the Channel C [9], which is considered as a dense multi-path environment, is drawn for each transmitted packet. The simulation results are depicted by averaging curves over 100 received packets. First we give the way that the parameters of the algorithm are chosen, then we analyse the performance of the algorithm, and finally we test its robustness over an extremely wide range of I/Q imbalance.

A. Parameters

The algorithm requires two parameters: the number of data symbols \( N \), and the number of iterations \( i \). A good criterion to choose these parameters is the conversion gain, defined as the power of the undesired complex down-conversion divided by the power of the desired complex down-conversion [6]. The conversion gain of the compensated signal after the \( i^{th} \) iteration is defined by:

\[
GC_{\text{comp}} = \frac{K_{21}^2}{K_{11}}, \tag{12}
\]

where \( K_{11} \) and \( K_{21} \) denote the remaining I/Q imbalance parameters after the \( i^{th} \) iteration. We restrict the packet error rate so that it does not exceed 10%. The corresponding uncoded target BER is \( 10^{-2} \) and is reached for the energy per bit to noise ratio \( (E_b/N_0) \) equal to 25 dB in the case of no I/Q imbalance (Fig. 2). Thus we propose to define the optimal number of data symbols \( N \) and iterations \( i \) required in order to minimize the conversion gain of the compensated signal for \( E_b/N_0 = 25 \text{ dB} \). We set the maximum tolerated I/Q imbalance to \( g = 1.5 \) and \( \phi = 15^\circ \), Channel C, and \( E_b/N_0 = 25 \text{ dB} \). For reference, we define the conversion gain of the uncompensated data symbols \( GC_{\text{uncomp}} \). All the curves saturate within \( i = 5 \), and the error floor is fixed by the value of \( N \). Moreover, it appears that for \( N \geq 5 \), there is no significant improvement of the \( GC_{\text{comp}} \) curves. The best trade off between complexity and performance is also obtained for \( i = 5 \) and \( N = 5 \).

B. Performance

We propose to compare the performance of the proposed algorithm to the performance in the case of no I/Q imbalance and to the performance in the case of I/Q imbalance but without any compensation scheme. Fig. 2 gives the uncoded BER curves versus the number of iterations for various values of \( N \) for \( g = 1.5 \) and \( \phi = 15^\circ \). Fig. 1 gives the average of \( GC_{\text{comp}} \) versus the number of iterations for various values of \( N \) for \( g = 1.5 \) and \( \phi = 15^\circ \) and \( E_b/N_0 = 25 \text{ dB} \). For reference, we define the conversion gain of the uncompensated data symbols \( GC_{\text{uncomp}} \). All the curves saturate within \( i = 5 \), and the error floor is fixed by the value of \( N \). Moreover, it appears that for \( N \geq 5 \), there is no significant improvement of the \( GC_{\text{comp}} \) curves. The best trade off between complexity and performance is also obtained for \( i = 5 \) and \( N = 5 \).

C. Operating range

We now test the proposed algorithm over an extremely wide range of I/Q imbalance, i.e. until \( g = 2 \) and \( \phi = 30^\circ \). The parameters of the algorithm are set to \( N = 10 \) and \( i = 10 \) in a similar way as in Subsection V-A. A good criterion for measuring the performance of the proposed scheme is the
imbalance is estimated and compensated for with the proposed
scheme, “I/Q Imbalance Comp.” refers to the proposed estimation and
compensation scheme for a receiver with I/Q imbalance.

Fig. 3. Loss in $E_b/N_0$ as a function of gain and phase mismatch for a
fixed uncoded BER of $10^{-2}$ when no compensation scheme is used and in
the case of Channel C.

$E_b/N_0$ gap between the uncoded BER curves of a receiver
with no I/Q imbalance and a receiver with I/Q imbalance. Figs.
3 and 4 give respectively the loss in $E_b/N_0$ as a function of the
gain and phase mismatches for a fixed uncoded BER of $10^{-2}$
when the I/Q imbalance is not corrected, and when the I/Q
imbalance is estimated and compensated for with the proposed
algorithm. Note that the loss in $E_b/N_0$ is not given when that
the uncoded BER curves saturate to a value higher than the
desired $10^{-2}$. We note that it is not possible to reach the target
BER bellow $10^{-2}$ even for small values of mismatch when
the I/Q imbalance is not compensated for. The performance
degradation attained with our algorithm is below 1.5 dB even
for extremely high I/Q imbalance.

VI. CONCLUSION

Due to low-cost requirements for consumer product applications,
the perfect match between I and Q branches at
the receiver is not possible in the analog domain. The I/Q
imbalance affects the received training symbols, as well as
the data symbols implying that: (i) the channel estimation and
the data symbols are erroneous, and (ii) the equalization is
performed coarsely. These problems lead to a saturation of
the BER curves even for small I/Q imbalance in a frequency
selective environment. We propose an iterative algorithm for
the joint channel estimation and I/Q imbalance compensation
in OFDM receivers. This algorithm uses both the training and
data received OFDM symbols in a decision-directed fashion.
Even with no change of the pilots symbols, it leads to sig-
nificantly improved performance, approaching the performance
of a receiver not affected by I/Q imbalance.

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