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The maximal regularity operator on tent spaces

Pascal Auscher, Sylvie Monniaux, Pierre Portal

En l’honneur des 60 ans de Michel Pierre

Abstract

Recently, Auscher and Axelsson gave a new approach to non-smooth boundary value problems with $L^2$ data, that relies on some appropriate weighted maximal regularity estimates. As part of the development of the corresponding $L^p$ theory, we prove here the relevant weighted maximal estimates in tent spaces $T^{p,2}$ for $p$ in a certain open range. We also study the case $p = \infty$.

1 Introduction

Let $-L$ be a densely defined closed linear operator acting on $L^2(\mathbb{R}^n)$ and generating a bounded analytic semigroup $(e^{-tL})_{t \geq 0}$. We consider the maximal regularity operator defined by

$$M_L f(t, x) = \int_0^t L e^{-(t-s)L} f(s, \cdot)(x) ds,$$

for functions $f \in C_0(\mathbb{R}_+ \times \mathbb{R}^n)$. The boundedness of this operator on $L^2(\mathbb{R}_+ \times \mathbb{R}^n)$ was established by de Simon in [14]. The $L^p(\mathbb{R}_+ \times \mathbb{R}^n)$ case, for $1 < p < \infty$, turned out, however, to be much more difficult. In [10], Kalton and Lancien proved that $M_L$ could fail to be bounded on $L^p$ as soon as $p \neq 2$. The necessary and sufficient assumption for $L^p$ boundedness was then found by Weis [17] to be a vector-valued strengthening of analyticity, called R-analyticity. As many differential operators $L$ turn out to generate R-analytic semigroups, the $L^p$ boundedness of $M_L$ has subsequently been successfully used in a variety of PDE situations (see [14] for a survey).

Recently, maximal regularity was used in a different manner as an important tool in [2], where a new approach to boundary value problems with $L^2$ data for divergence form elliptic systems on Lipschitz domains, is developed. More precisely, in [2], the authors establish and use the boundedness of $M_L$ on weighted spaces $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dtdx)$, for certain values of $\beta \in \mathbb{R}$, under the additional assumption that $L$ has bounded holomorphic functional calculus on $L^2(\mathbb{R}^n)$. This additional assumption was removed in [3, Theorem 1.3]. Here is the version when specializing the Hilbert space to be $L^2(\mathbb{R}^n)$.

Theorem 1.1. With $L$ as above, $M_L$ extends to a bounded operator on $L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^\beta dtdx)$ for all $\beta \in (-\infty, 1)$.

The use of these weighted spaces is common in the study of boundary value problems, where they are seen as variants of the tent space $T^{2,2}$ which occurs for $\beta = -1$, introduced by Coifman, Meyer and Stein in [6]. For $p \neq 2$, the corresponding spaces are weighted versions of the tent spaces $T^{p,2}$, which are defined, for parameters $\beta \in \mathbb{R}$ and $m \in \mathbb{N}$, as the completion of $C_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$\|g\|_{T^{p,2,m}(t^\beta dtdy)} = \left( \int_{\mathbb{R}_+} \left( \int_{\mathbb{R}^n} \frac{1}{t^m} \frac{1}{B(x,t^\frac{\beta}{2})} |g(t, y)|^2 t^\beta dy dt \right)^{\frac{p}{p-1}} dx \right)^{\frac{1}{p-1}},$$

the classical case corresponding to $\beta = -1$, $m = 1$, and being denoted simply by $T^{p,2}$. The parameter $m$ is used to allow various homogeneities, and thus to make these spaces relevant in the study of
differential operators \( L \) of order \( m \). To develop an analogue of \([2]\) for \( L^p \) data, we need, among many other estimates yet to be proved, boundedness results for the maximal operator \( \mathcal{M}_L \) on these tent spaces. This is the purpose of this note. Another motivation is well-posedness of non-autonomous Cauchy problems for operators with varying domains, which will be presented elsewhere. In the latter case, \( \mathcal{M}_L \) can be seen as a model of the evolution operators involved. However, as \( \mathcal{M}_L \) is an important operator on its own, we thought interesting to present this special case alone.

In Section 2 we state and prove the adequate boundedness results. The proof is based on recent results and methods developed in \([3]\), building on ideas from \([5]\) and \([8]\). In Section 3 we recall the relevant material from \([3]\).

2 Tools

When dealing with tent spaces, the key estimate needed is a change of aperture formula, i.e., a comparison between the \( T^{p,2} \) norm and the norm

\[
\|g\|_{T^{p,2}} := \left( \int_{\mathbb{R}^n} \left( \int_0^\infty \frac{1_B(x,u)}{t^n} \left| g(t,u) \right|^2 t^\frac{p}{2} du \right)^\frac{2}{p} dx \right)^{\frac{1}{2}},
\]

for some parameter \( \alpha > 0 \). Such a result was first established in \([3]\), building on similar estimates in \([2]\), and analogues have since been developed in various contexts. Here we use the following version given in \([3]\), Theorem 4.3.

**Theorem 2.1.** Let \( 1 < p < \infty \) and \( \alpha > 1 \). There exists a constant \( C > 0 \) such that, for all \( f \in T^{p,2} \),

\[
\|f\|_{T^{p,2}} \leq \|f\|_{T^{p,2}} \leq C(1 + \log \alpha)^{n/\tau} \|f\|_{T^{p,2}},
\]

where \( \tau = \min(p, 2) \) and \( C \) depends only on \( n \) and \( p \).

This is crucial in what follows, and has been shown to be optimal in \([3]\). Note, however, that it improves the power of \( \alpha \) appearing in the inequality from the \( n \) given in \([2]\) to \( n/\tau \).

Applying this to \( (t,y) \mapsto t^\frac{m(n+1)}{n} f(t^m, y) \) instead of \( f \), we also have the weighted result, where

\[
\|g\|_{T^{p,2,m}(\tau^2 dt dy)} := \left( \int_{\mathbb{R}^n} \left( \int_0^\infty \frac{1_B(x,u)}{t^n} \left| g(t,u) \right|^2 t^\frac{p}{2} du \right)^\frac{2}{p} dx \right)^{\frac{1}{2}}.
\]

**Corollary 2.2.** Let \( 1 < p < \infty \), \( m \in \mathbb{N} \), \( \alpha > 1 \), and \( \beta \in \mathbb{R} \). There exists a constant \( C > 0 \) such that, for all \( f \in T^{p,2,m}(\tau^2 dt dy) \),

\[
\|f\|_{T^{p,2,m}(\tau^2 dt dy)} \leq \|f\|_{T^{p,2,m}(\tau^2 dt dy)} \leq C(1 + \log \alpha)^{n/\tau} \|f\|_{T^{p,2,m}(\tau^2 dt dy)},
\]

where \( \tau = \min(p, 2) \) and \( C \) depends only on \( n \) and \( p \).

To take advantage of this result, one needs to deal with families of operators, that behave nicely with respect to tent norms. As pointed out in \([3]\), this does not mean considering R-bounded families (which means R-analytic semigroups when one considers \((tLe^{-tL})_{t \geq 0}\) as in the \( L^p(\mathbb{R}^n) \) case, but tent bounded ones, i.e., families of operators with the following \( L^2 \) off-diagonal decay, also known as Gaffney-Davies estimates.

**Definition 2.3.** A family of bounded linear operators \((T_t)_{t \geq 0} \subset B(L^2(\mathbb{R}^n))\) is said to satisfy off-diagonal estimates of order \( M \), with homogeneity \( m \), if, for all Borel sets \( E, F \subset \mathbb{R}^n \), all \( t > 0 \), and all \( f \in L^2(\mathbb{R}^n) \):

\[
\|1_E T_1 F f\|_2 \lesssim \left( 1 + \frac{dist(E,F)^m}{t} \right)^{-M} \|1_F f\|_2.
\]

In what follows \( \| \cdot \|_2 \) denotes the norm in \( L^2(\mathbb{R}^n) \).
As proven, for instance, in [3], many differential operators of order $m$, such as (for $m = 2$)
divergence form elliptic operators with bounded measurable complex coefficients, are such that
$(tL^{-1}k)^{t > 0}$ satisfies off-diagonal estimates of any order, with homogeneity $m$. This condition can,
in fact, be seen as a replacement for the classical gaussian kernel estimates satisfied in the case of
more regular coefficients.

3 Results

**Theorem 3.1.** Let $m \in \mathbb{N}$, $\beta \in (-\infty, 1)$, $p \in \left(\frac{2m}{m+\beta}, \infty\right) \cap (1, \infty)$, and $\tau = \min(p, 2)$. If
$(tL^{-1}k)^{t > 0}$ satisfies off-diagonal estimates of order $M > \frac{m}{m+\beta}$, with homogeneity $m$, then $M_L$ extends to a bounded operator on $T^{p,2,m}(t^p dt dy)$.

**Proof.** The proof is very much inspired by similar estimates in [2] and [3]. Let $f \in \mathcal{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$. Given $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$, and $j \in \mathbb{Z}_+$, we consider

$$C_j(x, t) = \begin{cases} B(x, t) & \text{if } j = 0, \\ B(x, 2t) \setminus B(x, 2^{j-1}t) & \text{otherwise.} \end{cases}$$

We write $\|M_L f\|_{T^{p,2}} \leq \sum_{k,j=0}^{\infty} I_{k,j} + \sum_{j=0}^{\infty} J_j$ where

$$I_{k,j} = \left( \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} \left| \int_0^{t-s} L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 t^\beta dy dt \right)^{\frac{1}{2}},$$

$$J_j = \left( \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} \left| \int_0^{t-s} L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 t^\beta dy dt \right)^{\frac{1}{2}}.$$

Fixing $j \geq 0$, $k \geq 1$ we first estimate $I_{k,j}$ as follows. For fixed $x \in \mathbb{R}^n$,

$$\int_0^\infty \int_{B(x, \frac{t}{t^\alpha})} \left| \int_{2^{-k-1}t}^{2^{-k}t} L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 t^\beta dy dt$$

$$\leq \int_0^\infty \int_{B(x, \frac{t}{t^\alpha})} \left| \int_{2^{-k-1}t}^{2^{-k}t} (t-s) L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 \frac{1}{t-s} t^\beta dy dt$$

$$\leq \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} \left( \int_{B(x, \frac{t}{t^\alpha})} \left| \int_{2^{-k-1}t}^{2^{-k}t} (t-s) L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 \frac{1}{t-s} t^\beta dy dt \right) t^\beta \frac{1}{t-s} 2^{-2k} ds dt$$

$$\leq 2^{-k} 2^{-2k M} \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} \left( \int_{B(x, \frac{t}{t^\alpha})} \left| \int_{2^{-k-1}t}^{2^{-k}t} (t-s) L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 \frac{1}{t-s} t^\beta dt \right) t^\beta \frac{1}{t-s} ds dt$$

$$\leq 2^{-k} \left( \frac{m+\beta}{m+\beta+1} \right)^{2k M} \int_0^\infty \int_{2^{-k-1}t}^{2^{-k}t} 2^{-k} \left( \int_{B(x, \frac{t}{t^\alpha})} \left| \int_{2^{-k-1}t}^{2^{-k}t} (t-s) L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right) f(s, \cdot)(y) ds \right|^2 \frac{1}{t-s} t^\beta dt \right) t^\beta \frac{1}{t-s} ds dt$$

In the second inequality, we use Cauchy-Schwarz inequality for the integral with respect to $t$, the
fact that $t - s \sim t$ for $s \in \mathcal{U}_{k+1} \left( \frac{1}{2^{k-1} t}, \frac{1}{2^{k-1} s} \right) \subset \left[ 0, \frac{t}{2} \right]$ and Fubini’s theorem to exchange the integral in $t$ and the integral in $y$. The next inequality follows from the off-diagonal estimate verified by $(t-s) L^{-1} \left( \frac{B(x, \frac{t-s}{t})}{t^\alpha} \right)$ and again the fact that $t - s \sim t$. By Corollary 2.2 this gives

$$I_{k,j} \lesssim (j + k) 2^{-k \left( \frac{m+\beta}{m+\beta+1} - \frac{m}{m+\beta} \right)} 2^{-j (mM - \frac{m}{m+\beta})} \| f \|_{T^{p,2,m}(t^p dt dy)},$$
where \( \tau = \min(p, 2) \). It follows that
\[
\sum_{k=1}^\infty \sum_{j=0}^\infty I_{k,j} \lesssim \|f\|_{T^{p,2}(\mathbb{R}^n; \mathbb{R}^n)} \text{ since } M > \frac{a}{m} \text{ and } \frac{a}{m} + 1 - \beta > \frac{2\kappa}{mT} \text{ (Note that for } p \geq 2, \text{ this requires } \beta < 1).
\]

We now turn to \( J_0 \) and remark that \( J_0 \leq (\int_{\mathbb{R}^n} J_0(x)^{\frac{p}{2}} \, dx)^{\frac{1}{p}} \), where
\[
J_0(x) = \int_0^\infty \int_0^t \left| I \left( \int_0^t e^{-((t-s)L)}(g(s,\cdot))(y) \, ds \right) \right|^2 \nu(y) \, dy \, dt
\]
with \( g(s,y) = 1_{B(x,4s^{\frac{1}{m}})}(y) f(s,y) \). The inside integral can be rewritten as
\[
M_L g(t,\cdot) - e^{-\frac{t}{2}L} M_L g(x,\cdot).
\]
As \( M_L \) is bounded on \( L^2(\mathbb{R}_+ \times \mathbb{R}^n; t^{\beta - \frac{m}{2}} \, dy \, dt) \) by Theorem 1.1 and \( (e^{-tL})_{t \geq 0} \) is uniformly bounded on \( L^2(\mathbb{R}^n) \), we get
\[
J_0(x) \lesssim \int_0^\infty \|1_{B(x,4s^{\frac{1}{m}})} f(s,\cdot)\|_2^{\frac{p}{2}} s^{\beta - \frac{m}{2}} \, ds.
\]

We finally turn to \( J_j \), for \( j \geq 1 \). For fixed \( x \in \mathbb{R}^n \),
\[
\int_0^\infty \int_0^\infty 1_{B(x,\frac{t}{m^{\frac{1}{m}}})}(y) \left( \int_0^t \left( \int_{\mathbb{R}^n} e^{-((t-s)^2)} f(s,\cdot)(y) \, dy \right)^2 \nu(s) \, ds \right) \, ds \, dy \, dt
\]
\[
\leq \int_0^\infty \int_0^\infty 1_{B(x,\frac{t}{m^{\frac{1}{m}}})}(y) \left( \int_{\mathbb{R}^n} e^{-((t-s)^2)} f(s,\cdot)(y) \, dy \right)^2 \nu(s) \, ds \, dy \, dt
\]
\[
\leq \int_0^\infty \int_0^\infty 1_{B(x,\frac{t}{m^{\frac{1}{m}}})}(y) \left( \int_{\mathbb{R}^n} e^{-((t-s)^2)} f(s,\cdot)(y) \, dy \right)^2 \nu(s) \, ds \, dy \, dt
\]
\[
\leq \int_0^\infty \int_0^\infty \left( 2^{j(m+2M-2)} \int_{\mathbb{R}^n} \left( \int_{\mathbb{R}^n} e^{-((t-s)^2)} f(s,\cdot)(y) \, dy \right)^2 \nu(s) \, ds \, dy \, dt
\]
\[
\leq \int_0^\infty \int_{\mathbb{R}^n} e^{-((t-s)^2)} f(s,\cdot)(y) \, dy \, ds \, dy \, dt
\]
where we have used Cauchy-Schwarz inequality in the second inequality, the off-diagonal estimates and the fact that \( s \leq t \) in the third, Fubini’s theorem and the fact that \( s \geq \frac{t}{2} \) in the fourth, and the change of variable \( \sigma = \frac{1}{t-s} \) in the last. An application of Corollary 2.2 then gives
\[
J_j \lesssim 2^{-j(m+2M-2)} \|f\|_{T^{p,2}(\mathbb{R}^n; \mathbb{R}^n)} = 2^{-j(m+2M-2)} \|f\|_{T^{p,2}(\mathbb{R}^n; \mathbb{R}^n)}
\]
and the proof is concluded by summing the estimates.

An end-point result holds for \( p = \infty \). In this context the appropriate tent space consists of functions such that \( |g(t,x)|^{2 \frac{dt}{t}} \) is a Carleson measure, and is defined as the completion of the
space $\mathcal{C}_c(\mathbb{R}_+ \times \mathbb{R}^n)$ with respect to

$$
\|g\|_{T^\infty,2} = \sup_{(x,r) \in \mathbb{R}^n \times \mathbb{R}_+} \int_0^r \int_0^t |g(t,x)|^2 \frac{dxdt}{t}.
$$

We also consider the weighted version defined by

$$
\|g\|_{T^{\infty,2,m}(t^\beta dtdy)} := \sup_{(x,r) \in \mathbb{R}^n \times \mathbb{R}_+} \int_0^r \int_0^r |g(t,x)|^2 t^\beta dxdt.
$$

**Theorem 3.2.** Let $m \in \mathbb{N}$, and $\beta \in (-\infty, 1)$. If $(tLe^{-tL})_{t \geq 0}$ satisfies off-diagonal estimates of order $M > \frac{n}{2m}$, with homogeneity $m$, then $M_L$ extends to a bounded operator on $T^{\infty,2,m}(t^\beta dtdy)$.

**Proof.** Pick a ball $B(z, r^\frac{n}{m})$. Let

$$
I_2 = \int_0^r \int_{B(z, r^\frac{n}{m})} |(M_Lf)(t,x)|^2 t^\beta dxdt.
$$

We want to show that $I_2 \lesssim r^\frac{n}{m} \|f\|_{T^{\infty,2}(t^\beta dtdy)}^2$. We set

$$
I_2^j = \int_0^r \int_{B(z, r^\frac{n}{m})} |(M_Lf_j)(t,x)|^2 t^\beta dxdt
$$

where $f_j(s, x) = f(s, x)1_{C_j(\epsilon, 4^j r^\frac{1}{m})} (x)1_{(0, r)}(s)$ for $j \geq 0$. Thus by Minkowsky inequality, $I \leq \sum I_j$.

For $I_0$ we use again Theorem 3.1 which implies that $M_L$ is bounded on $L^2(\mathbb{R}_+ \times \mathbb{R}^n, t^\beta dxdt)$. Thus

$$
I_0^2 \lesssim \int_0^r \int_{B(z, 4^j r^\frac{1}{m})} |f(t,x)|^2 t^\beta dxdt \lesssim r^\frac{n}{m} \|f\|_{T^{\infty,2,m}(t^\beta dtdy)}^2.
$$

Next, for $j \neq 0$, we proceed as in the proof of Theorem 3.1 to obtain

$$
I_j^2 \lesssim \sum_{k=1}^\infty \int_0^r \int_{2^{-k-1}r}^{2^{-k}r} 2^{-kt} \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 t^\beta ds dt
$$

$$
+ \int_0^r \int_{2^{-j}r}^{r} t(t-s)^{-2} \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s, \cdot)\|_{L^2}^2 t^\beta ds dt.
$$

Exchanging the order of integration, and using the fact that $t \sim t-s$ in the first part and that $t \sim s$
in the second, we have the following.

\[
I_2^j \lesssim \sum_{k=1}^{\infty} 2^{-k} 2^{-2jmM} r^{-2M} \int_0^r \int \frac{t^{\beta+2M-1}}{t^{-2+s}} \|f_j(s,.)\|_{L^2_s dt ds}^2
\]

\[
+ \int_0^r \int \frac{r(t-s)^{-2}}{t^{-s}} \left(1 + \frac{2^{jm}r}{t-s}\right)^{-2M} \|f_j(s,.)\|_{L^2_s s^\beta}^2 \, dt ds
\]

\[
\lesssim \sum_{k=1}^{\infty} 2^{-k} 2^{-2jmM} \int_0^r (2^k s)^{\beta} \|f_j(s,.)\|_{L^2_s}^2 ds + \int_0^1 \int (1 + 2^{jm} s)^{-2M} \|f_j(s,.)\|_{L^2_s s^\beta}^2 \, d\sigma ds
\]

\[
\lesssim 2^{-2jmM} \int_0^r \|f_j(s,.)\|_{L^2_s s^\beta}^2 ds,
\]

where we used $\beta < 1$. We thus have

\[
I_2^j \lesssim 2^{-2jmM} (2^{jm} \cdot \cdot \cdot)^n \|f\|_{T_{\infty,2}^{2n+2} (t^\beta dt dy)},
\]

and the condition $M > \frac{2n}{2m}$ allows us to sum these estimates.

Remark 3.3. Assuming off-diagonal estimates, instead of kernel estimates, allows to deal with differential operators $L$ with rough coefficients. The harmonic analytic objects associated with $L$ then fall outside the Calderón-Zygmund class, and it is common (see for instance [3]) for their boundedness range to be a proper subset of $(1, \infty)$. Here, our range $(\frac{2n}{n+m(1-\beta)}, \infty]$ includes $[2, \infty]$ as $\beta < 1$, which is consistent with [3]. In the case of classical tent spaces, i.e., $m = 1$ and $\beta = -1$, it is the range $(2, \infty]$, where 2, denotes the Sobolev exponent $\frac{2n}{n+2}$. We do not know, however, if this range is optimal.

Remark 3.4. Theorem 3.2 is a maximal regularity result for parabolic Carleson measure norms. This is quite natural from the point of view of non-linear parabolic PDE (where maximal regularity is often used), and such norm have, actually, already been used in the context of Navier-Stokes equations in [1], and, subsequently, for some geometric non-linear PDE in [2]. Theorem 3.3 is also reminiscent of Krylov’s Littlewood-Paley estimates [4], and of their recent far-reaching generalization in [5]. In fact, the methods and results from [4], on which this paper relies, use the same circle of ideas (R-boundedness, Kutan-Weis $\gamma$ multiplier theorem...) as [4]. The combination of these ideas into a “conical square function” approach to stochastic maximal regularity will be the subject of a forthcoming paper.

References


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