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Jumps and Synchronization in Anti-lock Brake Algorithms

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The aim of our paper is to provide a new class of anti-lock brake algorithms (that use wheel deceleration logic-based switchings) and a simple mathematical background that explains their behavior. These algorithms extend those proposed in our previous work [6], and consider cases where there might be discontinuities of road characteristics or where it is intended to synchronize the ABS strategies on several wheels of the vehicle.

Topics: Traction and Brake Control, Tire Property, Modeling and Simulation Technology.

INTRODUCTION

In the literature, one can distinguish two completely different kinds of anti-lock brake system designs: those based on logic switching from wheel deceleration information (see e.g. [8]) and those based on wheel slip regulation (see e.g. [7]). In our previous work [6], we proposed a new algorithm based on wheel deceleration thresholds and a method for analyzing the limit cycles that appear with this kind of strategies (which gives some elements for tuning the different parameters that are involved in the algorithm).

The aim of the present paper is to extend, in three new directions, our previous results [6]. Firstly, we improve the basic five-phase strategy by adding a phase during which the brake torque is increased slowly; and we analyse the consequences of this modification on the limit cycles of the algorithm. Secondly, we consider the problem of discontinuous transitions of road characteristics, and propose a new eleven-phase strategy that is robust to such discontinuous transitions. Thirdly, we consider the synchronization problem (between the front wheels) and the anti-synchronization problem (between the rear wheels), in order to obtain a coordinated behavior among the four independent ABS strategies applied to each wheel of the vehicle.

In this paper, we only present an overview on some ABS algorithms we have obtained recently. It is impossible, in the six pages of these proceedings, to treat all the mathematical problems associated with these algorithms (see [1] for a longer version).

1 WHEEL DYNAMICS

1.1 Tyre forces

The longitudinal tyre force F_x is often modelled by a relation

$$F_x(\lambda, F_z) = \mu(\lambda) F_z.$$

That is, by a function that depends linearly on vertical force F_z and nonlinearly on wheel slip

$$\lambda = \frac{R\omega - v_x}{v_x},$$

where ω denotes the angular velocity of the wheel and v_x the speed of the vehicle. It should be noted that this kind of models can only be used at high speeds.

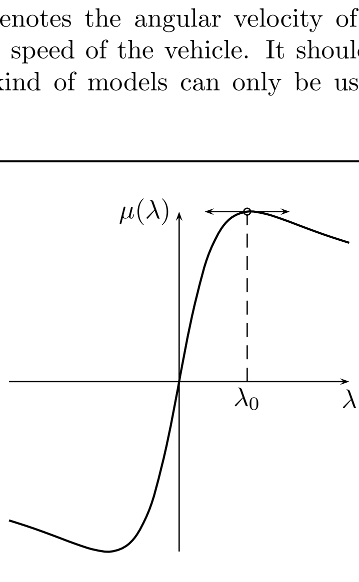

Figure 1: Tyre forces. The shape of the curve $\mu(\cdot)$.

The function $\mu(\cdot)$ will be described, for negative wheel slips, using a second order rational fraction

$$\mu(\lambda) = \frac{a_1\lambda - a_2\lambda^2}{1 - a_3\lambda + a_4\lambda^2}.$$

The coefficients a_i are all positive and depend on tyre characteristics, road conditions, tyre pressure, temperature, etc. They should thus be assumed to be unknown.

1.2 Wheel velocity

The angular velocity ω of a given wheel of the vehicle has the following dynamics:

$$I\dot{\omega} = -RF_x + T, \qquad (1)$$

where I denotes the inertia of the wheel, R its radius, F_x the longitudinal tyre force, and T the torque applied to the wheel.

The torque $T = T_e - T_b$ is composed of the engine torque T_e and the brake torque T_b. We will assume that during ABS braking the clutch pedal is kept engaged, and thus neglect the engine torque.

1.3 Wheel acceleration

In our simplified wheel dynamics model, the vehicle will be supposed to brake with the maximal constant deceleration a_x^* allowed by road conditions, which is $a_x^* = -\mu(\lambda_0)g$. In other words

$$\dot{v}_x = a_x^*.$$

The vertical forces F_z on the front and rear axles are assumed constant and equal to those one would have at equilibrium for this constant deceleration a_x^*. The front and rear wheel dynamics are then completely decoupled.

Even though the simplifications introduced in this model might seem a little bit excessive, a comparison of the ABS simulations obtained with this model, with those obtained with a more complex model, shows that the simulated limit cycles are are very similar with both models (see [1]).

Let $\lambda^* = -\lambda_0$ be the optimal negative slip rate, such that that $\mu'(\lambda^*) = 0$. If we define the wheel slip and wheel acceleration offsets by

$$\begin{aligned} x_1 &= \lambda - \lambda^* \\ x_2 &= R\dot{\omega} - a_x^*, \end{aligned}$$

we obtain the following control system:

$$\dot{x}_1 = \frac{1}{v_x}\left(x_2 - (\lambda^* + x_1)a_x^*\right) \qquad (2)$$

$$\dot{x}_2 = -\frac{a}{v_x}\bar{\mu}'(x_1)\left(x_2 - (\lambda^* + x_1)a_x^*\right) + u, \qquad (3)$$

where

$$a = \frac{R^2}{I}F_z \quad \text{and} \quad u = \frac{R}{I}\dot{T}.$$

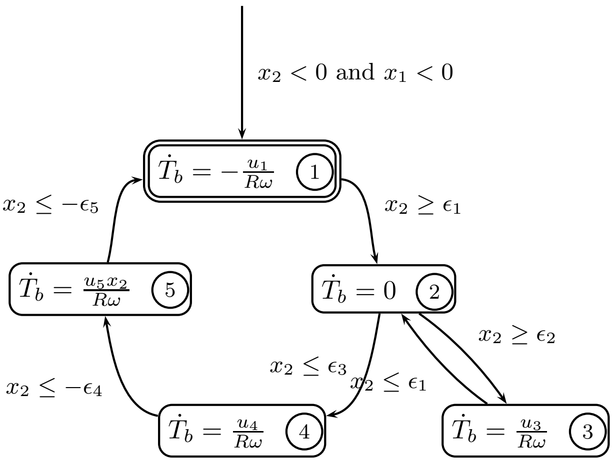

Figure 2: The five-phase ABS regulation logic.

The function $\bar{\mu}(\cdot)$ is defined as

$$\bar{\mu}(x) = \mu(\lambda^* + x) - \mu(\lambda^*).$$

Observe that, for a function $\mu(\cdot)$ that is a second order rational fraction, we have

$$\bar{\mu}(x) = \frac{x^2}{\bar{a}_1 - \bar{a}_2 x + \bar{a}_3 x^2}.$$

Other wheel deceleration models are available in the literature (see e.g. [2] and [5]).

2 THE FIVE-PHASE ALGORITHM

In this paper, our goal will be to keep the unmeasured variable x_1 in a small neighborhood of zero, with a control u that only uses the measured variable $y = R\dot{\omega} - a_x^*$; the function $\mu(\cdot)$ being unknown. Our approach can also be used when $y = R\dot{\omega}$.

2.1 Simplified first integrals

Consider a dynamical system

$$\dot{x} = f(x), \text{ where } x \in \mathbb{R}^n.$$

For this system, a *first integral* is a function of the state $I(x)$ that remains constant along the trajectories of the system. That is, such that

$$\frac{d}{dt}I(x(t)) = \sum_{i=1}^{n} \frac{\partial I}{\partial x_i}(x(t)) f_i(x(t)) = 0.$$

Thus, on any time interval $[t_0, t_1]$, we have

$$I(x(t)) = I(x(t_0)), \qquad (4)$$

for all $t \in [t_0, t_1]$.

In the particular case of two-dimensional dynamical systems, first integrals can be used to compute the phase-plane evolution of the system. Indeed, the evolution of any of the two variables of the system can be deduced from the other variable using equation (4).

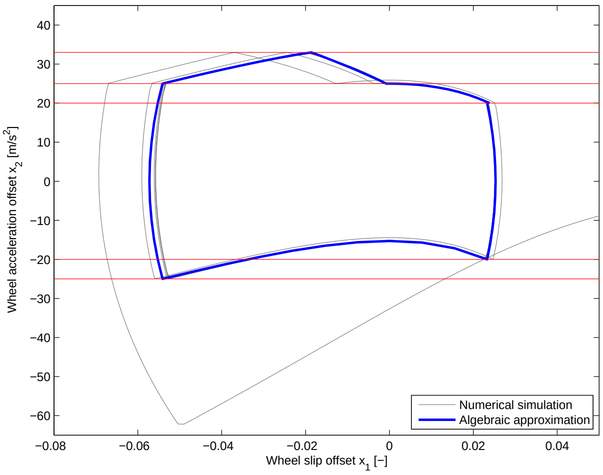

Figure 3: Limit cycle obtained for $\epsilon_1 = \epsilon_5 = 20$ and $\epsilon_3 = \epsilon_4 = 25$, in the case $u_5 = 0$, with $\epsilon_2 = 33$ and $u_3 = 12970$ (obtained with the formulas of [6]).

Constant torque When the control variable is such that $u = 0$, the torque applied to the wheel remains constant. Therefore, the torque itself is a first integral. By considering equation (1), it can be easily seen that

$$I_a(x) = x_2 + a\bar{\mu}(x_1)$$

is a first integral of the dynamical system defined by equations (2) and (3). In fact $I_a = RT/I$.

Large torque variations When $u \neq 0$, finding exact first integrals is difficult. Nevertheless, for controls having the following particular form $u = u_0/R\omega$ with u_0 large enough, an approximative description of the system's evolution can still be obtained if we consider the following function

$$I_b(x) = x_1 - \frac{1}{2u_0}x_2^2.$$

This function I_b is an approximative first integral, in the sense that for big enough values of u_0 the state of the system defined by equations (2) and (3) evolves inside a tube of radius $o(1/u_0)$ around the curves defined by a constant I_b.

Small torque variations When u is small, none of the previous first integrals I_a and I_b can be used to approximate the evolution of the system. Nevertheless, for controls having the following particular form $u = u_0 x_2/R\omega$ with u_0 small enough, an approximative description of the system's evolution can still be obtained if we consider the following function

$$I_c(x) = x_2 + a\bar{\mu}(x_1) - u_0 x_1,$$

which is almost constant along the system's trajectories. Observe that the first integral I_a can be obtained from I_c, if we take $u_0 = 0$.

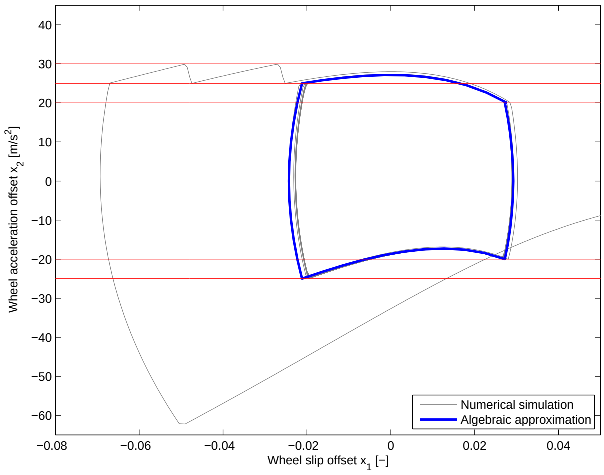

Figure 4: Limit cycle obtained for $\epsilon_1 = \epsilon_5 = 20$ and $\epsilon_3 = \epsilon_4 = 25$, in the case $u_5 = 200$, also with $\epsilon_2 = 33$ but with $u_3 = 10^5$ (obtained with the formulas of [1]).

2.2 The algorithm

The basic ABS regulation logics that we will consider is described on Figure 2. For this control law, the vertex of the graph on which the algorithm is evolving determines the control applied to the brakes. The transitions from one vertex of the graph to a different vertex are imposed by the guard conditions (the labels associated to each edge of the graph), which depend only on the value of the wheel deceleration offset $x_2 = R\omega - a_x$.

When a hybrid control law (based on the state of an automaton) is used to control a continuous dynamical system, the mathematical object that is obtained is called a *hybrid automaton*. We refer the reader to [4] for a detailed treatment of the mathematical questions related to this kind of objects ; and to our previous article [6] for a detailed analysis of the hybrid automaton generated by our control law.

2.3 Tuning the algorithm's parameters

Choosing the adequate values of the wheel deceleration thresholds ϵ_i is relatively easy. Firstly, some constraints are imposed on these parameters if we want to avoid the situations where the algorithm might block (see [6]). Secondly, choosing symmetric thresholds (that is, $\epsilon_5 = \epsilon_1$ and $\epsilon_4 = \epsilon_3$) usually gives better results and simplifies considerably the tuning of the control parameters u_i. Indeed, in the symmetric case, the stability of the cycles generated by the algorithm do not depend on u_1 nor on u_4.

Therefore, the only parameters that have to be tuned are ϵ_2, u_3, and u_5. To choice the optimal values of these parameters, two completely different methods are available (even though both methods are based on the analysis of limit cycles given by the

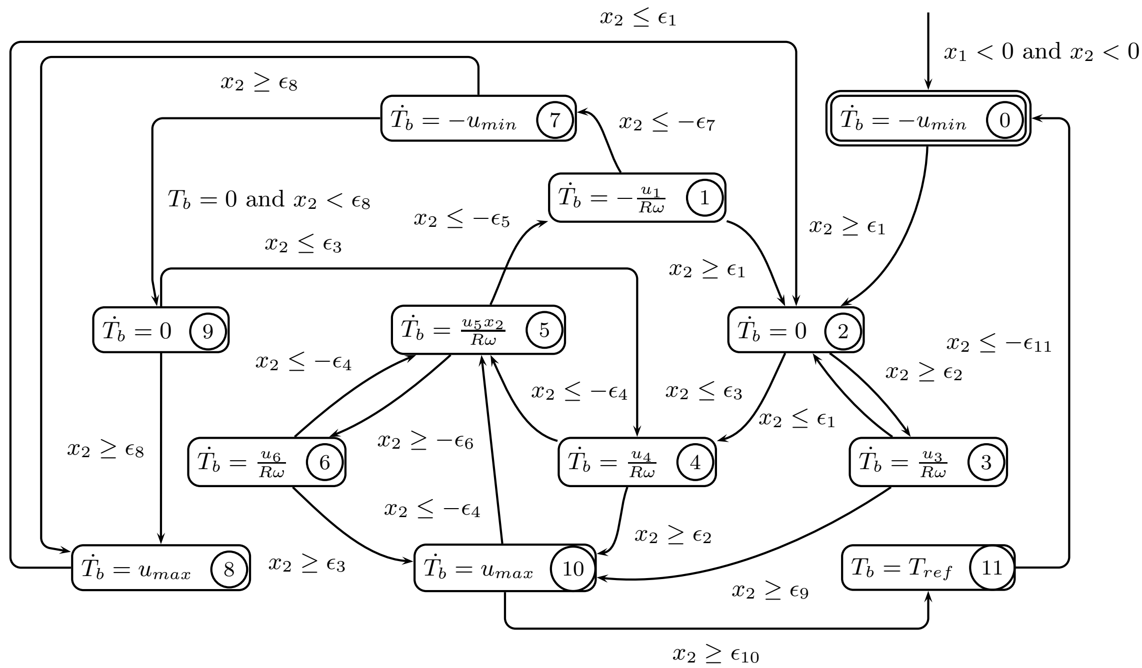

Figure 5: Our eleven-phase ABS regulation logics. The first five phases coincide with those of Figure 2.

Poincaré map of the system, which can be approximated using the first integrals I_a, I_b, and I_c).

A first method was proposed in [6]. It takes $u_5 = 0$. And the parameters ϵ_2 and u_3 are chosen in such a way that they minimise the amplitude of the generated limit cycle.

A second method is proposed in [1]. For high enough values of u_3, the limit cycle does not depend on u_3. Then, for any value of ϵ_2, there is a unique value of u_5 that minimizes the amplitude of the ABS limit cycle (see [1]). Since in this case the limit cycle has only four phases (like the algorithms of [3]), it does not depend on ϵ_2 (which thus can be tuned with the first method).

The limit cycles obtained using these two different methods are compared on Figures 3 and 4.

3 THE ELEVEN-PHASE ALGORITHM

Even though the five-phase strategy is robust with respect to variations of tyre characteristics, it is not robust with respect to discontinuous transitions of road characteristics (like, for example, a transition from a dry to a wet road). The aim of this section is to solve this problem, by introducing a new eleven-phase strategy, illustrated in Figure 5.

3.1 Changes of road characteristics

If at an instant t_0 there is a discontinuous change of tyre characteristics, the value of the friction coefficient $\mu(\cdot)$ will jump from $\mu_0^- = \mu(\lambda(t_0^-))$ to $\mu_0^+ = \mu(\lambda(t_0^+))$. And thus the value of the wheel acceleration offset x_2 will change from x_2^- to $x_2^+ \simeq x_2^- - \frac{R^2 F_z}{I} \left(\mu_0^+ - \mu_0^-\right)$. Though the wheel slip will still be a continuous function of time, the wheel slip offset might jump since the value of λ_0 can change for different road conditions.

3.2 From five to eleven phases

Robustness with respect to small variations of tyre characteristics can be easily obtained by adding a sixth phase to the algorithm (**Phase 6**). Indeed, depending on the phase during which the discontinuous transition happens, the five-phase strategy either works correctly or gets stuck in the stable zone during Phase 5 (a situation that cannot happen if the sixth-phase is added).

For larger variations of tyre characteristics, the main problem is to come back as soon as possible to a situation where only the first five phases will be active. In order to reach this goal, it is necessary to use the maximal and minimal torque variations allowed by the actuator, denoted by $\dot{T}_b = u_{max}$ and $\dot{T}_b = -u_{min}$, instead of a torque of the form $\dot{T}_b = \pm u_0/R\omega$.

The case $\mu_0^+ < \mu_0^-$, which corresponds to a transition from a dry to a wet or icy road, is handled by phases 7, 8, and 9. This situation is detected by the threshold $-\epsilon_7$. During **Phase 7** the brake torque is decreased as quickly as possible, until the threshold ϵ_8 is reached, which triggers **Phase 8** during which brake torque is increased at the maximal rate. Observe that unlike all other thresholds, the value of ϵ_8 depends on the road transition. It is given

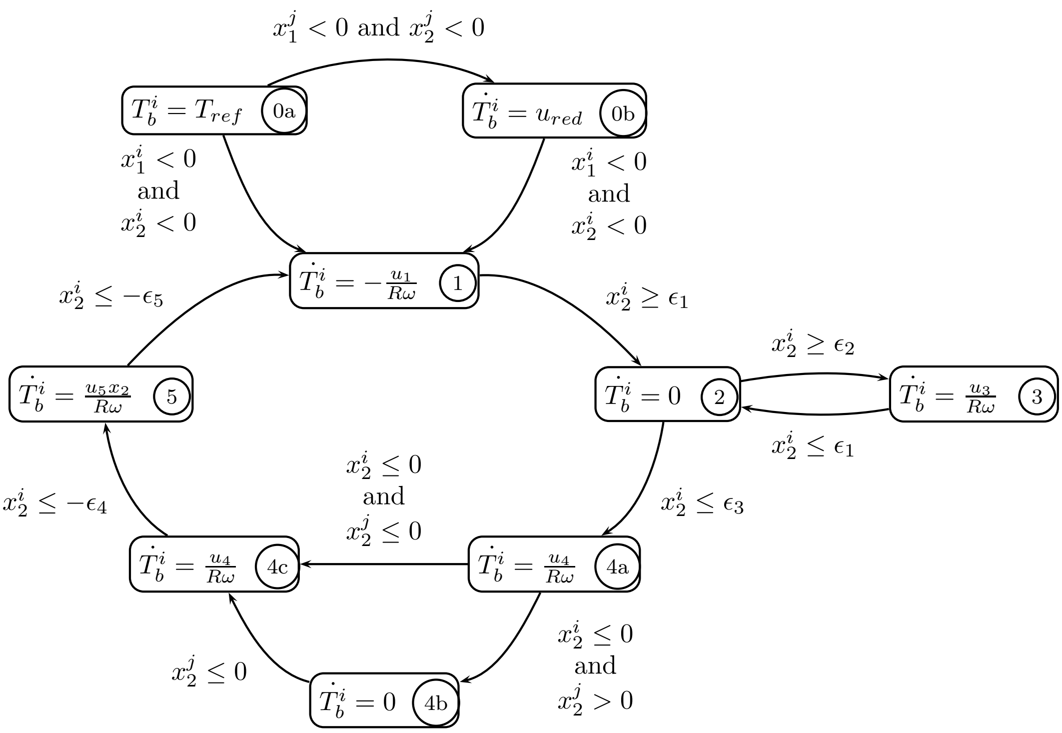

Figure 6: Synchronized ABS regulation logics for the front wheels. The index i represents the wheel to which the algorithm is applied and j the opposite wheel.

by

$$\epsilon_8 = \sqrt{\frac{u_{max} M(x_2)^2 + u_{min}\epsilon_1^2}{u_{max} + u_{min}}},$$

where $M(x_2)$ is the minimal value reached by the wheel acceleration offset x_2 during phase 7. Note that we must take $\epsilon_7 > \epsilon_5$, and that we obviously have $\epsilon_8 > \epsilon_1$. For a transition between a dry road and an icy road, the value of the applied brake torque might reach zero. In this case **Phase 9** is triggered until the threshold ϵ_8 or the stable zone of the tyre are reached.

The case $\mu_0^+ > \mu_0^-$, which corresponds to a transition from a wet or icy road to a dry road, is handled by **Phase 10**, during which brake torque is increased at the maximal rate ; and **Phase 11**, that just turns the ABS regulation off, if the transition is really severe.

4 SYNCHRONIZATION ALGORITHMS

4.1 Front-wheels Synchronization

When our original five-phase strategy is applied, independently, to two different wheels of the vehicle (for exemple, the front wheels), the difference between the applied brake torques has a random behavior. The two algorithms are not synchronized. Since the braking torques at the front wheels are not equal, their difference applies a force on the steering rack that propagates through the column up to the steering wheel. This has an undesirable effect : a torque perturbation on the steering wheel, which is unpleasant for most drivers.

In order to synchronize the ABS algorithms associated to the front wheels of the vehicle, we decompose the original fourth phase of the algorithm is three sub-phases (Figure 6). The first one (**Phase 4a**), does the same thing as in the five-phase strategy ; but is interrupted as soon as the wheel deceleration offset of the current wheel becomes negative $x_i \leq 0$. At this point, there are two cases. If the wheel deceleration offset of the opposite wheel is already negative $x_j \leq 0$, we do the same as in the five-phase strategy (**Phase 4c**). But if the wheel deceleration offset of the opposite wheel is still positive $x_j > 0$, then we “wait” in the same phase (by applying a constant brake torque) until it becomes also negative (**Phase 4b**).

4.2 Rear-wheels Anti-Synchronization

For the rear wheels of the vehicle, we apply the opposite strategy. Our aim is to anti-synchronize the rear wheels, in such a way that there is always one of the two rear wheels in the stable zone of the tyre (thus with a small wheel-slip), which can be useful for estimating the longitudinal velocity of the vehicle.

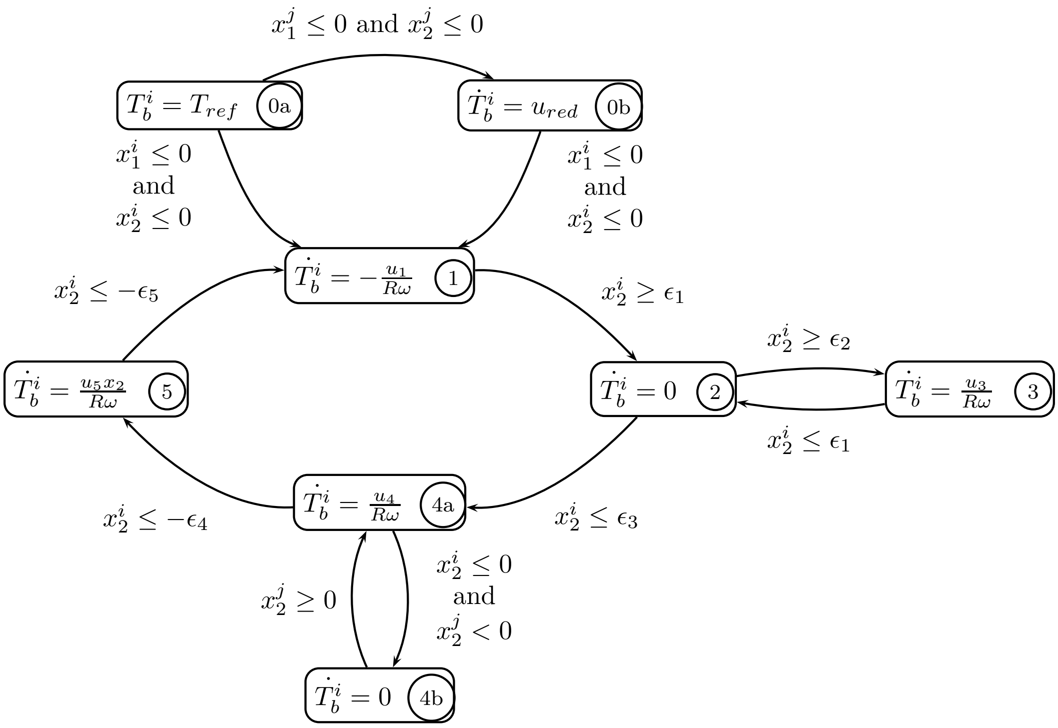

Figure 7: Anti-synchronized ABS regulation logics for the rear wheels. The index i represents the wheel to which the algorithm is applied and j the opposite wheel.

In order to anti-synchronize the ABS algorithms associated to the rear wheels of the vehicle, we decompose the original fourth phase of the algorithm is two sub-phases (Figure 7). The first one (**Phase 4a**), does the same thing as in the five-phase strategy ; but is interrupted as soon as the wheel deceleration offset of the current wheel becomes negative $x_i \leq 0$. At this point, there are two cases. If the wheel deceleration offset of the opposite wheel is already negative $x_j \leq 0$ then we "wait" in the same phase (by applying a constant brake torque) until it becomes positive (**Phase 4b**). Otherwise, the wheels are not synchronized, and thus we can continue the forth phase.

In fact, the result of our algorithm is that when a wheel is on the fourth phase (with a small wheel slip) is waits until the other arrives at the first phase (with a large wheel slip), in such a way that the two individual wheel algorithms are in phase opposition.

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