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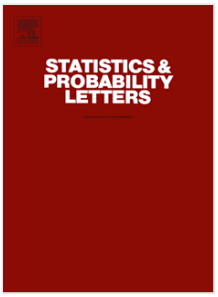
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The extremes of a random scenery as seen by a random walk in a random environment

Brice Franke* and Tatsuhiko Saigo†

April 28, 2008

Abstract

A random walk performs a motion in an iid-environment and observes an iid-scenery along its path. Assuming the scenery is in the domain of attraction of an extreme-value distribution, we prove a limit-theorem for the observed extremes. [1]

Keywords: extremes, random walk, random scenery, random environment

MSC: 60G70, 60G50

1 Introduction

The following model for a random walk in random environment can be found in the physical literature (see Anshelevic and Vologodskii (1980), Alexander et al. (1981), Kawazu and Kesten (1984)). The random

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environment is described by a family of positive iid random variables $\{\lambda_j; j \in \mathbb{Z}\}$ on a fixed probability space $(\Omega, \mathcal{F}, \mathbb{P})$. We denote by $\mathcal{A}$ the σ-algebra generated by the random variables $\{\lambda_j; j \in \mathbb{Z}\}$. Further, let $\{X(t); t \geq 0\}$ be a continuous-time random walk on $\mathbb{Z}$ with transition rates given by

$$\mathbb{P}(X(t+h) = j+1 | X(t) = j, \mathcal{A}) = \lambda_j h + o(h)$$

$$\mathbb{P}(X(t+h) = j-1 | X(t) = j, \mathcal{A}) = \lambda_{j-1} h + o(h)$$

$$\mathbb{P}(X(t+h) = j | X(t) = j, \mathcal{A}) = 1 - (\lambda_j + \lambda_{j-1})h + o(h)$$

as $h \to 0$. The resulting process is a birth-death process with random birth and death rates. It is a special case of a random walk in a random environment. The process $\{X(t); t \geq 0\}$ is symmetric in the sense that the probability of moving from j to $j+1$ is equal to that of moving from $j+1$ to j. This models the physical phenomenon that the permeability of the edge connecting the vertex j with the vertex $j+1$ does not depend on the direction of the motion. A lot of results have been obtained so far on the asymptotic behavior of the resulting random walk under suitable scaling. In this article we want to investigate how the resulting random walk explores the extremes of a random scenery.

Let $\{\xi(k); k \in \mathbb{Z}\}$ be a family of $\mathbb{R}$-valued iid random variables which are assumed to be independent of the process $\{X(t); t \geq 0\}$. The resulting stochastic process $\{\xi(X(t)); t \geq 0\}$ is called a random walk in random scenery in the literature.

In this letter we investigate the asymptotic behavior of the maxima

$$\Xi(t) := \max\{\xi(X(s)); 0 \leq s \leq t\}.$$

We assume that the distribution function F of the random variable $\xi(1)$ is in the domain of attraction of an extreme-value distribution G. Then there are two sequences $\{a_n > 0; n \in \mathbb{N}\}$ and $\{b_n \in \mathbb{R}; n \in \mathbb{N}\}$ such that for

$$M_n := \max\{\xi(1), ..., \xi(n)\}$$

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follows

$$\mathbb{P}\left((M_n - b_n)/a_n \leq x\right) = (F(a_n x + b_n))^n \longrightarrow G(x) \quad \text{as } n \to \infty.$$

It follows from the convergence of types theorem that the distribution G is either of Fréchet-, Gumbel- or Weibull-type (see Resnick (1987)). The characterization of the corresponding domains of attraction and suitable norming constants $\{a_n > 0; n \in \mathbb{N}\}$ and $\{b_n \in \mathbb{R}; n \in \mathbb{N}\}$ were investigated by Gnedenko (1943) and can be found in Resnick (1987). The distribution-function G can be used to define an extreme-value process $\{Z(t); t \geq 0\}$ with the following finite dimensional distributions

$$G_{t_1,\ldots,t_k}(x_1,\ldots,x_k) := G^{t_1}\left(\bigwedge_{i=1}^{k} x_i\right) G^{t_2-t_1}\left(\bigwedge_{i=2}^{k} x_i\right) \cdot \ldots \cdot G^{t_k-t_{k-1}}(x_k).$$

The process $\{Z(t); t > 0\}$ is a Markov process and has non-decreasing paths. Further, there exists a version of $\{Z(t); t \geq 0\}$ in the space

$$D(0,\infty) := \{\gamma : (0,\infty) \to \mathbb{R}; \gamma \text{ is right continuous and has left limits}\}.$$

It was proved by Lamperti (1964) that the rescaled maximum-process

$$Z^{(n)}(t) := (M_{[nt]} - b_n)/a_n = (\max\{\xi(1),\ldots,\xi([nt])\} - b_n)/a_n$$

converges in distribution toward Z with respect to the Skorohod-topology on $D(0,\infty)$ (see Resnick (1987) p.211).

Subsequently we will need some results on the random walk $\{X(t); t \geq 0\}$. The following convergence results are described in Kawazu and Kesten (1984):

W1. *If $c := \mathbb{E}[\lambda_0^{-1}] < \infty$, then for almost all environments the sequence of processes*

$$X^{(n)}(t) := n^{-1} X(n^2 t)$$

converges in distribution toward the process $\{c^{-1/2} B(t); t \geq 0\}$, where $\{B(t); t \geq 0\}$ is standard Brownian motion on $\mathbb{R}$. (see also Papanicolaou and Varadhan (1981))

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W2. *If there exists a slowly varying function L_1 such that*

$$\frac{1}{nL_1(n)}\sum_{j=1}^{n}\frac{1}{\lambda_j} \longrightarrow 1 \quad \textit{in probability as } n \to \infty,$$

then the sequence of processes

$$X^{(n)}(t) := \frac{1}{n}X(n^2 L_1(n)t)$$

converges in distribution toward the standard Brownian motion $\{B(t); t \geq 0\}$.

W3. *If there exists a slowly varying function L_2 such that the sequence of random variables*

$$\Lambda^{(n)} := \frac{1}{n^{1/\alpha}L_2(n)}\sum_{j=1}^{n}\frac{1}{\lambda_j}$$

converges in distribution toward a one-sided stable distribution Λ with index $0 < \alpha < 1$, then the sequence of processes

$$X^{(n)}(t) := \frac{1}{n}X(n^{(1+\alpha)/\alpha}L_2(n)t)$$

converges in distribution toward a continuous and self-similar process $\{X^*(t); t \geq 0\}$.

The self-similar process $\{X^*(t); t \geq 0\}$ in the previous statement can be constructed as follows:

To the stable distribution Λ corresponds a stable Lévy-subordinator $\{W^o(t); t \geq 0\}$ starting in zero. Let W^+ and W^- be two independent copies of W^o and define

$$W(t) := \begin{cases} W^+(t) & \text{if } t \geq 0 \\ W^-(-t) & \text{if } t < 0. \end{cases}$$

Let $\{B(\tau); \tau \geq 0\}$ be standard Brownian motion, which is independent of W^+ and W^-. We denote by $\{L(\tau, x); \tau \geq 0, x \in \mathbb{R}\}$ the local time of B and define

$$V_*(\tau) := \int_{\mathbb{R}} L(\tau, W(x))dx.$$

Since W and V_* are increasing we can define the following pseudo-inverse

$$W^{-1}(x) := \inf\{y \in \mathbb{R} : W(y) > x\} \quad \text{and} \quad V_*^{-1}(t) := \inf\{\tau \geq 0 : V_*(\tau) > t\}$$

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In Kawazu and Kesten (1984) the following representation for X^* was given

$$X^*(t) := W^{-1}(B(V_*^{-1}(t))).$$

All three situations which were studied by Kawazu and Kesten (1984) have in common the property that there exists a suitable increasing sequence $\{k_n; n \in \mathbb{N}\}$ and a suitable continuous process $\{Y(t); t \geq 0\}$ such that for $n \to \infty$ the sequence of processes

$$X^{(n)}(t) := \frac{1}{n} X(k_n t)$$

converges in distribution toward the process Y. Without loss of generality we assume that the processes Y and Z are independent. In the following we will use the following notation for the range of Y during the time interval $(t_1, t_2]$

$$Y(t_1, t_2] := \{Y(s); s \in (t_1, t_2]\}.$$

Moreover, we will denote by m the Lebesgue measure on $\mathbb{R}$. According to the three situations we will prove the following theorem in this letter.

Theorem 1 *The sequence*

$$\Xi^{(n)}(t) := (\max\{\xi(X(s)); 0 \leq s \leq k_n t\} - b_n)/\, a_n$$

converges in distribution toward the stochastic process

$$\{Z(m(Y(0, t])); t \geq 0\}.$$

Remarks: 1) We note that the results from Kawazu and Kesten (1984) were generalized in Kawazu (1989). He considered random walks in random environments defined by the following transition asymptotics

$$\mathbb{P}(X(t+h) = j+1 | X(t) = j, \mathcal{A}) = (\lambda_j/\eta_j)h + o(h)$$

$$\mathbb{P}(X(t+h) = j-1 | X(t) = j, \mathcal{A}) = (\lambda_{j-1}/\eta_j)h + o(h)$$

$$\mathbb{P}(X(t+h) = j | X(t) = j, \mathcal{A}) = 1 - ((\lambda_j + \lambda_{j-1})/\eta_j)h + o(h),$$

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where $\{\eta_j; j \in \mathbb{N}\}$ is an iid family of positive random variables satisfying suitable assumptions. It is not difficult to see that our results can be generalized to this situation after some straight forward modification of the scaling.

2) The process $\{\xi(X(s)); s \geq 0\}$ is stationary. There exist general results on the extremes of stationary sequences (see Leadbetter et al. (1983)). However those results need some special mixing-conditions called $D(u_n)$ and $D'(u_n)$. It can be proved that the condition $D(u_n)$ does not hold for the sequence $\{\xi(X(k_n)); n \in \mathbb{N}\}$ with $u_n = a_n x - b_n$.

3) In Franke and Saigo (2007) we proved similar theorems for a situation which is related to classical random walks $\{S_n; n \in \mathbb{N}\}$. The proofs presented there are based on random point processes and can be generalized to cover the present situation. However, the proof presented in the next section is more elementary and therefore interesting for a larger audience.

4) We remark that the distribution of $Z(m(Y(0,1])))$ has the following expression

$$\mathbb{P}(Z(m(Y(0,1]))) \leq x) = \mathbb{E}\left[(G(x))^{m(Y(0,1])}\right] = \int_0^\infty (G(x))^t \mathbb{P}_{m(Y(0,1])}(dt).$$

If we use the known expressions for the extreme-value distribution G, we see that the limit-distributions are mixtures of extreme-value distributions. Those distributions are in general not extreme-value distributions.

2 The Proof of Theorem 1

Obviously the distribution of $\Xi(t) = \max\{\xi(X(s)); 0 \leq s \leq t\}$ depends on the range of the underlying random walk $\{X(t); t \geq 0\}$. We therefore have to understand the asymptotic behavior of the process

$$R(t) := \mathrm{card}\{X(s); 0 \leq s \leq t\}.$$

Lemma 1 *For all $M \in \mathbb{N}$ and $0 < t_1 < ... < t_M$ the joint distributions of $(R(k_n t_1)/n, ..., R(k_n t_M)/n)$ converge in distribution toward the random vector $(m(Y(0,t_1]), ..., m(Y(0,t_M])))$.*

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Proof: Since the random walk $\{X(t); t \geq 0\}$ makes only jumps of size one, we immediately see that

$$R(t) = 1 + \max\{X(s); s \leq t\} + \min\{X(s); s \leq t\}.$$

Since the functions max and min are continuous with respect to the Skorohod-topology, we can use the continuous mapping theorem and the statements (W1), (W2) and (W3) to prove the convergence of $R(k_n t)/n$ toward

$$m(Y(0,t]) = \max\{Y(s); s \leq t\} + \min\{Y(s); s \leq t\}.$$

The same arguments prove the joint convergence of the vector-components. □

For $k \in \mathbb{N}$ we define the sequence of random variables

$$\tau_k := \inf\{t \geq 0 : R(t) \geq k\}.$$

The random variable τ_k is the waiting times until the process X has visited k sites. The number of new sites visited by $X^{(n)}$ during the time-interval $(t_1, t_2]$ is equal to

$$Q^{(n)}(t_1, t_2) := \text{card}\{X(s); k_n t_1 < s \leq k_n t_2, X(s) \notin \{X(r); r < s\}\}.$$

The process $X^{(n)}$ visits a new site during the time-interval $(t_1, t_2]$ if and only if there exists an integer l such that $\tau_l/k_n \in (t_1, t_2]$. This implies the following identity

$$Q^{(n)}(t_1, t_2) \quad = \quad \sum_l \mathbb{1}_{(t_1,t_2]}(\tau_l/k_n) = R(k_n t_2) - R(k_n t_1).$$

The following lemma states that conditioned on $\tau_1, ..., \tau_k$ the distribution of $\xi(X(\tau_1)), ..., \xi(X(\tau_k))$ equals the distribution of $\xi(1), ..., \xi(k)$.

Lemma 2 *For all $L \in \mathbb{N}$ and all measurable sets $B_k, A_k \subset \mathbb{R}, 1 \leq k \leq L$ one has that*

$$\mathbb{P}(\tau_k \in B_k, \xi(X(\tau_k)) \in A_k, 1 \leq k \leq L) = \mathbb{P}(\xi(k) \in A_k, 1 \leq k \leq L)\mathbb{P}(\tau_k \in B_k, 1 \leq k \leq L).$$

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Proof: We use the independence of the random walk and the scenery to prove

$$\begin{aligned}
&\mathbb{P}(\tau_1 \in B_1, \tau_2 \in B_2, \xi(X(\tau_1)) \in A_1, \xi(X(\tau_2)) \in A_2) \\
&= \sum_z \sum_{z' \neq z} \mathbb{P}(X(\tau_1) = z, \xi(z) \in A_1, \tau_1 \in B_1, X(\tau_2) = z', \xi(z') \in A_2, \tau_2 \in B_2) \\
&= \sum_z \sum_{z' \neq z} \mathbb{P}(X(\tau_1) = z, \tau_1 \in B_1, X(\tau_2) = z', \tau_2 \in B_2) \mathbb{P}(\xi(1) \in A_1, \xi(2) \in A_2) \\
&= \mathbb{P}(\tau_1 \in B_1, \tau_2 \in B_2) \mathbb{P}(\xi(1) \in A_1, \xi(2) \in A_2).
\end{aligned}$$

The general case follows similarly. □

For $t_1 < t_2$ we define the random variables $\Xi(t_1, t_2) := \max\{\xi(X(s)); t_1 < s \leq t_2\}$ and $R(t_1, t_2) := R(t_2) - R(t_1)$. The following lemma holds.

Lemma 3 *For all $k \in \mathbb{N}$, $x_1, ..., x_k \in \mathbb{R}$ and $t_1 < t_2 < ... < t_k$ we have that*

$$\mathbb{P}(\Xi(t_1) \leq x_1, ..., \Xi(t_k) \leq x_k) = \mathbb{E}\left[\left(F\left(x_1 \vee ... \vee x_k\right)\right)^{R(t_1)} \left(F\left(x_2 \vee ... \vee x_k\right)\right)^{R(t_1,t_2)} \cdot ... \cdot \left(F(x_k)\right)^{R(t_{k-1},t_k)} \right].$$

Proof: As before we show the case $k = 2$. The general case follows by the same arguments. We have

$$\begin{aligned}
\mathbb{P}(\Xi(t_1) \leq x_1, \Xi(t_2) \leq x_2) &= \mathbb{P}(\Xi(t_1) \leq x_1 \vee x_2, \Xi(t_1, t_2) \leq x_2) \\
&= \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} \mathbb{P}(\Xi(t_1) \leq x_1 \vee x_2, \Xi(t_1, t_2) \leq x_2, R(t_1) = m_1, R(t_1, t_2) = m_2) \\
&= \sum_{m_1=1}^{\infty} \sum_{m_2=1}^{\infty} (F(x_1 \vee x_2))^{m_1} (F(x_2))^{m_2} \mathbb{P}(R(t_1) = m_1, R(t_1, t_2) = m_2) \\
&= \mathbb{E}\left[(F(x_1 \vee x_2))^{R(t_1)} (F(x_2))^{R(t_1,t_2)} \right].
\end{aligned}$$

Here we have used the previous lemma and the fact that on the set

$$\{\omega \in \Omega; R(t_1, t_2) = m_2, R(t_1) = m_1\}$$

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the statement $\Xi(t_1,t_2) \leq x_2$ is equivalent to $\max\{\xi(X(\tau_i)); m_1 < i \leq m_1+m_2\} \leq x_2$. □

Proposition 1 *The finite-dimensional distributions of the processes* $\{\Xi^{(n)}(t); t \geq 0\}$ *converge toward the finite-dimensional distributions of* $\{Z(m(Y(0,t]); t \geq 0\}$.

Proof: We first note that for $u_n = a_n x + b_n$ we have

$$(F(u_n))^n \longrightarrow G(x) \quad \text{as } n \to \infty.$$

By Lemma 1 the random variables $R_1^{(n)} := n^{-1}R(k_n t)$ and $R_2^{(n)} := n^{-1}R(k_n t_1, k_n t_2)$ converge jointly in distribution toward the random variables $R_1 := m(Y(0,t_1])$ and $R_2 := m(Y(0,t_2]) - m(Y(0,t_1])$. Therefore, there exist for $i \in \{1,2\}$ random variables $\tilde{R}_i^{(n)}$ and $\tilde{R}_i$ with the same distribution as $R_i^{(n)}$ resp. R_i such that $\tilde{R}_i^{(n)}$ converges toward $\tilde{R}_i$ almost surely (see Billingsley (1986) p.343). It then follows that

$$((F(u_n))^n)^{\tilde{R}_i^{(n)}} \longrightarrow (G(x))^{\tilde{R}_i} \quad \mathbb{P}-\text{almost surely as } n \to \infty.$$

Moreover, by Lemma 3 it follows that

$$\begin{aligned}
\mathbb{P}(\Xi^{(n)}(t_1) \leq x_1, \Xi^{(n)}(t_2) \leq x_2) &= \mathbb{P}(\Xi(k_n t_1) \leq a_n x_1 - b_n, \Xi(k_n t_2) \leq a_n x_2 - b_n) \\
&= \mathbb{E}\left[(F(a_n(x_1 \vee x_2) - b_n))^{R(k_n t_1)} (F(a_n(x_2 - b_n))^{R(k_n t_1, k_n t_2)}\right] \\
&= \mathbb{E}\left[(F(a_n(x_1 \vee x_2) - b_n))^{n\tilde{R}_1^{(n)}} (F(a_n x_2 - b_n))^{n\tilde{R}_2^{(n)}}\right].
\end{aligned}$$

The dominated convergence theorem then yields

$$\mathbb{P}(\Xi^{(n)}(t_1) \leq x_1, \Xi^{(n)}(t_2) \leq x_2) \longrightarrow \mathbb{E}\left[(G(x_1 \vee x_2))^{\tilde{R}_1} (G(x_2))^{\tilde{R}_2}\right] \quad \text{as } n \to \infty.$$

The proposition now follows from the definition of the extreme-value process Z, which yields the identity

$$\mathbb{P}(Z(m(Y(0,t_1])) \leq x_1, Z(m(Y(0,t_2])) \leq x_2) = \mathbb{E}\left[(G(x_1 \vee x_2))^{R_1} (G(x_2))^{R_2}\right].$$

Of course the general statement holds with the same arguments. □

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Proof of Theorem 1: It remains to prove that the sequence $\{\Xi^{(n)}; n \in \mathbb{N}\}$ is tight in the J_1-topology on $D(0,\infty)$. We first note that the processes $\{\Xi^{(n)}(t); t \geq 0\}$ has the same distributions as the process $\left\{Z^{(n)}\left(R^{(n)}(t)\right); t \geq 0\right\}$, where $R^{(n)}(t) := \frac{1}{n}R(k_n t)$. Then it is sufficient to prove the tightness for the process $Z^{(n)} \circ R^{(n)}$.

To see the equivalence of the two sequences we note that by Lemma 3

$$\begin{aligned}
\mathbb{P}(\Xi^{(n)}(t_1) \leq x_1, \Xi^{(n)}(t_2) \leq x_2) &= \mathbb{P}(\Xi(k_n t_1) \leq a_n x_1 + b_n, \Xi(k_n t_2) \leq a_n x_2 + b_n) \\
&= \mathbb{E}\left[(F(a_n(x_1 \vee x_2) + b_n))^{R(k_n t_1)}(F(a_n x_2 + b_n))^{R(k_n t_1, k_n t_2)}\right] \\
&= \mathbb{E}\left[(F(a_n(x_1 \vee x_2) + b_n))^{nR^{(n)}(t_1)}(F(a_n x_2 + b_n))^{nR^{(n)}(t_1, t_2)}\right] \\
&= \mathbb{P}(Z^{(n)}(R^{(n)}(t_1)) \leq x_1 \vee x_2, Z^{(n)}(R^{(n)}(t_2)) \leq x_2).
\end{aligned}$$

Now we prove the tightness of the process $Z^{(n)} \circ R^{(n)}$. We use a criterion for tightness which can be found in Stone (1963). For a given D-valued process W, $N > 0$ and $c > 0$ we define the random variable

$$\Delta^{(2)}_{N,c}(W) := \sup_{0 \leq t_1 < t < t_2 \leq N:\ t_2 - t_1 \leq c} \min\left(|W(t_1) - W(t)|, |W(t) - W(t_2)|\right).$$

Further, we define the random variables

$$\Delta^{(1)}_{N,c}(W) := \sup_{0 \leq t_1 < t_2 \leq N:\ t_2 - t_1 \leq c} |W(t_2) - W(t_1)|$$

and

$$\Delta^{(0)}_{N}(W) := \sup_{0 \leq t \leq N} |W(t)|.$$

The criterion proved by Stone (1963) states that $Z^{(n)} \circ R^{(n)}$ is tight in $D(0,\infty)$, if and only if for all $\epsilon > 0$ and $N > 0$ one has

$$\mathbb{P}\left(\Delta^{(2)}_{N,c}(Z^{(n)} \circ R^{(n)}) > \epsilon\right) \longrightarrow 0 \quad \text{as } n \to \infty \text{ and then } c \to 0.$$

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The statement

$$\Delta_{N,c}^{(2)}\left(Z^{(n)} \circ R^{(n)}\right) > \epsilon$$

implies for all $\mu > 0$ and $M > 0$

$$\Delta_{N,c}^{(1)}\left(R^{(n)}\right) > \mu \quad \text{or} \quad \Delta_{M,\mu}^{(2)}\left(Z^{(n)}\right) > \epsilon \quad \text{or} \quad \Delta_{N}^{(0)}(R^{(n)}) > M.$$

Since $R^{(n)}$ converges in distribution toward the process $R(t) := m(Y(0,t])$, we can find for a given $\delta > 0$ a suitable $M > 0$ such that

$$\mathbb{P}\left(\Delta_{N}^{(0)}(R^{(n)}) > M\right) \leq \delta \quad \text{for all } n \in \mathbb{N}.$$

Further, since the process R is continuous, we have

$$\mathbb{P}\left(\Delta_{N,c}^{(1)}\left(R^{(n)}\right) > \mu\right) \longrightarrow 0 \ \text{ as } n \to \infty \ \text{ and then } c \to 0.$$

Moreover, the weak convergence of $\{Z^{(n)}; n \in \mathbb{N}\}$ was proved in Lamperti (1964). Therefore, we have that

$$\mathbb{P}\left(\Delta_{M,\mu}^{(2)}\left(Z^{(n)}\right) > \epsilon\right) \longrightarrow 0 \ \text{ as } n \to \infty \ \text{ and then } \mu \to 0.$$

The result now follows from those considerations. □

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3 References

Alexander S., Bernasconi J., Schneider W., Orbach R. (1981), Excitation dynamics in random one-dimensional systems, *Rev. Mod. Phys.*, **53**, 175-198.

Anshelevic V.V., Vologodskii A.V. (1980), Laplace operator and random walk on one-dimensional nonhomogenious lattice, *J. Stat. Phys.*, **25**, 419-430.

ACCEPTED MANUSCRIPT

Billingsley P. (1986): *Probability and measure*, Second edition, Wiley Series in Probability and Mathematical Statistics: Probability, New York.

Franke B., Saigo T. (2007): The extremes of random walks in random scenery, manuscript.

Gnedenko B. (1943): Sur la distribution limité du terme d'une série aléatoire, *Ann. Math.*, **44**, 423-453.

Kawazu K., Kesten H. (1984): On birth and death processes in symmetric random environment, *J. Stat. Phys.*, **37** 561-575.

Kawazu K. (1989): A one-dimensional birth and death process in random environment, *Japan J. Appl. Math.*, **6**, 97-109.

Kesten H., Spitzer F. (1979): A limit theorem related to a new class of self-similar processes, *Z. Wahrsch. Verw. Gebiete*, **50**, 5-25.

Lamperti J. (1964): On extreme order statistics, *Ann. Math. Stat.*, **4**, 1726-1737.

Leadbetter M., Lindgren G., Rootzén H. (1983): *Extremes and Related Properties of Random Sequences and Processes*, Applied Probability, Springer-Verlag, New-York.

Papanicolaou G., Varadhan S.R.S. (1981): Boundary value problems with rapidly oscillating random coefficients, *Coll. Math. Soc. János Bolyai, 27, Random Fields*, **2**, North-Holland, Amsterdam, 835-873.

Resnick S. (1987): *Extreme Values, Regular Variation and Point Processes*, Applied Probability, Springer-Verlag, New-York.

Stone C. (1963): Weak convergence of stochastic processes defined on semi-infinite time-intervals, *Proc. Amer. Math. Soc.*, **14**, 694-696.

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