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Piezoelectromechanical structures: new trends towards the multimodal passive vibration control

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ABSTRACT

An intelligent control system has the ability to learn about its environment, process the information to reduce uncertainty, plan, generate and execute actions to either control or reduce to a minimum the undesired motion of all or some of its parts. It generally incorporates sensors, actuators, a controller and a power supply unit. Most of the previous work has focused on active control in which electric power is supplied to the actuators that exert actions on the host structure to suppress its vibrations. Alternatively, undesired mechanical energy of a host structure could be converted into electrical energy that can be dissipated through a set of resistor. This does not require an external power unit and is a more economical means of controlling vibrations of a structure, but an effective transduction of mechanical energy into electric energy has to be guaranteed. Such an effective transduction can be achieved imposing to the electric controller to be resonant at all the mechanical resonance frequencies, and to mimic all the mechanical modal shapes, i.e. to be the analog of the host structure. In this paper we synthesize a completely passive electric circuit analog to an Euler beam, aimed for distributed vibration control.

Keywords: vibration damping, piezoelectric transducers, distributed control, synthesis of electric network, smart structures.

1. INTRODUCTION

Modern structures are subject to extensive vibrations that can reduce structural life and contribute to mechanical failure. Piezoelectric transducers in conjunction with appropriate circuitry, can be used as a mechanical energy dissipation device.

The direct and inverse piezoelectric effect has been discovered by the Curie brothers (Pierre and Jacques Curie) in 1880 and mathematically modeled by Voigt (for a brief description of the piezoelectric effect see e.g. IEEE[1]). Its technological applications were limited for a long period by the physical properties of piezoelectric ceramics. However more recently (in the last two decades) more performing transducers based on the piezoelectric effect were conceived (for more details on the novel concepts and architectures used in the design of piezoelectric transducers see e.g. Niezreski et al.[2]) and made them available for technical and industrial applications such as active and passive control of vibrations of flexible structures.

Among the first applications conceived for the developed family of transducers in the field of active vibrations control there was the so called "electronic damping" (see e.g. Forward and Swigert,[3] and Hanagud et al.[4]): a set of piezoelectric devices is placed on a host structure to sense and control the mechanical vibrations. The deformation of the sensing transducers results in electrical signals, which are conditioned by suitably designed feedback electronics and then applied to actuating transducers; while, the actuators convert the applied electrical energy into mechanical energy, transmitting mechanical control actions to the host structure. Such a concept proved to be effective as the available actuators can exert forces of several hundreds of newton as a response of voltage signals of several hundreds of volt, without losing their dielectric properties or undergoing destructive

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strain deformations. The common features of the control devices described in the framework of "electronic damping" are represented by

- the differentiation of the sensing and the actuation systems;
- the localization of PZT actuators at limited, selected sites of the vibrating structure.

Both of these features are limits to control efficiency. The first one implies the need for a coordinating active system that controls the actuators action in response to the input from the sensors. The second feature implies an optimal localization problem (for both actuators and sensors), the solution of which depends on the particular mechanical vibration mode to be considered. When active control systems are used, the piezoelectric transducers driving requires complex power-amplifiers and associated precise sensing electronics, resulting into the consumption of a significant amount of electrical power. Furthermore, the presence of an active controller can cause instability in the closed loop system, the plant of which is naturally passive (the vibrating structure). Spillover phenomena can also be introduced, inducing dramatic oscillations of the structure at high frequencies.

An interesting development of "electronic damping" has been proposed in Kader et al.,[5] where the design of optimal distributed electronic active controllers is addressed. The outlined approach in Kader et al.[5] includes a distributed array of piezoelectric elements uniformly positioned over a host structure and a distributed interconnecting (active) electronic circuit. The piezoelectric layer is employed to measure (sense) the deformation of the flexible structure and to exert a continuous action at every point. The distributed electronic circuit is aimed to extract from the sensor signals the complete state of the plant, to optimally condition this signals and to feed the actuators at high voltage. The resulting smart structure is able to efficiently suppress mechanical vibrations induced by broadband disturbance, nevertheless the intrinsic active nature of the controller and the complexity of the required circuitry drastically limits is technical feasibility and exploitation in industrial applications.

In Hagood and von Flotow[6] the possibility of damping mechanical vibrations by means of a piezoelectric transducer positioned on a structural element and shunted with completely passive electrical circuits is investigated. In particular two different shunting circuits are considered: a resistive (R) and a resonant (LR) one. By placing such an electrical impedance across the terminals of the piezoelectric transducer, the passive network is capable of damping structural vibrations. If a simple resistor is placed across the terminals of the transducer, the piezoelectric element will act as a viscoelastic damper. If the network consists of a an inductor-resistor circuit, the passive network combined with the inherent capacitance of the piezoelectric transducer creates a damped electromechanical beating. The resonance can be tuned so that the piezoelectric element acts as a tuned vibration absorber (paralleling the analysis of mechanical vibration absorbers exposed in Den Hartog[7]). The method proposed in Hagood and von Flotow[6] allows for an efficient single-mode control of structural vibrations, whenever the resonant circuit is tuned to the mechanical mode to be suppressed; nevertheless the efficiency of the electromechanical coupling strongly depends on the position of the transducer over the host structure. Moreover the technical feasibility of the passive piezoelectric controller proposed in Hagood and von Flotow[6] is limited, since too large inductors are required to produce low-frequency electrical resonance with the small inherent capacitance of the piezoelectric transducer. A lot of efforts have been devoted to simulate huge inductors by means of active electronic circuits, in particular in Fleming et al.[8] an implementation method using a digital signal processor is presented, while in Keun and In[9] an analog realization exploiting operational amplifiers and multipliers is addressed.

A generalization of the "piezoelectric shunt damping" technique proposed in Hagood and von Flotow[6] to the multimodal control of vibrations is presented in Hollkamp[10] and Fleming et al.,[11] where a complicated impedance is placed across the terminals of the piezoelectric transducer. Such a control methodology seems to present severe inconveniences, indeed the used inductances are still very high and the damping efficiency even for few modes is drastically reduced.

In order to cure these drawbacks without forsaking the advantages featured by passive control, in dell'Isola and Vidoli[12] it is proposed to position on the host structure an array of piezoelectric elements and to interconnect the electric terminals of each pair of adjacent transducers via a floating RL impedance, conceiving an archetype of a so called "piezoelectromechanical" structure. The main advantage of this strategy consists in the drastic reduction of employed optimal inductances, thus, making conceivable the realization of a truly passive network. The basic idea behind this strategy is to provide a synthetic support for low-speed electric signals, to be effectively coupled to mechanical waves. Nevertheless, in this framework a multimodal control is not guaranteed and it is only possible to effectively damp one particular vibration mode.

2. PROBLEM STATEMENT AND OBJECTIVE

The optimization problem of finding the best distributed passive electric network, to be piezoelectrically coupled to the vibrating host structure, to achieve the most effective multimodal energy transduction has been addressed for the first time in Vidoli and dell'Isola.[13] In Vidoli and dell'Isola,[13] the authors prove that in order to guarantee the maximum energy transfer between the mechanical and electric systems they should be governed by the same partial differential equations, i.e. the sought optimal distributed network should be the electric analogue of the host structure. Therefore, the so found electrical circuit exhibit the same modal characteristics as those characterizing the host structure, so a multiresonance electromechanical coupling can be settled. Nevertheless in Vidoli and dell'Isola,[13] no attention is paid towards the suppression of structural vibrations. In dell'Isola et al.[16] the optimization problem is tackled in the framework of electromechanical wave propagation in a smart beam, concerning the search of a distributed electric controller aimed for an optimal attenuation of wave propagation over any frequency range. The results presented in dell'Isola et al.[16] extend those showed Vidoli and dell'Isola,[13] assessing that the optimal continuous network for beam vibration damping should be governed by the same partial differential equations of the vibrating structure (i.e. elastica equation), but it should be endowed with an internal dissipation proportional to the rate of change of the electric curvature.

The problem of synthesizing a completely passive lumped electrical circuit governed by a discrete approximation of the elastica equation has been extensively analyzed in Alessandroni et al.[14]; nevertheless the proposed circuits have stern practical inconvenience, either due to negative inductors or multiport transformers: the former are typical active elements needing to be electronically simulated, the latter are in general very heavy and their weights can represent even a significant part of the mass of the overall smart structure. For a critical analysis of this control technique and comparisons among the electric controllers proposed in dell'Isola and Vidoli[12] and Alessandroni et al.[14] see Maurini et al..[15]

In this paper we will find a completely passive lumped electric circuit analog to a vibrating beam, which is constituted only by inductors, capacitors and elementary two port transformers. The proposed electric circuit will be synthesized following the following design steps:

- finite difference discretization of the constitutive and balance equations for a Timoshenko beam,
- mobility representation of a beam element,
- synthesis of a four port grounded circuit, the admittance matrix of which parallels the beam element mobility matrix,
- cascade connection of the so found networks to simulate the whole beam,
- neglect of beam shear deformability and rotatory inertia terms in the electric analogue to achieve the Euler beam electric analogue.

Once the beam electric analogue has been designed, suitably resistors will be placed in the circuit to guarantee the most efficient multimodal damping, as established in dell'Isola et al..[16]

3. FINITE DIFFERENCE APPROXIMATION OF THE GOVERNING EQUATIONS OF A TIMOSHENKO BEAM

The governing equations for the vibrations of a Timoshenko beam are:

$$\left\{ \begin{array}{c} M' + T = \Im\,\dot{\Omega} \\ T' = \rho\,\dot{S} \end{array} \right. , \qquad \left\{ \begin{array}{c} \dot{M} = K_M\,\Omega' \\ \dot{T} = K_T\,(S' - \Omega) \end{array} \right. , \tag{1}$$

where M indicates the bending moment, T the shear contact action, S the deflection velocity, Ω the angular velocity of the cross sections, $\Im$ the cross section moment of inertia, ρ the mass density per unit length, K_M the bending stiffness, K_T the shear stiffness, and the superscripts dot and prime denote respectively time and space derivatives. The first two partial differential equations in (1) indicate the couples and shear contact actions balance equations respectively, while the other two express the assumed linear constitutive equations. In order to derive an analog circuit for the beam, let us nondimensionalize the aforementioned governing equations introducing the scaling parameters M_0, T_0, t_0, u_0, ϑ_0 and l. Hence, the dimensionless set of governing equations becomes:

$$\left\{\begin{array}{c} \dfrac{M_0}{l}m' + T_0 t = \dfrac{\vartheta_0}{t_0^2}\Im\,\dot{\omega} \\ \dfrac{T_0}{l}t' = \dfrac{u_0}{t_0^2}\rho\,\dot{s} \end{array}\right. , \qquad \left\{\begin{array}{c} M_0\dot{m} = \dfrac{\vartheta_0}{l}K_M\,\omega' \\ T_0\dot{t} = K_T\,(\dfrac{u_0}{l}s' - \vartheta_0\omega) \end{array}\right. , \tag{2}$$

where each variable has been nondimensionalized.

Introducing a suitable finite differences approximation for the previous set of equations with respect of the space variable, it is straightforward to achieve the following set of first order ordinary differential equations:

$$\left\{\begin{array}{c} \dfrac{M_0}{l}\dfrac{m_{i+1}-m_i}{\delta} + T_0 t_{i+1} = \dfrac{\vartheta_0}{t_0^2}\Im\,\dot{\omega}_{i+1} \\ \dfrac{T_0}{l}\dfrac{t_{i+1}-t_i}{\delta} = \dfrac{u_0}{t_0^2}\rho\,\dot{s}_i \end{array}\right. , \qquad \left\{\begin{array}{c} M_0\dot{m}_i = \dfrac{\vartheta_0}{l}K_M\,\dfrac{\omega_{i+1}-\omega_i}{\delta} \\ T_0\dot{t}_{i+1} = K_T\,(\dfrac{u_0}{l}\dfrac{s_{i+1}-s_i}{\delta} - \vartheta_0\omega_{i+1}) \end{array}\right. , \tag{3}$$

where δ indicate the dimensionless spatial sampling step. Let us explicitly remark that the adopted finite differences schemes alternate between the forward and the backward rule; this mixed approach will permit us to obtain symmetric higher order schemes when dealing with discrete governing equations expressed in terms of the only kinematical descriptors.

4. SYNTHESIS OF THE ELECTRIC ANALOG OF A BEAM ELEMENT

Once a finite differences approximation for the impedance matrix of a beam element has been found one well established synthesis technique, see Alessadroni et al.,[14] requires to parallel the velocity with the voltage, and the contact actions with the currents, i.e. to regard the velocities at the beam element edges as *across* variables and the contact actions as *through* variables.

In particular the correspondence between the mechanical variables in (3) and the electrical variables describing the analog network is:

$$\left\{\begin{array}{c} (V_1, I_1) \longleftrightarrow \left(\dfrac{V_0 t_0}{\vartheta_0}\Omega_i, -\dfrac{I_0}{M_0}M_i\right) \\ (V_2, I_2) \longleftrightarrow \left(\dfrac{V_0 t_0}{u_0}S_i, -\dfrac{I_0}{T_0}T_i\right) \end{array}\right. , \qquad \left\{\begin{array}{c} (V_3, I_3) \longleftrightarrow \left(\dfrac{V_0 t_0}{\vartheta_0}\Omega_{i+1}, \dfrac{I_0}{M_0}M_{i+1}\right) \\ (V_4, I_4) \longleftrightarrow \left(\dfrac{V_0 t_0}{u_0}S_{i+1}, \dfrac{I_0}{T_0}T_{i+1}\right) \end{array}\right. . \tag{4}$$

where V_0 and I_0 denote, respectively, a characteristic voltage and current. By means of this analogy the impedance matrix representation for the beam element parallels the admittance matrix representation for the analog four port grounded network. The dimensionless mechanical impedance matrix of a beam element is defined by:

$$\left[\begin{array}{c} -\widetilde{m}_i \\ -\widetilde{t}_i \\ \widetilde{m}_{i+1} \\ \widetilde{t}_{i+1} \end{array}\right] = \mathbf{z}^m(\eta)\left[\begin{array}{c} \widetilde{\omega}_i \\ \widetilde{v}_i \\ \widetilde{\omega}_{i+1} \\ \widetilde{v}_{i+1} \end{array}\right],$$

where the $\widetilde{\bullet}$ denotes a bilateral Laplace transform and η denotes the dimensionless Laplace variable.

From Eqn. (3) it is immediate to obtain:

$$\mathbf{z}^m(\eta) = \begin{bmatrix} \frac{1}{\eta}\frac{K_M\vartheta_0}{(\delta l)M_0} & 0 & -\frac{1}{\eta}\frac{K_M\vartheta_0}{(\delta l)M_0} & 0 \\ 0 & \frac{1}{\eta}\frac{K_T u_0}{(\delta l)T_0} + \eta\frac{\rho u_0(\delta l)}{T_0 t_0^2} & \frac{1}{\eta}\frac{K_T\vartheta_0}{T_0} & -\frac{1}{\eta}\frac{K_T u_0}{T_0(\delta l)} \\ -\frac{1}{\eta}\frac{K_M\vartheta_0}{(\delta l)M_0} & \frac{1}{\eta}\frac{K_T u_0}{M_0} & \frac{1}{\eta}\left(\frac{K_M\vartheta_0}{(\delta l)M_0} + \frac{K_T\vartheta_0(\delta l)}{M_0}\right) + \eta\frac{\Im\,\vartheta_0(\delta l)}{M_0 t_0^2} & -\frac{1}{\eta}\frac{K_T u_0}{M_0} \\ 0 & -\frac{1}{\eta}\frac{K_T u_0}{T_0(\delta l)} & -\frac{1}{\eta}\frac{K_T\vartheta_0}{T_0} & \frac{1}{\eta}\frac{K_T u_0}{(\delta l)T_0} \end{bmatrix}. \quad (5)$$

The dimensionless mechanical impedance matrix in (5) $\mathbf{z}^m(\eta)$ can be decomposed in the Foster canonical form as follows, see Newcomb[17]:

$$\mathbf{z}^m(\eta) = \frac{1}{\eta}\mathbf{z}_0^m + \eta\mathbf{z}_\infty^m$$

with the residue matrices defined by:

$$\mathbf{z}_0^m = \begin{bmatrix} \frac{K_M\vartheta_0}{(\delta l)M_0} & 0 & -\frac{K_M\vartheta_0}{(\delta l)M_0} & 0 \\ 0 & \frac{K_T u_0}{(\delta l)T_0} & \frac{K_T\vartheta_0}{T_0} & -\frac{K_T u_0}{T_0(\delta l)} \\ -\frac{K_M\vartheta_0}{(\delta l)M_0} & \frac{K_T u_0}{M_0} & \frac{K_M\vartheta_0}{(\delta l)M_0} + \frac{K_T\vartheta_0(\delta l)}{M_0} & -\frac{K_T u_0}{M_0} \\ 0 & -\frac{K_T u_0}{T_0(\delta l)} & -\frac{K_T\vartheta_0}{T_0} & \frac{K_T u_0}{(\delta l)T_0} \end{bmatrix}; \quad \mathbf{z}_\infty^m = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{\rho u_0(\delta l)}{T_0 t_0^2} & 0 & 0 \\ 0 & 0 & \frac{\Im\,\vartheta_0(\delta l)}{M_0 t_0^2} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In order to synthesize an analog circuit for the entire transversely vibrating beam it is sufficient to cascade connect a number of elementary analog networks of the beam element, assuring the compatibility of the displacement field and the equilibrium of the contact actions. Thus the synthesis problem can be stated as:

Problem: *find a four port grounded network, the dimensionless admittance matrix of which is equal to the dimensionless impedance matrix* $\mathbf{z}^m$.

Hence, we are looking for an electrical circuit whose admittance matrix $\mathbf{Y}(s)$, see Fig. 1, should be equal to $\frac{1}{s}\frac{I_0}{V_0 t_0}\mathbf{z}_0^m + s\frac{I_0 t_0}{V_0}\mathbf{z}_\infty^m$ where s represents the Laplace variable. The strategy developed to solve the considered synthesis problem consists of the following steps:

1. synthesis of an electrical network whose admittance matrix $\mathbf{Y}_0(s)$ is equal to $\frac{1}{s}\frac{I_0}{V_0 t_0}\mathbf{z}_0^m$,
2. synthesis of an electrical network whose admittance matrix $\mathbf{Y}_\infty(s)$ is equal to $s\frac{I_0 t_0}{V_0}\mathbf{z}_\infty^m$,
3. parallel connecting of the aforementioned electrical networks for the design of the circuit, the admittance matrix of which is $\mathbf{Y}(s)$.

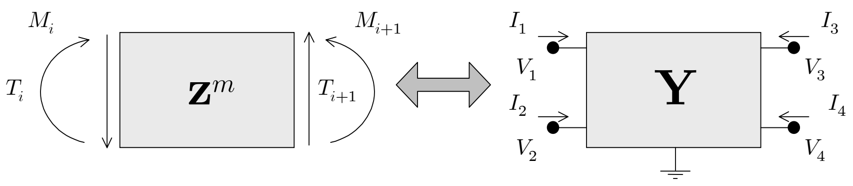

Figure 1. Analog circuit of a beam element utilizing the voltage-velocity analogy.

In order to guarantee the analog network to be reciprocal, see Newcomb,[17] it is necessary to require the symmetry of the admittance matrix $\mathbf{Y}(s)$, which yields the following condition on the scaling parameters:

$$u_0 T_0 = \vartheta_0 M_0. \tag{6}$$

This condition from a mechanical point of view establishes that the characteristic work done by the bending moment M_0 on the rotation ϑ_0, is equal to the characteristic work done by the shear contact action T_0 on the displacement u_0.

Furthermore, introducing the parameters α and γ, defined by:

$$\alpha = \frac{K_T}{K_M}\frac{u_0^2}{\vartheta_0^2}, \qquad \gamma = \frac{K_T\,(\delta l)^2}{K_M}$$

the residue at zero becomes:

$$\mathbf{z}_0^m = \frac{K_M \vartheta_0}{(\delta l)\, M_0}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & \alpha & \sqrt{\alpha\gamma} & -\alpha \\ -1 & \sqrt{\alpha\gamma} & 1+\gamma & -\sqrt{\alpha\gamma} \\ 0 & -\alpha & -\sqrt{\alpha\gamma} & \alpha \end{bmatrix}.$$

The analog network of the beam element can be synthesized connecting in parallel connection a capacitive network, whose admittance network is $s\dfrac{I_0 t_0}{V_0}\mathbf{z}_\infty^m$ and an inductive network, the admittance matrix of which is $\dfrac{1}{s}\dfrac{I_0}{V_0 t_0}\mathbf{z}_0^m$.

The capacitive network can be designed as two uncoupled capacitors C_1 and C_2 connected, respectively, at the second and third terminal of the grounded network; the capacitance of these two elements are given by:

$$C_1 = \frac{I_0}{V_0}\frac{\rho u_0}{T_0 t_0}(\delta l), \qquad C_2 = \frac{I_0}{V_0}\frac{I\vartheta_0}{M_0 t_0}(\delta l).$$

The ratio of the two capacitances is given by:

$$\frac{C_1}{C_2} = \frac{\rho}{\Im}\frac{u_0^2}{\vartheta_0^2}.$$

The design of the inductive circuit is much more involving, since this residue matrix of the residue in zero is not diagonal. In particular it is not even dominant (see Newcomb[17] and Panel discussion[18]), hence its transformerless synthesis is not even guaranteed in the multiport case. Let us decompose it as the sum of the two following matrices:

$$\mathbf{z}_0^m = \frac{K_M \vartheta_0}{(\delta l)\, M_0}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \frac{K_M \vartheta_0}{(\delta l)\, M_0}\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha & \sqrt{\alpha\gamma} & -\alpha \\ 0 & \sqrt{\alpha\gamma} & \gamma & -\sqrt{\alpha\gamma} \\ 0 & -\alpha & -\sqrt{\alpha\gamma} & \alpha \end{bmatrix}. \tag{7}$$

By means of this decomposition the synthesis problem has been drastically reduced to the design of a 3 port grounded network, whose admittance matrix is:

$$\mathbf{Y}_0^{red} = \frac{1}{s}\frac{I_0}{V_0 t_0}\frac{K_M \vartheta_0}{(\delta l)\, M_0}\begin{bmatrix} \alpha & \sqrt{\alpha\gamma} & -\alpha \\ \sqrt{\alpha\gamma} & \gamma & -\sqrt{\alpha\gamma} \\ -\alpha & -\sqrt{\alpha\gamma} & \alpha \end{bmatrix}.$$

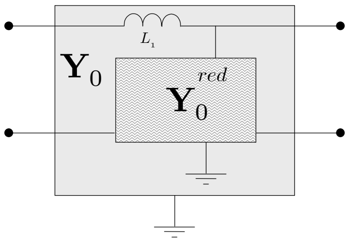

Figure 2. A first step towards the synthesis of $\mathbf{Y}_0(s)$.

In fact the first term in the RHS of (7) can be immediately synthesized as an inductor interconnecting the first and the third terminal, see Fig. 2. The value of the inductance is equal to:

$$L_1 = \frac{V_0 t_0 M_0}{I_0 K_M \vartheta_0} (\delta l).$$

Multiplying the inductance L_1 by the capacitance C_1 we get:

$$C_1 L_1 = \frac{\rho}{K_M} \frac{u_0^2}{\vartheta_0^2} (\delta l)^2.$$

The synthesis of a network governed by $\mathbf{Y}_0^{red}$ is still very tricky; in fact we cannot regard this matrix as the paramount (see Newcomb[17] and Panel discussion[18]) admittance matrix of a three port ungrounded network, since we are not dealing with multiport ungrounded networks. The reason why we are not interested in multiport ungrounded networks lies in the impossibility to guarantee that a certain pair of terminals behave as a port when interconnected with another pair of terminals as we will explain dealing with the design of the analog circuit for the entire beam. It is immediate to realize that the rank of $\mathbf{Y}_0^{red}$ is equal to one, hence it can be decomposed as:

$$\mathbf{Y}_0^{red} = \begin{bmatrix} 1 \\ \sqrt{\frac{\gamma}{\alpha}} \\ -1 \end{bmatrix} \left(\frac{1}{s} \frac{I_0}{V_0 t_0} \frac{K_M \vartheta_0}{(\delta l) M_0} \alpha \right) \begin{bmatrix} 1 & \sqrt{\frac{\gamma}{\alpha}} & -1 \end{bmatrix};$$

and the circuit can be designed following the standard synthesis technique explained in Newcomb,[17] as shown in Fig. 3 with the inductance given by:

$$L_{red} = \frac{V_0 t_0 T_0}{I_0 u_0} \frac{(\delta l)}{K_T}.$$

The ratio of the found inductance turns to be:

$$\frac{L_1}{L_{red}} = \frac{K_T}{K_M} \frac{u_0^2}{\vartheta_0^2},$$

while the product with the capacitance C_2 yields:

$$L_{red} C_2 = \frac{\mathfrak{I}}{K_T} (\delta l)^2 \frac{\vartheta_0^2}{u_0^2}.$$

The previous topology can be further simplified by noticing that the turns ratio of the first and the third transformers are opposite, as shown in Fig. 4.

In order to find the turns ratio of the used transformer and the value of the introduced inductance, let us find the admittance matrix of the network shown in Fig. 4 and compare it to $\mathbf{Y}_0^{red}$.

The constitutive equation of the inductor L_2 and of the ideal transformer yield:

$$\begin{cases} \tilde{I}_2 = -\tilde{I}_4 \\ -\dfrac{\tilde{V}_2 - \tilde{V}_4}{n} + \tilde{V}_3 = sL_2\,\tilde{I}_3 \\ n\tilde{I}_4 = \tilde{I}_3 \end{cases} ,$$

which establish the following admittance matrix:

$$\frac{1}{sL_2}\begin{bmatrix} \frac{1}{n^2} & -\frac{1}{n} & -\frac{1}{n^2} \\ -\frac{1}{n} & 1 & \frac{1}{n} \\ -\frac{1}{n^2} & \frac{1}{n} & \frac{1}{n^2} \end{bmatrix}.$$

Comparing the aforementioned admittance matrix to $\mathbf{Y}_\infty^{red}$, we obtain:

$$\frac{1}{s}\frac{I_0}{V_0 t_0}\frac{K_M \vartheta_0}{(\delta l)\,M_0}\begin{bmatrix} \alpha & \sqrt{\alpha\gamma} & -\alpha \\ \sqrt{\alpha\gamma} & \gamma & -\sqrt{\alpha\gamma} \\ -\alpha & -\sqrt{\alpha\gamma} & \alpha \end{bmatrix} \equiv \frac{1}{sL_2}\begin{bmatrix} \frac{1}{n^2} & -\frac{1}{n} & -\frac{1}{n^2} \\ -\frac{1}{n} & 1 & \frac{1}{n} \\ -\frac{1}{n^2} & \frac{1}{n} & \frac{1}{n^2} \end{bmatrix},$$

thus:

$$\begin{cases} \dfrac{1}{sL_2} = \dfrac{1}{s}\dfrac{I_0}{V_0 t_0}\dfrac{K_M \vartheta_0}{(\delta l)\,M_0}\gamma \\ \dfrac{1}{n^2} = \dfrac{\alpha}{\gamma} \end{cases} \Rightarrow \begin{cases} L_2 = \dfrac{V_0 t_0}{I_0}\dfrac{1}{K_T\,(\delta l)}\dfrac{M_0}{\vartheta_0} \\ n = \dfrac{\vartheta_0}{u_0}\,(\delta l) \end{cases}$$

The inductance L_{red} is related to L_2 by:

$$\frac{L_2}{L_{red}} = \frac{1}{(\delta l)^2}\frac{u_0^2}{\vartheta_0^2} = \frac{1}{n^2}$$

By inspection it is immediate to obtain the following set of relations between all the parameters so far introduced:

$$n^2 C_1 L_1 = \frac{\rho}{K_M}\,(\delta l)^4\,, \quad \frac{C_1}{C_2}n^2 = \frac{\rho}{\Im}\,(\delta l)^2\,, \quad \frac{L_1}{L_2} = \frac{K_T}{K_M}\,(\delta l)^2\,.$$

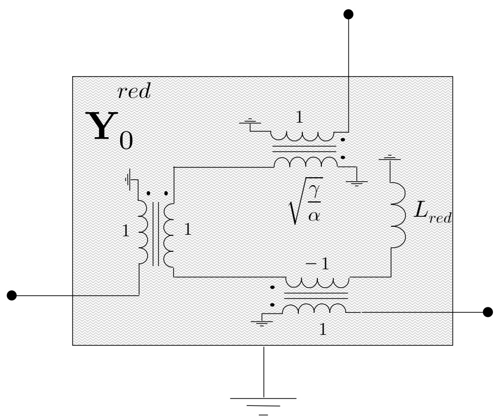

Figure 3. Direct design of $\mathbf{Y}_0^{red}$.

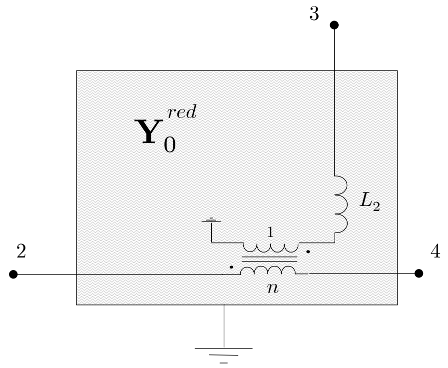

Figure 4. Minimal design of $\mathbf{Y}_0^{red}$.

The previous set of equations provides a group of conditions to be imposed on the employed circuit elements, completely independent on the arbitrarily chosen scaling parameters. Hence, it is immediate to see that for every possible choice of scaling parameters there are always three fixed constraints on the circuit elements, which depend only on the physical properties of the beam and on the sampling step of the mesh.

In order to synthesize the analog circuit for the transversely vibrating Timoshenko beam it is sufficient to cascade connect a number of analog circuit for the generic beam element. Indeed the electrical cascade connection correspond exactly to the mechanical conditions of continuity of the contact actions and the kinematical descriptors over the length of the beam.

5. ELECTRIC ANALOG OF AN EULER BEAM

In the proceeding of our discussion we will neglect the shear deformation and the rotatory inertia, these hypothesis stem from the aim to control and damp only low frequencies vibrations. For this particular model, the dimensionless governing equations (2) become:

$$\left\{\begin{array}{l} \frac{M_0}{l}m' + T_0 t = 0 \\ \frac{T_0}{l}t' = \frac{u_0}{t_0^2}\rho\,\dot{v} \end{array}\right. , \quad \left\{\begin{array}{l} M_0\dot{m} = \frac{\vartheta_0}{l}K_M\,\omega' \\ \frac{u_0}{l}v' = \vartheta_0\omega \end{array}\right. . \tag{8}$$

Deriving the first balance equation with respect to time and substituting the second balance equation, we get:

$$\frac{M_0}{l}m'' + T_0\left(\frac{lu_0}{T_0 t_0^2}\rho\,\dot{v}\right) = 0;$$

deriving the previous equation with respect to time and making use the two constitutive equations in (8) we finally get:

$$\frac{K_M t_0^2}{\rho l^4}v^{IV} + \ddot{v} = 0 \tag{9}$$

Hence, for the so called Euler beam the analog circuit becomes the circuit depicted in Fig. 5, with

$$C_1 L_1 n^2 = \frac{\rho}{K_M}(\delta l)^4 . \tag{10}$$

The governing equations of the analog circuit sketched in Fig. 5 in terms of the flux linkage ψ_i at the generic node i (defined as the time integral of the voltage drop across the $i-$th capacitor) can be easily written as:

$$\frac{\psi_{i+2} - 4\psi_{i+1} + 6\psi_i - 4\psi_{i-1} + \psi_{i-2}}{L_1 C_1 n^2} + \ddot{\psi}_i = 0,$$

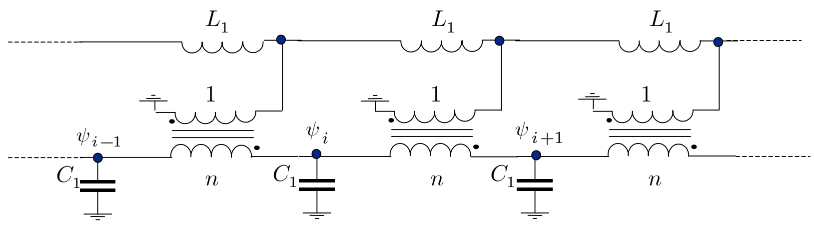

Figure 5. Circuit analog of an Euler beam.

which represent a discrete form of the elastica, once condition (10) is satisfied.

Hence we have shown that the previous completely passive circuit is the analog circuit of the elastica according to a symmetric finite difference approximation, provided that the used electrical elements fulfill condition (10).

In order to synthesize the analog circuit for the Euler beam using a finite difference approximation and exploiting the standard immittance matrices synthesis techniques it is necessary to study initially the Timoshenko beam and then impose the shear undeformability condition. In fact, as the shear rigidity goes to infinity the impedance matrix representation becomes not conceivable.

6. DESIGN OF A PROTOTYPE

The electric circuit in Fig. 5, represents the circuit analog of an Euler beam. Hence, interconnecting the piezoelectric elements positioned on the structure as the capacitors of the analog circuit, and inserting floating resistors across each pair of adjacent transducers, a completely passive realization of the optimal controller established in dell'Isola et al.[16] is found. In fact the array of floating resistors provide a centered finite difference approximation of the second spatial derivative $(\cdot)''$. One of the main feature of the proposed strategy lies in the technical feasibility of the controller as a truly passive electric circuit. Indeed for the particular smart beam designed in dell'Isola et al.,[16] relation (10) yield:

$$L_1 n^2 = \frac{\rho}{C_1 K_M} (\delta l)^4 = \frac{0.2462\, kg/m}{(184.9\ nF)\,(2.032\, Nm^2)} \left(\frac{25mm}{91.67\%}\right)^4 = .362H$$

which considering transformers characterized by a unitary turns ratio establishes an optimal inductance

$$L_1 = .362H.$$

The so found optimal inductance can be realized in a completely passive way, without exploiting any electronic active circuit.

7. CONCLUSIONS

In the present paper, the synthesis problem of finding a completely passive electric analog of an Euler beam has been tackled. The designed electric circuit is constituted only by capacitors, inductors and ideal transformers; its hardware realization exploiting truly passive electric elements has been proved.

The need of such an electric analog arises when designing distributed controllers for beam multimodal vibrations suppression, as shown in dell'Isola et al..[16] Indeed, uniformly positioning an array of piezoelectric elements on the host beam, and interconnecting their electrical terminal via the so found completely passive electric circuits, the multimodal beam vibration suppression is assured. The resulting smart structure is able to self-damp structural vibrations, dispensing with the use of any external power supply.

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