Numerical modelling of reinforced geomaterials by wires using the non smooth contact dynamics

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Abstract. In Civil Engineering soils may be reinforced by different structures. Wires will interest us. Mixed sand and wire, known as *TexSol*, may be modelled as a continuous medium with classical behaviour laws [6] or with more sophisticated ones taking into account remote interactions [1].

Our approach consists of a discrete model based on the Non Smooth Contact Dynamics. Different choices have been tested on some numerical examples to exhibit at the macroscopic scale the influence of the local models of interaction [5].

First of all we make some numerical tests to compare the mechanical behaviour of a *TexSol* and a sand sample. Then, we compute in both samples the stress tensors of the wires and the sand in order to understand the role of each component.

Our final goal is to define a micro-macro approach and a homogenized realistic behaviour law; if this study is only a first step, it is essential.

1 Motivations

The civil pieces of work needs planed stable floor. The environment configuration often forces civil engineers to raise huge embankments. Moreover, it can be interesting to reinforce them in order to assure a better embankment mechanical behaviour. A lot of different solutions can be used to reinforce soil but one interests us : the *TexSol* process.

Leflaive, Khay and Blivet from LCPC³, have created the *TexSol* in 1984 [4]. The *TexSol* is a heterogeneous material composed by mixed sand and wires network. This particularity gives to this material a better mechanical resistance than the sand without wires. Of course, the *TexSol* behaviour depends on sand and wire parameters and its frictional angle can be larger than sand one from 0° to 10° [3]. The wire is described by its linear density with a

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dtex unit ($1 \text{ dtex} = 0,1 \text{ g.km}^{-1}$), its ponderal content, and its stiffness. Classically, the wire density in a *TexSol* sample is included between 100 km.m^{-3} and 200 km.m^{-3} .

To make a *TexSol* bank, a machine named “*Texsoleuse*” is used. It works on throwing sand and, in the same time, injecting wire. The wire is deposited on the free plane of the sand with a random orientation. This machine carries out several passes to raise the bank. The Figure 1 is the *TexSol* microstructure representation. We find, in the literature, a lot of different continuous

Fig. 1. Schematic *TexSol* sections.

models. The model suggested in [1] is non local and includes remote interactions (corresponding to the wires effects) but also needs an identification of their parameters with macroscopic experiments. Villard proposes a simpler local model in [6]. This one couples a standard model of sand and an equivalent unilateral elastic stiffness contribution corresponding to the wire network. This last contribution is activated only on the traction directions because of the unilateral behaviour of wires. Our first work (exit from the scope of this paper) was to clearly define thermo-dynamic potentials of the Villard local model with both stress and strain formulations in order to identify the best-adapted one. But which micro-mechanisms are working? No continuous theory could ever answer this question.

We thus explore currently a new track using the distinct elements approach. Indeed, thanks to the computation power we have our days, it is possible to carry out some numerical experiments using only microstructural contact laws. Those contact laws must be able to account for the grain/grain, grain/wire and wire/wire interactions.

2 A Numerical discrete model for experiments

We use as a numerical simulation tool, the computer code LMGC90 which uses the Non Smooth Contact Dynamics [2].

2.1 Model of sand

The NSCD method is able to solve multi-body multi-contact problems with rigid and/or deformable bodies. However, our sand sample corresponds to a poly-disperse rigid spheres collection. This kind of problem can thus be computed by LMGC90. On a single contact problem, the principle is to evaluate

Fig. 2. Local/Global transformations.

external forces dynamic effects on the contactor point. To make such a transformation, we use $\mathbf{H}$ and $\mathbf{H}^*$ to move variables from the local contact frame to the global body one (cf. Figure 2) and vice-versa. In that way, the PFD⁴ is expressed in the local contact frame.

We thus consider $\underline{q}$, $\underline{r}$ respectively the Lagrange coordinates vector of the bodies and the contact reactions/torques vector⁵ and $\underline{U}$, $\underline{R}$ the relative velocity and the contact reactions in the contact local frame ($\underline{U} = \mathbf{H}^* \underline{q}$ and $\underline{r} = \mathbf{H} \underline{R}$). $\underline{F}$ are the external forces, $\mathbf{M}$ the mass matrix and h the time step :

$$\text{Global PFD : } \mathbf{M} d\underline{\dot{q}} = (\underline{F} + \underline{r}) dt \quad (1)$$

$$\text{Discrete Local PFD : } \underline{U}_{i+1} = \underline{U}_{free} + h \mathbf{W} \underline{R}_{i+1}$$

Smooth dynamic effects are included in the expression of the relative free (of contact) velocity $\underline{U}_{free} = \underline{U}_i + h \mathbf{H}^* \mathbf{M}^{-1} \underline{F}$. $\mathbf{W} = \mathbf{H}^* \mathbf{M}^{-1} \mathbf{H}$ is called the Delassus matrix. This local expression of PFD can “intersect” a normal contact condition (equation 2) modelling an inelastic shock.

$$0 \leq U_n \perp R_n \geq 0 \quad \Leftrightarrow \quad U_n \geq 0 \quad R_n \geq 0 \quad U_n R_n = 0 \quad (2)$$

Tangential reactions are computed with a frictional condition (Coulomb for example). One Gauss – Seidel loop computes all contact reactions, a convergence criterion (quadratic, maximum et cætera) decides or not to re-execute the loop until a good convergence.

⁴ The Principle of Fundamental Dynamics

⁵ Reactions imposed on a candidate particle by the neighbour ones

2.2 How making wires with discrete elements

The main difficulty is to model a continuous object with discrete elements. The “discrete wire” is described as a chain of beads [5]. Those are connected with a particular interaction law representing the wire resistance on the normal axis ($R_t = 0$ preserves the wire flexion). We have selected four laws, which can describe the wire behaviour.

- ***elastic wire*** and ***elastic rod*** : respectively unilateral and bilateral elastic laws⁶. Those laws are the most pertinent for small deformations of the sample without large sliding between grains or grains and wire but lead to numerical instabilities for large deformations. Such laws may be only used to validate numerically the local continuous model [6].
- ***rigid rod*** : the simplest law we can use. It imposes $U_n = 0$. Of course, this kind of law may produces some compression stresses in the wire which are not realistic.
- ***rigid wire*** : This law makes possible to free from disadvantages of the preceding one while keeping its advantages. Moreover, no compression component disturbs tensile stresses in the wire.

Unilateral laws must define a reference gap⁷ g_{ref} . This last one is a maximum length of the wire between two beads. Beyond this limit, the tensile stress is activated. Let us define a contact candidate particle. We try to solve the α contact (β are neighbour ones) without friction ($R_t^\alpha = 0$), t^- , t^+ the initial and final instants. Let us write a comparative study between a normal Signorini spheres contact law and a *rigid wire* law on a “quasi-inelastic shock” formalism.

<i>Spheres contact</i>	<i>Rigid wire interaction</i>
Predicted gap computation : $g_{pred}^\alpha = g^\alpha(t^-) + hU_{n free}^\alpha$	
• Case $g_{pred}^\alpha > 0 \Rightarrow R_n^\alpha = 0$	• Case $g_{pred}^\alpha < g_{ref}^\alpha \Rightarrow R_n^\alpha = 0$
• Case $g_{pred}^\alpha \leq 0 \Rightarrow (*)$	• Case $g_{pred}^\alpha \geq g_{ref}^\alpha \Rightarrow (*)$

$$\begin{array}{c}
 (*) : \textbf{Modified Inelastic Shock} \\
 \left. \begin{array}{l}
 U_{n cont}^\alpha = -\frac{g^\alpha(t^-)}{h} \\
 \tilde{U}_n^\alpha = U_n^\alpha(t^+) - U_{n cont}^\alpha \\
 \tilde{R}_n^\alpha = R_n^\alpha
 \end{array} \right| \begin{array}{l}
 U_{n cont}^\alpha = \frac{g_{ref}^\alpha - g^\alpha(t^-)}{h} \\
 \tilde{U}_n^\alpha = U_{n cont}^\alpha - U_n^\alpha(t^+) \\
 \tilde{R}_n^\alpha = -R_n^\alpha
 \end{array} \\
 \\
 hW^{\alpha\alpha}\tilde{R}_n^\alpha - \tilde{U}_n^\alpha = -U_{n free}^\alpha - \sum_{\alpha \neq \beta} hW^{\alpha\beta}\tilde{R}_n^\beta + U_{n cont}^\alpha \\
 0 \leq \tilde{U}_n^\alpha \perp \tilde{R}_n^\alpha \geq 0
 \end{array} \tag{3}$$

$U_{n cont}^\alpha$ is non null when a contact have to be established during the time step and is the contribution of the velocity to establish this contact. The

⁶ The tension is proportional to the gap

⁷ It is the minimum distance between two particles

inelastic shock in the second part of the time step leads to define new variables ($\tilde{U}_n^\alpha$, $\tilde{R}_n^\alpha$) on which are applied the Signorini conditions. Those two problems thus resume to a classical LCP⁸ thanks to adapted variable changes.

Let us notice that the wire and sand reactions are computed with two different interaction laws. We can thus separate those contributions in order to analyse how do the wire and the sand work independantly (cf. section 3).

3 A 2D numerical study

This study is a qualitative comparison between sand and *TexSol* [5]. Its aim is to understand the wire contribution towards mechanical solicitations. The two last rigid interaction laws will be used to model the wire but in a first step we start with the *TexSol* sample preparation.

Its wire network must be in a random orientation state (cf. section 1) and the 2D membrane effect must be minimized. Consequently, we define a wire bead diameter close to that of sand particles and the reference gap must be large enough to let pass the coarsest sand grain.

Once the wire generated, two solutions exist to add sand grains. The first superposes a grid of poly-disperse particles and let them deposit by LMGC90. The main problem with this solution is the computing time. Indeed, the NSCD method convergence is slow with weak contact reactions (characteristic of a deposit test). The second uses the Taboada 2D pre-processor. This one makes a geometrical deposit sample of sand with poly-disperse grains. A little LMGC90 deposit relaxes the sample and tightens grains around the wire.

Let us make a biaxial compression test on the final sample in order to compare interaction laws. We consider a 2000 particles *TexSol* sample with 300 for the wire. We carry out three simulations. One with a *rigid wire* interaction law between wire particles, another with a *rigid rod* interaction law and the last one without interaction law (sand). The Figure 3 represents the graph of support reaction according to the crushing percentage. Let us notice that the unilateral or bilateral *TexSol* behaviour is stiffer than sand one. But an accident happens to the bilateral law at the middle of the simulation. A brutal increase of the support reaction shows that a wire compression column has been formed. Sand particles hold it on and it returns a jump of vertical reaction. This bilateral law can makes us a mistake so we would rather use the *rigid wire* law.

The Figure 4 is a deformation state comparison between the *TexSol* and the sand at the same level of an upper side force. It also displays contact reaction chains of spheres contacts (red/grey one) and *rigid wire* interactions (blue/dark one). In the sand sample, reaction chains develop everywhere in every directions. In the *TexSol* sample, *rigid wire* tensile stresses concentrate sphere contact reactions in the centre of the sample. They work on the horizontal direction to prevent the sample from widening.

⁸ Linear Complementarity Problem

Fig. 3. Different material responses.

Fig. 4. Reaction chains in *TexSol* (left) and Sand (right).

Thus we interest in the *TexSol* stress tensor. A discrete material stress tensor does not express like continuous material one. We choose the Weber's definition (detailed in [5]). We thus introduce two complementary parts of the *TexSol* stress tensor, one on the wire and the other on sand. The unilaterality of the wire and *TexSol* is highlighted on the Figure 5 graph. Indeed, for the wire component, only one principle stress is positive ; the other one is close

Fig. 5. Evolution of stress tensors in the *TexSol*.

to zero. That is reflected in the global *TexSol* behaviour where one principle stress is equal to one of the sand alone and the other one is reduced by the wire component.

4 A first attempt in 3D modelling

The *TexSol* problem is naturally 3D and the preceding model has some drawbacks. Indeed membrane effects disturbing the material behaviour does not reveal real mechanisms of the wire. We thus have to carry out some 3D numerical study on samples carefully generated. The section 1 tells about the industrial process to raise a *TexSol* bank. Such a process cannot provide an isotropic material. Indeed, the wire is deposited layer by layer and is arranged on parallel planes. The equivalent elastic tensor does not have stiffness on the normal planes direction. It becomes an anisotropic tensor. A 3D pre-processor has been written to define the chain of beads with several rules.

- Each bead is defined with a constant interstice with respect to the previous one.
- The direction of a bead n in the $(O, \underline{x}, \underline{y})$ plane is given by the angle $\theta_n = \theta_{n-1} + \theta_{random}$ with $\theta_{random} \in [-\theta_{max}; \theta_{max}]$ and $\theta_0 = 0$.
- The direction of a bead n in the $\underline{z}$ axis is determinate by the angle $\varphi_n = \varphi_{up} + \varphi_{random}$ with $\varphi_{random} \in [-\varphi_{max}; \varphi_{max}]$ and $\varphi_{up} = cst$.
- **Skirting** of the chain : if the bead n intersects the chain then $\varphi_{random} \in [-\frac{2\pi}{3}; \frac{2\pi}{3}]$.
- **Rebound** of the chain : if the bead n gets out from lateral box limits then $\theta_{random} \in [-\frac{2\pi}{3}; \frac{2\pi}{3}]$.
- **Switch** of the chain : if the bead n gets out from vertical box limits then $\varphi_{up} = -\varphi_{up}$.

The raise angle is generally calculated to put the last bead at the top of the box. Sometimes it is impossible to define a bead position, so the wire is cut.

This wire model must have beads diameter around 20% than the smallest sand particle diameter. Indeed, a too large bead would make a rough wire

Fig. 6. 3D wire disposition : isometric (left) and lateral view (right).

and the friction parameter would be more difficult to control. But such a condition increases the number of particle to make a realistic sample and the LMGC90 deposit problem still exists in 3D. In fact, good sample would represent around 10000 particles. We are currently working on a 3D extension of the pre-processor which generates dense sample.

Fig. 7. Wire disposition (left) and dense *TexSol* (right).

Conclusion and perspectives

The distinct elements give us a new approach of the *TexSol* problem. They are able to show us which are the wire deformation mechanisms. Those 3D investigations will be soon compared with the continuous local model in order to determinate if it is the best adapted.

First of all, we have to optimise our simulation tools on several areas : samples preparation, deposit, compaction and mechanical test computation. Then we will be able to analyse 3D wires mechanical influence on the *TexSol*.

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