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# An empirical strategy to detect spurious effects in long memory and occasional-break processes

Luisa Bisaglia, Margherita Gerolimetto *

August 27, 2008



**Abstract**

Long range dependence and structural changes in level are intimely related phenomena and it is very difficult to separate the two effects. In this paper we present an empirical procedure to distinguish between long memory and occasional-break processes. An extensive Monte Carlo experiment illustrates the performance of the procedure and an application to real data is also included.

**key words**: Long memory, Occasional structural breaks, Break-free series

## 1 Introduction

Since the seminal papers of Granger and Joyeux (1980) and Hosking (1981) there has been a considerable interest in modelling strong persistence in time series. This interest is motivated by the analysis of many empirical time series whose autocorrelation function decreases to zero like a power function rather than exponentially and the spectral density diverges as the frequencies tend to zero.

Many authors have studied the problem of estimating the long memory parameter with both parametric and non/semi-parametric methods (see Beran, 1994, Percival and Walden, 2000 and Palma, 2007 for good reviews on this argument). However, recently (see, for example, Giraitis et al., 2001, Berkes et al., 2006 and the refences therein), it has been shown that inference on the long memory parameter and persistence tests are severely compromised in series which display occasional structural breaks, since these processes give the impression of persistence. In

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other words, neglecting structural changes causes a spurious over estimation of the long memory parameter, leading the reseacher to believe erroneously in a long memory data generating process (among the others, Granger and Hyung, 2004, Diebold and Inoue, 2001).

On the other hand, if an observed time series follows a fractionally integrated model, then change point tests may indicate the presence of change points, when, in fact, there are no change. Indeed, when the Data Generated Process (hereafter, DGP) is fractionally integrated of order $d$, $I(d)$, with $0 < d < 0.5$ and no change, many well-known structural change tests are likely to suggest that changes have spuriously take place (Kuan and Hsu, 1998).

From an empirical point of view, Morana and Beltratti (2004) analyse the realized variance process for the DM/US$ and Yen/US$ exchange rates. They find that while long memory is evident in the actual process, a structural break analysis reveals that this feature is partially explained by unaccounted changes in regime. Hsu (2005) analyses monthly G7 inflation rates and suggests that for Germany and Japan the long memory phenomenon may just be a consequence of structural change. The same findings apply to other countries, whose inflation rates may exhibit both long memory and structural change. Gil-Alana (2004) analyses annual data on German real GDP and shows that the series may be well described in terms of fractional models with a structural slope break due to World War II.

Moreover, the importance of distinguishing between long memory and occasional-break models is justified also in the perspective of forecasting (Diebold and Inoue, 2001, Gabriel and Martins, 2004).

With this paper we contribute in two directions. Firstly, we conduct an extensive Monte Carlo experiment to evaluate and compare the performance of various estimators of the long memory parameter under some occasional-break DGPs.

Secondly, we present an empirical strategy to detect the spurious effects caused by a misspecification of the model when the data show persistence. The procedure is based on the different behaviour of the long memory parameter estimators when applied to the original series and its break-free version. A further Monte Carlo experiment illustrates the performance of the strategy. We also apply the suggested strategy to the daily series of absolute S&P500 stock returns to show with an empirical example how to implement the procedure.

In literature, to our knowledge, there are only a few proposals to separate the long memory from the occasional breaks effects (Ohanissian et al., 2004, Dolado et al., 2004, Teyssiere and

Abry, 2006). However, they do not refer to the cases we consider.

The plan of the paper is as follows. In Section 2, we briefly introduce $I(d)$ processes. In Section 3 some models for occasional structural breaks are discussed. Section 4 describes a simple empirical strategy to deal with spurious effects. Section 5 describes our Monte Carlo experiments. In Section 6 we apply the proposed methodology to the S&P 500 absolute stock returns series and Section 7 concludes.

## 2 Structural breaks

Following Granger and Hyung (2004) we describe three models with occasional breaks in mean, hence the number of breaks that can occur in a specific period of time is somehow bounded.

More formally, we assume, as in Granger and Hyung (2004), that the probability of breaks, $p$, converges to zero slowly as the sample size increases, i.e. $p \to 0$ as $T \to \infty$, yet $lim_{T\to\infty} Tp$ is a non-zero finite constant.

This implies that letting $p$ decrease with the sample size, realization tends to have just a finite number of breaks (see also Diebold and Inoue, 2001).

The three model we consider are the following:

1. **Mean plus noise model** (Chen and Tiao, 1990, Engle and Smith, 1999)

$$\begin{aligned} y_t &= m_t + \epsilon_t, \qquad t = 1, ..., T, \\ m_t &= m_{t-1} + q_t \eta_t \end{aligned}$$

   where $\epsilon_t$ is a noise variable[1] and occasional level shifts, $m_t$, are controlled by two variables $q_t$ (date of breaks) and $\eta_t$ (size of jump). $\eta_t$ is an i.i.d. $N(0, \sigma^2_\eta)$ although the normality assumption can be dropped and $q_t$ is assumed to be an i.i.d. sequence of Bernoulli random variables such that $Pr(q_t = 1) = p$.

2. **Markov switching model** (Hamilton, 1989) The binomial model is characterized by sudden changes only, but the structural changes might also occur gradually. In this case a Markov switching model is more appropriate. Suppose $s_t$ is a latent random variable that can assume only values 0 or 1. $s_t$ is assumed to be a Markov chain, with transition probability $p_{ij} = Pr\left(s_t = j/s_{t-1} = i\right)$. Then, it is possible to use a switching model for $q_t$ such that $q_t = 0$ when $s_t = 0$ and $q_t = 1$ when $s_t = 1$. In this specification a regime with $s_t = 1$ represents a period of structural change, regardless of the value of $s_{t-1}$.

[1] Even if $\epsilon_t$ can be any short memory process, in this work we consider $\epsilon_t \sim i.i.d.N(0, \sigma^2_\epsilon)$

3. **Stocastic Permanent Break (STOPBREAK) model** (Engle and Smith, 1999)

   This model bridges the gap between transience and permanence of the shocks. In the STOPBREAK process the long run impact of each obervation is time varying and stochastic. The formulation is as follows:

$$\begin{aligned} y_t &= m_t + \epsilon_t \\ m_t &= m_{t-1} + q_{t-1}\epsilon_{t-1} \end{aligned}$$

   where $q_t = q\left(|\epsilon_t|\right)$ is non decreasing in $|\epsilon_t|$ and bounded by zero and one, so that bigger innovations have more permanent effects, and $\epsilon_t$ are i.i.d $N\left(0, \sigma_\epsilon^2\right)$, moreover $q_t = \frac{\epsilon_t^2}{\left(\gamma+\epsilon_t^2\right)}$ for $\gamma > 0$. Therefore in the STOPBREAK process permanent shocks can be identified by their larger magnitude. With this approach the effects of shocks can fluctuate between transient and permanent. Typically, realizations of this process show periods of apparent stationarity punctuated by occasional mean shifts.

To estimate and test for multiple breaks at unknown dates one of the most used methods is the Bai-Perron (hereafter BP) procedure (Bai and Perron, 1998, 2003). It is based on a type test for the null hypothesis of no change versus a prespecified number of changes and also an alternative of an arbitrary number of changes (up to some maximum) as well as a procedure that allows one to test for the null hypothesis of, say, $l$ changes against the alternative hypothesis of $l+1$ changes. The latter is particularly useful in that it allows a specific to general modeling strategy to determine consistently the appropriate number of changes in the data. The tests can be constructed allowing different serial correlation in the errors, different distributions for the data and the errors across segments or imposing a common structure.

# 3 Fractionally integrated processes

There are various definitions of long memory processes. In particular, long memory can be expressed either in the time domain or in the frequency domain. In the time domain, a stationary discrete time series is said to be long memory if its autocorrelation function decays to zero like a power function. This definition implies that the dependence between successive observations decays slowly as the number of lags tends to infinity. On the other hand, in the frequency domain, a stationary discrete time series is said to be long memory if its spectral density is unbounded at low frequencies.

In this paper we consider the fractionally integrated process, $I(d)$, independently introduced by Granger and Joyeux (1980) and Hosking (1981). Let $\epsilon_t$ be a white noise process such that $E[\epsilon_t^2] = \sigma^2$. The process $\{X_t,\ t \in \mathbf{Z}\}$ is said to be an $I(d)$ process with $d \in (-1/2, 1/2)$, if it is stationary and satisfies the difference equation

$$\Delta(B)\,(X_t - \mu) = \epsilon_t,$$

where $\Delta(B) = (1-B)^d = \sum_{j=0}^{\infty} \pi_j B^j$ with $\pi_j = \Gamma(j-d)/[\Gamma(j+1)\Gamma(-d)]$, $\Gamma(\cdot)$ is the gamma function and $\mu$ is the mean of the process.

In the following we will concentrate on $I(d)$ processes with $d \in (0, 1/2)$: for this range of values the process is stationary, invertible and possesses long-range dependence. Moreover, we will assume for convenience and without loss of generality that $\sigma^2 = 1$ and $\mu = 0$.
In literature several methods to estimate the long memory parameter $d$ have been proposed, see Palma (2007) for a comprehensive review.

To our knowledge, in the literature regarding the problem of discriminating between long memory and occasional-break processes only Gaussian semiparametric estimators like the Geweke-Porter Hudak (1983) method (hereafter GPH), the local Whittle method (Kunsch, 1987, Robinson, 1995b) or Lobato and Robinson's LM test (1998) are used.[2] There are no results on the performance of other estimation methods, for this reason in the Monte Carlo study in Section 5 of this paper we will consider:

1. The R/S or rescaled adjusted range method (Hurst, 1951, Mandelbrot 1972, 1975, Mandelbrot and Taqqu, 1979);
2. The aggregate variance method (Taqqu et al., 1995);
3. Higuchi method (Higuchi, 1988);
4. Whittle method (Fox and Taqqu, 1986)
5. Robinson's modified version of the periodogram method (Robinson, 1995a).

The GPH method is also included in the Monte Carlo experiment with the role of benchmark. Indeed, Granger and Hyung (2004), Perron and Qu (2006) and Smith (2005) show analitically

[2]We thank a referee for reminding us of the existence of a strain of literature that treats the estimation of $d$ with wavelets (e.g. Jensen, 1999, Wang, 1999, Craigmile et al., 2005). In this framework Teyssiere and Abry (2006) consider also the issue of distinguishing between long memory and structural breaks but the models they study are different from the models analyzed in this paper.

that, in hypothesis of occasional-break process, the GPH estimator is biased. Smith (2005) derives an approximation to this bias and proposes a modified version of the GPH estimator that takes into account this result. Bearing in mind the bias of the GPH estimator, we included also Robinson's version of the GPH (R-GPH, herafter) since it triggers out the first frequencies of the periodogram and its performance may be less affected by the presence of occasional breaks.[3]

# 4 An empirical strategy

As discussed in the Introduction, there are several results in literature about the spurious effects that may arise from the analysis of both stationary data with long memory and non stationary data with structural breaks.

When a series gives the impression of persistence and the estimated value of the long memory parameter ($\hat{d}$) is bigger than zero, there might be the suspicion that this result is due to spurious long memory, provoked by neglected structural breaks.

To overcome this uncertainty it is possible to use an empirical procedure based on the estimation of $d$ after detecting and eliminating the true break points from the series. Indeed this allows (Granger and Hyung, 2004) examining the effects of removing break components on the estimated value of $d$.

More specifically, the following is the simple scheme of the strategy:

- Estimate $d$ in the original series $x_t$, obtaining $\hat{d}$
- Estimate the break dates in $x_t$ with the BP procedure
- Obtain the break-free series $x'_t = x_t - m_t$, where $m_t$ is the sample mean of each regime
- Estimate $d$ in $x'_t$, obtaining $\hat{d}'$

If after filtering out the break points, $\hat{d}'$ is still approximately close to $\hat{d}$, then long memory should be a true feature of the series and the structural breaks are spurious. If on the contrary, after deleting the break points from the series, $\hat{d}'$ tends to zero we can conclude that the long memory is spurious, caused by neglected occasional breaks.[4]

[3]Originally, we had considered in our experiment also Smith's estimator but we did not present the results in the tables since its performance seemed to be approximately between the GPH and R-GPH.

[4]For the estimators that possess the asymptotic distribution (e.g. Whitlle and GPH) it is possible to test the null

Because of its empirical nature, this strategy cannot answer the question of whether long memory or structural changes are present for all imaginable frameworks. However it gives helpful indications in some important cases.

# 5 Simulation study

In this Section we describe our twofold Monte Carlo experiment. On the one hand it is devoted to evaluate the performance (in terms of bias and standard errors) of the estimation methods listed in Section 2 in hypothesis of DGP with occasional breaks in mean (in the tables the acronyms rs, av, hi, gph,r-gph, wh stay respectively as R/S, aggregate variance, Higuchi, GPH, R-GPH, Whittle method). On the other hand it aims to show the behaviour of the empirical strategy to distinguish between long memory and occasional-break DGP described in the previous Section.

The functions we use are written in R language (R Development Core Team, 2006) and are available upon request by the authors.

The following are the models we consider in the simulations:[5]

1. DGP1: mean plus noise model, with $p = 0.01$, $0.05$, $0.1$ and $\sigma^2_\eta = 0.01$, $0.05$, $0.1$;
2. DGP2: Markov switching model, with $(p, q) = (0.95, 0.95;\ 0.95, 0.99;\ 0.99, 0.95;\ 0.99, 0.99;\ 0.999, 0.999)$ and $\sigma^2_\eta = 0.1$. In this case the initial state $s_1$ is generated by a Bernoulli random variable with $p = 0.5$;
3. DGP3: STOPBREAK model, with $\gamma = (10^{-5}, 10^{-1}, 1, 10, 10^3)$, following Diebold and Inuoe (2001), in order to make comparisons.
4. DGP4: $I(d)$ model, with $d = 0.1, 0.2, 0.3, 0.4, 0.45$. The error term in normally distributed with zero mean and unit variance.

For each model we consider $\sigma^2_\epsilon = 1$ and $s = 1000$ independent realizations. Thus for a given estimation method we obtain $s = 1000$ estimated values for $d$. The considered sample sizes are $T = 500$, $1000$ for the first three DGPs and $T = 500$, $1000$, $2000$ for DGP4.[6] All series are generated with 200 additional values to obtain random starting values. The number

hypothesis of true long memory ($\hat{d} = \hat{d}'$) by extending to this special context the results obtained by Beran and Terrin (1996) and further revised by Horváth and Shao (1999).

[5]We did not consider processes with both long memory and structural breaks since in this case our procedure would clearly detect both phenomena.

[6]For DGP1, DGP2, DGP3 we did not consider $T = 2000$ because of the computational burden of the procedure that would have required to much time.

of frequencies included in the GPH estimator is set $m = \sqrt{T}$ as suggested by Geweke and Porter-Hudack (1983), whereas the number of frequencies triggered out in the R-GPH method is set $l = (m - \tau)/J$ where $\tau = J = 4$ as in Lee and Robinson (1996).

In tables 3-6 our results are presented. In each cell of the tables we report in the first row the estimation (standard error in parenthesis) of the long memory parameter $d$ from the original series, whereas in the second row the same estimation from the break-free series.

From the tables we can obtain a conspicuous amount of information regarding the two aims of the simulation study.

Relatively to the first objective, the observation of the first row of the cells in tables 3-5 reveals that long memory is erroneously found whatever estimation method is used. Hence not only the GPH (as already highlighted in the literature) but more in general all the considered estimators are severely upward biased. Whittle and R-GPH estimator exhibit a slightly better performance, in the sense that $\hat{d}$ is often much closer to zero over all the considered DGPs.

Moreover, the perfomance of all methods becomes poorer as the sample size, $T$, and the jump size, $\sigma_\eta^2$, increase. Indeed, as the series length grows, $Tp$ also grows and more breaks are present, that emphasize the non stationary behaviour of the series. Consequently the degree of similarity between occasional-break and long memory processes grows. Similarly, an increase of $\sigma_\eta^2$ strenghtens the bias of $d$ as the size of the jumps is bigger.

Relatively to the second objective, the comparison between the first and the second row of each cell in tables 3 and 4 reveals a clear change in the estimate before and after removing the break points. The result is particularly interesting when Whittle and R-GPH method are used since in that case the break-free estimation, $\hat{d}'$, is approximately around zero. This gives the indication that the long memory found in the original series is in fact spurious and the true DGP should be with occasional breaks.

Some more comments have to be made for table 5 where the variation in the estimation of $d$ after removing the break points is not so evident as in DGP1 and DGP2. In the STOPBREAK process $q_t$ is $\frac{\epsilon_t}{\gamma+\epsilon_t}$ therefore if $\gamma \to 0$, $q_t \to 1$ and the process is almost $I(1)$, whereas if $\gamma \to \infty$, $q_t \to 0$ and the process is $I(0)$. As a consequence, when $\gamma$ does not tend to zero all methods work, since the process is $I(0)$ and it is easier to recognize the non long memory feature. For the smallest values of $\gamma$ the process is so close to $I(1)$ that it is very complicate to identify the DGP. Indeed, in this case it may be more adequate to model the first difference of the series rather than looking for structural breaks.

On the other hand, it appears that the empirical procedure is also able to recognize when the DGP is truly long memory. Table 6 shows that the break-free estimation ($\hat{d}'$) obtained with Whittle and R-GPH methods, is approximately stable around the original estimation $\hat{d}$ and this result improves when $T$ increases.

We observe that there is a slight tendence to underestimate $d$ in the break-free series. This is expected because the implementation of the procedure means subtracting from the series the sample means of each estimated regime and somehow "compressing" the series that looks less persistent. However, when we use Whittle or R-GPH method this is only a negligible effect, whereas for the other estimation methods this phenomenon affects much more the estimates that are often negative. These results are consistent with the findings of Granger and Hyung (2004) about the GPH method.

# 6 An example on a real time series

In this Section we illustrate on a real time series how our procedure can be applied in practice. In particular, we apply the empirical strategy to the time series of absolute stock returns of the S&P 500 from January 2, 1988 to June 15, 2005, with 4292 daily observations. The series of returns is constructed by differencing the log-price index of S&P 500.

In several papers this series has been found to possess features typical of long memory data. In particular, Granger and Ding (1995, 1996), Lobato and Savin (1998) suggest a fractionally integrated model.[7] However, more recently, attempts have been made to explain strong persistence in this series with unaccounted structural breaks (Granger and Hyung, 2004) and the problem is still partially open. On the one hand the only method used to estimate the long memory parameter is the GPH that has been shown to be biased. On the other hand structural change tests are severely biased in presence of long memory. Therefore we take this series as a concrete example of a real problem where our strategy can be used to detect spurious effects.

Figure 1 presents the plot of absolute stock returns as well as the breaks detected by BP procedure. Figure 2 shows the sample autocorrelation function for absolute stock returns that exhibits a slow hyperbolic decay indicating the presence of long memory.

In table 1 are reported the estimates of the long memory parameter in the whole sample

[7] The series considered by these authors is in fact the S&P 500 from January 4, 1928 to October 30, 2002.

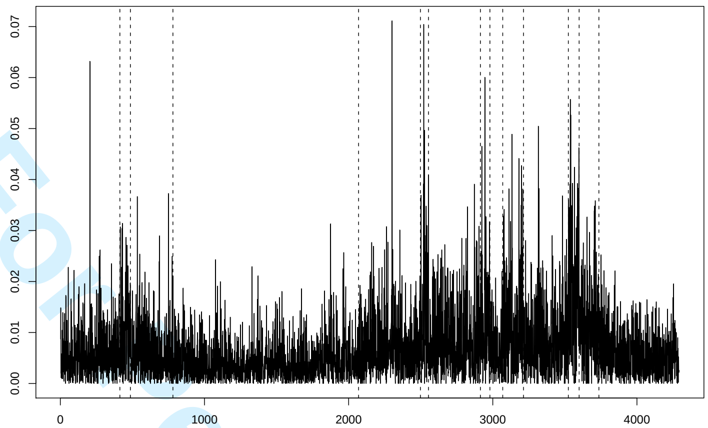


Figure 1: S&P 500 daily absolute returns and estimated breaks

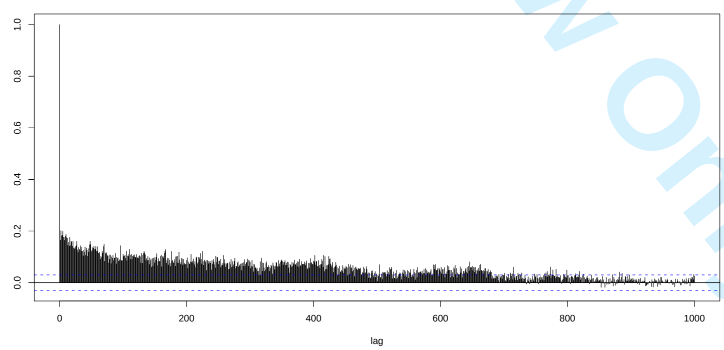


Figure 2: Autocorrelation for S&P 500 daily absolute returns

Table 1: Estimation of $d$ in the original and break-free series

| | rs | av | hi | gph | r-gph | wh |
|---|---|---|---|---|---|---|
| $\hat{d}$ | 0.815 | 0.397 | 0.506 | 0.534 | 0.437 | 0.160 |
| $\hat{d}'$ | 0.014 | -0.107 | -0.131 | -0.161 | 0.050 | 0.015 |

Table 2: Estimation results for $d$ in the subsamples with Whittle method

| *subseries* | **1:411** | **412:484** | **485:779** | **780:2067** | **2068:2496** | **2497:2552** | **2553:2912** |
|---|---|---|---|---|---|---|---|
| $\hat{d}$ | 0.000 | 0.000 | 0.000 | 0.048 | 0.079 | 0.044 | 0.000 |
| *subseries* | **2913:2978** | **2979:3067** | **3068:3211** | **3212:3522** | **3523:3597** | **3598:3734** | **3735:4292** |
| $\hat{d}$ | 0.042 | 0.000 | 0.062 | 0.006 | 0.045 | 0.000 | 0.008 |

before and after the detection of the break points. We can observe that, after removing the break points from the series, the estimated value of the long memory parameter move towards zero whatever estimation method is used. However, as expected from our Monte Carlo study, $\hat{d}'$ is very close to zero when computed with Whittle method.

Our conclusion is that the series of S&P 500 daily absolute returns is characterized by structural breaks and not by long memory. To give some more evidence on this we estimate (with Whittle method) the long memory parameter in the subseries obtained between breaks and, as expected, they are found to be short memory (table 2).

# 7 Conclusions

In this article we focus on the issue of distinguishing between a time series exhibiting long-range dependence and one with short memory but suffering from occasional structural breaks. Nowadays this is a very interesting topic to which an entire special number of the *Journal of Econometrics* (vol. 129, issue 1-2, 2005) is devoted.

Firstly, we investigate with an extensive Monte Carlo experiment about the performance of various estimators of the long memory parameter under some occasional-break DGPs. Our findings are that not only the GPH (as well known in literature), but also other estimators of $d$ are severely biased when applied to a series that looks persistent in case of neglected structural breaks.

Then, we present an empirical strategy based on the different behaviour of the estimators of the long memory parameter when applied to the original series and its break-free version. From the findings of another simulation study, the procedure, combined to Whittle or R-GPH method, seems in general able to signal the presence of spurious effects. This is a rather crucial point since they are responsible for leading the researcher to erroneously believe that the DGP is long memory when it is in fact with occasional breaks in mean.

It is important to observe that the strategy we describe cannot for all possible circumstances answer the question of whether long memory or structural changes cause the persistent appearance of the series. However it gives indications in various interesting situations.

Many topics are still worthy of investigation, for example the idea of studying models characterized both by long memory and structural breaks. In this direction is the very interesting paper by Gil-Alana (2008) who proposes a simple procedure for determining fractional integration and structural breaks in a unified treatment. With a different approach Ko and Vannucci (2006) describe a wavelet-based bayesian procedure to estimate and locate multiple change points in the long memory parameter of Gaussian autoregressive fractionally integrated moving average models with unknown number of change points.

# Acknowledgments

We thank two anonymous Referees for the insightful comments.

Table 3: Estimation results for $d$: DGP1, T=500, 1000

| | | **T=500** | | | **T=1000** | | |
|---|---|---|---|---|---|---|---|
| | $p$ | 0.01 | 0.05 | 0.1 | 0.01 | 0.05 | 0.1 |
| $\sigma_\eta^2 = 0.01$ | rs | 0.091 (0.094) | 0.204 (0.125) | 0.285 (0.139) | 0.168 (0.117) | 0.343 (0.148) | 0.438 (0.147) |
| | | 0.063 (0.089) | 0.067 (0.095) | 0.074 (0.093) | 0.074 (0.085) | 0.075 (0.088) | 0.091 (0.079) |
| | av | 0.030 (0.088) | 0.136 (0.106) | 0.208 (0.111) | 0.112 (0.099) | 0.263 (0.103) | 0.331 (0.087) |
| | | -0.002 (0.078) | 0.011 (0.079) | 0.021 (0.083) | 0.023 (0.072) | 0.041 (0.077) | 0.078 (0.075) |
| | hi | 0.190 (0.157) | 0.337 (0.134) | 0.380 (0.118) | 0.284 (0.143) | 0.399 (0.104) | 0.431 (0.085) |
| | | 0.126 (0.178) | 0.094 (0.216) | 0.065 (0.210) | 0.112 (0.186) | 0.028 (0.177) | 0.019 (0.154) |
| | gph | 0.086 (0.183) | 0.230 (0.188) | 0.317 (0.196) | 0.156 (0.152) | 0.359 (0.162) | 0.450 (0.166) |
| | | 0.031 (0.173) | 0.059 (0.173) | 0.052 (0.184) | 0.035 (0.136) | 0.069 (0.157) | 0.046 (0.156) |
| | r-gph | 0.017 (0.356) | 0.037 (0.347) | 0.063 (0.351) | 0.006 (0.321) | 0.050 (0.324) | 0.083 (0.340) |
| | | 0.030 (0.351) | 0.005 (0.338) | 0.012 (0.332) | -0.028 (0.338) | -0.009 (0.322) | 0.014 (0.325) |
| | wh | 0.019 (0.024) | 0.055 (0.045) | 0.089 (0.056) | 0.033 (0.030) | 0.091 (0.050) | 0.135 (0.055) |
| | | 0.014 (0.021) | 0.015 (0.021) | 0.017 (0.022) | 0.011 (0.014) | 0.016 (0.018) | 0.029 (0.024) |
| | $p$ | 0.01 | 0.05 | 0.1 | 0.01 | 0.05 | 0.1 |
| $\sigma_\eta^2 = 0.05$ | rs | 0.194 (0.138) | 0.376 (0.157) | 0.474 (0.154) | 0.325 (0.155) | 0.538 (0.152) | 0.604 (0.153) |
| | | 0.057 (0.099) | 0.054 (0.093) | 0.044 (0.084) | 0.067 (0.093) | 0.054 (0.081) | 0.049 (0.071) |
| | av | 0.129 (0.121) | 0.284 (0.109) | 0.357 (0.083) | 0.244 (0.113) | 0.392 (0.073) | 0.426 (0.057) |
| | | 0.001 (0.085) | 0.012 (0.082) | 0.011 (0.082) | 0.027 (0.083) | 0.043 (0.077) | 0.049 (0.075) |
| | hi | 0.327 (0.144) | 0.424 (0.095) | 0.446 (0.080) | 0.391 (0.112) | 0.453 (0.072) | 0.462 (0.067) |
| | | 0.084 (0.218) | -0.028 (0.157) | -0.064 (0.122) | 0.021 (0.174) | -0.062 (0.106) | -0.094 (0.086) |
| | gph | 0.203 (0.194) | 0.466 (0.208) | 0.599 (0.193) | 0.352 (0.179) | 0.607 (0.168) | 0.719 (0.164) |
| | | 0.047 (0.171) | 0.036 (0.186) | 0.025 (0.202) | 0.041 (0.149) | 0.021 (0.145) | -0.010 (0.144) |
| | r-gph | 0.041 (0.351) | 0.118 (0.356) | 0.221 (0.356) | 0.054 (0.334) | 0.189 (0.333) | 0.300 (0.342) |
| | | 0.034 (0.336) | 0.027 (0.332) | 0.079 (0.334) | -0.001 (0.328) | 0.062 (0.300) | 0.163 (0.338) |
| | wh | 0.054 (0.048) | 0.141 (0.069) | 0.203 (0.072) | 0.085 (0.051) | 0.199 (0.064) | 0.254 (0.068) |
| | | 0.013 (0.020) | 0.019 (0.024) | 0.022 (0.026) | 0.015 (0.018) | 0.036 (0.025) | 0.058 (0.030) |
| | $p$ | 0.01 | 0.05 | 0.1 | 0.01 | 0.05 | 0.1 |
| $\sigma_\eta^2 = 0.1$ | rs | 0.256 (0.161) | 0.474 (0.160) | 0.546 (0.156) | 0.416 (0.164) | 0.602 (0.146) | 0.648 (0.156) |
| | | 0.056 (0.098) | 0.046 (0.089) | 0.027 (0.082) | 0.067 (0.089) | 0.044 (0.064) | 0.055 (0.059) |
| | av | 0.181 (0.133) | 0.350 (0.093) | 0.398 (0.069) | 0.311 (0.105) | 0.425 (0.056) | 0.446 (0.048) |
| | | -0.002 (0.087) | 0.011 (0.088) | -0.004 (0.085) | 0.031 (0.078) | 0.043 (0.073) | 0.056 (0.074) |
| | hi | 0.372 (0.133) | 0.444 (0.077) | 0.458 (0.069) | 0.425 (0.094) | 0.465 (0.059) | 0.468 (0.062) |
| | | 0.053 (0.216) | -0.057 (0.133) | -0.103 (0.106) | -0.004 (0.148) | -0.101 (0.084) | -0.113 (0.073) |
| | gph | 0.282 (0.216) | 0.587 (0.198) | 0.703 (0.195) | 0.446 (0.185) | 0.719 (0.160) | 0.808 (0.144) |
| | | 0.034 (0.182) | 0.012 (0.192) | -0.042 (0.206) | 0.061 (0.162) | -0.005 (0.153) | 0.013 (0.136) |
| | r-gph | 0.065 (0.359) | 0.210 (0.354) | 0.319 (0.369) | 0.097 (0.337) | 0.291 (0.325) | 0.399 (0.338) |
| | | 0.002 (0.356) | 0.054 (0.351) | 0.103 (0.362) | 0.034 (0.334) | 0.171 (0.311) | 0.301 (0.336) |
| | wh | 0.080 (0.062) | 0.199 (0.077) | 0.262 (0.076) | 0.125 (0.063) | 0.250 (0.066) | 0.312 (0.068) |
| | | 0.012 (0.018) | 0.021 (0.026) | 0.027 (0.029) | 0.017 (0.021) | 0.057 (0.033) | 0.097 (0.036) |

Table 4: Estimation results for $d$: DGP2, T=500, 1000

| **T=500** | *(p,q)* | *(0.95,0.95)* | *(0.95,0.99)* | *(0.99,0.95)* | *(0.99,0.99)* | *(0.999,0.999)* |
|---|---|---|---|---|---|---|
| | rs | 0.679 (0.178) | 0.558 (0.222) | 0.694 (0.189) | 0.651 (0.208) | 0.439 (0.341) |
| | | -0.031 (0.068) | 0.015 (0.084) | -0.064 (0.055) | -0.025 (0.081) | -0.027 (0.079) |
| | av | 0.455 (0.046) | 0.382 (0.119) | 0.464 (0.040) | 0.434 (0.087) | 0.285 (0.235) |
| | | -0.082 (0.088) | -0.042 (0.084) | -0.121 (0.079) | -0.085 (0.087) | -0.084 (0.091) |
| | hi | 0.474 (0.057) | 0.463 (0.074) | 0.477 (0.052) | 0.468 (0.065) | 0.328 (0.234) |
| | | -0.185 (0.090) | -0.084 (0.134) | -0.236 (0.067) | -0.154 (0.121) | -0.132 (0.180) |
| | gph | 0.884 (0.177) | 0.706 (0.265) | 0.927 (0.165) | 0.861 (0.221) | 0.610 (0.458) |
| | | -0.238 (0.193) | -0.054 (0.192) | -0.254 (0.165) | -0.166 (0.183) | -0.157 (0.213) |
| | r-gph | 0.563 (0.346) | 0.372 (0.390) | 0.619 (0.343) | 0.566 (0.357) | 0.408 (0.464) |
| | | 0.079 (0.332) | 0.061 (0.359) | 0.209 (0.345) | 0.144 (0.370) | 0.129 (0.381) |
| | wh | 0.436 (0.089) | 0.282 (0.123) | 0.499 (0.082) | 0.419 (0.121) | 0.308 (0.234) |
| | | 0.053 (0.039) | 0.021 (0.026) | 0.098 (0.049) | 0.051 (0.046) | 0.066 (0.059) |
| **T=1000** | *(p,q)* | *(0.95,0.95)* | *(0.95,0.99)* | *(0.99,0.95)* | *(0.99,0.99)* | *(0.999,0.999)* |
| | rs | 0.713 (0.180) | 0.691 (0.176) | 0.723 (0.207) | 0.749 (0.186) | 0.534 (0.341) |
| | | -0.069 (0.053) | -0.018 (0.076) | -0.091 (0.038) | -0.046 (0.063) | -0.038 (0.076) |
| | av | 0.469 (0.035) | 0.447 (0.060) | 0.474 (0.034) | 0.467 (0.040) | 0.342 (0.212) |
| | | -0.139 (0.072) | -0.060 (0.079) | -0.177 (0.058) | -0.117 (0.074) | -0.120 (0.089) |
| | hi | 0.475 (0.054) | 0.473 (0.059) | 0.479 (0.052) | 0.480 (0.046) | 0.371 (0.204) |
| | | -0.246 (0.064) | -0.142 (0.098) | -0.281 (0.049) | -0.200 (0.089) | -0.156 (0.167) |
| | gph | 0.942 (0.128) | 0.838 (0.177) | 0.963 (0.133) | 0.942 (0.150) | 0.705 (0.421) |
| | | -0.124 (0.134) | -0.070 (0.154) | -0.139 (0.143) | -0.110 (0.154) | -0.092 (0.155) |
| | r-gph | 0.611 (0.305) | 0.464 (0.353) | 0.654 (0.302) | 0.615 (0.324) | 0.474 (0.406) |
| | | 0.421 (0.339) | 0.171 (0.375) | 0.485 (0.354) | 0.421 (0.395) | 0.413 (0.398) |
| | wh | 0.472 (0.075) | 0.348 (0.095) | 0.532 (0.071) | 0.469 (0.086) | 0.367 (0.219) |
| | | 0.099 (0.041) | 0.031 (0.028) | 0.164 (0.037) | 0.102 (0.046) | 0.099 (0.073) |

Table 5: Estimation results for $d$: DGP3, T=500, 1000

| **T=500** | $\gamma$ | $10^{-5}$ | $10^{-1}$ | 1 | 10 | $10^{3}$ |
|---|---|---|---|---|---|---|
| | rs | 0.719 (0.308) | 0.731 (0.330) | 0.714 (0.285) | 0.709 (0.191) | 0.044 (0.085) |
| | | -0.093 (0.043) | -0.092 (0.041) | -0.089 (0.046) | -0.059 (0.060) | 0.037 (0.086) |
| | av | 0.476 (0.030) | 0.478 (0.033) | 0.475 (0.033) | 0.463 (0.041) | -0.012 (0.075) |
| | | -0.204 (0.058) | -0.207 (0.058) | -0.192 (0.060) | -0.116 (0.084) | -0.018 (0.074) |
| | hi | 0.476 (0.051) | 0.479 (0.049) | 0.479 (0.047) | 0.475 (0.052) | 0.076 (0.132) |
| | | -0.279 (0.042) | -0.282 (0.042) | -0.278 (0.042) | -0.230 (0.074) | 0.069 (0.131) |
| | gph | 0.989 (0.162) | 0.994 (0.164) | 0.997 (0.157) | 0.924 (0.154) | 0.011 (0.172) |
| | | -0.615 (0.155) | -0.609 (0.155) | -0.508 (0.143) | -0.231 (0.172) | 0.017 (0.162) |
| | r-gph | 0.747 (0.328) | 0.734 (0.318) | 0.749 (0.319) | 0.624 (0.327) | -0.005 (0.344) |
| | | -0.213 (0.374) | -0.247 (0.362) | -0.066 (0.363) | 0.267 (0.368) | -0.017 (0.353) |
| | wh | 0.982 (0.023) | 0.958 (0.033) | 0.825 (0.059) | 0.485 (0.082) | 0.012 (0.018) |
| | | 0.695 (0.051) | 0.657 (0.051) | 0.472 (0.056) | 0.080 (0.046) | 0.009 (0.017) |
| **T=1000** | $\gamma$ | $10^{-5}$ | $10^{-1}$ | 1 | 10 | $10^{3}$ |
| | rs | 0.729 (0.348) | 0.707 (0.339) | 0.731 (0.302) | 0.728 (0.199) | 0.061 (0.084) |
| | | -0.057 (0.046) | -0.055 (0.048) | -0.010 (0.045) | 0.128 (0.060) | 0.044 (0.081) |
| | av | 0.478 (0.029) | 0.478 (0.029) | 0.478 (0.046) | 0.473 (0.031) | 0.011 (0.073) |
| | | -0.163 (0.049) | -0.163 (0.053) | -0.104 (0.061) | 0.111 (0.069) | -0.001 (0.068) |
| | hi | 0.477 (0.052) | 0.476 (0.052) | 0.479 (0.046) | 0.477 (0.054) | 0.147 (0.144) |
| | | -0.271 (0.036) | -0.267 (0.039) | -0.229 (0.046) | -0.087 (0.068) | 0.137 (0.155) |
| | gph | 0.997 (0.125) | 1.001 (0.128) | 1.001 (0.127) | 0.962 (0.125) | 0.045 (0.142) |
| | | -0.590 (0.111) | -0.597 (0.118) | -0.591 (0.122) | -0.279 (0.133) | 0.023 (0.138) |
| | r-gph | 0.735 (0.303) | 0.720 (0.293) | 0.723 (0.296) | 0.661 (0.293) | 0.0134 (0.330) |
| | | -0.188 (0.349) | -0.213 (0.354) | -0.177 (0.356) | 0.354 (0.316) | -0.008 (0.321) |
| | wh | 0.987 (0.015) | 0.965 (0.025) | 0.834 (0.049) | 0.514 (0.069) | 0.010 (0.015) |
| | | 0.825 (0.033) | 0.793 (0.033) | 0.656 (0.036) | 0.324 (0.045) | 0.009 (0.014) |

Table 6: Estimation results for $d$: DGP ARFIMA(0,d,0), T=500, 1000, 2000

| **T=500** | $d$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.45 |
|---|---|---|---|---|---|---|
| | rs | 0.117 (0.089) | 0.201 (0.103) | 0.272 (0.111) | 0.360 (0.120) | 0.393 (0.129) |
| | | 0.075 (0.087) | 0.048 (0.077) | 0.000 (0.069) | -0.033 (0.056) | -0.041 (0.054) |
| | av | 0.068 (0.074) | 0.153 (0.075) | 0.216 (0.074) | 0.289 (0.073) | 0.317 (0.069) |
| | | 0.024 (0.076) | 0.011 (0.078) | -0.037 (0.073) | -0.080 (0.069) | -0.095 (0.074) |
| | hi | 0.074 (0.107) | 0.184 (0.109) | 0.262 (0.117) | 0.369 (0.104) | 0.417 (0.098) |
| | | 0.017 (0.122) | -0.029 (0.147) | -0.138 (0.089) | -0.194 (0.072) | -0.208 (0.071) |
| | gph | 0.117 (0.162) | 0.197 (0.167) | 0.315 (0.175) | 0.408 (0.178) | 0.481 (0.166) |
| | | 0.095 (0.165) | 0.101 (0.172) | 0.080 (0.183) | -0.014 (0.178) | -0.085 (0.183) |
| | r-gph | 0.082(0.350) | 0.165 (0.335) | 0.241 (0.371) | 0.284 (0.369) | 0.371 (0.340) |
| | | 0.079 (0.347) | 0.162 (0.336) | 0.217 (0.364) | 0.244 (0.372) | 0.311 (0.358) |
| | wh | 0.093 (0.036) | 0.194 (0.033) | 0.291 (0.036) | 0.396 (0.039) | 0.446 (0.036) |
| | | 0.079 (0.037) | 0.148 (0.041) | 0.215 (0.044) | 0.291 (0.046) | 0.332 (0.047) |
| **T=1000** | $d$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.45 |
| | rs | 0.101 (0.089) | 0.199 (0.099) | 0.286 (0.109) | 0.354 (0.118) | 0.399 (0.114) |
| | | 0.062 (0.077) | 0.021 (0.067) | -0.017 (0.054) | -0.034 (0.048) | -0.032 (0.042) |
| | av | 0.063 (0.067) | 0.155 (0.071) | 0.235 (0.071) | 0.292 (0.068) | 0.325 (0.062) |
| | | 0.027 (0.067) | -0.007 (0.066) | -0.056 (0.067) | -0.089 (0.061) | -0.099 (0.060) |
| | hi | 0.080 (0.111) | 0.181 (0.110) | 0.275 (0.106) | 0.359 (0.109) | 0.401 (0.105) |
| | | 0.024 (0.118) | -0.085 (0.122) | -0.169 (0.079) | -0.216 (0.062) | -0.220 (0.060) |
| | gph | 0.106 (0.139) | 0.201 (0.124) | 0.306 (0.134) | 0.404 (0.132) | 0.458 (0.138)) |
| | | 0.088 (0.139) | 0.096 (0.141) | 0.059 (0.153) | -0.010 (0.153) | -0.097 (0.153) |
| | r-gph | 0.089 (0.354) | 0.121 (0.330) | 0.198 (0.350) | 0.288 (0.326) | 0.316 (0.352) |
| | | 0.087 (0.355) | 0.108 (0.334) | 0.186 (0.347) | 0.243 (0.318) | 0.277 (0.340) |
| | wh | 0.094 (0.025) | 0.196 (0.025) | 0.296 (0.027) | 0.399 (0.027) | 0.449 (0.026) |
| | | 0.087 (0.024) | 0.165 (0.027) | 0.242 (0.031) | 0.333 (0.032) | 0.379 (0.032) |
| **T=2000** | $d$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.45 |
| | rs | 0.110 (0.084) | 0.196 (0.092) | 0.279 (0.096) | 0.369 (0.110) | 0.407 (0.117) |
| | | 0.069 (0.072) | 0.052 (0.066) | 0.058 (0.060) | 0.091 (0.065) | 0.112 (0.062) |
| | av | 0.074 (0.067) | 0.159 (0.068) | 0.232 (0.065) | 0.304 (0.065) | 0.333 (0.059) |
| | | 0.041 (0.065) | 0.044 (0.065) | 0.054 (0.067) | 0.095 (0.072) | 0.120 (0.069) |
| | hi | 0.083 (0.104) | 0.182 (0.109) | 0.273 (0.106) | 0.361 (0.105) | 0.414 (0.092) |
| | | 0.028 (0.112) | -0.046 (0.115) | -0.095 (0.076) | -0.093 (0.069) | -0.085 (0.071) |
| | gph | 0.110 (0.113) | 0.198 (0.108) | 0.303 (0.117) | 0.396 (0.120) | 0.479 (0.112) |
| | | 0.093 (0.111) | 0.117 (0.112) | 0.099 (0.121) | 0.063 (0.126) | 0.018 (0.122) |
| | r-gph | 0.088 (0.355) | 0.144 (0.348) | 0.218 (0.365) | 0.263 (0.331) | 0.332 (0.341) |
| | | 0.086 (0.355) | 0.140 (0.347) | 0.206 (0.364) | 0.240 (0.337) | 0.300 (0.358) |
| | wh | 0.097 (0.018) | 0.198 (0.017) | 0.298 (0.018) | 0.399 (0.018) | 0.450 (0.017) |
| | | 0.093 (0.018) | 0.185 (0.018) | 0.280 (0.019) | 0.378 (0.019) | 0.428 (0.019) |

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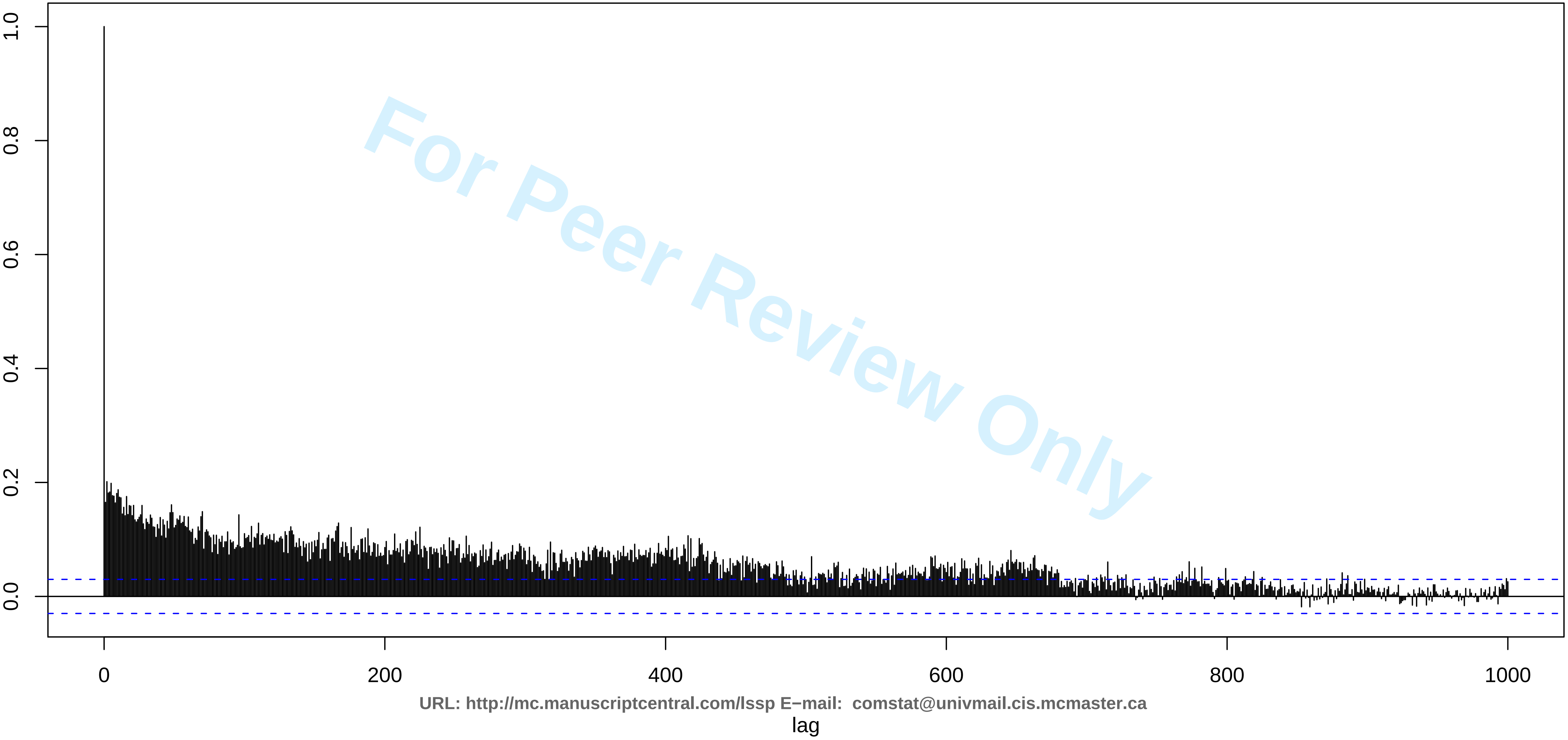
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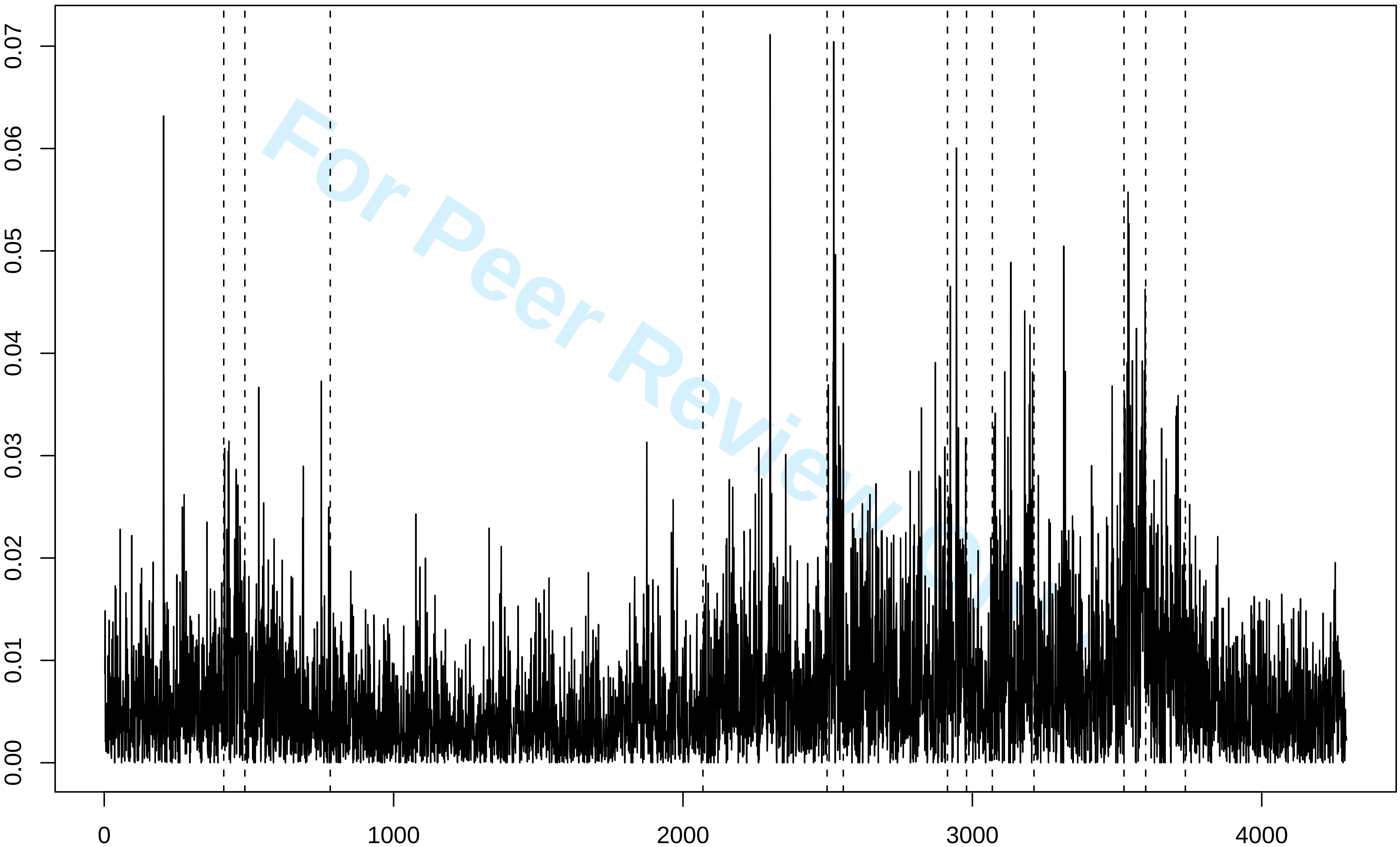
0.8
0.6
0.4
0.2
0.0
0
200
400
600
800
1000
lag


0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0
1000
2000
3000
4000