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Dislocation bowing and passing in persistent slip bands

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Abstract: In response to a recent paper by Mughrabi and Pschenitzka (2005), an estimate is given of the saturation stress in cyclic plasticity, assuming only the bowing of screw dislocations and the simultaneous passing of them at a critical separation determined by their cross-section for mutual annihilation by cross-slip. Although, as Mughrabi and Pschenitzka show, the simple addition of the two stresses is a poor approximation, if the bowing stress is made equal to the passing stress, the equations governing the saturation stress and the associated average plastic strain in a persistent slip band are scarcely altered from earlier work (Brown 2004). A comparison between theory and experiment shows good quantitative agreement. Finally, an estimate is made of the alternating internal stress in a band, and reasons suggested for why it does not contribute to the saturation stress.

1. Introduction

In persistent slip bands formed by subjecting a ductile material to cyclic plastic strain, screw dislocations shuttle backwards and forwards between walls of densely-packed edge dislocation dipoles. The maximum stress achieved in any one hysteresis loop saturates after several hundred cycles, becoming constant with further cycling and defining the endurance limit in conventional fatigue tests. There are several possible contributions to the endurance limit: (i) the stress required to make screw dislocations of opposite sign pass one another, which is the same as the stress required to split apart screw dislocation dipoles stranded between the walls; (ii) the stress required to bow the screw dislocations between the walls; (iii) internal stress resulting from inhomogenous plastic deformation, caused by the resistance to plastic flow of the walls being much greater than that of the inter-wall, dislocation-free, material; (iv) friction stress which might come from dislocation debris left between the walls by the shuttling screw dislocations. Brown [1] neglected all but the first two of these, and demonstrated that quantitative formulae could be found which correlate the observed wall spacing with both the saturation stress and the characteristic plastic strain carried by the persistent slip bands. The key assumptions made were that the bowing stress must equal the passing stress, and that the two contributions to the saturation stress should add together. Recently, Mughrabi and Pschenitzka [2] have queried the the latter assumption, and in a new calculation taking into account all four of the contributions listed above are able to reach agreement with data for the saturation stress of copper at room temperature. There is no doubt that they are right to insist that the simple additive assumption of Brown [1] must be modified. An attempt is made here to find improved analytical formulae which enable a quantitative account to be given of the saturation stress and the plastic strain amplitude in persistent slip bands.

2. Combined Bowing and Passing of Screw Dislocations

Fig 1(a) shows the configuration analysed by Brown [1]. Rigid screw dislocations of opposite sign must escape from one another while at the same time they deposit edge dislocations in the walls, between which they are confined. The stress required to accomplish this is

$$\tau = \frac{2E_{edge}}{bd} + \frac{\mu b}{4\pi h} \qquad (1)$$

The first term gives the contribution from bowing an isolated screw dislocation of Burgers vector b between walls of spacing d, and the second the contribution from splitting an infinitely long screw dislocation dipole of height h - that is to say, making the screw dislocations pass one another on slip planes of spacing h simultaneously with the bowing. The minimum value of h, h_c, at which the screw dislocations annihilate one another by cross-slip, determines the maximum stress achievable, and hence the endurance limit. Equation (1) can be easily derived by imagining the rigid screw dislocations to move forward by a virtual displacement δx. The work done by the total stress includes depositing edge dislocations of energy per unit length[1] E_{edge} at the walls, as well as simultaneously overcoming the maximum splitting force of the straight screw dipole. However, the approximation of rigid dislocations has been rightly criticised by Mughrabi and Pschenitzka [2]. Fig 1(b) shows their proposed configuration. A truncated elliptical arc of dislocation αβγδ is pushed by the applied stress between two walls. Simultaneously its partner of opposite sign is pushed in the opposite direction. The two dislocations attract one another and if their curvature is not great can be thought of as forming a dipole of width x and of height determined by the spacing between their two slip planes, h. As the stress is increased from zero, the elliptical segments become more and more curved, until they become tangent to the walls, whereupon they escape one another. This is the critical stress for flow. An equation can be written for the stress to maintain the separation x:

$$\tau(x) = \frac{4T_{screw}x}{b\left(d^2+\kappa^2x^2\right)} + \frac{\mu b}{2\pi}\frac{x}{x^2+h^2}. \qquad (2)$$

The applied stress τ acts both to bow the dislocation, whose line tension is T_{screw}, and to split the dipole. The ratio of major to minor axes of the ellipse is κ. As in equation (1), the first term in equation (2) is the bowing term, and the second the dipole-splitting (or screw-passing) term. Equation (2) shows that as the separation between the screw dislocations increases, the stress required to maintain them in position goes through two maxima, one mostly due to the bowing, at $x = d/\kappa$, and the other mostly due to the passing, at $x = h$. The maximum passing stress occurs when the two screw dislocations are close enough to annihilate one another by cross-slip, which again defines a critical minimum dipole height, h_c. Mughrabi and Pschenitzka plot the stress as a function of separation and show that for equal maximum passing stress and bowing stress, using the experimentally observed values of bowing stress and h_c, the two

[1] Table 1 shows the formulae used for dislocation line energies and tensions.

maxima in the combined stress given by equation (2) are equal to one another and are only about 17% greater than the stress due to either bowing or passing acting alone. That is to say, in equation (1), only about 17% of the bowing resistance impedes the passing, or equivalently, only about 17% of the passing resistance impedes the bowing.

Mughrabi and Pschenitzka's pioneering calculation leading to equation (2) is a considerable improvement upon the rigid dislocation approximation leading to equation (1). However, despite allowing the passing dislocations to be curved, it allows them only one 'degree of freedom' – namely the separation x. The separation determines both the dipole interaction term and the bowing term. This limitation is particularly evident if one thinks of the bowing term, which if it were to act alone, must have a maximum value equal to the Orowan stress, and depend upon the energy - not the tension - of the trailing edge dislocations $\alpha\beta$ and $\gamma\delta$ in Fig. (1b). These dislocations pull the elliptical sections towards one another, but their effect is not taken into account. For this reason, it seems worthwhile to attempt a more elaborate calculation, in anticipation of accurate computer models which one hopes will produce a definitive solution to the problem.

Fig. 1(c) shows a configuration with three degrees of freedom: the separation x, as before, but now allowed to depend upon position x' in the channel, together with independently determined values of the radii of curvature of the dislocations near the walls and in the centre of the channel. A further refinement is to improve the model of the wall, which consists of edge dislocation dipoles. Consider the three regions separately under the action of the applied stress:

(i) AB and EF: here, the edge dislocation which is part of the bowing configuration experiences zero force because its closely-spaced neighbour suffers an equal and opposite force, and the two dislocations comprising the dipole find an equilibrium, unsplittable, configuration. All dislocations AB and EF are straight.

(ii) BC and DE: here, the crossing dislocations pull the edge dipoles apart and feel not only the applied stress, but attraction for each other. As shown inset in the diagram, this region is modelled by three touching circles, rather as in a dislocation node. One expects the radii of curvature r_{BC}, r_{DE} to be very small. An estimate is given below.

(iii) CD: here, because the dislocations are on the point of passing one another, they feel the applied stress pushing them forward, and the passing stress near its maximum pulling them back. Because they are screw dislocations, they are stiff and have a large radius of curvature, r_{CD}. An estimate is given below. As a result of the curvature, not all the length CD experiences the maximum passing stress, but some average $\langle \tau_{pass} \rangle$.

At the position of unstable equilibrium, the variation of energy with respect to any variable defining the shape is zero; furthermore, its second derivative with respect to one of them is negative. The unstable dislocation thus advances at constant shape. We assume that the instability arises from a small displacement δx. We have

$$2E_{edge}\delta x + \langle \tau_{pass} \rangle b(d - 2r_{BC})\delta x = \tau b d \delta x. \qquad (3)$$

We now estimate r_{BC}. It is very small, so it need not be calculated very carefully. Without other dislocations, the force maintaining the curvature is $\tau b = T/r$. The appropriate tension T might be for a pure edge dislocation (that is, the one on the left-hand of the inset diagram) or for a mixed screw-edge (45^0) dislocation. We choose the latter. Let us imagine that the dislocations in the node-like triple-circle construction are distant on average from one another by the diameter of the inscribed circle, about $r_{BC}/3$. The approximate equation for equilibrium including the dislocation interactions is now

$$\frac{1}{r_{BC}}\left(T_{45}-\frac{3\mu b^2}{2\pi(1-\nu)}\right)=\tau b=\frac{T_{eff}}{r_{BC}}\ . \tag{4}$$

Table 1 shows values for these quantities. One finds that the effective line tension is very small indeed, $T_{eff}\approx 0.1\mu b^2$. There is even the possibility that the effective tension is negative, pulling the circles into sharp cusps. However, what is certain is that the radius of curvature is very small, less than a few percent of the channel width. The 'sharp corner' approximation in Figs. 1(a) and (b) is very good indeed.

We now estimate the average passing stress – the dipole splitting stress – felt by the curved screw segments. At the centre of the channel, they experience the maximum stress. Our aim is to find an approximate expression for the interaction stress between the two gently curved segments. If we expand the passing stress near its maximum as a function of the distance t measured from where the screw would be if it were straight – see the inset to Fig. 1(c) – we find for the interaction stress

$$\tau(t)\approx\frac{\mu b}{4\pi h}-\frac{\mu b t^2}{8\pi h^3} \tag{5}$$

But the curved screw dislocations have a position given by

$$t\approx\frac{1}{2}\frac{x'^2}{r_{CD}}, \tag{6}$$

where x' measures the distance from the centre of the channel (see Fig. 1(c)). Our average passing stress is now given by

$$\langle\tau_{pass}\rangle\approx\frac{\mu b}{4\pi h}\frac{2}{d}\int_0^{d/2}\left(1-\frac{x'^4}{8r_{CD}^2h^2}\right)dx'=\frac{\mu b}{4\pi h}\left(1-\frac{d^4}{640r_{CD}^2h^2}\right) \tag{7}$$

The radius of curvature at the centre of the channel is given by

$$\frac{T_{screw}}{r_{CD}} = \tau - \tau(0) = \tau - \frac{\mu b}{4\pi h}. \qquad (8)$$

If equations (8), (7), and (4) are substituted into equation (3), we get an equation for the applied stress maintaining the dislocation in its position of unstable equilibrium:

$$\frac{2E_{edge}}{bd} + \frac{\mu b}{4\pi h}\left(1 - \frac{b^2 d^4\left(\tau - \frac{\mu b}{4\pi h}\right)^2}{640 h^2 T_{screw}^2}\right)\left(1 - \frac{2T_{eff}}{\tau d b}\right) = \tau. \qquad (9)$$

Although this equation looks formidable, the final bracket differs negligibly from unity, and we can write

$$\tau = \frac{\mu b}{4\pi h} + \alpha \frac{2E_{edge}}{bd} \qquad (10)$$

to find

$$1 - \alpha - \alpha^2 \frac{\mu b^2}{2E_{edge}} \frac{1}{640\pi}\left(\frac{d}{h}\right)^3 \left(\frac{E_{edge}}{T_{screw}}\right)^2 = 0. \qquad (11)$$

Equation (11) makes a great deal of physical sense. If the screw tension is much greater than the edge energy, the screw dislocations are rigid and straight, giving $\alpha = 1$, and the flow stress is the simple addition of the passing stress and the Orowan stress, as in equation (1). The approximations cannot work if the screw tension is much less than the edge energy. Putting in numerical values for copper from

Table 1, the ratio E_{edge}/T_{screw} is about equal to ½. The ratio of d to h is about equal to 30, perhaps somewhat less. One finds the coefficient of α^2 in equation (11) to be about 2, giving $\alpha \approx 0.5$. In fact, for values of the coefficient between ½ and 4, α varies slowly from 0.7 to 0.4. It doesn't depend strongly upon the assumed values of the parameters.

If we return to equation (2), and the results of Mughrabi and Pschenitzka [2], we see that they have calculated the value $\alpha = 0.17$.

Now we shall write

$$\tau = \frac{2E_{edge}}{bd} + \frac{\mu b}{4\pi h} - (1-\alpha)\frac{2E_{edge}}{bd} = \tau_{Orowan} + \tau_{pass} \quad (12)$$

where we recognise that the Orowan or bowing stress must always be overcome, but the passing stress can be reduced from its value for infinite straight screw dislocations by bowing: the reduction amounting to some 50% of the Orowan stress, if the calculation above is correct.

The various models for combined bowing and passing thus produce a range of values for α. However, in all of them a bowing term can be recognised, and a passing term reduced from its value for rigid screw dislocations.

3. Equations for persistent slip

In cyclic plasticity, saturation is accompanied by the development of dislocation walls with an equilibrium spacing. Edge dislocation dipoles, a majority of them of vacancy type, are randomly disposed on their slip planes within the walls. The walls are thought to act as zones where intruder edge dislocations either annihilate with a partner of opposite sign, or become tightly bound into a dipole, while at the same time releasing another which was, before its release, less well bound.

Equilibrium of the wall spacing requires the bowing stress and the passing stress to be equal. This is a general argument relating to dissipative structures. The force causing the wall spacing to increase originates in bowing. When the dislocations are bowing, spacings slightly smaller than average are pulled together by line tension so the overall spacing increases. However, the force causing the wall spacing to decrease comes from screw dipoles straddling the walls and pulling all the walls, of whatever spacing, together. If the passing stress is larger than the bowing stress, the screw dipoles pull the walls together. But if the bowing stress is larger than the passing stress, the walls are on average pulled apart. They must be equal to produce an equilibrium wall spacing. At a certain minimum dipole height h_c the screw dipoles vanish by mutual cross-slip. This produces the maximum passing stress, and hence the maximum bowing stress. The combination of the two is the saturation stress.

Note that the vanishing of the screw dipoles is unlikely to occur exactly at peak load, when the screws are being forced apart by the applied stress, but in the course of unloading. The equilibrium wall structure is a product of the statistical exchange and shuffling of edge dislocations throughout the cycle, not just at its peak.

If we equate τ_{Orowan} with τ_{pass} in equation (12), we find an equation relating the critical dipole height to the wall spacing:

$$h_c = \frac{\mu b^2}{8\pi E_{edge}(2-\alpha)} d_c \ . \qquad (13)$$

This equation, with $\alpha = 0.5$, produces 46nm for the critical spacing. This value agrees satisfactorily with the experimental value of 55nm observed by Mughrabi, Ackermann and Herz [3] under conditions in which there is neutron damage which preserves the screw dislocations for observation by electron microscopy by contributing a friction stress to both the bowing and passing process[2].

Equations for the saturation stress and plastic strain amplitude become:

$$\tau_c = \frac{4E_{edge}}{bd_c} = \frac{\mu b}{2\pi h_c(2-\alpha)}. \tag{14}$$

Because the **bowing of the screws affects the minimum spacing they can have,** the cross-section for annihilation by cross-slip, h_c as it appears in Brown [4], is replaced by $h_c(2-\alpha)$ and the equation for the plastic strain amplitude e_p in that paper is unchanged:

$$e_p = \frac{\pi(1+p)^2}{(1-p)^2}\frac{3d_c}{3d_c+d_w}\frac{\tau_c}{\mu} \approx 4\pi\frac{\tau_c}{\mu}, \tag{15}$$

where the probability of penetration of a wall is p, equal to 1/3 at saturation. The term involving the wall separation and width is an attempt to correct the equation for the partial permeability of the walls; it **differs by only a few percent from** unity.

Thus the cross-section for cross-slip, calculable in principle from atomistic models, controls the critical slip plane spacing, which controls in turn the Orowan stress and the saturation stress.

4. Comparison with Experiment

Fig. 2 shows a plot of the observed saturation stress for a variety of materials as a function of the calculated Orowan stress. As shown by Tippelt, Bretschneider and Hähner [5], the wall-width d_w is independent of temperature, and for both Ni and Cu is about equal to

[2] Equation (13) differs by 50% from the earlier equation of Brown [1].

0.16 μm. The logarithm which appears in Table 1 is thus very nearly equal to 2π. To an excellent approximation we have

$$\tau_{Orowan} = \frac{\mu b}{(1-\nu)d}\ , \qquad (16)$$

and this is what is plotted as the abscissa in Fig. 2. The elastic constants which accurately give the energies of edge and screw dislocations are taken from those listed by Bacon [6]. The data for Mg and AgCl are taken from Kwadjo and Brown [7] and Ogin [8], as quoted by Brown [9]. The temperature dependence of the elastic constants, which produces something like a 10% reduction in the slope of the graph, is taken from the tables by Simmons and Wang [10].

The resulting best-fit line has a slope of 1.9 ± 0.1, to be compared with the expected value of 2.0. It has an intercept of 1.9 ± 2.6 MPa, consistent with zero. The most comprehensive data for a single metal are those for Ni presented in Fig. 6 of the paper by Bretschneider, Holste and Tippelt [11]. They find a small positive intercept of 5.6 MPa, and a slope which agrees with Eq. (16) to within 4%. There can be no doubt that Eq. (14) gives very precise agreement with experiment, somewhat better than the earlier equations of Brown [1], which were based on the assumption of impenetrable walls.

The small positive intercept for the Ni results perhaps comes from the data obtained at higher temperatures. This might be affected by the onset of sessile vacancy loop formation between walls, as observed at similar homologous temperatures in Mg by Kwadjo and Brown [7]. It seems unlikely that the saturation stress in the absence of diffusion can depend on a term arising from mobile interwall debris. According to Kratochvíl and co-workers [12, 13] (see also Kubin and Kratochvíl [14]), debris in the form of glissile dipole loops **unconnected from the primary dislocations** is swept into the walls by the bowing dislocations, leaving the channels clear. This process**, together with the pulling into walls of edge dipoles connected to primary**

dislocations, is the primary cause of pattern formation in cyclic plasticity.

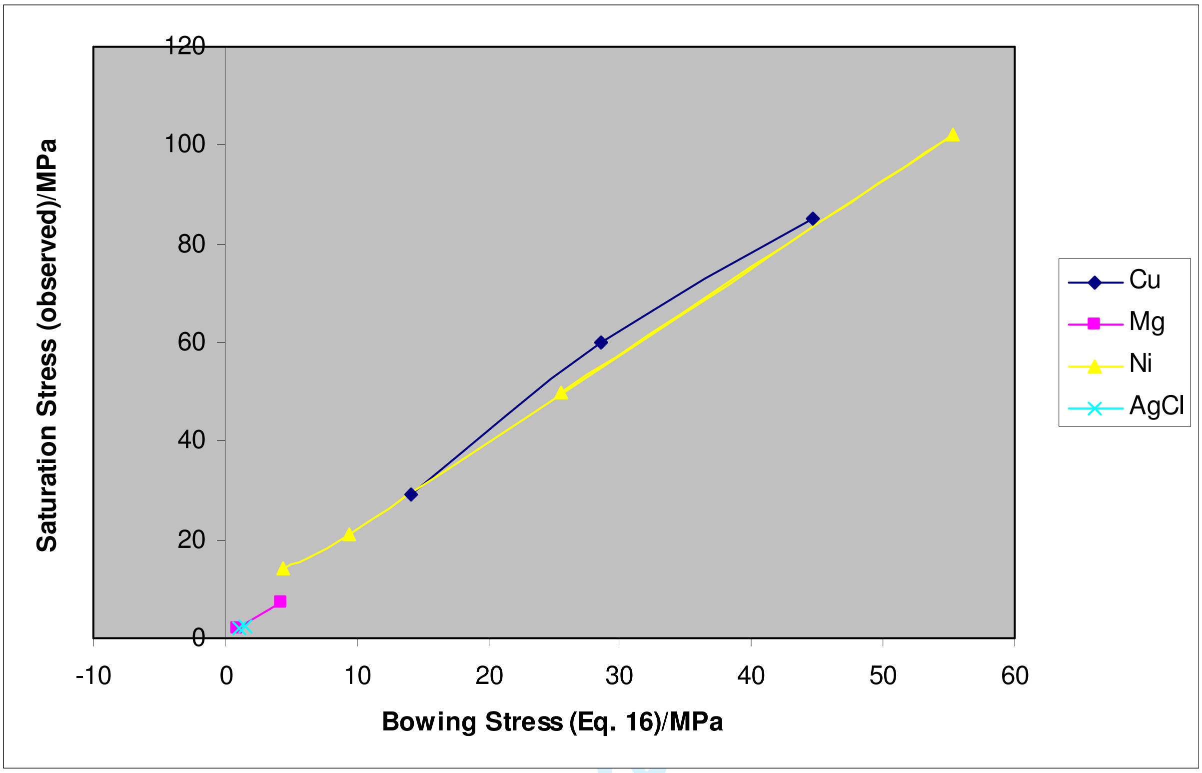

Fig. 2

One cannot understand the phenomenon of persistent slip without an explanation for the limited plastic strain borne by the shuttling screw dislocations. Because this strain is limited, the volume fraction of persistent slip bands must increase in proportion to the imposed overall cyclic plastic strain: Winter's law [15]. Although the local strain is very inhomogeneous, the mean strain is well-defined and if it is determined by the mean free path for annihilation of the screw dislocations by cross-slip, it is given by Eq. (15). Bretschneider *et al.* [11] in their Fig. 7 show all the data for Ni and Cu. It can be represented by

$$e_p \approx (12.7 \pm 0.1)\frac{\tau_{sat}}{\mu} + 10^{-3}. \qquad 17$$

This result is practically identical to that found independently by Brown [1] on the basis of the Cu data alone. From Eq. (16), one expects a slope somewhat less than $4\pi \approx 12.6$. The agreement with experiment is adequate, although again there appears to be a small intercept. If one attempts to include an estimate of the full correction for the wall width one finds a somewhat increased slope of the line with a reduced intercept. At the moment, the work by Bretschneider *et al*. [11] seems to mark the limit of what one can sensibly do with the data.

The important point here is that the simplest theory agrees with experiment to better than 10%. It provides an outline explanation for the endurance limit in fatigue and Winter's law in cyclic plasticity. Although it is fascinating to see how closely more refined theories can be made to produce exact quantitative agreement with experiment, studies of cyclic plasticity are not the best way to determine the numerical value of π!

5. Internal Stress

It seems clear that the bowing and passing of screw dislocations in persistent slip bands can account for the saturation stress without a contribution from internal stress. Yet the walls impede many glide dislocations, so on average at the maximum forward plastic strain they must contain elastic shear stresses which promote forward flow, balanced by weaker back stresses in the channels. Why do these back stresses not contribute to the saturation stress?

The answer can be given that if the walls are deformable, the resulting distribution of internal stress does not contribute to the flow stress. Recent authors – see Nabarro and Duesbery [16] – are in agreement that this is true for all composite models, whatever the detailed configuration of the deformable obstacles. However, one must be careful: is the inclusion of a bowing term consistent with the concept of a deformable dipole wall? How can a deformable wall exclude the bowing dislocations?

The concept of a statistically deformable wall [4] circumvents this problem. Dislocations encountering the wall may penetrate it, or be absorbed into the dipole structure. The wall does not have a well-defined flow stress, but acts as a filter on dislocations which intrude into it. Consider dislocations AB and EF in Fig. 1(c). They are lodged in dipoles in the wall, and because they have entered it in a definite sense, driven by the applied stress, the dipoles produce an elastic polarisation and thus a long-range image stress. This can be calculated if we regard each dipole as a sheared inclusion whose interior cannot be penetrated: there is a large 'forward' stress inside the dipole (in the space between its dislocations) and a balancing 'back' stress elsewhere. The accommodation factor γ for a sheared, ribbon-like inclusion of circular cross-section can be found from Brown and Clarke [17] and it is given by $\gamma = 1/4(1-\nu)$. This also gives **exactly** the image stress due to a dipole at the centre of a cylindrical body. The dipoles have an effective width approximately equal to the wall-width – these are the intruder dislocations stuck in the wall – and their spacing perpendicular to the slip plane, H, is determined by the engineering plastic strain amplitude per quarter cycle: $e_p = b/H$. However, about 1/3 of them penetrate the wall and so do not contribute to the image stress. Thus the image stress due to the dipoles is given by

$$\langle \tau \rangle_M = \frac{2}{3}\frac{\mu b w}{4(1-\nu)Hd} = \gamma_{average}\,\mu f_{wall}\, e_p, \qquad 18$$

where $\gamma_{average} = (2/3)(1/4(1-\nu)) \approx 1/4$.

An electron microscopist, viewing the wall as an inclusion with volume fraction f_{wall}, will estimate the 'mean stress in the matrix' – that is, the internal stress between the walls – using the Eshelby equation with an average accommodation factor of about ¼, whereas an entirely equivalent view is that the wall produces an image stress because it is an assembly of dipoles due to the intruder dislocations stuck near its periphery.

The engineering plastic strain amplitude in the persistent slip band is related to the saturation stress by Eq. 15, so we have finally

$$\langle\tau\rangle_M = 4\pi\gamma_{average} f_{wall} \tau_c \approx 0.3\tau_c . \qquad 19$$

The residual alternating back stress is thus about one-third of the saturation stress. This is in approximate agreement with a variety of results. For Cu, Mughrabi and Ungar [18] present data which support an internal stress about equal to ¼ the saturation stress. Work by Hecker and co-workers [19, 20, 21] and theoretical work by the same team [22] supports a value somewhat smaller, between 0.1 and 0.2. Lisiecki and Pedersen [23], on the basis of observations of wall configurations, suggest an average accommodation factor of about 1/3. A further study by Pedersen and Winter [24] confirms this value, and suggests that a higher value – about 0.45 – is appropriate for the thicker bundles of dipoles in the matrix structure. The estimate given here does not depend upon the detailed arrangement of the dipoles in the wall, so the agreement with Lisiecki and Pedersen's analysis is fortuitous. All one can say is that the dipoles exert very little elastic influence upon one another, so the walls can easily be wavey and slanted, as noted by them. In Ni, X-ray measurements suggest a value of 0.3 [21]. These authors also show beautifully how the internal stress builds up over one cycle. On reversal of the strain, it takes about a quarter of a cycle for the internal stress to reverse, after which it remains constant. This is exactly what one expects for **Mughrabi's model of** a deformable wall, in which no work-hardening takes place. It is also what one expects for a statistically deformable wall in which the dislocation content comes into equilibrium, with as many intruder dislocations getting stuck in the wall as are liberated from it.

Although the estimates given above for the magnitude of the internal stress at the end of a stroke seem very reasonable, nagging doubts remain about why these stresses seem to contribute nothing to the saturation stress. How can the wall be penetrable, yet offer resistance to the bowing dislocation? In part, this comes about because under the applied stress the dipoles AB and FE in Fig. 1(c) feel no force except for that due to the line tension from the bowing and passing screws. But there is another possible reason, as follows.

Consider a dispersion-hardened metal containing a random array of undeformable obstacles. In this case, the uncorrelated position of the obstacles precludes simultaneous bowing and passing of the dislocations: only bowing is important. The virgin flow stress is accurately equal to the stress required to bow single dislocations between the obstacles, without the addition of a term due to the mean stress in the matrix, often called the 'back stress'. Yet once the first dislocations have swept through the field of obstacles, leaving Orowan loops around each one, the alloy has suffered a microstrain approximately equal to the Burgers vector divided by the diameter of the obstacle: this may be as large as a few percent strain if the obstacles are of nanometre size. Thus the microstrain establishes a back stress. But because the back-stress due to the obstacles is set up only when the Orowan loops around them are completed, it does not contribute to the flow stress. From the point of view of elasticity, the bowing dislocation produces a long-range stress field equal to that of a straight dislocation, together with a shorter range field due to its excursions from straightness. These excursions comprise uncompleted loops of both signs. Once the unstable position of the bowing dislocation has collapsed, there is the field of the straight dislocation, now forced against another array of obstacles, together with that of the Orowan loops left behind, systematically of one sign. The field of the loops, inversely proportional to the cube of the distance from each one, communicates with the external surfaces of the alloy to produce the image field and hence the long-range back stress. Until the loops are deposited, there is no back-stress. It is interesting to think about the effect of the bowing dislocation as it passes through its unstable configuration: it generates a violent pulse of lattice vibrations which after multiple reflections at the surfaces establish the image stress. Before the pulse, the external surface is scarcely affected by the plasticity about to occur; after the pulse its shape is changed by the long-range stress field of the internally stored Orowan loops. Another way of looking at this is to think about the energy storage per unit volume in the alloy. The back stress can be regarded as the derivative of the irreversible elastic energy storage with respect to strain, or equally well as due to the average stress felt by a dislocation which cannot sample the obstacle interiors. Until the Orowan loops are completed, there is no irreversible elastic energy storage, nor is there an average back stress.

If the saturated hysteresis loop of the metal undergoing cyclic plasticity is entirely in the microstrain region, then the back stress will not contribute to the saturation stress. The highly inhomogeneous microscopic behaviour of slip, resulting probably from the statistical nature of the penetration of the walls by intruder dislocations, is an important part of this picture. One must imagine that early in one cycle intense slip takes place on planes where the obstacles can be easily penetrated, followed by slip on the more difficult planes, until the cross-slip of the screw dislocations exhausts the available strain.

The cross-slip of the screw dipoles of critical spacing is most likely to occur in the unloaded state, when the screw dislocations of opposite sign are not pushed apart by the applied stress [25]. Thus the completion of dipole formation will take place as the specimen is unloaded. On this view, the back stress, observed in the unloaded state, does not contribute to the saturation stress.

There is scope for experiments on acoustic emission in the saturated state, and for further fascinating theoretical work to clarify the picture of how the pulse of lattice vibrations degenerates mostly into heat while at the same time communicates with the surfaces to establish the back stress and to store a small fraction of its kinetic energy. Theoretical developments on the statistical nature of the penetration of the walls are also badly needed.

6. Conclusions

(i) Details of the shape of the screw dislocations as they simultaneously bow between the walls and pass one another seem not to affect the basic equations (14) and (15) which accurately give the saturation stress and the average plastic strain in a persistent slip band in terms of the equilibrium wall spacing. **The picture proposed by Brown [4] of the annihilation of screw dislocation dipoles by the thermally activated migration of acute jogs along them, with an activation energy of about 100meV, is unaffected by the shape of the screws in the critical configuration, because the cross-slip occurs in the unloaded state.**

(ii) The saturation stress is the sum of the bowing stress and the passing stress reduced from that for infinite straight screws. The factor by which it is reduced can be written as $(1-\alpha)$ times the bowing stress, where estimates of α are 1 ([1], no reduction); 0.17 [2]; 0.5 (this work). We anticipate that detailed computer models will produce an accurate value. The value of α determines the relationship between the smallest stable screw dislocation dipole and the wall spacing (equation 13).

(iii) On the basis of the statistically deformable dipole model of a persistent slip band, reasonable estimates can be given of the residual alternating internal shear stress (the back stress, not the fibre stress) in it.

(iv) It is suggested that the reason the back stress does not contribute to the saturation stress may be that the saturated hysteresis loop in cyclic plasticity is in the microstrain region, analogous to the microstrain leading to the flow stress in a dispersion-hardened alloy. Further work is required to clarify this.

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E_{edge}	$\dfrac{\mu b^2 \ln(d_w/b)}{4\pi(1-\nu)}$
E_{screw}	$\dfrac{\mu b^2 \ln(d/b)}{4\pi}$
τ_{Orowan}	$\dfrac{2E_{edge}}{bd}$
T_{edge}	$\dfrac{\mu b^2 (1-2\nu)\ln(d/b)}{4\pi(1-\nu)}$
T_{screw}	$\dfrac{\mu b^2 (1+\nu)\ln(d/b)}{4\pi(1-\nu)}$
T_{45}	$\dfrac{\mu b^2 (2-\nu)\ln(d/b)}{8\pi(1-\nu)}$
κ	$\left(\dfrac{T_{screw}}{T_{edge}}\right)^{1/3}$

Table 1. Formulae for dislocation line energies and line tensions used in the text. For Cu at room temperature, $\mu = 42.1 GPa$ and $\nu = 0.43$.

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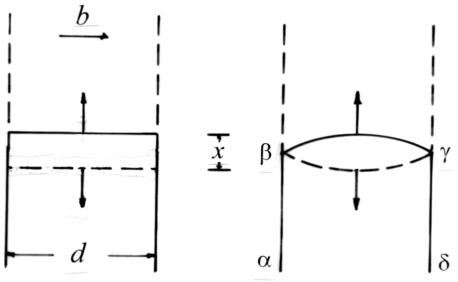
<u>Captions for Figures:</u>

Figure 1. Solid lines represent dislocations in the plane of the paper; broken lines represent dislocations on a neighbouring slip plane displaced by *h*; dotted lines represent dislocations on yet another closely-spaced plane. The dislocations bow in a channel of width *d*. Arrows represent the forces on the lines due to the applied stress.

(a) Combined bowing and passing according to Brown [1]. The screw dislocations are assumed to be rigid.

(b) Combined bowing and passing according to Mughrabi and Pschenitzka [2]. The bowing screws are bent into arcs of an ellipse.

(c) Combined bowing and passing in a configuration allowing three degrees of freedom. Circles show an enlarged view near CD and near the centre. Symbols are explained in the text.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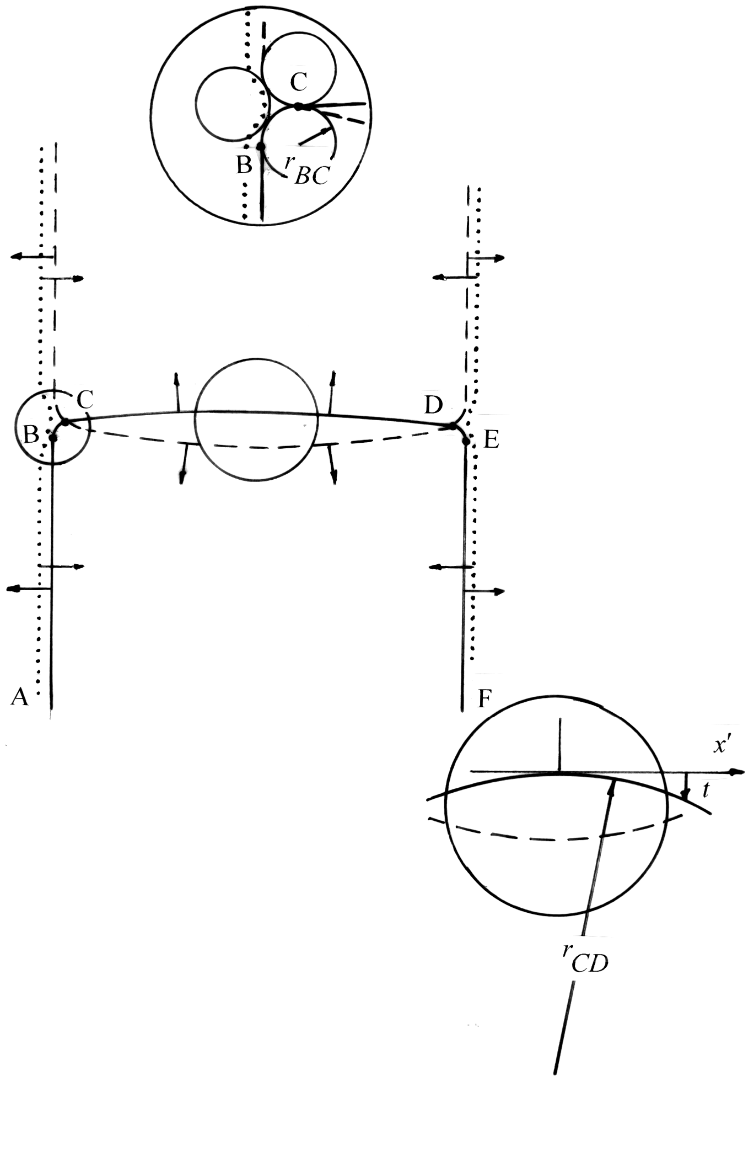
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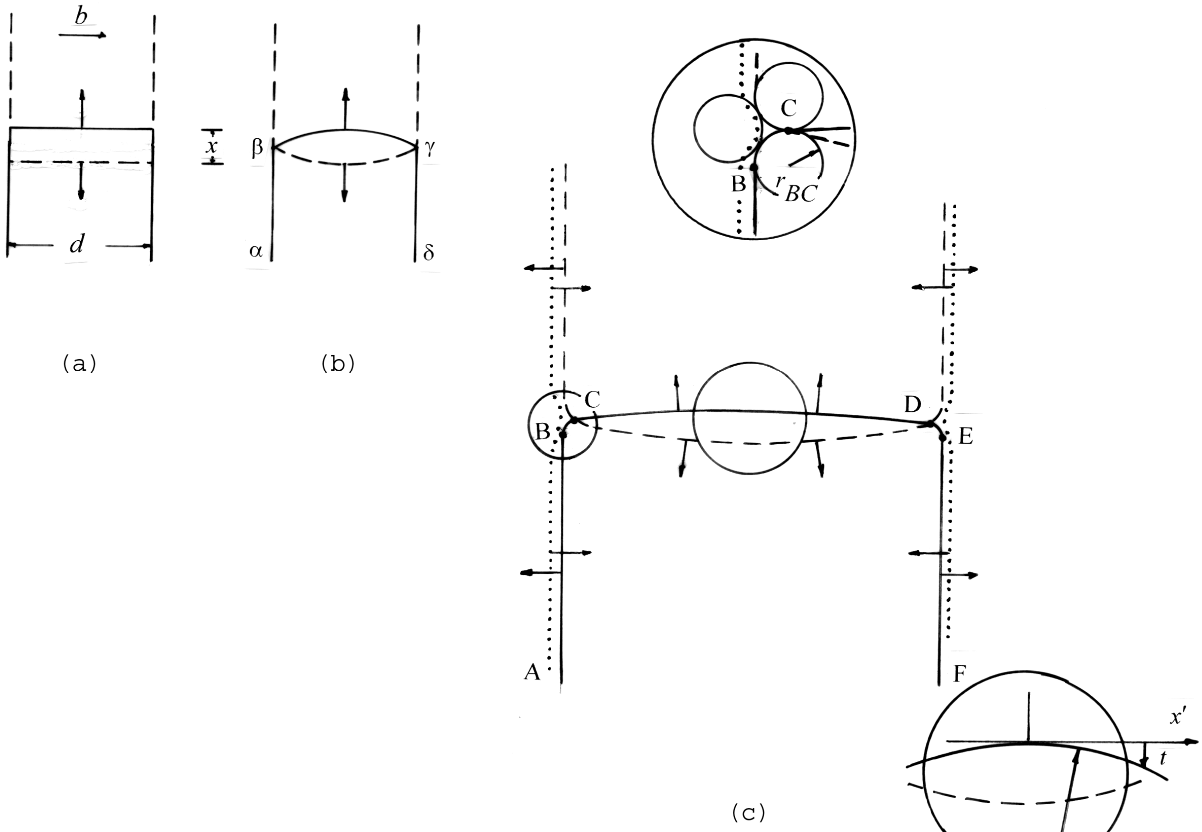

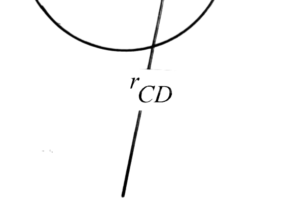

Fig. 1 Solid lines represent dislocations in the plane of the paper; broken lines represent dislocations on a neighbouring slip plane displaced by h; dotted lines represent dislocations on yet another closely-spaced plane. The dislocations bow in a channel of width d. Arrows represent the forces on the lines due to the applied stress.

(a) Combined bowing and passing according to Brown (2000). The screw dislocations are assumed to be rigid.

(b) Combined bowing and passing according to Mughrabi and Pschenitzka (2005). The bowing screws are bent into arcs of an ellipse.

(c) Combined bowing and passing in a configuration allowing three degrees of freedom. Circles show an enlarged view near CD and near the centre. Symbols are explained in the text.