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Scattering by an Oscillating Resonance

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We consider the scattering of a particle by an oscillating rectangular barrier at energies matching with the above barrier resonances. A resonance associated to the barrier has an energy which oscillates in time. With appropriately defined parameters it is possible, using a calculation entirely based on the time-independent scattering amplitudes, to show that the transmission probability is enhanced when the wave packet describing the particle arrives on the device at the time when the resonance energy matches with the energy of the particle.

Keywords: (Resonance scattering, Oscillating barrier, Floquet Theory, Wave packet propagation)

1 Introduction

Boundary conditions are of primary importance in the Multichannel Quantum Defect Theory (MQDT), the development of which owes much to Christian Jungen [1, 2]. Such conditions are present both in the molecular context and the one-dimensional study of semiconductor devices [3]. Among the models which are considered in this field are oscillating barriers, either laterally [4], or vertically [5–8]. The present paper deals with the latter case, the barrier being of rectangular shape. A rectangular potential barrier has a very striking property [9, 10]: the transmissivity of a particle is unity when its energy matches the energy of a resonance above the barrier. The reflectivity is completely quenched. An elementary proof can be given along the lines used to describe the Fabry-Perot interferometer: each potential discontinuity splits an incident wave into a reflected and a transmitted wave. At the resonance energy there is a complete destructive interference of all the reflected waves. In this paper we consider an oscillating rectangular barrier. In the low-frequency limit the above-barrier resonances are oscillating in phase with the barrier. Oscillating barriers in the high-frequency limit have been treated by Gilary *et al.* [11]. We are using a method already presented to treat the case of a wave packet with an energy equal to the top energy of the unmodulated barrier [12]. In this case the purpose was to show that the transmission probability of an incident wave packet depends on the height of the barrier at the time of the collision. Our purpose now is to show that transmission probability is enhanced if during the collision there is temporary matching between the incident energy and the energy of the moving resonance. The method amounts to an expansion of the incident wave packet over the basis of scattering wave functions and the use of the sectorial forms of these functions in both asymptotic regions. This is developed in section 2. In section 3 we recall briefly how the calculation of the time-independent scattering amplitudes is to be formulated. Section 4 is devoted to the derivation of the transmitted and reflected wave packets which leads to an estimate of the transmission and reflection probabilities. Section 5 presents the model and some numerical results.

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2 Sector analysis of wave packets

Dynamics of wave packets is a very valuable tool to study the collision of a particle with a target. There are two different formulations which will be called choice I and choice II. Consider a one-dimensional scattering problem with a target positioned close to the origin of the x axis. In the asymptotic sector with $x < 0$ the k -normalized scattering wave functions have the form

$$X_{1k}(x) = \frac{1}{\sqrt{2\pi}} [e^{ikx} + r(k)e^{-ikx}] \quad (1)$$

and:

$$X_{2k}(x) = \frac{1}{\sqrt{2\pi}} \tilde{t}(k) e^{-ikx} \quad (2)$$

while in the asymptotic sector with $x > 0$ they are

$$X_{1k}(x) = \frac{1}{\sqrt{2\pi}} t(k) e^{ikx} \quad (3)$$

$$X_{2k}(x) = \frac{1}{\sqrt{2\pi}} [e^{-ikx} + \tilde{r}(k)e^{ikx}] \quad (4)$$

The first class of solutions (X_1) corresponds to an incident wave coming from the left and reflected by the potential arrangement. The second class (X_2) corresponds to an incident wave coming from the right and which has been transmitted. The tilda over the scattering amplitudes is used to differentiate the second class from the first. For each class k is positive.

Choice I

The wave packet is formulated as an expansion over exact scattering states, for instance over the scattering solutions of the first class X_{1k}

$$F(x, 0) = \int_0^{+\infty} dk c_1(k) X_{1k}(x) \quad (5)$$

where $c_1(k)$ is an arbitrary well-behaved function of k . Since the $X_{1k}(x)$'s are eigenfunctions of the Hamiltonian, the time evolution is simply

$$F(x, t) = \int_0^{+\infty} dk c_1(k) X_{1k}(x) e^{-i\frac{\hbar k^2 t}{2m}} \quad (6)$$

Such a choice has been made by Cohen-Tannoudji *et al.* [10] to discuss the evolution of a wave packet approaching a step potential.

Choice II

The wave packet is formulated as a given function of the coordinate. A method for its time evolution consists in a preliminary expansion over a complete set of scattering eigenfunctions. It is this choice which constitutes the object of this section. Consider for instance an initial normalized wave packet centered at position x_0 in the asymptotic sector with $x < 0$ and initial momentum $k_0 > 0$

$$F(x, 0) = \frac{1}{(\pi\sigma_x^2)^{1/4}} \exp\left[-\frac{(x - x_0)^2}{2\sigma_x^2} + ik_0x_0\right] \quad (7)$$

We expand the wave packet in the basis of the scattering wave functions X_{1k} and X_{2k} , $k > 0$ which, according to an argument developed by Kriman *et al.* [13], make up a complete orthonormal set of wave functions. The argument is based on the fact that the plane wave is an eigenfunction of the Lippmann-Schwinger equation with the Green function G_0 of the free particle, while a scattering wave function is an eigenfunction of the Lippmann-Schwinger equation with the Green function G of the particle interacting with the target. Orthonormality of one class of functions guarantees the orthonormality of the other class. The development in the complete set of scattering wave functions reads

$$F(x, 0) = \int_0^{+\infty} dk a_1(k) X_{1k}(x) + \int_0^{+\infty} dk a_2(k) X_{2k}(x) \quad (8)$$

where $a_1(k)$ and $a_2(k)$ are determined by the choice of the initial wave packet (*i.e.* the gaussian function given in Eq.(7)). We assume for simplicity equality of the left and right asymptotic thresholds of the potential. The expressions for the amplitudes $a_1(k)$ and $a_2(k)$ are

$$a_1(k) = \int_{-\infty}^{+\infty} dx X_{1k}^*(x) F(x, 0) \quad (9)$$

$$a_2(k) = \int_{-\infty}^{+\infty} dx X_{2k}^*(x) F(x, 0) \quad (10)$$

the $*$ being used for the complex conjugate. Integration over the entire x axis is allowed if the function $F(x, 0)$ is sufficiently small for x outside the pre-collision sector. We use for the scattering wave functions their form in this sector. The integrals are straightforward and give

$$a_1(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx [e^{-ikx} + r^*(k)e^{+ikx}] F(x, 0) = \alpha(k) + r^*(k)\beta(k) \quad (11)$$

$$a_2(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx \tilde{t}^*(k)e^{ikx} F(x, 0) = \tilde{t}^*(k)\beta(k) \quad (12)$$

with

$$\alpha(k) = \frac{1}{(\pi\sigma_k^2)^{1/4}} \exp[-i(k - k_0)x_0] \exp\left[-\frac{(k - k_0)^2}{2\sigma_k^2}\right] \quad (13)$$

and

$$\beta(k) = \frac{1}{(\pi\sigma_k^2)^{1/4}} \exp[i(k+k_0)x_0] \exp\left[-\frac{(k+k_0)^2}{2\sigma_k^2}\right] \quad (14)$$

We note the relation $\beta(k) = \alpha(-k)$. σ_k is $1/\sigma_x$.

2.1 Transmitted wave packet

Assuming that a sufficiently long time t has elapsed so that a transmitted wave packet can be found in the asymptotic sector with $x > 0$, this packet has the form

$$F^T(x, t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk a_1(k) t(k) e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk a_2(k) [e^{-ikx} + \tilde{r}(k)e^{ikx}] e^{-i\frac{\hbar k^2 t}{2m}} \quad (15)$$

This means that both functions $X_{1k}(x)$ and $X_{2k}(x)$ of Eq.(8) are replaced by their local forms, with the time-dependent factor appropriate for an eigenfunction of the Hamiltonian. Sector expansions have already been used by Kim and Lee [14] and Stoica and Dragoman [15] to develop an analytical treatment of resonant tunneling through a static double-barrier. Using Eqs.(11,12) the wave packet can also be written

$$F^T(x, t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \alpha(k) t(k) e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \beta(k) \tilde{t}^*(k) e^{-ikx} e^{-i\frac{\hbar k^2 t}{2m}} + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \beta(k) [r^*(k)t(k) + \tilde{t}^*(k)\tilde{r}(k)] e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} \quad (16)$$

Considerable simplification of this expression can be made. Take the last integral. According to Messiah [9], equating the Wronskians of the scattering solutions $X_{1k}(x)$ and $X_{2k}(x)$ in the two asymptotic sectors gives, for equal thresholds, the relations

$$\frac{i}{2} W[X_{1k}(x), X_{2k}(x)] = k \tilde{t}(k) = k t(k) \quad (17)$$

$$\frac{i}{2} W[X_{1k}(x), X_{2k}^*(x)] = -k r(k) \tilde{t}^*(k) = k t(k) \tilde{r}^*(k) \quad (18)$$

The complex conjugate of the bracket in the last integral is $[r(k) t^*(k) + \tilde{t}(k) \tilde{r}^*(k)]$. Because $\tilde{t}(k) = t(k)$, the bracket is zero according to Eq.(18). Consider now the first two integrals. Taking the complex conjugate of $X_{1k}(x)$ and changing k into $-k$ produces a wave function with again a plane wave incident from the left. This shows that the amplitudes obey the two relations

$$t^*(-k) = t(k) \quad ; \quad r^*(-k) = r(k) \quad (19)$$

We can then replace in the second integral $\tilde{t}^*(k)$ by $t^*(k)$ which in turn is $t(-k)$. Since $\beta(k)$ is $\alpha(-k)$ the two integrals can be combined to give

$$F^T(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk \alpha(k) t(k) e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} \quad (20)$$

or also (cf. Eq.(3))

$$F^T(x \rightarrow +\infty, t) = \int_{-\infty}^{+\infty} dk \alpha(k) X_{1k}(x) e^{-i\frac{\hbar k^2 t}{2m}} \quad (21)$$

The meaning is rather simple. Only the plane wave decomposition of the initial wave packet matters. Each plane wave is to be replaced in the asymptotic sector with $x > 0$ by the local form of the scattering wave functions of the first class. This looks very much like the well-known statement in scattering theory about the fact that an initial wave packet made of free waves will develop in time as a wave packet where each free wave is replaced by an exact scattering solution. We note however that in Eq.(21) the integral extends over all values of k . This differentiates this result from the formulation with choice I where k is restricted to positive values. It has been suggested by Kouri *et al.* [16], that the adjunction of the $X_{1k}(x)$'s ($k < 0$) to the $X_{1k}(x)$'s ($k \geq 0$) produces a complete set. This difference in the range of relevant wave numbers is of minor importance if $\alpha(k)$ is strongly peaked around $k_0 > 0$. In other words if σ_k is small and therefore σ_x large. In such a case choice I and choice II should lead to practically identical results.

2.2 Reflected wave packet

We assume again that time has evolved sufficiently for a reflected wave packet to appear in the asymptotic sector with $x < 0$. This wave packet is developed with the local forms of the scattering solutions of both classes in this sector

$$\begin{aligned} F^R(x, t) = & \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk a_1(k) [e^{ikx} + r(k)e^{-ikx}] e^{-i\frac{\hbar k^2 t}{2m}} \\ & + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk a_2(k) \tilde{t}(k) e^{-ikx} e^{-i\frac{\hbar k^2 t}{2m}} \end{aligned} \quad (22)$$

With the explicit forms given to $a_1(k)$ and $a_2(k)$ in Eqs.(11) and (12), we obtain

$$\begin{aligned} F^R(x, t) = & \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \alpha(k) e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} \\ & + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \alpha(k) r(k) e^{-ikx} e^{-i\frac{\hbar k^2 t}{2m}} \\ & + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \beta(k) r^*(k) e^{ikx} e^{-i\frac{\hbar k^2 t}{2m}} \\ & + \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} dk \beta(k) [r^*(k)r(k) + \tilde{t}^*(k)\tilde{t}(k)] e^{-ikx} e^{-i\frac{\hbar k^2 t}{2m}} \end{aligned} \quad (23)$$

Using $\tilde{t}(k) = t(k)$, the bracket in the last integral is equal to one (conservation of flux). Because $r^*(k) = r(-k)$ and $\beta(k) = \alpha(-k)$, the second and third integrals can be combined into a single integral over k from $-\infty$ to $+\infty$. The first and last integrals can also be combined in such an integral. The wave packet has finally the form

$$F^R(x, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk \alpha(k) [e^{ikx} + r(k) e^{-ikx}] e^{-i\frac{\hbar k^2 t}{2m}} \quad (24)$$

We note that in fact the first term in the bracket is not relevant in the study of reflection since this is nothing but the free propagation of the initial packet, which by assumption is already beyond the interaction region. Finally the reflected wave packet can be written

$$F^R(x \rightarrow -\infty, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk \alpha(k) r(k) e^{-ikx} e^{-i\frac{\hbar k^2 t}{2m}} \quad (25)$$

The wave packet, just as in the case of transmission, involves only its plane wave analysis represented by the amplitude $\alpha(k)$ and the local form of the scattering solutions of the first class.

3 Floquet scattering by an over-barrier resonance

We consider an oscillating rectangular barrier. The wave equation is

$$\left[-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_0(x) + V_1(x) \cos(\omega t + \phi) \right] \Psi(x, t) = i\hbar \frac{\partial \Psi(x, t)}{\partial t}. \quad (26)$$

$V_0(x)$ and $V_1(x)$ are constant (say V_0 and V_1) within the barrier and zero otherwise. V_0 is positive. The solutions in the two potential-free regions, say $j = 1$ on the left of the barrier and $j = 3$ on the right of the barrier, are combinations of free waves. We write them as

$$\begin{aligned} \Psi_k^{(j)}(x, t) = & \frac{1}{\sqrt{2\pi}} \sum_n t_n^{(j)}(k) \exp[i(k_n^{(j)} x - \frac{E_n t}{\hbar})] \\ & + \frac{1}{\sqrt{2\pi}} \sum_n r_n^{(j)}(k) \exp[-i(k_n^{(j)} x + \frac{E_n t}{\hbar})]; \quad (j = 1, 3) \end{aligned} \quad (27)$$

with $E_n = E + n\hbar\omega = \hbar^2 k_n^{(j)2}/2m$. The energy E can be identified with the incident energy of the particle. The functions are clearly of Floquet form [17, 18], that is the product of the exponential $\exp[-iEt/\hbar]$ by a periodic function of time. The mixing of waves with different energies is necessary to account for a change in the number of quanta accompanying the particle when it meets the target. The summation index n is limited in practice to a range $-N \leq n \leq N$, where N is the maximum number of quanta effectively gained or lost by the particle. This number depends on the strength of the coupling between the particle and the target. The solutions within the barrier (sector $j = 2$) have been given many times in the literature [5, 6, 19]. The most general form of the solution is

$$\begin{aligned} \Psi_k^{(2)}(x, t) = & \left\{ \sum_n t_n^{(2)} \exp[i(k_n^{(2)}x - \frac{E_n t}{\hbar})] \right. \\ & \left. + \sum_n r_n^{(2)} \exp[-i(k_n^{(2)}x + \frac{E_n t}{\hbar})] \right\} \exp[-i \frac{V_1 \sin(\omega t + \phi)}{\hbar \omega}]. \end{aligned} \quad (28)$$

$k_n^{(2)}$ is $\hbar^{-1}[2m(E_n - V_0)]^{\frac{1}{2}}$. With appropriate matching conditions to ensure continuity in space and time [6, 20, 21] it is possible to determine all the quantities $r_n^{(1)}$ and $t_n^{(3)}$, assuming that among the $t_n^{(1)}$ only $t_0^{(1)}$ is non-zero. In particular the total transmissivity¹ is given by

$$T(E) = \sum_n (k_n^{(3)} / k_0^{(1)}) |t_n^{(3)}|^2 \quad (29)$$

the sum being restricted to the open channels, that is to the channels satisfying the condition $E + n\hbar\omega > 0$. This is a time-independent quantity even though the potential is time-dependent. It corresponds to the scattering of a wave packet with a well-defined energy extending over a large coordinate region, so that many oscillations take place while the wave packet is travelling across the potential. The time-independent scattering amplitudes $r_n^{(1)}$ and $t_n^{(3)}$ are sufficient to study the case where a wave packet is narrow enough in coordinate space to "feel" the instantaneous configuration of the target at the time of the collision. This was shown in various ways for the case of an oscillating barrier [12] when the energy of the wave packet is equal to the top of the unmodulated barrier.

4 Transmitted and reflected wave packets

We choose the normalized wave packet of gaussian form already studied in section 2

$$\Psi(x, 0) = \frac{1}{(\pi\sigma_x^2)^{1/4}} \exp[-\frac{(x - x_0)^2}{2\sigma_x^2} + ik_0 x]. \quad (30)$$

k_0 is the average wave number, which can be identified with $k_0^{(1)}$. Its plane wave expansion is done through the relation

$$\Psi(x, 0) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dk e^{ikx} \alpha(k), \quad (31)$$

$\alpha(k)$ has already been given in Eq.(13). We extend now the argument developed in section 2 to Floquet solutions of a wave equation with a time dependent periodic Hamiltonian. To every plane wave incident at energy E , either coming from the left or from the right, there corresponds a Floquet wave function. These stationary wave functions make up a complete orthonormal set. The Floquet Hamiltonian takes the place of the Hamiltonian of ordinary scattering theory. There is the possibility to develop the initial wave packet over this complete set and in our case we are interested by the fate of the wave packet after it has collided with the target. We therefore replace for the transmitted or reflected wave packets the plane wave by the *local forms* of the scattering wave functions in either the asymptotic post-collision (with $x > 0$) or pre-collision (with $x < 0$) sectors. This gives

¹We reserve the word transmissivity for the transition probability of a monoenergetic beam.

$$\Psi^T(x, t) = \frac{1}{\sqrt{2\pi}} \sum_n \int dk \alpha(k) t_n^{(3)}(k) \exp[-i \frac{\hbar k_n^{(3)2}}{2m} t] \exp[+i k_n^{(3)} x], \quad (32)$$

and

$$\Psi^R(x, t) = \frac{1}{\sqrt{2\pi}} \sum_n \int dk \alpha(k) r_n^{(1)}(k) \exp[-i \frac{\hbar k_n^{(1)2}}{2m} t] \exp[-i k_n^{(1)} x]. \quad (33)$$

It is to be noted that the contributions of closed channels for which $k_n = i\kappa_n, \kappa_n > 0$ are automatically eliminated as x increases in $\Psi^T(x, t)$ and as x decreases in $\Psi^R(x, t)$. The difference with previous expressions (Eqs.(20) and (25)) resides in the fact that there is now a summation over the "photon" index n . The transmission and reflection probabilities are identified with the integrated wave packets

$$T(t) = \int dx |\Psi^T(x, t)|^2 \quad (34)$$

and

$$R(t) = \int dx |\Psi^R(x, t)|^2. \quad (35)$$

For t large enough for the wave packets to be well separated from the target, these probabilities are expected not to depend on t . This has been proved in [12]. The conditions to recover the partial and total probabilities of Floquet scattering have also been discussed. The requirement is essentially that in the double summations implied in Eqs(32-35) there are no interfering terms corresponding to energies differing at least by one quantum. Stated differently, the energy spread of the initial wave packet must be smaller than one quantum.

5 Model and results

The model has to satisfy several conditions. The coordinate extension of the wave packet has to be sufficiently small to avoid the averaging over one or more periods of the oscillation during the encounter between the packet and the barrier. However this could lead to such an energy spread of the wave packet that the resonance would always be within this energy range. Our model is, as in our previous study [12], a barrier of height $V_0 = 11$ eV and width $L = 10$ Å. The mass of the particle is the free electron mass m_0 . The transmissivity of this barrier is shown in Figure 1. There are four resonances below 18 eV. We choose to study the second resonance at energy $E_R \sim 12.50$ eV, of full width at half maximum 1.1 eV. The modulation amplitude is $V_1 = 0.30$ eV and the phase ϕ of the modulated potential is zero. The resonance energies of a rectangular barrier of height H are given by $H + p^2 \hbar^2 \pi^2 / 2mL^2$, with p an integer [9]. When the barrier is in upward position the resonance moves to $E_+ = E_R + V_1 = 12.8$ eV. When the barrier is in downward position, the resonance goes to $E_- = E_R - V_1 = 12.2$ eV. We can also say that the entire transmissivity profile is moving with the barrier. The preliminary calculation of all the amplitudes present in Eq.(27) is made with 25 channels, (*i. e.* with $N = 12$). Only the amplitudes of the 11 central channels ($N = 5$) need to be retained. This is enough to keep all amplitudes of significant magnitude. However, a calculation with only $N = 5$ does not produce amplitudes which are small enough for the extreme values of n , that is $n = -5$ and $n = +5$. The coordinate extension of the incoming wave packet is $\sigma_x = 100$ Å.

Figure 2 gives the example of an initial wave packet positioned in such a way that it takes one period of oscillation T to reach the barrier with the left edge placed at $x = -L/2$. This position x_0 is given by $-\bar{v}T$ with the velocity $\bar{v} = \hbar k_0/m$. The energy spread of the wave packet is 0.55 eV, estimated from $k_0 \pm \sigma_k$. The spread of the wave packet is less than the energy range $E_+ - E_-$. This is a favourable circumstance for observing a modulation of the transmission probability. This actually is the case, despite the fact that the resonance width is of the order of $E_+ - E_-$. The frequency of oscillation is taken equal to 0.05 eV.

Figure 3 gives the transmission probability as a function of the time it takes for the wave packet to reach the target (in units of the period of oscillation $T = 2\pi/\omega$). This is done for three choices of the peak energy $\bar{E}$ of the wave packet: $\bar{E} = E_R$, $\bar{E} = E_+$ and $\bar{E} = E_-$. A simple and very interesting conclusion emerges from these three calculations. When $\bar{E} = E_R$, there are two maxima during one period. This is due to the fact that the resonance passes twice during a period at the position it takes when there is no modulation. On the other hand, when $\bar{E}$ is either $\bar{E} = E_+$ or $\bar{E} = E_-$ the resonance reaches $\bar{E}$ only once in a period. The difference in the minima observed in the transmission probability of the upper panel when the travel time is either T or $1.5T$ has its explanation in Figure 1 which shows that the transmissivity differs on the two sides of the resonance, with less transmissivity on the left than on the right. At time $t = 0$ the barrier is in high position. After 1 period the barrier is again in high position. The wave packet is testing the transmissivity on the low side of the resonance. At time $t = 1.5T$, the barrier is in low position. The wave packet is affected by the high side of the profile, where the transmissivity is larger. Another way to give this explanation is shown in Figure 4, where the transmissivity profiles for the three heights of the barrier V_0 , $V_0 + V_1$ and $V_0 - V_1$ are displayed together with the incident wave packet centred at $\bar{E} = E_R$. For a sufficiently small frequency (say with a transit time smaller than the period [12]), the particle goes through the barrier with the height it assumes at the time of the collision. The difference in transmissivities $E = \bar{E} = 12.5$ eV for the upward and downward positions of the barrier is clearly visible on Figure 4, with values approximately equal to 0.5 and 0.7 respectively. This agrees with the upper panel of Figure 3.

Figure 5 shows how, for the case $\bar{E} = E_R$, the transmission probability varies as we modify the frequency of oscillation, all other parameters being the same. The modulation of this probability decreases, as the frequency takes larger values. This is as expected. An averaging process is at work: several oscillations may take place during the collision.

6 Conclusions

This study illustrates the power of the procedure based on the time-independent Floquet scattering amplitudes in the asymptotic regions to study the motion of a wave packet. This is an extension to the Floquet world of a well-known method used for time-independent potentials. The expansion of the wave function at time $t = 0$ in the basis of stationary wave functions allows to write the wave function at time t in a trivial manner: there is just to multiply each component by its energy-dependent exponential. However we take advantage here of the sectorial nature of the problem: only the form taken by the scattering eigenfunctions in the asymptotic sectors are needed. The expressions (32) and (33) for the transmitted and reflected wave packets are of general applicability. Every particular problem has its own way to yield these amplitudes. This is a substitute for the methods based on a step-by-step propagation of the wave packet [22]. Other cases would be of interest: e. g. in-phase or out-of-phase oscillations of two barriers. The former case can also present resonances with unit transmissivity. However the oscillation of a resonance position is much less than that of an over-barrier resonance. It is only about a tenth of that of the barriers top. It is therefore more difficult to find appropriate parameters to observe an effect similar to that described in the present note. The latter case could serve as a model for some recent realizations of the electron turnstile [7, 8].

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Figure 1. Transmissivity of the barrier without any modulation of its height. The second resonance at energy $E = 12.5$ eV is chosen for this study.

Figure 2. An initial wave packet positioned in such a way that it takes one period of oscillation to reach the barrier represented by the thick vertical line at $x = 0$. The width of the wave packet is much larger than the width of the barrier. This is needed to produce an energy spread of the packet narrow enough to discriminate between possible positions of the resonance at the time of the encounter. In this example the peak energy of the wave packet is $\bar{E} = 12.5$ eV and the frequency of oscillation is $\omega = 0.05$ eV, so that x_0 is $-1734 \text{ \AA}$.

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Figure 3. Transmission (T) and reflection (R) probabilities of the oscillating rectangular barrier as a function of the time it takes for the wave packet to reach the barrier (in units of the oscillation period). The frequency of oscillation is 0.05 eV. Upper panel: the peak energy of the incident wave packet is equal to the second resonance energy of the barrier when the modulation is zero. Middle panel: the energy of the incident wave packet is equal to the resonance energy of the barrier when the height is maximum. Lower panel: the energy of the incident wave packet is equal to the resonance energy of the barrier when the height is minimum. Note that in the upper panel there are two maxima during one period of the oscillation, while in the two lower panels there is only one maximum during one period. This is due to the fact that in the off-resonance case the resonance passes only once at the energy of the incident wave packet during a period, while in the on-resonance case it passes twice during that same time.

Figure 4. Thick solid line: the incident wave packet centered at the second resonance energy of the unmodulated barrier (in arbitrary units). Solid, dotted and dashed lines: transmissivities around the second resonance for three instantaneous positions of the barrier with heights V_0 , $V_0 + V_1$ and $V_0 - V_1$, respectively.

Figure 5. Transmission probability of the oscillating rectangular barrier as a function of the time it takes for the wave packet to reach the barrier (in units of the oscillation period) for three different frequencies of oscillation ω . Solid line: $\omega = 0.05$ eV. Dotted line: $\omega = 0.1$ eV. Dashed line: $\omega = 0.2$ eV. The peak energy of the wave packet is equal to the second resonance energy of the static barrier.

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PROBABILITIES

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