A computational comparison of cyclical and non-cyclical control for stochastic production-inventory systems

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Abstract
Cyclical production planning is popular in real-life because of its organizational benefits. We study the application and performance of cyclical production planning in a complex stochastic production-inventory system with job shop routings. We propose a decision-support system that allows computing cost efficient cyclical production plans. Insights on the applicability and performance of cyclical production planning are obtained from an extensive simulation study in which the cyclical approach is compared with a state-of-the-art non-cyclical approach.

Keywords: stochastic production/inventory systems; cyclical production planning

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1. Introduction

The use of cyclical production schedules has become increasingly popular in real-life production systems because of the many organizational benefits that can be associated with their application. The benefits have been described extensively in the literature, see e.g. Campbell and Mabert (1991), Bowman and Muckstadt (1995) and Schmidt et al. (2001). Firstly, cyclical production plans are significantly simpler to describe, understand and implement than many other scheduling methods. There is no need for dispatching decisions, since the sequence is fixed. The fixed sequence also makes the communication between different work centers on the shop floor easier. Furthermore, cyclical production planning facilitates related planning activities such as work force planning, raw material delivery, shipment of finished products and scheduling of preventive maintenance. Next, cyclical production planning allows concentrating efforts to reduce setup times on the combination of operations that occur in the sequence. Finally, the use of cyclical production plans provides discipline and stability on the shop floor.

Cyclical production planning approaches can also be found in production and inventory research. In particular proposed solutions to the classical Economic Lot Scheduling Problems (ELSP) are often based on cyclical production schedules. The classical ELSP investigates a production/inventory (PI) system consisting of a single work center in which several items are produced to stock. The PI system is characterized by deterministic demand rates, production times and setup times. The available production capacity is limited. The goal of the ELSP is to determine a production schedule that is feasible with respect to the available production capacity and that minimizes the sum of the setup and inventory holding costs. The ELSP is known to be NP-hard. This implies that finding an optimal solution for realistic problem sizes in reasonable time is very unlikely. Therefore most research has been based on heuristics to constrain the search space. The most popular heuristic is to use cyclical production schedules. Well-known early work on the ELSP include Maxwell (1964) and Elmaghraby (1978), Silver et al. (1998) review the most important contributions. Several extensions of the ELSP have been studied. Firstly, the Capacitated Lot Sizing Problem (CLSP) extends the ELSP by considering time-varying deterministic demand, see Silver et al. (1998) for a review. Jans and Degraeve (2004) present new lower bounds for the CLSP and computational results on the comparison with other lower bounds. Secondly, the ELSP has been extended to situations with more than one work center. Several researchers have investigated cyclical schedules for flow shop production systems, see e.g. El-Nadjawi and Kleindorfer (1993) and Dobson and Yano (1994) and for job shop production systems, e.g. Ouenniche and Boctor (1998). Thirdly, the Economic Lot Scheduling Problem has been studied under stochastic demand, and stochastic setup and processing times. Sox et al. (1999) review the literature on this variant. A more extensive review of the literature on production-inventory systems can be found in Van Nyen (2005c, p. 9-15).

We can find many applications of cyclical production planning in industries where a set of products is produced on a single machine. Reports on applications in flow shop or job shop production situations are rare. This may be due to an apparent disadvantage of cyclical production for these types of production. For a flow shop or a job shop production system, constraining the search space to cyclical production schedules may create unavoidable machine idle time on top of the machine idle time that results from the difference between available capacity and capacity needed for production and setups. As a result, there are problems for which feasible solutions exist, but for which no cyclical solution exists. This in particular will pertain to problems with high capacity utilization on one or more of the machines. Thus, if high capacity utilization is demanded, many of such problems can only be solved using non-cyclical schedules. However, non-cyclical schedules result in highly variable production cycle times, and in the loss of the many organizational advantages of cyclical production mentioned above. These drawbacks may be partly compensated by the flexibility of non-cyclical production to include new customer orders at any point in time.

In view of the pros and cons of cyclical and non-cyclical production schedules, a quantitative investigation into the effects of cyclical and non-cyclical production on production costs for job-shop production systems is justified. This is the aim of this paper.

We consider production situations where a number of products are produced to stock in a job shop facing stationary stochastic demand. This type of manufacturing system can be found in the middle of supply chains, e.g. in the capital goods manufacturing industry where raw materials (steel, plastics, etc.) are transformed into parts or components for assembly. Real-life examples include parts manufacturers for the aircraft assembly industry (Stoop and Bertrand, 1997), or for the off-highway vehicles industry (Lambrecht et al., 1998).

<Insert Figure 1: Production-inventory system>

Our research approach is as follows. We first develop an approach that tries to find a cyclical production schedule (if it exists) that minimizes total costs under a service constraint, for an arbitrary job shop production system with stochastic processing times and setup times, operating under stationary stochastic demand. Since the problem is analytically intractable we use heuristic techniques to search the design space for high quality solutions. Next we generate a test bed of problem instances that contains sufficient variety to allow for the interpretation of the results in the light of the problem instance characteristics. We apply our approach to each instance and use simulation to estimate the total costs that would result from applying the solution to the problem instance. We compare the total costs from the cyclical approach with the total costs that would result from applying a state-of-the-art non-cyclical cost minimizing approach under a service constraint. See Van Nyen et al. (2005a) for an account of this non-cyclical approach and an analysis of its performance. We focus on work-in-process carrying costs, setup costs and inventory carrying costs, and neglect the costs and benefits that can be attached to the organizational advantages of the use of cyclical production schedules.

The reason for this is that estimating the organizational benefits of cyclical and non-cyclical production schedules for each specific situation seems to be straightforward, whereas the impact of the application of cyclical production schedules on work-in-process, set-up and inventory costs as compared to non-cyclical schedules is much less straightforward. Thus when observing the cost differences between the cyclical and the non-cyclical approach in this research, one should bear in mind that there may exist organizational effects that are not accounted for. In view of the many organizational benefits of the cyclical approach, we may expect that the comparison made in this paper is a worst case test scenario for the cyclical approach.

The rest of this paper is organized as follows. First, in Section 2, we present a formal model of the PI system studied in this paper. Section 3 gives an account of the approach that has been developed to find a cost effective cyclical production schedule. Section 4 presents the test bed of problem instances and gives a short description of the non-cyclical approach that is used as a performance benchmark. Section 5 presents performance differences between the cyclical and the non-cyclical approach and discusses these differences in the light of the characteristics of the problem instances. Conclusions are given in Section 6.

2. Modeling of the production inventory system

In the PI system, K products $(k = 1,\dots, K)$ are produced to stock, and the demand is satisfied from stock. The demand for the products is a stationary renewal process. The demand interarrival times A_k are stochastic variables with a known expectation $E[A_k]$ and squared coefficient of variation (scv) $c^2[A_k]$. The demand size (number of product units requested per demand) is equal to one. Demand that cannot be satisfied directly from stock is backordered. The production-inventory management has to ensure that a target fill rate β_k is attained. The fill rate is a service measure that is used frequently in real-life and can be defined as the fraction of the demand that is satisfied directly from the inventory (Silver et al., 1998).

The inventory management system generates replenishment orders for the different products in order to satisfy the demand. Every time a replenishment order for product k is generated, a fixed cost o_k is incurred. The items for product k in the final stock point have a unit value of v_k. A carrying charge, denoted as r, is incurred per amount of money that is kept in stock per unit of time. The stock replenishment orders are made-to-order by the production system. Therefore, the replenishment orders are equivalent to production orders.

The production orders are manufactured in the production system that consists of J functionally organized work centers. We assume there is ample supply of raw material. Each of the products requires a specific serial sequence of production steps, which results in a job shop routing structure. Each production step of a product requires a different work center. The production orders for different products compete for capacity at the different work centers. Before the production of an order for product k at a work center j can start, a machine setup has to be performed. This machine setup takes a certain time L_{jk} and cost s_{jk}. The setup costs over the entire routing of product k are denoted as s_k. The setup times and costs are sequence and lot size independent. After the setup, the processing of the production order starts. The mean time required to process the entire production order is proportional to the size of the production order. The production time for one unit of product k on work center j is given by P_{jk}. The manufacturing process is subject to variability: setup times and processing times are stochastic variables with a known expectation and scv. When the production of the entire batch is completed at a certain work center, the batch is transferred to the next work center in the routing of the product. The transfer batch size equals the production batch size. When the processing of the entire batch is finished at the last work center in the routing of a product, the batch is transferred to the stock point. We assume that the transfer times are negligible. Table 1 summarizes the notation that is used throughout this paper.

Table 1 (ctd.): notation used

3. Cyclical Production Planning

In this section, we present the details of the Cyclical Production Planning (CPP) approach used in this study. We propose a heuristic decision support system (DSS) for determining cost efficient control decisions. We proceed as follows. First, in Section 3.1 we give general characteristics of our approach. In Section 3.2 we propose a heuristic for solving the deterministic version of the control problem. This heuristic is embedded in Section 3.3 in a DSS that allows determining the common cycle length, production schedule and order-up-to levels for the stochastic PI control problem.

3.1. Description of CPP approach

Our implementation of CPP is based on the common cycle approach (Elmaghraby, 1978), which assumes that: (i) all products have the same cycle length R; (ii) all products are manufactured exactly once during this common cycle; (iii) the common cycle repeats itself and the processing sequence, i.e. the order in which the products are produced on the work centers, is fixed and remains the same during every cycle.

Then the PI system operates as follows. At the end of the common cycle (at time R, $2R$, $3R$, ...), replenishment orders are generated for all products according to an order-up-to policy characterized by the order-up-to level S_k. The replenishment orders are manufactured by the production system according to the cyclical production schedule, which is determined such that the total costs are minimized. The production schedule prescribes the sequence in which the orders are produced on the work centers, but not the precise starting times. Because of the fixed processing sequences, it is possible to perform the setup for the next product in the sequence as soon as the previous operation is finished. The same steps are repeated at the end of each common cycle.

3.1.1. Deterministic PI system

The analysis of the deterministic problem starts with specifying a cost model for the total relevant costs. After this, a formal problem description is presented. Finally, we present a heuristic to determine a cost efficient schedule and common cycle length.

3.1.2. Cost components

1. Setup and ordering costs. Every product k is ordered and produced once in a common cycle of length R. Therefore, the total ordering and setup costs per time unit for product k are given by:

$$SC_k(R) = \frac{o_k + s_k}{R}$$

2. Work-in-process inventory holding costs. Little's law states that the average amount of work-in-process is equal to the average throughput time multiplied by the arrival rate. The arrival rate of demand for product k is equal to $\frac{1}{E[A_k]}$. Now we derive expressions for the expected throughput time $E[T_{jk}(R)]$. We derive separate expressions for the first operation in the routing of a product and for the remaining operations. We assume that for the first operation in the routing of a product, work-in-process costs are only incurred for the processing of the order (no waiting time costs). This assumption is justified since the repetitive nature of CPP allows for the just-in-time delivery of raw material and components. The period during which work-in-process costs are incurred is equal to the expected production time, which is given by:

$$E\left[T_{\mu(k,1),k}(R)\right]=\frac{R}{E\left[A_k\right]}E\left[P_{\mu(k,1),k}\right]$$

For the remaining operations in the routing of product k, work-in-process costs are incurred over the entire throughput time, which consists of the waiting time and the processing time. The throughput time of operation o can be expressed as:

$$E\left[T_{\mu(k,o),k}(R)\right]=d_{\mu(k,o),k}+\frac{R}{E\left[A_k\right]}E\left[P_{\mu(k,o),k}\right]-d_{\mu(k,o-1),k}-\frac{R}{E\left[A_k\right]}E\left[P_{\mu(k,o-1),k}\right]$$

for $o=2,\ldots,J_k$

Using Little's law, the work-in-process costs incurred by product k per unit of time can be computed as:

$$WIPC_k(R)=R\frac{E\left[P_{\mu(k,1),k}\right]}{E^2\left[A_k\right]}v_{\mu(k,1),k}r+R\sum_{o=2}^{M_k}\left(\frac{E\left[P_{\mu(k,o),k}\right]}{E^2\left[A_k\right]}-\frac{E\left[P_{\mu(k,o-1),k}\right]}{E^2\left[A_k\right]}\right)v_{\mu(k,o),k}r$$

$$+\sum_{o=2}^{M_k}\frac{d_{\mu(k,o),k}-d_{\mu(k,o-1),k}}{E\left[A_k\right]}v_{\mu(k,o),k}r$$

3. Final inventory holding costs. We use -for consistency- the same target fill rate β_k in the deterministic model as in the stochastic model. Van Nyen (2005c, p.115-116) shows that in steady state the final inventory holding cost for product k per unit of time is given by:

$$FIC_k(R)=\frac{\beta_k^2R}{2E\left[A_k\right]}v_kr$$

3.1.3. Formal problem statement for deterministic model

In this section we formulate the problem of finding the optimal processing sequences and common cycle length, under the assumption of deterministic demand, setup times and processing times. We present a Non Linear Program (NLP) that consists of the objective function discussed above and the constraints that are required to obtain a feasible cyclical production plan. In this NLP, the decision variables are the common cycle length R and the starting times of the operations $d_{j,k}$ at the work center. The NLP is similar to the model proposed by Ouenniche and Boctor (1998). However, we make several modifications and improvements, see Van Nyen (2005c, p.112-120) for more details. The problem can be formulated as follows:

Minimize:

$$\frac{1}{R}\sum_{k=1}^{K}(o_k+s_k)+R\sum_{k=1}^{K}\left[\begin{array}{c}\frac{\beta_k^2}{2E[A_k]}v_k r+\frac{E[P_{\mu(k,1),k}]}{E^2[A_k]}v_{\mu(k,1),k} r\\ +\sum_{o=2}^{M_k}\left(\frac{E[P_{\mu(k,o),k}]}{E^2[A_k]}-\frac{E[P_{\mu(k,o-1),k}]}{E^2[A_k]}\right)v_{\mu(k,o),k} r\end{array}\right]$$

$$+\sum_{k=1}^{K}\sum_{o=2}^{J_k}\frac{d_{\mu(k,o),k}-d_{\mu(k,o-1),k}}{E[A_k]}v_{\mu(k,o),k} r$$

Subject to:

1. $d_{\mu(k,o-1),k}+\frac{E[P_{\mu(k,o-1),k}]}{E[A_k]}R\le d_{\mu(k,o),k}$ for $k=1,\ldots,K$ and $o=2,\ldots,J_k$
2. $d_{j,k}+\frac{E[P_{jk}]}{E[A_k]}R+E[L_{jl}]-d_{j,l}\le\Omega(2-\delta_{k,n,j}-\delta_{l,n+1,j})$

   for $j=1,\ldots,J$ and $k,l\in K(j)$ and $k\ne l$ and $n=1,\ldots,K_j-1$
3. $d_{j,k}+\frac{E[P_{jk}]}{E[A_k]}R-d_{j,l}+E[L_{jl}]\le R$

   for $j=1,\ldots,J$ and $k,l\in K(j)$ and $k\ne l$
4. $d_{j,k}\ge E[L_{jk}]\delta_{k,1,j}$ for $j=1,\ldots,J$ and $k\in K(j)$
5. $\sum_{k\in K(j)}\delta_{k,n,j}=1$ for $j=1,\ldots,J$ and $n=1,\ldots,K_j$
6. $\sum_{n=1}^{K_j}\delta_{k,n,j}=1$ for $j=1,\ldots,J$ and $k\in K(j)$
7. $R=\xi\Phi$
8. $\delta_{k,n,j}\in\{0,1\}$ for $k=1,\ldots,K$ and $j\in J(k)$ and $n=1,\ldots,K_j$
9. $d_{j,k}\ge 0$ for $j\in J(k)$ and $k=1,\ldots,K$
10. int $\xi\ge 1$

This mathematical program states that the total relevant costs have to be minimized subject to ten sets of constraints. Constraints (1) state that the processing of a production order can only start after the processing of the order on the previous work center in the routing of the product is entirely finished. Constraints (2) ensure that there is no overlap in the production and setup phases of two successive orders in the production sequence of a work center. The symbol Ω denotes a large positive number. Constraints (3) ensure that the schedule is feasible with respect to capacity by keeping the time interval between the start of the setup of any product l on work center j and the completion of every other product k on work center j smaller than or equal to the common cycle length. Constraints (4) impose that the work center has to be set up before the production of the first product in the production sequence of the work center can start. Constraints (5) ensure that every position in the production sequence of a work center is occupied by precisely one product. Constraints (6) make sure that every product is assigned to exactly one position in the production sequence of the work centers in its routing. Constraints (7) stipulate that the common cycle length is an integer multiple of certain time interval Φ. Constraints (8) - (10) are the integer and non-negativity constraints.

3.2. Heuristic for deterministic model

NLPs are extremely hard to solve and require a prohibitive amount of computation time even for medium sized problems. Therefore, we present a heuristic to approximately solve the NLP. If a common cycle length R' is given, then the first two terms in the objective function are fully determined. The remaining problem is to determine a feasible production schedule that minimizes the third cost component. Also, if R' is given, then the processing times of all production orders are known. Therefore, the problem reduces to a variant of the standard job shop scheduling problem. This observation allows us to decompose the problem in a series of job shop scheduling problems.

The NLP can approximately be solved with a two-level procedure shown in Figure 2. At the first level, the common cycle length is varied. At this level we assume that the total relevant costs are convex in the common cycle length, implying that there exists a single local optimum. At the second level, a job shop scheduler determines the starting times of the operations, for a given common cycle length. The job shop scheduler used in the implementation is based on random sampling. The scheduler randomly generates a large number of feasible schedules. The schedule with the lowest cost and that satisfies all constraints is selected. For pseudo code and more details of the heuristic, see Van Nyen (2005c, p.120-128).

<Insert Figure 2: Two-level procedure for solving NLP >

3.3. Stochastic PI system

In this section, we propose a procedure to generate cyclical production plans for the stochastic version of the PI problem. The DSS is used to determine the common cycle length, processing sequences and order-up-to levels such that the total relevant costs are (approximately) minimized, while target fill rates are satisfied. The procedure is based on the heuristic for the deterministic model and a simulation model. The deterministic model is used to determine processing sequences and a common cycle length, which are tested for feasibility in the simulation model.

The control decisions (common cycle length, production sequences and order-up-to levels) for each instance of the stochastic PI system can be found with the procedure represented in Figure 3. This procedure consists of three phases.

Phase 1. In the *optimization phase*, the deterministic model is used to determine the common cycle length and processing sequences. The procedure starts with slack-time multiplier $\gamma = 0$. If the deterministic optimization problem can be solved, then go to Phase 2. Else, stop (no solution is found).

Phase 2. In the *checking phase*, a simulation model is used to verify whether the proposed solution for the deterministic problem is feasible in the stochastic environment. A solution is feasible if the work-in-process in the production system is not continuously increasing during the simulation run. If the work-in-process is continuously increasing, this indicates that there is insufficient production capacity. The checking phase is necessary because the feasibility of a schedule can not be guaranteed by keeping the utilization (due to production and setups) of the work centers below 1. By strictly following the processing sequences, production capacity may be wasted. For example, it may happen that work center j is idle, while the next product k in the processing sequence is still being processed at another work center and at the same time the product that has to be produced after product k in the processing sequence of work center j is already available at work center j. Since the processing sequence must be followed strictly, work center j will remain idle until the previous operation of product k is entirely finished. It is not possible to determine in advance how much production capacity will be lost due to this effect. Therefore, we rely on simulation to check whether a proposed solution is feasible. See Van Nyen (2005c, p.106-107 and p.128-132) for more details.

Some capacity idle time will already be present in the deterministic schedule because it is not possible to make completely tight schedules, e.g. as a consequence of the complex routing structure. However, as discussed above, this idle time may be insufficient to ensure feasibility under stochastic circumstances. If simulation reveals that a solution is not feasible, the procedure returns to Phase 1 where a new NLP with extra capacity slack is solved. Capacity slack can be defined as planned idle time in the schedule that can be used as a hedge for the capacity loss due to stochasticity. We can ensure that at least a fraction γ of free capacity is available in the schedule, by replacing the set of constraints (3) in the NLP with:

3.-new $$d_{j,k}+\frac{E[P_{jk}]}{E[A_k]}R-d_{j,l}+E[L_{jl}]\leq(1-\gamma)R$$

for $j=1,\ldots,J$ and $k,l\in K(j)$ and $k\neq l$

The solution of the modified deterministic problem is checked again with simulation for feasibility under stochastic conditions. This iterative process continues with increasing values for γ, until a feasible solution is found for the stochastic setting, or until it is decided that it is not possible to find a feasible cyclical production plan for this particular instance of the stochastic PI system. If a feasible solution is found, then Phase 3 is started. Else, the procedure stops without a feasible solution.

Phase 3. Once a feasible solution for the stochastic problem is found, discrete event simulation is used to *tune* the order-up-to levels with a technique developed by Gudum and De Kok (2002) and to *estimate* the performance of the proposed cyclical production plan. The procedure stops with a feasible solution.

For more details on the procedures embedded in the DSS we refer to Van Nyen (2005c, p.132-134).

<Insert Figure 3: Outline of DSS for CPP >

4. Experimental design

In the main simulation study, we investigate the performance of the CPP approach for a wide range of variants of the PI system. A selection of 240 instances of the PI system is generated, which represents in a systematic manner a subset of the PI systems that may be encountered in real-life.

We study a small-scale model that consists of 10 products and 5 work centers. The 10 products, 5 work center model represents the complexity that can be found in real-life production systems and has been used as a model for research on similar PI systems, see e.g. Ouenniche and Boctor (1998). We assume that the demand occurs according to a Poisson process and that the setup times and processing times are exponentially distributed. This is in line with assumptions commonly made in simulation research on job shops; see e.g. Kanet and Hayya (1982) or Bertrand (1983).

We investigate the effect of four factors -net utilization, setup costs, setup times and fill rates- on the performance of cyclical production schedules.

- The net utilization strongly determines the profitability of job shop like manufacturing systems. Therefore, companies typically try to operate at high levels of net utilization. However, at high levels of capacity utilization cyclical scheduling approaches may become infeasible. Therefore, it is worthwhile to investigate the performance under different levels of net utilization.
- Job shop like manufacturing systems are typically characterized by high setup times and high setup costs, because of technological or organizational restrictions. Setup times and costs are considered because the differences between the control approaches may have a strong impact on the setup frequency. We explicitly take into account both aspects of setups because they impact the decision-making process through different mechanisms.
- The final factor in the study is the target fill rate. When cyclical schedules are feasible, they typically lead to stability in the production cycle time. It is well-known that cycle time variance has a large impact on service levels. Therefore it is interesting to investigate the performance of cyclical and non-cyclical approaches for different target service levels.

Table 2 summarizes the four factors and their levels considered in the main simulation study. The number of combinations that can be made with the levels of the four factors equals 3 x 2 x 4 x 2 = 48. We furthermore generated five different basic problem configurations. A basic problem configuration determines several characteristics of a problem instance such as: the routing structure, the processing times, and inventory holding costs. 48 variants of each of these five basic problem configurations are generated leading to a total of 240 instances. The generation process is described in more detail in Van Nyen et al. (2005a, p.431) and Van Nyen (2005c, p.71-72).

Table 2: Factors and levels in simulation study

To evaluate the value of CPP, its performance for each of these 240 instances is compared with the performance obtained with a state-of-the-art non-cyclical control approach. This non-cyclical control approach, referred to as Coordinated Batch Control (CBC), uses a periodic review, order-up-to (R_k, S_k) control policy for the inventory management of product k, for $k = 1,\ldots,K$ and immediate release of replenishment orders to the shop floor. On the shop floor, orders are processed on a FCFS basis. The review period R_k determines the timing of order arrivals at the production system. Furthermore, R_k also determines the production batch size since there is no grouping of orders at the production facility and no transfer batching. The CBC strategy aims to minimize the system-wide costs by cleverly setting the review periods of all products, taking into account the effects of batch sizes on the distribution of the production cycle times. Setting the review periods is done using a heuristic procedure that is based on an approximate queueing model. We refer to Van Nyen et al. (2005a-b) for details on this strategy and its stand-alone performance.

5. Evaluating the performance of cyclical production planning

In this section we present and discuss the performance obtained with the application of the cyclical production planning method outlined in the previous sections, to the 240 instances in the test bed. Performance is measured as total relevant cost (work-in-process costs, ordering cost, set-up cost and inventory costs), while customer service is guaranteed by setting the order-up-to levels such that the target fill rate is achieved. We compare the CPP performance with the performance obtained with the application of CBC, a non-cyclical integrated approach. We start with an overview of the overall cost differences between the two approaches; then we analyze the differences in costs buildup between the two approaches, and finally we identify the characteristics of the problem instances where CPP was observed to perform better then CBC.

5.1. Overall overview of cost differences

We present the simulation results on the performance of CBC and CPP in terms of the realized total relevant costs, denoted respectively as TRC^{CBC} and TRC^{CPP}. In particular, we use as a measure the relative performance difference:

$$d^{TRC} = \frac{TRC^{CPP} - TRC^{CBC}}{TRC^{CBC}} \times 100\%$$

<Insert Figure 4: Relative cost difference between CPP and CBC strategy (all instances)>

Figure 4 presents a histogram of the values of d^{TRC} for the 240 problem instances in the simulation study. We can see that in about 62% of the instances, CPP outperforms CBC. The cost difference in these cases is typically between 0 and 10%, the largest improvement over CBC realized by

CPP is 14%. In the remaining 38% of the problem instances, CPP realizes higher costs than CBC. The cost difference in these cases can be very high: up to 705%. Furthermore, in 7% of the problem instances no feasible solution was found for the CPP approach. CPP infeasibility was strongly related to net capacity utilization and basic problem configuration. This indicates that CPP is only applicable under certain conditions, and care should be taken when selecting products and product routings to be combined in a production system.

An analysis of variance reveals that the following factors have a significant effect on d^{TRC} at the 95% confidence level: (i) net utilization; (ii) setup costs; (iii) interaction between setup costs and net utilization; (iv) basic problem configuration (which determines the characteristics of a problem instance).

5.2 *Differences in costs build-up*

In this section, we try to explain the differences in performance. In particular, we search for the mechanisms through which the four significant factors mentioned above influence d^{TRC}.

The main decision variable in both approaches is related to the frequency with which products are produced. Under CPP a common cycle time is chosen with the objective to minimize costs; under CBC a review period is chosen for each product with the objective to minimize costs. Set-up and ordering costs, inventory costs and work-in-process carrying costs are all related to the cycle time and the review periods. Thus we may expect that the differences in total relevant costs between the two approaches are related to differences in the cycle time decisions taken under the approaches. We have defined the variable d^{R} as relative difference between the common cycle time length under CPP and the average of the review periods under CBC. We have plotted the total relative total costs difference d^{TRC}, and the relative costs

differences for setup and ordering d^{SC}, work-in-process d^{WIPC} and inventory d^{FIC} against d^{R}. The results are shown in the Figures 5a through 5d.
In order to obtain a clear figure we left out the 13 observations with a very high d^{TRC} (>100%). We also omitted the instances for which no feasible solution was found for the CPP strategy.

<Insert Figure 5: Relative difference in costs between CPP and CBC as a function of relative difference between common cycle length and average review period: (a) total relevant costs - (b) setup costs - (c) work-in-process costs - (d) final inventory holding costs>

We first observe that for all instances $d^{R} > 0$; thus the common cycle time under CPP is always larger, sometimes much larger, than the average review period under CBC.

Figure 5-a shows that d^{TRC} increases approximately linearly as a function of d^{R}. For $d^{R} < 50\%$, thus, CPP outperforms the CBC. For larger values of d^{R}, it is CBC that outperforms CPP. The analysis of Figures 5 b-d explains why CPP outperforms CBC (and vice versa).

Figure 5-b shows the relative difference in setup costs between CPP and CBC as a function of d^{R}. Since the common cycle used by CPP is for all instances larger than the average of the review periods of CBC, the CPP approach performs less setups than CBC, so $d^{SC} < 0$. As d^{R} increases, there are less setups performed by CPP compared to CBC which causes d^{SC} to decrease.

Figure 5-c shows that the difference in work-in-process costs is increasing in d^{R}. For $d^{R} < 50\%$, $d^{WIPC} < 0\%$ for most instances. At first thought, it may seem strange that larger cycle times, and thus larger batch sizes, lead to lower work-in-process costs. However, because of the repetitive nature of the production schedule in the CPP approach, the delivery of the raw materials

can be done just-in-time. Therefore, work-in-process costs are only incurred from the moment that the production of the first operation is started under the CPP approach. Moreover, the cyclical production schedule ensures that the order throughput times, measured from the moment that the production of the first operation is started until the completion of the final operation, are highly predictable. CBC on the other hand has the disadvantage that the waiting times are strongly variable and on average can be considerably higher, mainly because order arrivals are not synchronized in time and a myopic sequencing rule (FCFS) is used on the shop flow. Under CBC, average order throughput times can be much larger than the review period, which is not the case under CPP. This explains why the work-in-process costs can be lower for CPP, even if the average size of the production orders is larger than in case of CBC. This shows one of the main advantages of CPP over CBC: by using a cleverly chosen, fixed sequence at each work center, the work-in-process inventories can be kept relatively small. Obviously, as d^R further increases, this advantage of CPP over CBC is dominated by the effect of the batch size on work-in-process. Eventually, the work-in-process costs of CPP become much larger than those of CBC.

Figure 5-d shows that the final inventory costs increase almost linearly in d^R. For some instances with $d^R < 30\%$, CPP has lower final inventory costs than CBC. Since the common cycle length of CPP is always larger than the average of the review periods of CBC ($d^R > 0\%$), the cycle stock is always larger for CPP than for CBC. Therefore, the lower final inventory costs for some instances have to be caused by the need for smaller safety stocks. Since both approaches are facing the same demand patterns and target fill rate, the difference in safety stocks has to be caused by differences in the replenishment lead times. To verify this, we have collected data about the average and the standard deviations of the order throughput times for each of the two approaches. Figure 6-a and 6-b shows the frequency diagram for $d^{E[T_k]}$ and $d^{c^2[T_k]}$, which are the relative differences in the average and scv of the throughput times between CPP and CBC.

The frequency diagram shows that in 80% of the instances CPP results in higher average throughput times than CBC. This is mainly caused by the fact that all replenishment orders are generated at the beginning of the common cycle so that most products have to wait a considerable time before their production can start.

Figure 6-b shows that in all cases the variability in throughput time (measured by the scv) is much smaller under CPP than under CBC: in 85% of the instances the standard deviation under CBC is more than 9 times the standard deviation under CBC. This is in line with what we might expect. Under CPP the throughput time is largely under control due to the fixed schedule and the fixed cycle. Under CBC, on the other hand, there is hardly any control on the throughput time variability. In conclusion, the results show that CPP tends to lead to larger average throughput times but also to (much) smaller variance in the throughput time.

The relative increases in $E[T_k]$ and the relative decrease in $c^2[T_k]$ for CPP have opposing effects on the safety stocks. Detailed inspection of the data showed that, when the difference in the expected throughput times $d^{E[T_k]}$ is moderate, the small value of $d^{c^2[T_k]}$ results in smaller safety stocks under CPP. This holds in particular when the target fill rate is high.

<Insert Figure 6: Relative difference in expectation $E[T_k]$ and scv $c^2[T_k]$ of throughput times between CPP and CBC >

5.3 When is cyclical production planning advisable?

In the previous subsection we have seen the strong and weak points of CPP and CBC and the very different build-up of costs for these two approaches. In particular CPP performs well if a feasible production plan can be achieved with a small cycle time; a cycle time that is not much larger than the average review period under CBC. Such situations frequently occur if the net capacity utilization is low and/or setup costs are high. For these instances, there is a lot of "spare" capacity so that CPP can make a feasible cyclical production schedule without large increases in the common cycle length. Let us now consider what happens if the net utilization increases and/or the setup costs decrease. First, if the net utilization increases, then the total utilization including setup times increases as well. Secondly, as the setup costs decrease, it is cost efficient to increase the ordering frequency. This results in a larger number of setups so that the total utilization of the work centers increases. As the total utilization of the work centers increases, there is less free capacity which makes it harder to find a feasible cyclical production schedule, since under CPP some production capacity will be wasted due to the fixed processing sequences. To find a feasible schedule, the cycle time must be increased such that the reduction in setup time compensates for the capacity losses caused by the fixed processing sequences. For some problem instances, the capacity losses may be so high that no feasible schedule can be found.

In the simulations, we observed strong differences between the different basic problem configurations. For some configurations, it is much easier to find a feasible cyclical production schedule than for others. In particular, the amount of capacity that is wasted under CPP depends strongly on the specific characteristics of a problem instance, e.g. the routing structure, batch processing times, etc. As a result, d^R strongly depends on the basic problem configuration.

The results of this research suggest that the use of cyclical production schedules is advisable for companies that can work at moderate levels of capacity utilization (which of course depends on capacity costs, set-up costs and several other factors not considered here) or that have some freedom in the selection of products, processes and resources so that unfavorable configurations can be avoided. Such freedom may exist if e.g. the manufacturer can choose which products to produce, or which processes to use for producing a product. The possibility to adapt the available hours of capacity per machine might be of use for accommodating unfavorable problem configurations.

6. Conclusions

In this paper we have investigated the performance of cyclical production planning for the control of integrated production inventory systems under stochastic conditions. We have developed a decision-support system for determining the common cycle length, production sequences and order-up-to levels that approximately minimize the total relevant costs under a service level constraint. We have designed a test bed of 240 randomly generated problem instances that differ in routing structure, capacity utilization, set-up times, set-up costs and service level. We used systematic computer simulation to measure the performance of the cyclical production planning for each of the instances and compare it to the performance obtained with a state-of-the-art non-cyclical production planning approach that is used as a benchmark.

The results of this exploratory simulation study reveal that cyclical production has very different operational characteristics as compared to non-cyclical production. The applicability of cyclical production planning is very dependent on the problem configuration, in particular the processing routings of the products and the capacity utilization of the machines. Our results suggest that for a specific subset of basic problem configurations and moderate capacity utilization, cyclical production planning can achieve a specified service target at lower total costs than a cost-efficient non-cyclical approach. This is due to the relatively small variance in production throughput time that is typical for cyclical planning approaches. This leads to less safety stock than under non-cyclical planning as long as the average throughput time is not too high. For a part of the basic problem configurations and at high capacity utilization however CPP requires such long cycle times that the advantage of the small variance in production throughput times is cancelled out by the disadvantage of long average throughput times.

Further research is needed to investigate the performance of more advanced cyclical planning systems, such as systems based on power-of-two types of schedules, and the impact of capacity flexibility and product selection on the performance. Our research reveals that the performance of CPP depends strongly on the basic problem configuration. Therefore, it would be interesting to identify which characteristics of the basic problem configuration determine whether cost-efficient cyclical production plans can be made.

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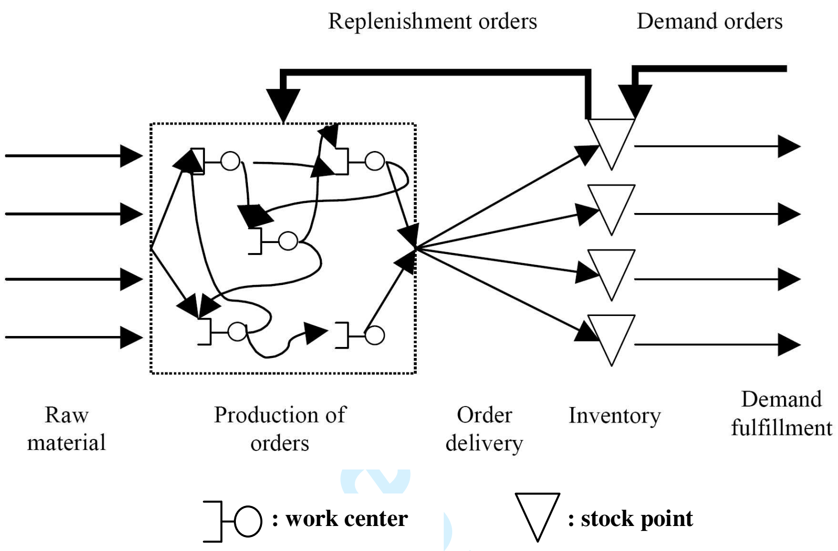
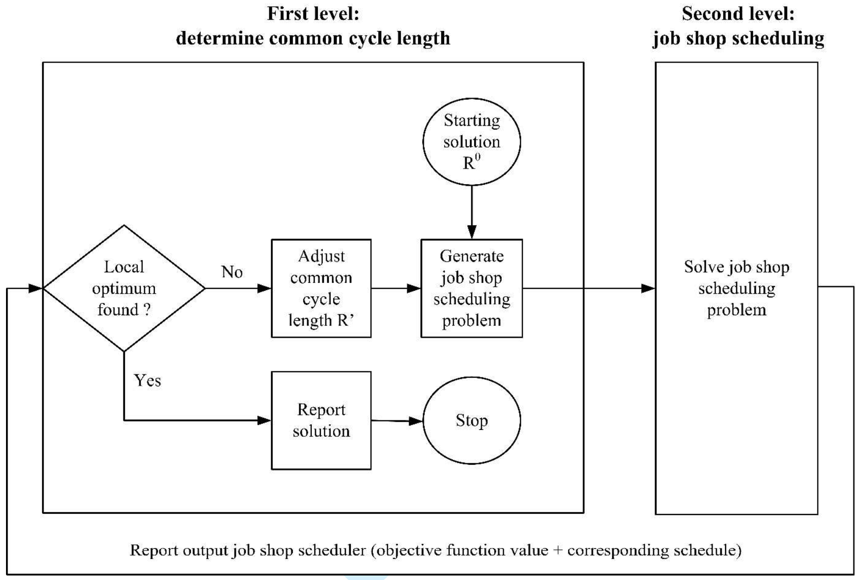
Figures

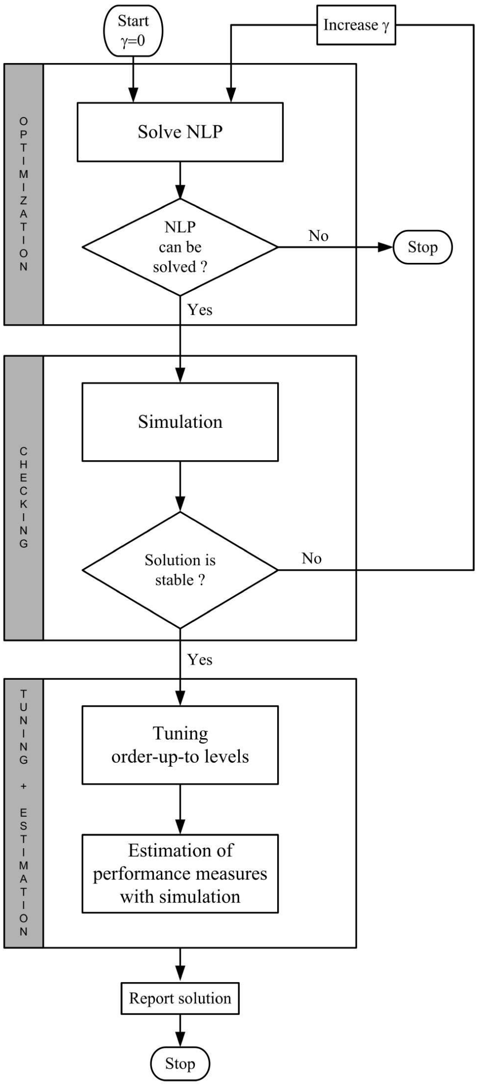
<Figure 1: Production-inventory system>



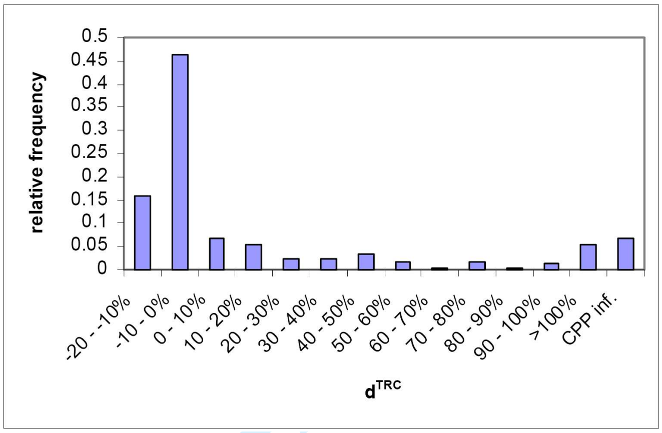
< Figure 2: Two-level procedure for solving NLP >



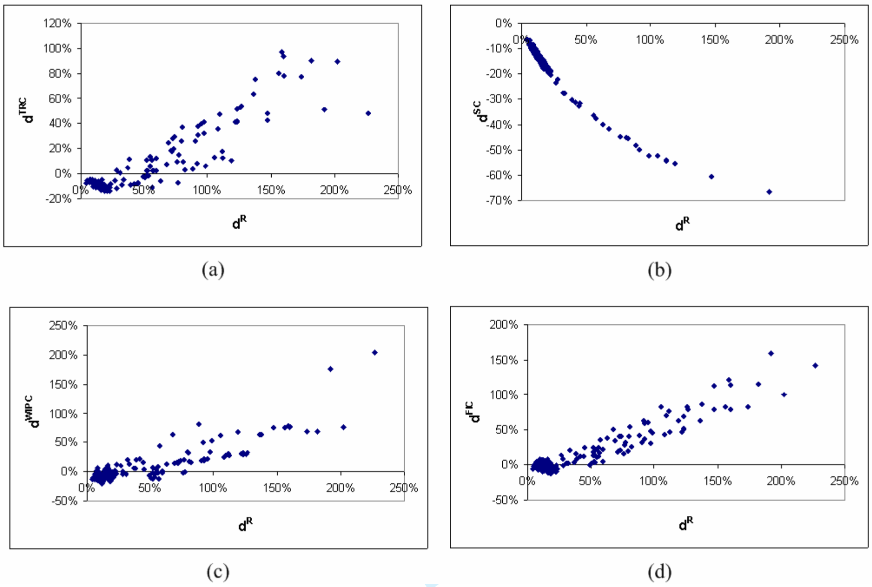
<Figure 3: Outline of DSS for CPP >



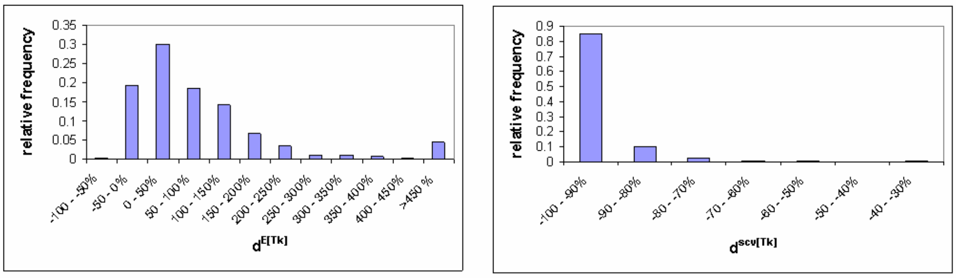
< Figure 4: Relative cost difference between CPP and CBC strategy (all instances)>



<Figure 5: Relative difference in costs between CPP and CBC as a function of relative difference between common cycle length and average review period: (a) total relevant costs - (b) setup costs - (c) work-in-process costs - (d) final inventory holding costs>



< Figure 6: Relative difference in expectation $E[T_k]$ and scv $c^2[T_k]$ of throughput times between CPP and CBC >



(a) (b)