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Evaluation of Maintenance Policies for Equipment Subject to Quality Shifts and Failures

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Abstract

We develop an economic model for the optimization of maintenance procedures in a production process with two quality states. In addition to deteriorating with age, the equipment may experience a jump to an out-of-control state (quality shift), which is characterized by lower production revenues and higher tendency to failure. The times to quality shift and failure are allowed to be generally distributed random variables. We consider two types of maintenance: minimal maintenance (MM) that upgrades the quality state of the equipment without affecting its age and perfect preventive maintenance (PM) that fully upgrades the equipment to the as-good-as-new condition. We derive the expression for the expected profit per time unit and we investigate, through a large number of numerical examples, the type of the optimal solution. It is concluded that in practically every case the optimal maintenance policy is an extreme one: it either calls for immediate MM as soon as a quality shift occurs (active policy) or it allows operation in the out-of-control state until the time of a scheduled PM action (passive policy).

1. Introduction

Equipment maintenance is a very important operation in almost every production system. An appropriate preventive maintenance (PM) policy not only reduces the probability of equipment failure but also improves the working condition of the equipment resulting in lower production costs and/or higher product quality. The failure rate of the equipment is typically assumed to increase with time or usage (after the initial infant mortality period) and consequently age-based PM models have been widely studied in the literature. These models usually assume that the operating condition of the equipment remains stable throughout production and no deterioration mechanism exists other than complete failure. Sometimes, though, the equipment may deteriorate to a less desirable working condition before failing altogether. This inferior condition may be associated with both higher operating or quality costs and increased failure rates. For example, consider the cutting process of metal frames using power saws. From time to time, the cutting disc loses its balance and consequently its ability to produce perfectly flat bur-free cuts on the metal parts. In addition to the lower quality of the produced metal parts, the misbalance leads to higher central axle fatigue and higher probability of breaking down (failure). Similarly, misbalance is a common malfunction in many electric and electronic devices where a fan is used for the cooling process of the device. When this happens, the consequences are poor cooling, higher electricity consumption, and higher proneness to failure of the electric motor due to overheating.

In cases like the above condition-monitoring can provide useful information regarding the operating state of the equipment and consequently can lead to PM actions that protect the equipment more effectively against failures. Several condition-based PM models have been developed to address the problem of properly maintaining equipment with multiple operating states; see the surveys by Pierskalla and Voelker (1976) and Valdez-Flores and Feldman (1989). However, the great majority of these models focus on the investigation of replacement or perfect PM policies that restore the equipment to an as-good-as-new state, while relatively few contributions have been made to the field of imperfect PM. Imperfect PM is assumed to improve the working condition of the equipment but not necessarily to an as-good-as-new state. Various imperfect PM models have been suggested in the literature, some of which are applicable to multi-state production processes/equipment. See the survey by Pham and Wang (1996) for more details. Here we are mainly interested in cases where an imperfect PM upgrades the operating state of the equipment by reducing its failure rate without affecting its age. A special case of this type of imperfect PM is minimal maintenance (MM), which improves the equipment condition by one state only.

The objective of this paper is to study a maintenance policy, including both perfect and imperfect maintenance actions, which is appropriate for a production process (equipment) with two operating states, namely an "in-control" state and an "out-of-control" state in Statistical Process Control terminology, and a failure state. The two distinct operating states are characterized by different operating and quality-related costs and by different failure rates; the in-control state has generally lower operating/quality cost per time unit and lower failure rate for the same equipment age compared to the out-of-control state.

Production processes with multiple operating states (quality states) are typically encountered in the context of statistical process control, where a common practice against quality deterioration is to bring the process back to its in-control state after the detection of a quality shift to an out-of-control state. In this paper, we combine quality adjustments to upgrade the process quality state with conventional maintenance actions to deal with failures. More specifically, the proposed maintenance policy comprises the following types of maintenance actions:

- Perfect corrective maintenance (CM) upon failure: it restores the failed equipment to the as-good-as-new state.
- Perfect preventive maintenance (PM) at some critical age: it restores the working equipment to the as-good-as-new state.
- Minimal maintenance (MM) applied only when the process is out-of-control: it upgrades the equipment from the out-of-control to the in-control state without affecting the equipment age.

The main contribution of the present paper is that by allowing for generally distributed times to quality shifts and to failures (with non-decreasing failure rates) it greatly expands the model's realism and applicability to actual production systems compared to existing models that require at least one of the above time distributions to be exponential. An additional contribution lies in the results; it is documented that in practically all cases it suffices to consider and compare only extreme quality maintenance policies (active or passive), which are easy to understand and implement.

Section 2 presents a brief literature review while Section 3 describes the problem in detail and introduces the necessary notation. In section 4 we develop the mathematical model while in section 5 we discuss the form of the optimal policy. Section 6 provides numerical examples and a systematic discussion of the effect of several model parameters on the optimal maintenance policy. The last section summarizes the basic results and suggests directions for future research.

2. Literature Review

The academic literature is replete with models of maintenance policies comprising various types of actions (imperfect maintenance, minimal repair etc.). In this brief literature review we restrict our attention to models for production processes (equipment) with multiple distinct operating states and at least one failure state, which assume perfect failure restoration and include the notion of imperfect PM. More specifically we consider imperfect PM that upgrades the operating state of the equipment by reducing its failure rate without affecting its age. One of the most important distinguishing features among these models is the type of the failure mechanism. We therefore start the review with models based on purely Markovian deterioration mechanisms (exponential distribution of the time to failure) and next we proceed to the presentation of more general non-Markovian models.

Purely Markovian deterioration

The pioneering work of Derman (1963) concerns a repair-replacement policy for a multi-state Markovian deteriorating production process. The model allows several alternative maintenance decisions at every operating state of the process, which do not necessarily restore the equipment to an as-good-as-new state but they affect the state transition probabilities. In addition, Derman (1963) considers multiple inoperative states where replacement is the only feasible action. The objective is to find the maintenance policy that maximizes the expected time between replacements. It is shown that this kind of problem can be expressed through a linear programming formulation.

Özekici and Günlük (1992) study a similar problem but with deterministic maintenance effects (every maintenance action leads to a certain process state) and a more general cost function. The objective is to select the most appropriate maintenance action for each process state so as to minimize the expected cost. Özekici and Günlük (1992) provide structural properties of the optimal maintenance policy under various cost structures.

A slightly different approach to modeling of imperfect maintenance (repair) for production processes with multiple operating states and an absorbing failure state has been proposed by Chiang and Yuan (2001). The process is periodically inspected and: a) nothing is done if the equipment is found to be within a group of "good" states, or b) the equipment is repaired to a better state if found to be within a group of "intermediate" states, or c) the equipment is replaced if found to be within a group of "inferior" states including the failure state. The optimal policy is derived by minimizing the expected long-run cost rate.

A Markovian model which explicitly combines quality control with maintenance procedures has been developed by Tagaras (1988); it concerns a production process subject to quality shifts and failures, assuming multiple out-of-control states. The objective is to optimize both quality control schemes and PM procedures. In addition to CM (restoration) following a failure and periodic PM, quality adjustments are carried out whenever the process is found to operate in an out-of-control state.

Non-Markovian deterioration

Moustafa et al. (2004) study a multi-state semi-Markovian deteriorating system, allowing both replacement and minimal maintenance. They assume that minimal maintenance upgrades the equipment condition by one state. They use a control limit policy with two threshold states in a way similar to Chiang and Yuan (2001), but they show that this type of policy is not always optimal.

Makis and Fung (1995, 1998) study the integrated problem of determining the optimal quality control schedule and the optimal production quantity in a production process with two quality states and a single failure state. The 1995 model allows for periodic preventive replacement as well. In both models the time to quality shift is assumed to be exponentially distributed while the time to failure is assumed to be generally distributed. As soon as a quality shift is detected the process is restored to the in-control state with the same equipment age. It is worth noting that the failure time distribution is assumed to be independent of the actual quality state of the process and consequently restorations of quality shifts do not constitute PM actions against failures.

The production process studied in this paper is similar to that of Makis and Fung (two quality states and a single failure state) but we focus on maintenance procedures rather than on the determination of production quantities. In addition, we consider general (not necessarily Markovian) deterioration mechanisms not only for failure but for quality shifts as well. Furthermore, we allow the failure time distribution to depend not only on the equipment age but also on its state.

Before concluding this brief literature review it should be noted, for the state of completeness, that there exist a number of papers combining statistical process control (SPC) with preventive maintenance procedures. Although these papers share some degree of similarity with our work, they approach the problem mostly from a quality-oriented point of view, since they only consider quality deterioration mechanisms. One of the earliest models in this field is that of Rahim and Banerjee (1993). Their objective is the integrated optimization of the SPC parameters and the PM time in production processes subject to quality shifts, which result in an inferior-quality, yet operating, state. Other relevant models have been developed more recently by Cassady et al.

(2000), Lee and Rahim (2001) and Linderman et al. (2005). Ben – Daya and Rahim (2000) study a similar problem incorporating the notion of imperfect preventive maintenance. However, these papers do not take into account the possibility of a complete failure that would enforce an immediate cease of operation, which is a typical element of practically all maintenance problems and models.

3. Problem definition, assumptions and notation

We consider a production process that may operate in one of two possible quality states; in-control state or state 0 and out-of-control state or state 1. Regardless of the actual quality state of the process, the equipment may suffer a failure at any time, resulting in complete stoppage of operation. The failure rate of the process in both quality states is a non-decreasing function of the equipment age. The failure rate in the out-of-control state is assumed to be higher than that of the in-control state for the same equipment age. Apart from that, the two quality states also differ in terms of production revenues; the out-of-control state is assumed to be less profitable than the in-control state. Equal failure rates or equal production revenues are easily treated as special cases. Quality shifts from the in-control to the out-of-control state may occur at any time but they are immediately observed and consequently the actual quality state of the process is always known with accuracy. Note that the assumption of continuous and accurate knowledge of the actual process quality state is not characteristic of typical SPC problems. However, it is often realistic since there are many cases where the quality shift is directly observable or even self announced, e.g. through some distinct noise. Besides, even when a quality shift has to be detected through inspection, the inspection process is sometimes performed on a continuous basis and is very accurate; e.g. inspection by means of on - line sensors.

Since the failure rate increases with the age of the equipment due to physical deterioration even in the in-control state, it is reasonable to preventively maintain the equipment when it reaches some critical age (t_{m0}), beyond which the probability of failure is unacceptably high. PM upgrades the equipment in the as-good-as-new condition and totally renews the process. Although the frequency of failures can be radically decreased through PM, the equipment will inevitably fail occasionally and production will be interrupted. CM is implemented as soon as a failure occurs and the equipment is restored again to the as-good-as-new condition.

From the above description it is clear that the production-maintenance process consists of a series of independent and stochastically identical cycles. Each cycle begins with the process in the as-good-as-new condition (in-control state and zero equipment age) and terminates either with a PM at t_{m0} or with a CM following a failure, whichever occurs first. Within each cycle the process may shift to the out-of-control state leading to decreased production revenues and/or increased failure rate. To avoid production under these poor conditions it is possible to stop the process and bring the equipment back to the in-control state without affecting the equipment age. In other words, an MM can take place to improve the quality state of the process.

It is worth noting that such MM actions are usually assumed to be performed immediately after the process is detected to operate at or above some threshold state. Our model is more flexible than that, since the process may intentionally be allowed to operate for some time in the out-of-control state before MM is performed. In general, MM is not necessarily preferable to operation in the out-of-control state. Whether MM is worthwhile or not depends on the tradeoff between its cost on one hand and the lower revenues and higher failure rate associated with out-of-control operation on the other. We consider the following three alternative situations/policies:

1. There is a critical age of the equipment, t_{m1} ($0<t_{m1}<t_{m0}$), beyond which operation in the out-of-control state is uneconomical due to the unacceptably high failure rate. That is, if a quality shift occurs at time $t>t_{m1}$ then MM is performed immediately and the process continues its operation in the in-control state. If a quality shift occurs at time $t<t_{m1}$, the process is allowed to continue in the out-of-control state until t_{m1} and it is only then restored to the in-control state (unless a failure occurs before t_{m1}).
2. Operation in the out-of-control state is so costly that the process is not allowed to operate at all in this state. In such cases MM is always performed as soon as a quality shift occurs ($t_{m1}=0$). Borrowing from the automatic control terminology, a policy with $t_{m1}=0$ will be called "Active Quality Maintenance" policy, or simply AQM.
3. The cost of MM is too high relative to its benefits and consequently the equipment is not restored to the in-control state earlier than t_{m0} ($t_{m1}=t_{m0}$). A policy with $t_{m1}=t_{m0}$ will be called "Passive Quality Maintenance" policy, or simply PQM.

Note that when $t_{m1}<t_{m0}$ (cases 1 and 2 above) the process may shift to the out-of-control state more than once in each production cycle and as a result MM will be performed several times in a cycle.

To summarize, the proposed maintenance policy is characterized by two critical times (equipment ages) t_{m1} and t_{m0} ($0 \le t_{m1} \le t_{m0}$) and the objective is to find the optimal values of t_{m0} and t_{m1} that maximize the expected profit per time unit. The notation that will be used to develop the optimization model is presented below:

$f(t)$ density function of the time to quality shift

$F(t)$ cumulative distribution function of the time to quality shift; $\bar{F}(t) = 1 - F(t)$

$h(t)$ $= \dfrac{f(t)}{\bar{F}(t)}$ (quality shift rate)

$\varphi_i(t)$ density function of the time of failure (equipment age) if the process is in state i (i=0,1) at t=0; note that the density function of the time to failure if a quality shift occurs at time t_s is $\varphi_1(t)/\bar{\Phi}_1(t_s)$ for $t>t_s$

$\Phi_i(t)$ cumulative distribution function of the time of failure in state i; $\bar{\Phi}_i(t) = 1 - \Phi_i(t)$

$h_i(t)$ $= \dfrac{\varphi_i(t)}{\bar{\Phi}_i(t)}$ (failure rate in state i)

t equipment age; t=0 at the beginning of each cycle

t_{m0} scheduled preventive maintenance time in the in-control state

t_{m1} scheduled minimal maintenance time if a quality shift occurs at $t<t_{m1}$

Z expected time to perform corrective maintenance

Z_P expected time to perform preventive maintenance

Z_M expected time to perform minimal maintenance

R_i expected net revenue per time unit of operation in state i

W cost of corrective maintenance

W_P cost of preventive maintenance

W_M cost of minimal maintenance

n Expected number of MM actions in a cycle

E(T) expected cycle length

E(P) expected cycle profit

EPT expected profit per time unit

Note that R_0 and R_1 take into account the potential cost of low quality items and the operating cost. Consequently it is reasonable to assume that $R_1 \leq R_0$.

4. Model development

In order to formulate the EPT function we first develop the expressions for the expected cycle length E(T) and the expected cycle profit E(P). Each cycle begins with zero equipment age (t=0) and terminates either with repair after failure before t_{m0} or with preventive maintenance at t_{m0}. In both cases the process may never shift to the out-of-control state, may shift just once or may shift several times. Thus, the total duration of a cycle consists of the following sub-periods:

- operating time in the in-control state (random variable T_0)
- operating time in the out-of-control state, if a quality shift occurs prior to failure and before t_{m1} (random variable T_1)
- repair time, if a failure occurs before t_{m0}
- preventive maintenance time, if the equipment reaches age t_{m0} without failure
- time for minimal maintenance actions.

Similarly, the expected profit per cycle consists of the following components:

- production net revenue in the in-control state

- production net revenue in the out-of-control state, if a quality shift occurs prior to failure and before t_{m1}
- repair cost, if a failure occurs before t_{m0}
- preventive maintenance cost, if the equipment reaches age t_{m0} without failure
- cost of minimal maintenance actions.

Operating time in the in-control state

Once the equipment reaches age t_{m1} (either in the in-control or in the out-of-control state) it is not allowed to operate in the out-of-control state in the remainder of the cycle, although quality shifts may still occur (but will be immediately removed through MM). Consequently, the expected in-control period during the cycle can be divided into two parts; one before t_{m1} and one after t_{m1}.

Regarding the first part, the in-control period lasts until t_{m1} only if neither a failure nor a quality shift occurs by that time; otherwise, it lasts until some time $t<t_{m1}$. Thus, the expected duration of the in-control period before t_{m1} is

$$E(T_0 \text{ before } t_{m1}) = t_{m1}\bar{\Phi}_0(t_{m1})\bar{F}(t_{m1}) + \int_0^{t_{m1}} t\varphi_0(t)\bar{F}(t)dt + \int_0^{t_{m1}} tf(t)\bar{\Phi}_0(t)dt .$$

Noting that $\varphi_0(t)dt = -d\bar{\Phi}_0(t)$ and integrating $\int_0^{t_{m1}} t\bar{F}(t)d\bar{\Phi}_0(t)$ by parts yields

$$E(T_0 \text{ before } t_{m1}) = \int_0^{t_{m1}} \bar{\Phi}_0(t)\bar{F}(t)dt . \tag{1}$$

The in-control period after t_{m1} lasts either until t_{m0} or until an equipment failure before t_{m0}. Thus, the expected length of the in-control period after t_{m1} provided that the equipment survives until t_{m1} is

$$E(T_0 \text{ after } t_{m1}) = \left(t_{m0} - t_{m1}\right)\frac{\bar{\Phi}_0\left(t_{m0}\right)}{\bar{\Phi}_0\left(t_{m1}\right)} + \int_{t_{m1}}^{t_{m0}} \left(t - t_{m1}\right)\frac{\varphi_0\left(t\right)}{\bar{\Phi}_0\left(t_{m1}\right)}dt .$$

Again, using $\varphi_0\left(t\right)dt = -d\bar{\Phi}_0\left(t\right)$ and integrating by parts yields

$$E(T_0 \text{ after } t_{m1}) = \int_{t_{m1}}^{t_{m0}} \frac{\bar{\Phi}_0\left(t\right)}{\bar{\Phi}_0\left(t_{m1}\right)}dt . \qquad (2)$$

The probability that the equipment will survive until t_{m1} without a failure is

$$p_{t_{m1}} = \bar{\Phi}_0\left(t_{m1}\right)\bar{F}\left(t_{m1}\right) + \int_0^{t_{m1}} f\left(t\right)\bar{\Phi}_0\left(t\right)\frac{\bar{\Phi}_1\left(t_{m1}\right)}{\bar{\Phi}_1\left(t\right)}dt . \qquad (3)$$

The first term in the right hand side of (3) expresses the probability that neither a failure nor a quality shift occurs before t_{m1} (the process remains in the in-control state until t_{m1}), while the second term expresses the probability that a quality shift occurs prior to t_{m1}, yet no failure occurs until t_{m1} (the operating time in the out-of-control state is strictly positive).

The total expected time that the process operates in the in-control state in a cycle is given by

$$E\left(T_0\right) = E(T_0 \text{ before } t_{m1}) + p_{t_{m1}} E(T_0 \text{ after } t_{m1}) ,$$

which, combining (1) through (3) and simplifying results in

$$E\left(T_0\right) = \int_0^{t_{m1}} \bar{\Phi}_0\left(t\right)\bar{F}\left(t\right)dt + \bar{F}\left(t_{m1}\right)\int_{t_{m1}}^{t_{m0}} \bar{\Phi}_0\left(t\right)dt + \int_0^{t_{m1}} f\left(t\right)\bar{\Phi}_0\left(t\right)\frac{\bar{\Phi}_1\left(t_{m1}\right)}{\bar{\Phi}_1\left(t\right)}dt \int_{t_{m1}}^{t_{m0}} \frac{\bar{\Phi}_0\left(t\right)}{\bar{\Phi}_0\left(t_{m1}\right)}dt . \qquad (4)$$

Operating time in the out-of-control state

The process spends some time in the out-of-control state if and only if a quality shift occurs prior to t_{m1}. Then, the operating time in the out-of-control state lasts until t_{m1} if no failure occurs by that time or until an equipment failure before t_{m1}. Thus, the expected period that the process operates in the out-of-control state in a cycle is given by:

$$E(T_1) = \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t) \int_t^{t_{m1}} \frac{(t' - t)\varphi_1(t')}{\bar{\Phi}_1(t)} dt' dt + \int_0^{t_{m1}} (t_{m1} - t) f(t) \bar{\Phi}_0(t) \frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)} dt. \quad (5)$$

Integrating by parts the second integral of the first term in the right hand side of (5) yields

$$E(T_1) = \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t) \int_t^{t_{m1}} \frac{\bar{\Phi}_1(t')}{\bar{\Phi}_1(t)} dt' dt. \quad (6)$$

Probability of preventive maintenance and probability of failure

Preventive maintenance is only performed whenever the equipment reaches age t_{m0} without a failure. The probability that the equipment will reach age t_{m1} without a failure, $p_{t_{m1}}$, is given by (5), while the probability that the equipment will not fail in the time interval from t_{m1} to t_{m0}, provided that it survived until t_{m1}, is $\bar{\Phi}_0(t_{m0})/\bar{\Phi}_0(t_{m1})$. The fact that the process may shift (once or more) to the out-of-control state after t_{m1} has no effect on the failure probability of the process since the equipment is immediately restored to the in-control state.

Thus, the probability of preventive maintenance in a cycle is

$$p_{PM} = p_{t_{m1}} \frac{\bar{\Phi}_0(t_{m0})}{\bar{\Phi}_0(t_{m1})} = \left[\bar{\Phi}_0(t_{m1})\bar{F}(t_{m1}) + \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t) \frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)} dt \right] \frac{\bar{\Phi}_0(t_{m0})}{\bar{\Phi}_0(t_{m1})}$$

$$= \bar{\Phi}_0(t_{m0})\bar{F}(t_{m1}) + \frac{\bar{\Phi}_0(t_{m0})}{\bar{\Phi}_0(t_{m1})} \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t) \frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)} dt. \quad (7)$$

The probability of failure in a cycle is simply $1\text{-}p_{PM}$.

Expected number of minimal maintenance actions

The earliest time that a minimal maintenance can take place is t_{m1}; this happens if the equipment reaches age t_{m1} in the out-of-control state. Therefore, the expected number of minimal maintenance actions by t_{m1}, denoted n_1, is equal to the probability of that event

$$n_1 = \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t)\frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)}dt . \tag{8}$$

The equipment is also minimally maintained every time a quality shift occurs after t_{m1}. The expected number of minimal maintenance actions after t_{m1}, provided that the equipment survived until t_{m1}, is denoted n_2 and computed by the following lemma, the extensive proof of which is presented in the Appendix.

Lemma 1 *The expected number of minimal maintenance actions after t_{m1} in a cycle, provided that the equipment survives until t_{m1}, is*

$$n_2 = \int_{t_{m1}}^{t_{m0}} \frac{\bar{\Phi}_0(t)}{\bar{\Phi}_0(t_{m1})}h(t)dt . \tag{9}$$

The expected total number of minimal maintenance actions throughout a cycle is $n = n_1 + p_{t_{m1}} n_2$. Combining equations (3), (8) and (9) and simplifying we obtain

$$n = \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t)\frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)}dt + \bar{F}(t_{m1})\int_{t_{m1}}^{t_{m0}} \bar{\Phi}_0(t)h(t)dt + \int_0^{t_{m1}} f(t)\bar{\Phi}_0(t)\frac{\bar{\Phi}_1(t_{m1})}{\bar{\Phi}_1(t)}dt \int_{t_{m1}}^{t_{m0}} \frac{\bar{\Phi}_0(t)}{\bar{\Phi}_0(t_{m1})}h(t)dt . \tag{10}$$

Expected profit per time unit

The total expected cycle length is given by

$$E(T) = E(T_0) + E(T_1) + Z_P p_{PM} + Z(1 - p_{PM}) + Z_M n ,$$

while the total expected profit per cycle is given by

$$E(P) = R_0E(T_0) + R_1E(T_1) - W_P p_{PM} - W\left(1 - p_{PM}\right) - W_M n .$$

Finally, since the process is a renewal reward process, the expected profit per time unit can be expressed as the ratio of the expected profit per cycle to the expected cycle length:

$$EPT\left(t_{m0}, t_{m1}\right) = \frac{E(P)}{E(T)} .$$

Note that the model holds for any value of t_{m1} in the interval $[0, t_{m0}]$ including the special cases $t_{m1}=0$ (immediate MM after a quality shift; active quality maintenance policy) and $t_{m1}=t_{m0}$ (the equipment is allowed to operate in the out-of-control state until t_{m0} without any MM intervention; passive quality maintenance policy). In the latter case, if no failure occurs and the equipment succeeds to survive until t_{m0} operating in the out-of-control state, the scheduled PM at t_{m0} is combined with an MM, with total cost $W_M + W_P$ and total time $Z_M + Z_P$, so that the next production cycle begins with the process in a perfect condition (equipment as-good-as-new and in-control).

5. What is (usually) the type of the optimal policy?

The maintenance model developed in the previous section allows t_{m1} to be anywhere in the range $[0, t_{m0}]$. For given t_{m0}, the model essentially compares the cost of MM to upgrade the process quality to the in-control state against the benefits of such an action and returns the optimal value of t_{m1}. However, the maintenance management of most actual production systems either adopts an Active Quality Maintenance (AQM) policy, whereby the equipment is never allowed to operate in the out-of-control state ($t_{m1}=0$) or it employs a Passive Quality Maintenance (PQM) policy, whereby the equipment is treated the same regardless of its quality state ($t_{m1}=t_{m0}$). The choice between AQM and PQM is typically made empirically; if the out-of-control state is considered unacceptable then AQM is adopted, while if process interruptions are considered costly and undesirable then PQM is preferred.

To investigate the actual type of the optimal policy and evaluate the quality of AQM and PQM policies we solved a large number of numerical examples with different problem parameters. Specifically, we solved 170 examples with all process parameters randomly selected from a wide range of values. In addition, we have allowed both Weibull and Gamma distributions to describe the process failure and quality shift mechanisms. The density functions of these distributions are presented in Table 1. Note that ensuring that the failure rate when operating in the out-of-control state with equipment age t is at least as large as the failure rate in the in-control state with the same equipment age, requires $c_0=c_1$ and $\lambda_0 \le \lambda_1$ for both distribution types. The allowable values of all process parameters are shown in Table 2.

[Insert Tables 1 and 2 about here]

The optimization is performed by means of an exhaustive search over all possible t_{m0} and t_{m1} values, under the constraint $0 \le t_{m1} \le t_{m0}$, to ensure that the global optimum of EPT is obtained. For computational simplicity we restrict our numerical investigation to integer values of t_{m0} and t_{m1} with initial values equal to zero. The optimization algorithm computes the expected profit for all t_{m0} and t_{m1} values and the search procedure stops as soon as t_{m0} and t_{m1} reach some threshold values beyond which their effect on the expected profit is insignificant. The convergence of EPT is assured by the fact that the probability of failure will eventually reach unity for large values of t_{m0}. The computational time required for finding the optimal solution typically varies between 10 and 30 minutes on a Pentium IV 1.8 GHz personal computer, depending on the parameters of the example.

The main findings are summarized below:

- In all 170 cases, the optimal policy is either AQM (t_{m1}=0) or PQM (t_{m1}=t_{m0}). In the vast majority of the cases examined (146 out of 170) the optimal policy is AQM.
- Among the 24 cases where AQM is not optimal, the percentage loss that would result from using the best possible AQM instead of the optimal PQM is 7.3% on average, ranging between 0.04% and 33.6%. Among the 146 cases where PQM is not optimal, the percentage loss that would result from using the best possible PQM is 6.2% on average, ranging between 0.02% and 31.8%.
- When the failure times follow Gamma distributions, the optimal solution usually dictates that the equipment should not undergo PM at all ($t_{m0} \to \infty$). This can be explained by the fact that the failure rate of a Gamma distribution is stabilized as the equipment age grows large (similar to the memoryless exponential distribution) and it may be better to continue operation from some point on rather than maintain the equipment.

It is important to mention here that although the optimal solution in all numerical examples solved in the course of this research (including the 170 examples of this section and the 48 examples of the next section) is obtained for an extreme value of t_{m1} (either 0 or t_{m0}), in general the optimal solution is not necessarily unique. In some cases the expected profit function EPT is so flat within a range of t_{m1} values (from t_{m1}=0 to some critical t_{m1} value) that practically all t_{m1} in this range can be considered optimal. In these cases it may be preferable to choose the largest t_{m1} in the "optimal" range so as to minimize downtimes due to MM actions.

Also note that our extensive numerical investigation of the behaviour of EPT has shown that an extreme value of t_{m1} (either $t_{m1}=0$ or $t_{m1}=t_{m0}$) is always optimal, even when there are multiple optimal solutions with $0<t_{m1}<t_{m0}$. Although a formal proof of such a property, if it is indeed true in general, remains elusive, the practical implication is that it is sufficient to search for the optimal solution only between the two extreme policies AQM and PQM. In this way the computational requirements are significantly reduced since it suffices to optimize two single-variable functions instead of searching over a two-dimensional decision space.

6. The effect of process parameters on the optimal policy

To investigate systematically the effect of the process parameters on the optimal policy we have solved 48 problems, differing substantially in key model features such as the relative costs of operation and maintenance activities and the equipment proneness to quality shifts and failures.

Specifically, we express the quality shift mechanism and the failure mechanism in both quality states (i=0,1) by Weibull distributions of the equipment age. The parameters λ, λ_1, R_1 and W_P are examined at 2 levels each as shown in Table 3, while the pair W_M, Z_M is examined at three levels as follows:

(a) $W_M=0.25W_P$ and $Z_M=0.25Z_P$

(b) $W_M=0.25W_P$ and $Z_M=0.75Z_P$

(c) $W_M=0.75W_P$ and $Z_M=0.25Z_P$

The remaining parameters are set equal to the following values in all 48 cases: $c_0=c_1=2$, $c=1.5$, $\lambda_0=0.004$, $R_0=300$, $Z=Z_P=1.0$, $W=800$. The 48 cases are numbered 1a, 1b, 1c to 16a, 16b, 16c where a, b or c indicates the W_M, Z_M combination.

[Insert Table 3 about here]

Table 4 shows the optimal critical equipment ages t_{m1}, t_{m0} and the corresponding maximum expected profit per time unit, EPT, for the 48 cases. Note that, exactly as in the 170 examples of the previous section, in all 48 cases the optimal policy is either AQM (34 cases) or PQM (14 cases). In 12 of the 14 cases where a passive quality maintenance (PQM) policy is optimal, it is optimal to only use corrective maintenance upon failure and never resort to PM or MM actions ($t_{m1} = t_{m0} = \infty$). The last 4 columns of Table 4 present the optimal AQM and the optimal PQM policies along with the percentage losses when using each policy. Obviously, when one of these policies is optimal its respective percentage loss is zero.

[Insert Table 4 about here]

The effects of the parameters on the type of the optimal policy as well as on the savings that can be achieved by using the proposed model as opposed to blindly following the AQM or the PQM policy are summarized as follows:

- Large R_1 ($R_1 \leq R_0$) and/or small λ_1 ($\lambda_1 \geq \lambda_0$) implies that state 1 (out-of-control) is not that inferior to state 0 (in-control) in terms of profit and/or failure rate. Consequently it may be more economical to allow operation in state 1 rather than upgrade the process quality to state 0. Thus, PQM tends to outperform AQM in such cases. The percentage loss associated with the use of an AQM policy when PQM is optimal increases as R_1 increases and/or as λ_1 decreases. This is because increasing R_1 or decreasing λ_1 increases the expected profit per time unit for $t_{m1} = t_{m0}$ (PQM policy), while the expected profit per time unit for $t_{m1} = 0$ remains unaltered (AQM policy); consequently the two solutions diverge.

- Combinations b (W_M=0.25W_P, Z_M=0.75Z_P) and c (W_M=0.75W_P, Z_M=0.25Z_P) are characterized by a higher direct or indirect (downtime) cost of MM than combination a (W_M=0.25W_P, Z_M=0.25Z_P). Consequently, a PQM policy is more likely to be optimal in combinations b and c. The percentage loss due to incorrectly adopting an AQM policy (while PQM is optimal) increases when moving from combination a of W_M and Z_M to combination b or c, while the percentage loss associated with incorrect adoption of a PQM policy decreases. This is because the increased cost of MM in combinations b and c tends to have a greater negative impact on the solutions with $t_{m1} = 0$ than on those with $t_{m1} = t_{m0}$.
- Large W_P and/or large λ signify more expensive or more frequent MM actions and consequently a PQM policy is more likely to be optimal. The percentage loss associated with incorrect use of an AQM policy increases as W_P and/or λ increase, due to the negative effect of both these parameters on the total cost of MM actions. However, the percentage loss associated with incorrect use of a PQM policy can either increase or decrease as W_P and/or λ increase.

We have also studied the isolated effect of W_P, keeping W_M constant; in all cases the type of the optimal policy remained unaltered.

Finally, the effect of failure time variability has also been investigated using $c_0 = c_1 = 3$ instead of $c_0 = c_1 = 2$ in all 48 cases and modifying the values of λ_0 and λ_1, so as to keep the mean times to failure unaltered for both quality states. This investigation has not revealed any systematic effect of the failure time variability on the type of the optimal policy nor on the percentage loss associated with consistently adopting AQM or PQM.

7. Conclusions

In this paper we have studied maintenance procedures in a production process subject to both quality shifts and failures. The equipment deteriorates continuously due to the ageing process and at the same time it may experience a jump to an inferior quality state (out-of-control state) upon the occurrence of an assignable cause. Transitions to the out-of-control state have a dual impact on the process; they result in lower production quality implying lower production revenues but they also increase the failure rate of the process.

The proposed maintenance policy is a combination of an age-based preventive maintenance policy at some critical age t_{m0}, with additional minimal maintenance actions, which upgrade the process quality to the in-control state. Such MM actions (quality adjustments) are commonly used in quality control to eliminate the negative effects of assignable causes. In our model restoring the process to the in-control state after the occurrence of an assignable cause not only improves production quality but also decreases the failure rate of the process. In contrast to typical quality control models, the MM considered here is not necessarily implemented immediately after quality shifts but may intentionally be postponed until some critical equipment age.

Our investigation showed that in practically every case the optimal maintenance policy either calls for immediate MM as soon as a quality shift occurs (active quality maintenance, AQM) or allows operation in the out-of-control state and an MM is only implemented along with preventive maintenance at predetermined times (passive quality maintenance, PQM). Thus, it suffices to consider only the optimal AQM and

PQM policies, compare them and use the one that is most effective in each particular case. Nevertheless, using the wrong extreme policy may result in significant loss. Our numerical investigation has shown that such losses can be as high as about 30% of the optimal expected profit.

There are some interesting extensions of the proposed maintenance model that are worth studying, such as the case of incomplete information about the quality state of the process. In addition, full integration of quality control procedures with equipment maintenance in deteriorating production processes under general (non-restrictive) assumptions would be of great practical interest.

Appendix: Proof of Lemma 1

We first determine the probability that exactly n (n=1, 2,…..) quality shifts occur in the interval (t_{m1}, t_{m0}) provided that the equipment survives until time/age t_{m1}. We start with the derivation for n=1 and then generalize for higher values of n.

The process shifts exactly once from the in-control to the out-of-control state (n=1) if either one of the following two scenarios materializes:

a) The process shifts to the out-of-control state at time t_1 ($t_{m1}<t_1<t_{m0}$) prior to failure, an MM is immediately implemented upgrading the process quality to the in-control state and operation continues without intervention (neither a failure nor a quality shift occurs) until t_{m0}.

b) The process shifts to the out-of-control state at time t_1 ($t_{m1}<t_1<t_{m0}$) prior to failure, an MM is immediately implemented upgrading the process quality to the in-control state and operation continues until the occurrence of a failure at time t before t_{m0}. No other quality shifts occur between t_1 and t.

Under these scenarios the probability of a quality shift at t_1 prior to failure provided that the equipment survives until t_{m1} is

$$p_{qs} = \int_{t_{m1}}^{t_{m0}} \frac{f(t_1)}{\bar{F}(t_{m1})} \frac{\bar{\Phi}_0(t_1)}{\bar{\Phi}_0(t_{m1})} dt_1 . \tag{A1}$$

The probability that neither a failure nor a quality shift occurs in (t_1, t_{m0}) is

$$p_a = \frac{\bar{\Phi}_0(t_{m0})}{\bar{\Phi}_0(t_1)} \frac{\bar{F}(t_{m0})}{\bar{F}(t_1)}, \tag{A2}$$

while the probability of failure in (t_1, t_{m0}) provided that the equipment operates continuously in the in-control state is

$$p_b = \int_{t_1}^{t_{m0}} \frac{\varphi_0(t)}{\bar{\Phi}_0(t_1)} \frac{\bar{F}(t)}{\bar{F}(t_1)} dt . \tag{A3}$$

Combining (A1) through (A3) the probability that the process shifts exactly once from the in-control to the out-of-control state during the interval (t_{m1}, t_{m0}) is given by

$$P_1 = p_{qs}(p_a + p_b)$$

$$= \int_{t_{m1}}^{t_{m0}} \frac{f(t_1)}{\bar{F}(t_{m1})} \frac{\bar{\Phi}_0(t_1)}{\bar{\Phi}_0(t_{m1})} \int_{t_1}^{t_{m0}} \frac{\varphi_0(t)}{\bar{\Phi}_0(t_1)} \frac{\bar{F}(t)}{\bar{F}(t_1)} dt dt_1 + \int_{t_{m1}}^{t_{m0}} \frac{f(t_1)}{\bar{F}(t_{m1})} \frac{\bar{\Phi}_0(t_1)}{\bar{\Phi}_0(t_{m1})} \frac{\bar{\Phi}_0(t_{m0})}{\bar{\Phi}_0(t_1)} \frac{\bar{F}(t_{m0})}{\bar{F}(t_1)} dt_1 . \tag{A4}$$

Using $h(t_1) = f(t_1)/\bar{F}(t_1)$ and simplifying yields

$$P_1 = \frac{1}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})} \left[\int_{t_{m1}}^{t_{m0}} h(t_1) \int_{t_1}^{t_{m0}} \varphi_0(t)\bar{F}(t) dt dt_1 + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0}) \int_{t_{m1}}^{t_{m0}} h(t_1) dt_1 \right].$$

Reversing the order of integration in the double integral results in

$$P_1 = \frac{1}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})} \left[\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t) \int_{t_{m1}}^{t} h(t_1) dt_1 dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0}) \int_{t_{m1}}^{t_{m0}} h(t_1) dt_1 \right]$$

$$= \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\overline{F}(t)\left[H(t)-H(t_{m1})\right]dt + \overline{\Phi}_0(t_{m0})\overline{F}(t_{m0})\left[H(t_{m0})-H(t_{m1})\right]}{\overline{F}(t_{m1})\overline{\Phi}_0(t_{m1})}$$

where $H(t)=\int_0^t h(x)dx$.

Similarly, the process shifts exactly twice (n=2) from the in-control to the out-of-control state during the interval (t_{m1}, t_{m0}) whenever a quality shift occurs at time t_1 ($t_{m1}<t_1<t_{m0}$) prior to failure (probability p_{qs}), an MM is immediately implemented upgrading the process quality to the in-control state and then the process shifts exactly once more (at time/age $t_2>t_1$) to the out-of-control state during the rest of the cycle (t_1, t_{m0}). Since the probability of that last event is analogous to P_1 of (A4) but refers to the interval (t_1, t_{m0}), it is denoted $P_1(t_1)$. Thus, properly adapting (A4) to the interval (t_1, t_{m0}) leads to the following expression for the probability P_2 of the event n=2:

$$P_2 = p_{qs}P_1(t_1) = \int_{t_{m1}}^{t_{m0}} \frac{f(t_1)}{\overline{F}(t_{m1})}\frac{\overline{\Phi}_0(t_1)}{\overline{\Phi}_0(t_{m1})}\int_{t_1}^{t_{m0}} \frac{f(t_2)}{\overline{F}(t_1)}\frac{\overline{\Phi}_0(t_2)}{\overline{\Phi}_0(t_1)}\int_{t_2}^{t_{m0}} \frac{\varphi_0(t)}{\overline{\Phi}_0(t_2)}\frac{\overline{F}(t)}{\overline{F}(t_2)}dtdt_1dt_2 +$$

$$\int_{t_{m1}}^{t_{m0}} \frac{f(t_1)}{\overline{F}(t_{m1})}\frac{\overline{\Phi}_0(t_1)}{\overline{\Phi}_0(t_{m1})}\int_{t_1}^{t_{m0}} \frac{f(t_2)}{\overline{F}(t_1)}\frac{\overline{\Phi}_0(t_2)}{\overline{\Phi}_0(t_1)}\frac{\overline{\Phi}_0(t_{m0})}{\overline{\Phi}_0(t_2)}\frac{\overline{F}(t_{m0})}{\overline{F}(t_2)}dt_2dt_1$$

$$= \frac{\int_{t_{m1}}^{t_{m0}} h(t_1)\int_{t_1}^{t_{m0}} h(t_2)\int_{t_2}^{t_{m0}} \varphi_0(t)\overline{F}(t)dtdt_1dt_2 + \overline{\Phi}_0(t_{m0})\overline{F}(t_{m0})\int_{t_{m1}}^{t_{m0}} h(t_1)\int_{t_1}^{t_{m0}} h(t_2)dt_2dt_1}{\overline{F}(t_{m1})\overline{\Phi}_0(t_{m1})}.$$

Reversing the order of integration in both integrals and then using

$$\int_a^b h(t)\left[H(t)-H(a)\right]dt = \left[\frac{\left[H(t)-H(a)\right]^2}{2}\right]_a^b = \frac{\left[H(b)-H(a)\right]^2}{2}$$

we get

$$P_2 = \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\int_{t_{m1}}^{t} h(t_1)\int_{t_{m1}}^{t_1} h(t_2)dt_2 dt_1 dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\int_{t_{m1}}^{t_{m0}} h(t_2)\int_{t_{m1}}^{t_2} h(t_1)dt_1 dt_2}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}$$

$$= \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\int_{t_{m1}}^{t} h(t_1)\left[H(t_1)-H(t_{m1})\right]dt_1 dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\int_{t_{m1}}^{t_{m0}} h(t_2)\left[H(t_2)-H(t_{m1})\right]dt_2}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}$$

$$= \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\frac{\left[H(t)-H(t_{m1})\right]^2}{2}dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\frac{\left[H(t_{m0})-H(t_{m1})\right]^2}{2}}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}.$$

Extending the preceding analysis to n>2 we arrive (by induction) at the following general expression for the probability P_n that the process shifts exactly n times from the in-control to the out-of-control state during the interval (t_{m1}, t_{m0}):

$$P_n = \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\frac{\left[H(t)-H(t_{m1})\right]^n}{n!}dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\frac{\left[H(t_{m0})-H(t_{m1})\right]^n}{n!}}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}.$$

The expected number of MM actions in the interval (t_{m1}, t_{m0}) is

$$n_2 = \sum_{n=1}^{\infty} nP_n$$

$$= \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\sum_{n=1}^{\infty}\left[n\frac{\left[H(t)-H(t_{m1})\right]^n}{n!}\right]dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\sum_{n=1}^{\infty}\left[n\frac{\left[H(t_{m0})-H(t_{m1})\right]^n}{n!}\right]}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}$$

.

Using successively the facts that

$$\sum_{n=1}^{\infty} n\frac{x^n}{n!} = x\sum_{n=1}^{\infty}\frac{x^{n-1}}{(n-1)!} = xe^x$$

and $e^{-H(t)} = \bar{F}(t)$

the expression for n_2 becomes

$$n_2 = \frac{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\bar{F}(t)\left[H(t)-H(t_{m1})\right]e^{\left[H(t)-H(t_{m1})\right]}dt + \bar{\Phi}_0(t_{m0})\bar{F}(t_{m0})\left[H(t_{m0})-H(t_{m1})\right]e^{\left[H(t_{m0})-H(t_{m1})\right]}}{\bar{F}(t_{m1})\bar{\Phi}_0(t_{m1})}$$

$$= \frac{1}{\bar{\Phi}_0(t_{m1})}\left\{\int_{t_{m1}}^{t_{m0}} \varphi_0(t)\left[H(t)-H(t_{m1})\right]dt + \bar{\Phi}_0(t_{m0})\left[H(t_{m0})-H(t_{m1})\right]\right\}.$$

Finally, using $\varphi_0(t)dt = -d\bar{\Phi}_0(t)$ and integrating by parts results in the simple expression of Lemma 1:

$$n_2 = \int_{t_{m1}}^{t_{m0}} \frac{\bar{\Phi}_0(t)}{\bar{\Phi}_0(t_{m1})} h(t)dt .$$

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Table 1. Density functions for quality shift and failure mechanisms

Distribution	Quality shift mechanism	Failure mechanism in state i (i = 0,1)
Weibull	$f(t)=\lambda c t^{c-1} e^{-\lambda t^{c}}$, t>0, c>0, λ>0	$\varphi_i(t)=\lambda_i c_i t^{c_i-1} e^{-\lambda_i t^{c_i}}$, t>0, c_i>0, λ_i>0
Gamma	$f(t)=\lambda^{c} t^{c-1} e^{-\lambda t} / \Gamma(c)$, t>0, c>0, λ>0	$\varphi_i(t)=\lambda_i^{c_i} t^{c_i-1} e^{-\lambda_i t} / \Gamma(c_i)$, t>0, c_i>0, λ_i>0

Table 2. Range of process parameter values

Process parameter		**Range of values**
R_0		100 -1000
R_1		0 - R_0
W		100 - 1000
W_P		0.1W - W
W_M		0.1W_P - W_P
$Z = Z_P$		0.5 - 1.5
Z_M		0.1Z - Z
φ_i: Weibull (mean μ_i)	c_i	1.5 - 4 (0.5)
	μ_0	15 - 60
	μ_1	10 - μ_0
f: Weibull (mean μ)	c	1.5 - 4 (0.5)
	μ	15 - 60
φ_i: Gamma (mean μ_i)	c_i	2 - 3 - 4
	μ_0	15 - 60
	μ_1	10 - μ_0
f: Gamma (mean μ)	c	1 - 2 - 3 - 4
	μ	15 - 60

Table 3. Parameter values for the numerical investigation

λ	λ_1	R_1	W_P
0.02	0.004	200	200
0.05	0.009	250	600

Table 4. Optimal solutions and evaluation of the AQM and PQM policies

Set	Parameters				MM	Optimum Policy			AQM	Loss	PQM	Loss
	λ_1	λ	R_1	W_P		t_{m1}	t_{m0}	EPT	t_{m0} (t_{m1}=0)	(%)	t_{m1}=t_{m0}	(%)
					a	0	13	224.80	13	0	10	7.4
1	0.004	0.02	200	200	b	0	12	218.73	12	0	10	6.2
					c	0	12	218.20	12	0	10	6.1
					a	0	34	211.13	34	0	∞	9.3
2	0.004	0.02	200	600	b	∞	∞	191.42	24	1.4	∞	0
					c	0	30	203.50	30	0	∞	5.9
					a	0	13	224.80	13	0	12	3.3
3	0.004	0.02	250	200	b	0	12	218.73	12	0	12	1.9
					c	0	12	218.20	12	0	12	1.8
					a	0	34	211.13	34	0	∞	1.0
4	0.004	0.02	250	600	b	∞	∞	209.08	24	9.7	∞	0
					c	∞	∞	209.08	30	2.7	∞	0
					a	0	12	215.45	12	0	8	11.6
5	0.004	0.05	200	200	b	0	10	201.49	10	0	9	8.5
					c	0	10	201.40	10	0	9	8.3
					a	0	24	189.12	24	0	∞	9.0
6	0.004	0.05	200	600	b	∞	∞	172.16	14	19.2	∞	0
					c	0	20	173.91	20	0	∞	1.0
					a	0	12	215.45	12	0	11	4.2
7	0.004	0.05	250	200	b	14	14	202.43	10	0.5	14	0
					c	14	14	202.39	10	0.5	14	0
					a	∞	∞	199.44	24	5.2	∞	0
8	0.004	0.05	250	600	b	∞	∞	199.44	14	30.2	∞	0
					c	∞	∞	199.44	20	12.8	∞	0
					a	0	13	224.80	13	0	9	8.9
9	0.009	0.02	200	200	b	0	12	218.73	12	0	9	7.6
					c	0	12	218.20	12	0	9	7.4
					a	0	34	211.13	34	0	∞	10.1
10	0.009	0.02	200	600	b	∞	∞	189.89	24	0.6	∞	0
					c	0	30	203.50	30	0	∞	6.7
					a	0	13	224.80	13	0	10	5.8
11	0.009	0.02	250	200	b	0	12	218.73	12	0	10	4.2
					c	0	12	218.20	12	0	10	4.1
					a	0	34	211.13	34	0	∞	4.4
12	0.009	0.02	250	600	b	∞	∞	201.77	24	6.4	∞	0
					c	0	30	203.50	30	0	∞	0.8
					a	0	12	215.45	12	0	7	14.2
13	0.009	0.05	200	200	b	0	10	201.49	10	0	7	11.1
					c	0	10	201.40	10	0	7	10.8
					a	0	24	189.12	24	0	∞	13.2
14	0.009	0.05	200	600	b	∞	∞	164.10	14	15.2	∞	0
					c	0	20	173.91	20	0	∞	5.6
					a	0	12	215.45	12	0	8	8.7
15	0.009	0.05	250	200	b	0	10	201.49	10	0	9	4.7
					c	0	10	201.40	10	0	9	4.6
					a	0	24	189.12	24	0	∞	2.3
16	0.009	0.05	250	600	b	∞	∞	184.82	14	24.7	∞	0
					c	∞	∞	184.82	20	5.9	∞	0