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Modelling and control of wind turbines

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Abstract: As the world is currently facing an energy and climate crisis, the development and utilization of alternative sources of energy has become an important challenge. From all types of renewable energy sources, wind turbines proved to be one of the cleanest and most reliable solutions for energy production. Wind energy conversion systems have in the last decades been subject of a strong interest as they could offer a viable source of electrical energy. This is why, it is important to focus on complex algorithms that meet with multiple objectives such as speed regulation, blade load and mode stabilization with simultaneously maximizing energy capture. This paper starts with a state of the art of wind turbines and their problematic and continues with the presentation of a polynomial control method designed for the third functioning zone of a wind turbine.

Keywords: Wind power, renewable energy, turbine, pitch control.

1. INTRODUCTION

Wind energy has proved to be an important source of clean and renewable energy, as no fossil fuels are burnt in order to produce electrical energy. The advantages of wind turbines usage made necessary the design of different control systems in order to improve wind turbines behavior and make them more reliable and efficient.

As the wind is the energy source, it is of great importance to be able to generate predictions regarding the wind behavior. The stochastic nature of the wind determines the necessity of a wind turbine to be able to work under different wind velocities that determine the functioning regimes of the turbine. For each of these regimes, certain characteristics are to be considered.

In order to keep a wind turbine’s performance within these conditions, controllers must be designed and implemented. Due to the unpredictable nature of the wind, the controllers used must be robust enough as to be able to perform properly even when wind bursts appear.

The output power of the turbine strongly depends in a non-linear form on the wind speed, the rotation speed of the turbine and the pitch angle of the blades. The designed controller must be able to adjust the torque of the generator and as well the pitch angle of the blades in order to adapt the rotational speed of the turbine which moves the rotor.

It is well known that wind speed and direction vary according to geographical area and surface aspect. This is why it is very difficult to use the exact wind speed value, measured at a given moment in time. This leads to the conclusion that a variable speed turbine is a multivariable system for which a multi-objective command is required.

This paper focuses on variable speed turbines control methods due to the advantages obtained with such turbines. Among these advantages, the generation of more energy at a given wind speed and lack of grid voltage fluctuations are the most important and determined this paper’s research direction.

The form, in which the delivered energy is controlled, depending on the wind speed, determines the working region of the variable speed turbine. We can classify these working regions into three different areas as shown in Figure 1.

![Figure 1. The working regions of a variable speed turbine](image-url)
The first region corresponds to the minimum operational wind speed of the turbine. That is, if the wind speed is lower than a certain threshold (usually around 5 m/s), then it is not worth to turn on the turbine because the energy consumed is higher than the one produced. (This area is also known as First Partial Load Area).

The second region covers from its minimum operational speed up to the speed where the maximum safe electric power can be given by the generator (around 14 m/s). In this area, the power delivered to the generator is controlled by adjusting the torque that is given to the generator. This area is also known as the Second Partial Load zone.

The third region goes from the wind speed where the maximum safe electrical power is given up to the cut off speed, which is the speed at which it is no longer safe to run the wind turbine (around 25 m/s). This is the area in which the pitch angle of the blades is adjusted in order to maintain the maximum power output without exceeding the threshold (Full Load Area). This paper will only focus on the third operational regime.

2. THEORETICAL BACKGROUND

The wind turbine cannot generate unlimited power due to its physical limitations. Some of these limitations are the tolerable rotational speed of the shafts and the maximum power the generator can produce before getting damaged. In Figure 2 one can observe a simplified energy conversion system scheme [1].

![Energy conversion system](image)

As it can be observed, the energy conversion system is decomposed in several sub-components that are to be analyzed and modeled for a more suitable control of the system. The wind speed in a fixed point in space is characterized in the frequencies domain and is represented in Figure 3. The curve illustrated in Figure 3 models the Van der Hoven spectrum and it highlights the presence of two components of the wind speed. One of the components is the average value, \( v_m(t) \) and it has a low variation. The second component is the turbulent component, \( v_t(t) \). The turbulent speed is the high frequency component and its power spectrum model is given by von Karman expression [2] [3]:

\[
\Phi_v(\omega) = \frac{K}{\left(1 + (T_v \cdot \omega)^2\right)^{5/6}}
\]

and it represents a suitable approximation on equation (1).

Control algorithms use a linearization of the turbulent component of the wind, which is given by a first order filter on which the white noise, \( m_w(t) \) is applied. [3]:

\[
v_t = -\frac{1}{T_v} \cdot v_t(t) + m_w(t)
\]

This equation shows that the turbulent component can be modeled as a linear state variable of the system and helps in generating, in a simple way, the control law.

The power spectrum that corresponds to the linear model is:

\[
\Phi_v(\omega) = \frac{K}{\left(1 + (T_v \cdot \omega)^2\right)}
\]

The time constant \( T_v \) of the model (2) and the white noise variance \( m_w(t) \) depend on the average wind speed \( v_m \) and on the characteristics of the turbine location. [2]

\[
T_v = L/v_m
\]

\[
\sigma_m = k_{\sigma,v_m}
\]

The mechanical power received by the turbine, \( P_{\text{aero}} \), depends on the air density, wind speed and power coefficient \( C_p \).

\[
P_{\text{aero}} = \frac{1}{2} \cdot \rho \cdot \pi \cdot R^2 \cdot v^3 \cdot C_p
\]

Where \( R \) is the radius of the area covered by the blades, \( v \) is the wind speed and \( \rho \) is the air density.

The power coefficient, \( C_p \) is a non-linear function of the blade pitch angle \( \beta \) and the \( \lambda \) parameter, which is the ratio between the peripheral speed of the blades and wind speed. In Figure 4, one can observe the power coefficient variation with the relative rotational speed \( \lambda \).

\[
\lambda = \frac{\omega_T \cdot R}{v},
\]

where \( \omega_T \) is the rotational speed of the rotor.

The power coefficient \( C_p(\lambda, \beta) \) is calculated in this paper as:

\[
C_p(\lambda, \beta) = c_1 \cdot \left(\frac{C_2}{\lambda^2} - c_3 \cdot \beta - c_4 \cdot \lambda^2\right) \cdot e^{-\frac{c_5}{\lambda}} + c_6 \cdot \lambda
\]
\[
\frac{1}{\lambda} = \frac{1}{\lambda + 0.08 \cdot \beta + 0.035}, \quad \text{and the coefficients } c_1 \text{ to } c_6
\]
are: \( c_1 = 0.5176 \), \( c_2 = 116 \), \( c_3 = 0.4 \), \( c_4 = 5 \), \( c_5 = 21 \) and \( c_6 = 0.0068 \). The variation of this coefficient with tip ratio speed and pitch angle is given in Figure 4.

![Figure 4. The power coefficient of a variable speed turbine](image)

The power coefficient is very important because it gives information upon the aerodynamic efficiency of the turbine. Given the fact that a wind turbine has three functioning regimes, each having distinct characteristics, lead to the idea that the control objectives should be different for each area. The first, region 1, includes the time when the turbine is starting up.

In the second partial load area, the main objective is to maintain the turbine at the maximum yield. While functioning in this region, it is desirable that the turbine captures as much power as possible from the wind. This region accounts for more than 50% of yearly energy capture for a typical modern turbine, so this is why, control in this region is extremely important.

One frequently used control method for this region states that the control system must act on the electromagnetic torque in order to adapt the rotational speed of the rotor, \( \omega_T \), to the wind speed and to have an optimized power coefficient \( C_p(\lambda, \beta) \). Therefore, the specific speed given by \( \lambda_{opt} = \frac{(\omega_T R)}{v} \) is also optimized. [4]

This control solution is based on an estimation of wind speed starting on an estimation of the aerodynamic torque of the rotor. This solution is proposed in [7] and allows the calculation of the torque estimator by taking into consideration the inertial forces of the shaft transmission system (Figure 5).

Thus, the referential rotational speed of the rotor is:

\[
\omega_{T,ref} = \sqrt{\frac{C_{aero}}{k}}, \quad \text{where } \dot{C}_{aero} \text{ is the aerodynamic torque.}
\]

This is called standard control method for region 2 and it has been proven that it presents a few shortcomings, caused by the turbulence in the wind and by the difficulty of determining the gain \( k \).

![Figure 5. Block Diagram representing a control method in Partial Load II area](image)

Usually, the aerodynamic torque that drives the wind turbine rotor and thus the generator is given by:

\[
C_{aero} = \frac{1}{2} \cdot \frac{\rho \cdot R^2 \cdot \pi^3 \cdot C_p(\lambda, \beta)}{\omega_T}
\]

Finally, in the Full Load area, the control system has to maintain the output power value to the nominal value of the generator. Through this, the rotational speed of the turbine is equal to its nominal value, while the pitch angle and electromagnetic torque are varied in order to obtain:

\[
C_p(\lambda, \beta) = \frac{P_{nom}}{\frac{P}{2} \cdot \pi \cdot R^2 \cdot v^3}
\]

This area corresponds to high wind speed values and important mechanical solicitation of the system. In the next section we present in detail a control method developed for this region, a method that is based on a digital controller.

### 3. CONTROL METHODS

#### 3.1 RST polynomial control for the full-load area of a wind turbine

Given the complexity of a wind turbine system, many control methods have been proposed and implemented, each having both advantages and disadvantages.

The control law that we will refer to has the advantage that it can be implemented on a digital computer (microprocessor, microcontroller). We chose to use a three branched RST controller. The controller will be designed using the poles placement method.

We will insist after the computation of the controller on the importance of shaping the sensitivity functions. The poles placement with shaping of sensitivity functions is a general methodology of digital control design that allows one to take into account simultaneously robustness and performances specifications for the closed loop of the system.

This is also a model based control method, for one need to know the discrete time model of the plant.

Therefore, we will continue with the analytic continuous mathematical model of the turbine and with the determination of the discrete model.

The dynamic characteristics of a wind power plant are determined by components such as: the drive train, the generator, the blades and the tower bending.

The equation that describes the rotor motion is given by:

\[
J_1 \cdot \dot{\omega} = T - T_s
\]

(9)
Where, \( J_t \) is the rotor inertia, \( \omega \) represents the angular speed of the rotor, \( T \) is the aero-dynamical torque and \( T_s \) is the reaction torque that appears in the drive shaft system. The power is regulated by adjusting the shaft rotational velocity. The faster the shaft turns the more power the generator can give as output. The equation that models the generator’s motion is:

\[
J_g \cdot \ddot{\theta} = T_m - T_g
\]  
(10)

Where \( J_g \) is the generator inertia, \( \ddot{\theta} \) is the angular acceleration of the generator rotor, \( T_m \) is the torque driving the generator’s rotor and \( T_g \) is the electrical torque produced in the generator (it includes losses). \[5\] \[6\]

The drive train is modelled by a spring coefficient \( K_s \) and a damping coefficient \( D_s \) that provide a spring damping model as:

\[
\gamma \cdot \dot{\gamma} + \dot{\gamma} = T_s
\]  
(11)

where \( \gamma \) is the torsion of the drive train. Also we assumed that all blades have the same pitch angle, and this is known as “collective pitch”. The blade servo is modelled as a first order system with \( T_{bs} \) as a time constant:

\[
T_{bs} \cdot \dot{\beta} + \beta = \beta_r
\]  
(12)

The control method proposed must ensure the desired behavior of the closed loop system, so in such a manner that maximum power output is obtained and a reducing in structural loads and fatigue is achieved. As turbine towers grow in height, tower oscillations cannot be ignored. In this situation, a model of the wind turbine with distributed parameters is required and a description in terms of mass and stiffness distribution. The tower is then affected by an aero-dynamic torque \( T \) and a thrust represented by the generalized force \( F \).\[6\]

The first mode of the tower bending is described by:

\[
M_T \cdot \ddot{z} = F - D_T \cdot \dot{z} - K_T \cdot z
\]  
(13)

where \( z \) is the displacement of the nacelle in the direction perpendicular to the rotor disc. The turbine’s mass is given by \( M_T \), the damping factor by \( D_T \) and a spring constant \( K_T \). After linearization and entering in the Laplace complex domain, the model results in the form:

\[
A \cdot \Delta \psi = B \cdot \Delta \beta_{ref} + C \cdot \Delta \nu
\]  
(14)

where \( A\), \( B\) and \( C \) are polynomials in complex variable \( s \) (Laplace domain). The polynomials \( A\), \( B\) and \( C \) have all the degree equal to 5. Here, it can be seen that the angular position \( \Delta \psi \) is the output of the system, the reference \( \Delta \beta_{ref} \) is the control signal and the wind speed \( \Delta \nu \) is the disturbance that has to be compensated by the controller. The goal was to maintain a constant electrical voltage, produced by the turbine, and this can be expressed in terms of constant angular velocity of the turbine rotor. Therefore, we used the angular position of the rotor as a control signal.

From the equation above, closed loop transfer functions with respect to wind speed change and reference signal respectively, can be obtained:

\[
H_p(s) = \frac{\Delta \psi(s)}{\Delta \beta_{ref}(s)}
\]  
(15)

\[
H_v(s) = \frac{\Delta \psi(s)}{\Delta \nu(s)}
\]  
(16)

As it can be observed from the equations 15 and 16, the model of the open loop system has the degree equal to 5. As it was mentioned above, the controller proposed for analyze in this paper is the RST controller. This type of controller is a structure with two freedom degrees and it allows the designer to specify performances both with reference trajectory tracking (reference variation) and with regulation. Once we obtained the transfer function of the system, one can determine the step response of the open loop system. The open loop response is depicted in Figure 6. As it can be observed, the step response of the system presents a significant overshoot and oscillatory aspect. Therefore, the control law will have to eliminate all these inconvenient.
imposed the following tracking pair of poles:

$$\omega_0 = 2.51 \text{ rad/s} \quad \text{and} \quad \zeta = 0.053,$$

$$\omega_0 = 1.2119 \text{ rad/s} \quad \text{and} \quad \zeta = 0.308.$$  
In the first stage, we have computed a controller with imposed tracking performances. As the overshoot of the system increases with the decrease of damping, the time rise also decreases with the decrease of the damping \(\zeta\). We imposed the following tracking pair of poles:

$$\omega_0 = 2.1214 \text{ rad/s} \quad \text{and} \quad \zeta = 0.053,$$

$$\omega_0 = 1.2119 \text{ rad/s} \quad \text{with} \quad \zeta = 0.8.$$  
For the disturbance rejection problem we imposed \(\omega_0 = 1.2119 \text{ rad/s} \quad \text{with} \quad \zeta = 0.8,\)

$$\omega_0 = 2.1214 \text{ rad/s} \quad \text{with} \quad \zeta = 0.053, \quad \omega = 2.51 \text{ rad/s}.$$  
We wanted a robust controller for our system and so we improved this classical RST controller by imposing a pole with multiplicity 5 (desired polynomial \(P_5(q^{-1}) = (1-0.5q^{-1})^5\)). As a general rule, the auxiliary poles are chosen in order to be faster than the dominant poles of the system. The introduction of these poles in the closed loop system reduces the stress on the actuators in the transient for the disturbance rejection. [8]

It is well known that if the feed forward channel contains the reference model, then the steady-state error is eliminated and as a plus, one obtains a significant attenuation of the disturbance effect on the output of the system. Therefore we factorized the R and S polynomials as:

$$S(q^{-1}) = S'(q^{-1})H_S(q^{-1})$$

$$R(q^{-1}) = R'(q^{-1})H_R(q^{-1})$$

where \(H_S(q^{-1})\) and \(H_R(q^{-1})\) are the fixed parts. \(H_S(q^{-1}) = 1 - q^{-1}\) (An integrator) \(H_R(q^{-1}) = 1 + q^{-1}\) (Open loop behavior to avoid disturbance amplification).

### 3.2 Robustness evaluation

In order to assure robustness of the closed loop system, one must take into account the input sensitivity of the system, \(S_{ip}\). The analysis of this function allows evaluating the influence of a disturbance on the plant input. The zeroes of the inverse of this function define the poles of the closed loop system. Therefore, in order to have stability in closed loop, the zeros of this function must be inside the unit circle. [8]

The input sensitivity function is defined as:

$$S_{ip}(z^{-1}) = \frac{A(z^{-1})S(z^{-1})}{1 + H_{oe}(z^{-1})}$$

The robustness of the closed loop system can be evaluated by looking at the opened loop Nyquist plot. The minimal distance between this graphic and the critical point [-1,j0] is the modulus margin, \(\Delta M\), and it is a measure of the nominal systems closed loop robustness. [9]

Fig.10. Modulus margin on Nyquist plots of the opened loop transfer function. [10]

From the definition we can find the following formula for the modulus margin:

$$\Delta M = \max \left| 1 + H_{oe}(z^{-1}) \right|_{\text{min}} = \left| S_{ip}(z^{-1}) \right|_{\text{min}}$$

$$= \left( \left| S_{ip}(z^{-1}) \right|_{\text{min}} \right)$$

This gives the upper margin of the output sensitivity function and proves that sensitivity functions can be used in designing robust controllers. [9]

For a robust system it is necessary to have a modulus margin greater or equal to 0.5 (-6 dB) implying a maximum of 6dB for the output sensitivity function.
As previously said, we first computed a controller by imposing tracking and regulation performances in order to have a good response of the system. This controller proved not to be robust, and this can be seen by analyzing the input sensitivity function. One can see that in this case, the closed loop system is not robust as the sensitivity function has a maximum value greater than 6dB \( (S_{sp}(e^{i\omega})_{\text{max}} = 6.19dB) \) which leads to a modulus margin of \( \Delta M = 0.693 \). In Figure 11 one can observe the \( S_{sp} \) magnitude variation, and also the fact that the maximum value exceeds 6dB.

4. CONCLUSIONS

This paper has presented a modeling technique and a control method proposed for variable speed wind turbines. Given the fact that the obtained model for such a turbine is nonlinearly dependant on wind speed, three different operation regimes are to be considered. Each regime has its own particularities and this leads to specific demands for wind turbine control. The difficulties in wind turbine control involve both the necessity of maintaining the output of the generator at a value which must correspond to maximization of captured energy and reducing mechanical oscillations of the structure that supports the turbine.

All this makes the controller design a very difficult task. In this paper we proposed a RST control approach. This method has shown a good regulation of rotor speed and a good response of the pitch angle.

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