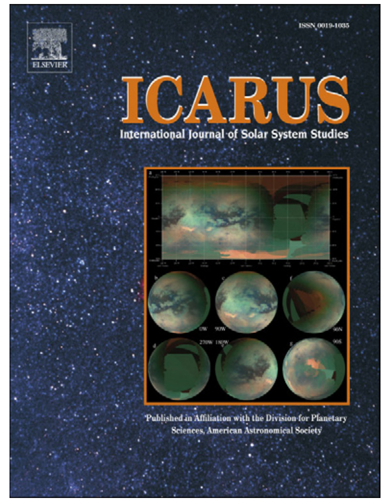
The effect of gravitational and pressure torques on Titan's length-of-day variations

T. van Hoolst, N. Rambaux, Ö. Karatekin

Accepted Manuscript



The effect of gravitational and pressure torques on Titan's length-of-day variations

T. Van Hoolst, N. Rambaux, Ö. Karatekin

PII: S0019-1035(08)00397-7
DOI: 10.1016/j.icarus.2008.11.009
Reference: YICAR 8817

To appear in: *Icarus*

Received date: 23 July 2008
Revised date: 15 October 2008
Accepted date: 3 November 2008

Please cite this article as: T. Van Hoolst, N. Rambaux, Ö. Karatekin, The effect of gravitational and pressure torques on Titan's length-of-day variations, *Icarus* (2008), doi: 10.1016/j.icarus.2008.11.009

The effect of gravitational and pressure torques on Titan's length-of-day variations

T. Van Hoolst

Royal Observatory of Belgium, Ringlaan 3, B-1180 Brussels, Belgium

N. Rambaux

Universit Pierre et Marie Curie Paris 6 - IMCCE Observatoire de Paris, France

Jet Propulsion Laboratory, California Institute of Technology

Ö. Karatekin

Royal Observatory of Belgium, Ringlaan 3, B-1180 Brussels, Belgium

Number of manuscript pages: 36

Number of manuscript tables: 2

Number of manuscript figures: 2

Preprint submitted to Elsevier Science *28 November 2008*

Proposed running head: Length-of-day variations of Titan

Editorial Correspondence to:

Tim Van Hoolst

Royal Observatory of Belgium

Ringlaan 3

B-1180 Brussels

Belgium

Email: tim.vanhoolst@oma.be

Phone: +322 373 0668

Fax: +322 374 9822

Abstract

Cassini radar observations show that Titan's spin is slightly faster than synchronous spin. Angular momentum exchange between Titan's surface and the atmosphere over seasonal time scales corresponding to Saturn's orbital period of 29.5 year is the most likely cause of the observed non-synchronous rotation. We study the effect of Saturn's gravitational torque and torques between internal layers on the length-of-day (LOD) variations driven by the atmosphere. Because static tides deform Titan into an ellipsoid with the long axis approximately in the direction to Saturn, non-zero gravitational and pressure torques exist that can change the rotation rate of Titan. For the torque calculation, we estimate the flattening of Titan and its interior layers under the assumption of hydrostatic equilibrium. The gravitational forcing by Saturn, due to misalignment of the long axis of Titan with the line joining the mass centers of Titan and Saturn, reduces the LOD variations with respect to those for a spherical Titan by an order of magnitude. Internal gravitational and pressure coupling between the ice shell and the interior beneath a putative ocean tends to reduce any differential rotation between shell and interior and reduces further the LOD variations by a few times. For the current estimate of the atmospheric torque, we obtain LOD variations of a hydrostatic Titan that are more than 50 times smaller than the observations indicate when a subsurface ocean exists and more than 100 times smaller when Titan has no ocean. Moreover, Saturn's torque causes the rotation to be slower than synchronous in contrast to the Cassini observations. The calculated LOD variations could be increased if the atmospheric torque is larger than predicted and or if fast viscous relaxation of the ice shell could reduce the gravitational coupling, but it remains to be studied if a two order of magnitude increase is possible and if these effects can explain the phase difference of the predicted rotation variations. Alternatively, the large differences with the observations may suggest that non-hydrostatic effects in Titan are important. In particular, we show that the amplitude and phase of the calculated rotation variations are similar to the observed values if non-hydrostatic effects could strongly reduce the equatorial flattening of the ice shell above an internal ocean.

Key words: Titan, interiors, rotational dynamics, ice

1 Introduction

Cassini radar observations over the last few years have shown that the rotation rate of Saturn's moon Titan differs from synchronous rotation and leads to a shift of 0.36° per year in apparent longitude (Lorenz et al. 2008). This shift is thought to be due to long-periodic exchanges of angular momentum between Titan's surface and atmosphere. By using their general circulation model of the atmosphere, Tokano and Neubauer (2005) predicted periodic length-of-day (LOD) variations of a solid Titan with main period of 14.74 yrs and an amplitude about a factor 5 or more smaller than the observed shift. However, if Titan has an ocean and the rotation of the shell could be considered decoupled from the interior, the atmosphere would only force rotation variations of the shell and the amplitude of the surface rotation variations would be close to the observed value (Tokano and Neubauer 2005). Therefore, Lorenz et al. (2008) take the Cassini radar data as evidence for the existence of an internal ocean. Noyelles (2008) suggests that the apparent faster rotation could also be possible for an entirely solid Titan if a long period wobble were to be resonantly excited, which would require fine-tuning of Titan's moment of inertia.

In the above theoretical predictions of the LOD variations, two gravitational effects on the spin of Titan are neglected. First, Saturn exerts a gravitational torque on Titan, which tends to reduce periodic deviations from synchronous rotation. Due to the proximity of Saturn, this torque can be larger than the atmospheric torque causing the LOD variations. Second, the shell cannot be considered to perform LOD variations decoupled from the interior beneath an ocean because of several interactions between the different internal layers of Titan. Although electromagnetic and viscous interactions at the boundaries of the internal ocean can most likely be neglected (Tokano and Neubauer 2005, Lorenz et al. 2008), the gravitational interaction between the shell and the interior is important (Karatekin et al. 2008).

Both gravitational torques exist by virtue of the flattened form of Titan and can only influence the rotation rate of Titan when the equatorial principal moments of inertia of Titan are different (non-zero equatorial flattening). Due to static tides generated by Saturn, which stretch Titan in the direction to Saturn and cause a contraction in the direction perpendicular to Saturn, the almost synchronously rotating Titan has a significant equatorial flattening that is of the same order of magnitude as the polar flattening. Karatekin et al. (2008) showed that the internal gravitational torque reduces the amplitude of the main periodic rotation variation of Titan due to the atmosphere by about an order of magnitude.

Besides gravitational effects, ocean pressure also influences the LOD variations of Titan. Variations in the gravitational field due to relative rotation of different internal layers and to orientation changes with respect to the direction to Saturn induce an additional pressure field in the putative subsurface ocean. This incremental pressure acts on the aspherical boundaries of the ocean and so exerts a torque on both the ice shell and the interior. Pressure counteracts the gravitational torques due to Saturn and internal misalignment and therefore reduces the total torques.

The plan of the paper is as follows. In Sect. 2, we study the effect of the gravitational torque of Saturn on Titan's LOD variations if Titan rotates rigidly, which is a good approximation for an entirely solid Titan without internal ocean. The effects of an ocean on rotation are introduced in Sect. 3. We explain how internal gravitational torques and pressure torques can be determined and derive differential equations that govern the LOD variations of Titan. LOD variations are calculated for different models of the internal structure of Titan, and we determine the relative importance of the different torques. The results are compared to the observations in the discussion Sect. 4 and conclusions are presented in the final section.

2 Saturn's gravitational torque

For the calculation of Saturn's gravitational torque on Titan, we neglect Titan's small obliquity, recently determined from Cassini radar images to be about 0.3° (Stiles et al. 2008), and assume that Titan's rotation axis is perpendicular to the orbital plane around Saturn. We also assume principal axis rotation with Titan rotating about its shortest (polar) principal axis, which corresponds to the largest principal moment of inertia C. The polar component of Saturn's gravitational torque on Titan $\Gamma_{\rm grav}$, which can change Titan's rotation rate, can then be expressed as (e.g. Murray and Dermott 1999)

$$\Gamma_{\rm grav} = \frac{3}{2}(B-A)\frac{GM_{\rm S}}{d^3}\sin 2\psi = K_{\rm grav}\left(\frac{a}{d}\right)^3 \sin 2\psi \qquad (1)$$

where $A < B$ are the two principal moments of inertia of Titan in the equatorial plane, G is the universal gravitational constant, $M_{\rm S}$ the mass of Saturn, d the distance between the mass centers of Titan and Saturn, and ψ the angle between the long axis of Titan (associated with the smallest moment of inertia A) and the direction from Titan to Saturn ($\psi = f - \phi$, where f is Titan's true anomaly and ϕ the orientation angle between the long axis of Titan and the line of apsides). We denote the strength of the gravitational coupling by $K_{\text{grav}} = 3(B-A)GM_{\text{S}}/(2a^3) \approx 3n^2(B-A)/2$, where a is the semi-major axis of Titan's orbit, and n Titan's mean motion.

The main geophysical parameter in the gravitational torque is the equatorial moment of inertia difference $B-A$ of Titan. We calculate it by assuming that Titan is in hydrostatic equilibrium so that Clairaut theory can be used. For a given spherical reference model of the interior structure of Titan with density profile $\rho(r_0)$, where r_0 is the radial coordinate from the center of the model, the equatorial flattening β of the model due to the static tides generated by Saturn can be determined from Clairaut's second-order differential equation

$$\frac{d^2\beta}{dr_0^2} + \frac{6}{r_0}\frac{\rho}{\overline{\rho}}\frac{d\beta}{dr_0} - \frac{6}{r_0^2}\left(1-\frac{\rho}{\overline{\rho}}\right)\beta = 0, \tag{2}$$

and the associated boundary condition

$$\frac{d\beta}{dr_0}(R) = \frac{1}{R}\left[\frac{15}{2}q - 2\beta(R)\right] \tag{3}$$

(Van Hoolst et al. 2008, for Clairaut theory, see, e.g. Jeffreys 1952, Moritz 1990). Here, $R = 2575$ km is the mean radius of Titan, $q = (\omega^2R^3)/(GM)$ the ratio of the centrifugal acceleration to the gravitational acceleration, $M = 1.346 \times 10^{23}$ kg the total mass of Titan, and $\overline{\rho}$ is the mean density in a sphere of radius r_0. The flattening $\beta(r_0) = [a(r_0) - b(r_0)]/a(r_0)$, where $a(r_0) > b(r_0)$ are the lengths of the two principal axes in the equator plane of the ellipsoidal surface with mean radius r_0. The equatorial moment of inertia difference $B-A$

can then be calculated by integration over the radial coordinate:

$$B - A = \frac{8\pi}{15} \int_0^R \rho(r_0') \frac{d(\beta r_0'^5)}{dr_0'} dr_0'. \tag{4}$$

We consider two models of the interior structure of Titan, representative of two different classes of models, for the numerical evaluation of our results (see Table 1). The first model (Sohl et al. 2003) has an outer ice shell of density 917 kg m^{-3} and thickness 68.7 km. Beneath the ice shell is a 223.76 km thick ammonia-water ocean with a density of 950 kg m^{-3}. The deep interior consists of a high-pressure (HP) ice layer with a density of 1310 kg m^{-3} and a rock-iron core with a density of 3813.31 kg m^{-3}. The interface between the core and the HP ice layer is at a radial coordinate of 1670.807 km. The model has a mean moment of inertia factor $I/(MR^2) = 0.304$, typical for this kind of strongly differentiated interior structure models (Sohl et al. 2003). The second model is chosen to have a low-density interior (Fortes et al. 2007) and has a mean moment of inertia $I/(MR^2) = 0.360$. We follow Fortes et al. (2007) and choose a core consisting of hydrated rock, a high-pressure ice mantle, an ocean of aqueous ammonium sulfate, and a shell of low-pressure ice, methane clathrate and ammonium sulfate. The densities of the layers are $\rho_s = 1065$ kg m^{-3}, $\rho_o = 1350$ kg m^{-3}, $\rho_m = 1400$ kg m^{-3}, and $\rho_c = 2325.1$ kg m^{-3}. The interface radii are $r_o = 2495$ km, $r_m = 2350$ km, $r_c = 2124.03$ km. Subscripts s, o, m, and c denote the ice shell, ocean, ice mantle, and core, respectively.

For model 1, $B - A = 2.68\,10^{31}$ kg m^2 and $K_{\text{grav}} = 8.35\,10^{20}$ Nm, whereas for model 2, we have $B - A = 4.07\,10^{31}$ kg m^2 and $K_{\text{grav}} = 1.27\,10^{21}$ Nm. Models 1 and 2 can be considered close to end-member models of the interior structure of Titan in terms of the degree of differentiation. Therefore, we consider the above values representative for a hydrostatic Titan and conclude that $B - A$

is on the order of 10^{31} kg m^2 and that the strength of the gravitational torque by Saturn K_{grav} is typically 10^{21} Nm.

Saturn's gravitational torque changes in time with the angle ψ between the long axis of Titan and the direction to Saturn and with the distance d between Saturn and Titan. The magnitude of the torque can be estimated from the maximum value of the angle ψ, which changes for two reasons. First, during the orbital motion (with a period of 15.945 days) the long axis of a synchronously rotating Titan is approximately oriented towards the empty focus of the elliptical orbit instead of to Saturn and ψ changes between 0 and $2e$ (Murray and Dermott 1999), where $e = 0.0292$ is the eccentricity of the orbit. On Titan's equator, this deviation of the direction of the long axis from the direction to Saturn corresponds to about 150km. As seen from Saturn, Titan therefore not always shows exactly the same face and periodically exposes regions with slightly larger and smaller longitude than the half of its surface it would show if the orbit were circular. These variations are equivalent to the geometrical librations of the Moon. The maximum torque at maximum misalignment $2e$ and distance a for the models considered is $\Gamma_{\mathrm{grav}} = 1.48\,10^{20}$ Nm.

The atmosphere of Titan also changes the angle ψ by changing the rotation rate (or equivalently LOD) and therefore changing the orientation of the long axis ϕ as a function of time. Cassini radar observations show that the orientation change with respect to synchronous rotation is 0.36° per year over the last years (Stiles et al. 2008). By assuming this to be due to the atmosphere with main period at 14.74 yr (half the orbital period of Saturn), we can estimate the amplitude of the periodic variation in ψ. The smallest atmosphere-induced amplitude, corresponding to a current rotation rate close to its maximum value, is about 0.015 rad and leads to variations of the orientation of the long axis at the equator with an amplitude of about 40 km. The strength of the torque associated with this maximum misalignment and at distance a is $2.51\,10^{19}$ Nm for model 1 and $3.84\,10^{19}$ Nm for model 2.

Instead of using the orientation angle ϕ for the study of small changes in the spin of Titan, we introduce the small rotation angle $\gamma = \phi - M_a$, where M_a is the mean anomaly and write $\psi = f - \gamma - M_a$. By expanding Expression (1) for the torque as a series in mean anomaly M_a and eccentricity e by using well-known expansions for $(a/d)^3$, $\cos f$, and $\sin f$ (Cayley 1861), we have

$$\Gamma_{\text{grav}} = -\frac{3}{2}(B-A)n^2 \sum_{k=-\infty}^{+\infty} X_{-k}^{-3,2}(e) \sin\left[2\gamma + (2+k)M_a\right], \tag{5}$$

where the eccentricity functions $X_{-k}^{-3,2}(e)$ are Hansen coefficients (Hansen 1855, Hughes 1981), and $n = 4.5607\,10^{-6}\text{s}^{-1}$ is the mean motion of Titan's orbit. We linearize this expression in eccentricity and angle γ. Since the Hansen coefficients $X_k^{n,m}$ are of order $e^{|k-m|}$ in eccentricity (e.g. Plummer 1918), only Hansen coefficients $X_{-k}^{-3,2}(e)$ in Eq. (2) with index k equal to -1, -2, and -3 have to be retained. These are $X_1^{-3,2}(e) = -e/2$, $X_2^{-3,2}(e) = 1$, $X_3^{-3,2}(e) = 7e/2$, up to first order in eccentricity, and we then have

$$\Gamma_{\text{grav}} = 6en^2(B-A)\sin M_a - 3n^2(B-A)\gamma. \tag{6}$$

The first term in the right-hand side corresponds to the periodic forcing at the main libration period of 15.945 days. The second term in the right-hand side will have an impact on the LOD variations at 14.74 yrs.

The changes in the rotation angle γ due to atmospheric forcing and the gravitational torque of Saturn can be obtained from the angular momentum equation

$$C\ddot{\gamma} + 3n^2(B-A)\gamma = 6en^2(B-A)\sin M_a + \Gamma(t), \tag{7}$$

where C is the polar moment of inertia of Titan. The time-variable atmospheric torque is principally semi-annual $\Gamma(t) = \Gamma_{\rm A} \sin(\omega_{\rm A} t + f_{\rm A})$ with an amplitude $\Gamma_{\rm A} = 1.6 \times 10^{17}$ N m (Tokano and Neubauer 2005, Karatekin et al. 2008). Time is measured from September 1980, the frequency $\omega_{\rm A} = 2\pi/14.74$ years $= 1.35 \times 10^{-8}$ s^{-1} and the phase $f_{\rm A} = 5.11$ rad. The atmospheric torque is about 200 times smaller than Saturn's gravitational torque at maximum misalignment due to the LOD variations.

Eq. (7) is the equation for a forced oscillator. At the main atmospheric forcing period, the solution for the variations in the rotation angle can be expressed as $\gamma = g \sin \omega_{\rm A} t$, with

$$g = \frac{\Gamma_{\rm A}}{3n^2(B - A) - \omega_{\rm A}^2 C}. \tag{8}$$

For model 1, the amplitude of the rotation variation is $9.87\,10^{-5}$ rad (see Table 2). This corresponds to a maximum orientation shift of about 0.0024° per year, which is more than 100 times smaller than the observed rotation variation. For model 2, the LOD variations are even smaller ($g = 6.46\,10^{-4}$ rad) because of the larger flattening of the model and thus the larger restoring gravitational torque.

For a spherical Titan, the gravitational torque of Saturn on Titan vanishes and the amplitude of the rotation variation would be

$$g = -\frac{\Gamma_{\rm A}}{\omega_{\rm A}^2 C}. \tag{9}$$

We have $g = -3.23\,10^{-3}$ rad for model 1, and $g = -2.73\,10^{-3}$ rad for model 2. From comparison with the values of the general solution above, it follows that the Saturn torque reduces the LOD variations by a factor of about 30-40. An important further difference is that the rotation angle variations are 180° out of phase with the atmospheric torque for a spherical Titan (see the minus sign in Eq. 9), whereas they are in phase when gravitational coupling with Saturn is taken into account. It follows that Saturn forces Titan's rotation angle variations to be in phase with the atmospheric torque. As for any forced harmonic oscillator, they are in phase because the normal (or free) frequency is larger than the forcing frequency ($\omega_{\mathrm{A}} = 1.35 \times 10^{-8}$ s^{-1}). The free frequency ω_f is given by

$$\omega_f = n\sqrt{\frac{3(B-A)}{C}} \tag{10}$$

(see, e.g., Murray and Dermott 1999) and is equal to 7.85×10^{-8} s^{-1} for model 1 and 8.89×10^{-8} s^{-1} for model 2.

The atmospheric torque model (Tokano and Neubauer 2005) shows that the atmospheric torque is negative over the last few years with minimum in 2009 (Karatekin et al. 2008). Therefore, landmarks are predicted to be shifted westwards with maximum in 2009, and the rotation is predicted to be slower than synchronous until 2009, contrary to the Cassini observations (Stiles et al. 2008). Both the phase difference and the too small amplitude suggest that a solid Titan without ocean cannot be reconciled with the observed rotation variations. In the next section, we study whether an ocean can change this situation.

3 Internal coupling

When a subsurface ocean exists in Titan, the shell can perform rotation variations different from those of the interior. However, the shell cannot be considered to be rotationally decoupled from the interior since it is coupled with the underlying ocean and interior through viscous, electromagnetic, pressure and gravitational torques. The viscous torques due to friction forces on the boundaries of the internal ocean can most likely be neglected because of the small ocean viscosity [Lorenz et al. 2008, Tokano and Neubauer 2005]. We also neglect electromagnetic coupling between the ocean and the shell and between the ocean and the interior because of the absence of a self-generated magnetic field in Titan. The coupling between the three internal regions considered is then due to the gravitational force and the pressure force of the liquid on the ocean boundaries. The effect of internal gravitational coupling on Titan's rotation has been studied by Karatekin et. (2008) and will be readdressed here. We first explain how these couplings can be taken into account and then study the LOD variations of Titan's shell.

3.1 Internal gravitational and pressure coupling

When Titan has a subsurface ocean, the differential rotations of the ocean and interior (consisting of a solid mantle and core) with respect to the shell have to be taken into account in the angular momentum equation for Titan. Since we only consider variations in the rotation rate of Titan, and not in its orientation, we only need to study components of physical quantities along the polar rotation axis, which we take as the z-axis. The z-component of the angular momentum equation for Titan can be written as

$$C_s\ddot{\phi}_s + C_o\ddot{\phi}_o + C_i\ddot{\phi}_i = \Gamma_t. \tag{11}$$

Here, the rotation angles ϕ_j, with respect to a direction fixed in inertial space, of the shell, ocean and interior are used, and C_j is the polar principal moment of inertia of the shell ($j = s$), ocean ($j = o$) and interior ($j = i$). The z-component of the total torque on Titan Γ_t is the sum of the atmospheric torque Γ and the external Saturn torque Γ_{grav}. The latter can be expressed as as the sum of the three torques of Saturn on the three internal layers of Titan (see Eq. 1):

$$\Gamma_{\mathrm{grav}} = \frac{3}{2}\frac{GM_{\mathrm{S}}}{d^3}\left[(B_s - A_s)\sin 2\psi_s + (B_o - A_o)\sin 2\psi_o + (B_i - A_i)\sin 2\psi_i\right], \tag{12}$$

with ψ_j the angle between the long axis of Titan's layer j (associated with the smallest moment of inertia A_j of layer j) and the direction from Titan to Saturn ($\psi_j = f - \phi_j$).

To be able to solve Eq. (11), we introduce angular momentum equations for the ocean and the interior. We have

$$C_o\ddot{\phi}_o = \Gamma_o, \tag{13}$$
$$C_i\ddot{\phi}_i = \Gamma_i. \tag{14}$$

The z-components of the total torques on the ocean (Γ_o) and interior (Γ_i) are due to Saturn and to internal gravitational forces and pressure forces on the boundary of the region considered. Internal gravitational coupling arises when the principal axes of the shell are not aligned with those of the interior. Such a misalignment occurs when the atmosphere or Saturn's gravitational torque forces the shell and interior to change their rotation. As a consequence, the internal gravitational field in Titan changes and different internal regions exert a torque on each other. Variations in the gravity field also cause pressure changes in the ocean that give rise to pressure torques.

The z-component of the total torque on the ocean Γ_o due to internal gravitational coupling, external gravitational coupling by Saturn, and pressure P, can be expressed as

$$\Gamma_o = -\int_{V_o}(\vec{r}\times\rho_o\nabla\Phi)_z\,dV - \int_{V_o}\left(\vec{r}\times\rho_o\nabla W^{\mathrm{ext}}\right)_z dV + \int_{S_o}(\vec{r}\times\hat{n})_z\,PdS, \tag{15}$$

where Φ is Titan's gravitational potential, W^{ext} the external gravitational potential due to Saturn, $\hat{n}$ is the outward unit normal on the ocean surface, V_o the volume of the ocean and S_o its boundary, which consists of the boundary with the interior below and the boundary with the ice shell above. The pressure torque can be calculated by using the Navier-Stokes equation for the ocean, which we assume to be inviscid. In a reference frame rotating with the shell rotation rate $\vec{\omega}_s$, we have

$$\nabla P = -\rho_o \nabla \Phi - \rho_o \nabla W^{\text{ext}} - \rho_o \nabla \Psi - \rho_o \frac{d\vec{v}_o}{dt} - 2\rho_o \vec{\omega}_s \times \vec{v}_o - \rho_o \dot{\vec{\omega}}_s \times \vec{r}, \quad (16)$$

where Ψ is the centrifugal potential and $\vec{v}_o$ the ocean velocity with respect to the reference frame considered. By assuming the perturbed flow in the ocean to be essentially a small rigid rotation (or a Poincaré flow, Poincaré 1910, Mathews et al. 1991, Dehant and Mathews 2007), and rewriting the torque expression as a surface integral over the boundary of the ocean by using a consequence of Gauss' theorem

$$\int_V \vec{r} \times \nabla f dV = \int_S f \vec{r} \times \hat{n} dS, \quad (17)$$

the total torque on the ocean reduces to zero in an approximation correct up to the first order in the small differential rotations of the shell, ocean and interior with respect to equilibrium synchronous rotation (see Hinderer et al. 1982 and Mathews et al. 2001 for details of this calculation in the case of the torque on the Earth's fluid outer core, see also Van Hoolst 2007). Here, it is assumed that the boundaries between the interior, the ocean, and the shell are ellipsoidal and we neglect any topography. The angular momentum equation for the ocean then simplifies to

$$C_o \ddot{\phi}_o = 0, \quad (18)$$

which means that the ocean does not take part in the rotation variations of Titan.

The z-component of the torque on the interior can be expressed as

$$\Gamma_i = -\int_{V_i} \left(\vec{r} \times \rho_i \nabla \Phi\right)_z dV - \int_{V_i} \left(\vec{r} \times \rho_i \nabla W^{\text{ext}}\right)_z dV - \int_{S_i} \left(\vec{r} \times \hat{n}\right)_z P dS, \quad (19)$$

where V_i is the volume of the interior (mantle+core) and S_i the boundary between the interior and the ocean. By using the Navier-Stokes equation for the ocean in the pressure term and transforming the surface integral to a volume integral by applying Gauss' theorem (17), we have

$$\Gamma_i = -\int_{V_i} (\rho_i - \rho_o) \left[\vec{r} \times \nabla \left(\Phi + W^{\text{ext}}\right)\right]_z dV. \quad (20)$$

Eq. (20) shows that the ocean pressure torque counteracts the gravitational torques and that the total torque on the interior is smaller than the sum of the gravitational torques. For a constant density interior, the volume integral can easily be transformed to a surface integral by re-applying Identity (17). We then have

$$\Gamma_i = -\rho_i \left(1 - \frac{\rho_o}{\rho_i}\right) \int_{S_i} \left(\vec{r} \times \hat{n}\right)_z \left(\Phi + W^{\text{ext}}\right) dS. \quad (21)$$

In this case, the ocean pressure reduces the total torque on the interior with respect to the gravitational torque by a factor $1 - \rho_o/\rho_i$. The last four, inertial terms in the Navier-Stokes Eq. (16) do not contribute to the torque because, as can most easily be seen from the surface integral in Eq. (21), they are orthogonal to a spherical harmonic of degree and order 2 and the z-component of the vectorial product $\vec{r} \times \hat{n}$ is proportional to a spherical harmonic of degree and order 2 for an ellipsoidal boundary.

An expression for the internal gravitational torque on the interior beneath the ocean due to Titan's self-gravity follows from the internal gravitational potential due to the ice shell, the ocean, and the interior, and is different from zero only for misalignment of the principal axes of the internal regions considered. For a homogeneous interior, it is easiest to calculate the surface integral (21), for a differentiated interior the volume integral (20) must be calculated. Since the z-component of the vectorial product $\vec{r}\times\hat{n}$ is proportional to a spherical harmonic of degree two and order two (Van Hoolst and Dehant 2002):

$$(\vec{r}\times\hat{n})_z = \frac{1}{3} r_i \beta_i P_2^2(\cos\theta)\sin 2\lambda', \tag{22}$$

where P_2^2 is the associated Legendre function of degree two and order two and θ is co-latitude, only those terms in the gravitational potential proportional to a spherical harmonic of degree two and order two contribute to the gravitational torque. Here, r_i is the mean radius and β_i the equatorial flattening of the interior. The angle λ' is measured in a coordinate frame fixed to the interior and differs from the angle λ in the shell reference frame by the orientation difference between the interior and the shell. Following Jeffreys (1952, see also Buffett 1996), the internal gravitational potential in the interior due to the ocean and the shell can be expressed as

$$\begin{aligned}\Phi(r,\theta,\lambda) = &-4\pi G \int_{r_i}^{R} \rho(r_0')r_0' dr_0' \\ &-\frac{2\pi G}{15} r^2 \int_{r_i}^{R} \rho(r_0')\frac{\partial\beta(r_0')}{\partial r_0'} dr_0' P_2^2(\cos\theta)\cos 2\lambda. \end{aligned}\tag{23}$$

The first term represents the spherically-symmetric part of the gravitational potential and does not contribute to the torque. The gravitational field generated by the mass of the interior doesn't contribute either to the torque as it is aligned with the mass of the interior. Therefore, only the ocean and ice shell contribute. However, the ocean mass in a thin layer with spherical outer surface around the interior also doesn't contribute since it has the same orientation as the interior.

By substituting Expression (23) for the gravitational potential into Eq. (20) and by taking into account the ellipsoidal form and the difference in orientation of the interior beneath the ocean and the shell, the z-component of the gravitational torque modified by ocean pressure $\Gamma_{\Phi,P}$ on the interior due to misalignment of the interior and the shell can be expressed as

$$\Gamma_{\Phi,P} = \frac{4\pi G}{5}\left[\overline{\rho\beta} + \rho_o\beta_m\right]\left[(B_i - A_i) - (B_i' - A_i')\right]\sin 2(\phi_s - \phi_i), \tag{24}$$

where

$$\overline{\rho\beta} = \int_{r_m}^{R} \rho(r_0')\frac{d\beta}{dr_0'}dr_0'. \tag{25}$$

Eq. (24) is equal to the internal gravitational torque expression for the Earth given by Xu et al. (2000, see also Buffett 1996), although these authors do not mention explicitly that it also includes the effect of pressure. Here, $B_i' - A_i'$ is the moment of inertia difference for the volume of the interior with a constant density ρ_o. Applied to our Titan interior models with an interior divided into four homogeneous layers, we have

$$\begin{aligned}\Gamma_{\Phi,P} &= \frac{4\pi G}{5}\frac{8\pi}{15}\left[\rho_s\beta_s + (\rho_o - \rho_s)\beta_o\right] \\ &\quad\times\left[(\rho_\mathrm{m} - \rho_o)\,\beta_m r_m^5 + (\rho_\mathrm{c} - \rho_\mathrm{m})\,\beta_c r_c^5\right]\sin 2(\phi_s - \phi_i) \\ &= K_\mathrm{int}\sin 2(\phi_s - \phi_i).\end{aligned} \tag{26}$$

The same expression has been derived by Van Hoolst et al. (2008) for the internal gravitational torque on a spherical region consisting of the interior beneath the ocean and a small liquid layer around the interior and bounded by a spherical surface. Here, β_j is the equatorial flattening of the outer surface of the shell (subscript $j = s$), ocean ($j = o$), ice mantle ($j = m$) and core

($j = c$) of Titan with principal axes $a_j > b_j > c_j$. The torque depends on twice the difference in angles $\phi_s - \phi_i$ since it is maximum when the axes make an angle of 45°. The strength of the torque depends on the equatorial flattening of the shell and interior.

For model 1, the coupling strength $K_{\text{int}} = 1.34\,10^{20}$ Nm, whereas $K_{\text{int}} = 2.60\,10^{20}$ Nm for model 2. The internal gravitational coupling strength is about a factor 5-6 smaller than the strength of Saturn's gravitational torque. To compare the magnitude of the torques, an estimate of the misalignment of shell and interior is needed. We use an estimated maximum misalignment between shell and interior $\phi_s - \phi_i$ equal to 0.015 rad, equal to the maximum angle considered for the estimate of Saturn's gravitational torque associated with observed LOD variations (see Sect. 2). The ratio between the maximum external and internal gravitational torques is therefore equal to the ratio $K_{\text{grav}}/K_{\text{int}}$, or about 5-6. For model 2 with the largest torque, the interior gravitational torque at maximum misalignment is $7.80\,10^{18}$ Nm.

For the rotational dynamics of the shell and interior, a better comparison of the internal gravitational torque applied by the shell and ocean on the interior is with Saturn's torque applied on the interior, instead of on the whole body of Titan. The gravitational torque of Saturn on the interior can be obtained as in Sect. 2 for the torque on the entire body of Titan. Without pressure effect, the Saturn torque on the interior is obtained by replacing the moment of inertia difference $B - A$ by the moment of inertia difference of the interior $B_i - A_i$ in Eq. (1). Since the ocean pressure contribution is given by a similar integral expression as for Saturn's gravitational torque in which the interior density is replaced by the opposite of the ocean density (see Eq. 20), the z-component of Saturn's torque modified by pressure $\Gamma_{S,P}$ can be expressed as

$$\Gamma_{S,P} = \frac{3}{2}[(B_i - A_i) - (B_i' - A_i')]\frac{GM_{\mathrm{S}}}{d^3}\sin 2\psi_i. \tag{27}$$

For a homogeneous interior, this equation simplifies to

$$\Gamma_{S,P} = \frac{3}{2}\left(1 - \frac{\rho_o}{\rho_i}\right)(B_i - A_i)\frac{GM_{\mathrm{S}}}{r^3}\sin 2\psi_i, \tag{28}$$

showing that the pressure effect then reduces the external gravitational potential by a factor $1 - \rho_o/\rho_i$.

For model 1, the strength of Saturn's gravitational torque on the interior, modified by pressure, $3n^2[(B_i - A_i) - (B_i' - A_i')]/2 = 2.52\,10^{20}$ Nm, and $3n^2[(B_i - A_i) - (B_i' - A_i')]/2 = 2.82\,10^{20}$ Nm, for model 2. The pressure-modified gravitational torque exerted by Saturn on Titan's interior at maximum LOD misalignment is then $7.56\,10^{18}$ Nm for model 1 and $8.46\,10^{18}$ Nm for model 2. Therefore, the maximum internal and external gravitational torques on the interior are of the same order of magnitude.

3.2 LOD variations of Titan

The rotation variations of the shell and interior can be determined by solving the angular momentum Eqs. (11), (14) and (18). By substituting the angular momentum for the ocean (Eq. 18) into the angular momentum equation for Titan (Eq. 11), we have

$$C_s\ddot{\phi}_s + C_i\ddot{\phi}_i = \Gamma_t, \tag{29}$$

which can be solved together with angular momentum Eq. (14) for the interior.

By introducing the small rotation angles $\gamma_j = \phi_j - M_a$ for the layers j, and substituting torque expressions (12), (26) and (27) into the angular momentum equations (14) and (29), we have

$$
\begin{aligned}
C_s\ddot{\gamma}_s + C_i\ddot{\gamma}_i = \Gamma(t) + \frac{3}{2}\frac{GM_\mathrm{S}}{d^3}\,[(B_s-A_s)\sin 2(f-M_a-\gamma_s) \\
+(B_i-A_i)\sin 2(f-M_a-\gamma_i) \\
+(B_o-A_o)\sin 2(f-M_a-\gamma_o)] \qquad (30)\\
C_i\ddot{\gamma}_i = K_\mathrm{int}\sin 2(\gamma_s-\gamma_i) \\
+\frac{3}{2}\frac{GM_\mathrm{S}}{d^3}\,[(B_i-A_i)-(B_i'-A_i')])\sin 2(f-M_a-\gamma_i). \qquad (31)
\end{aligned}
$$

As in Sect. 2, we use a series expansion of the Saturn torque in eccentricity and only retain the lowest-order terms, we then have (see Eq. 5)

$$C_s\ddot{\gamma}_s + C_i\ddot{\gamma}_i + 2K_s\gamma_s + 2K_i\gamma_i = \Gamma_\mathrm{A}\sin\omega_\mathrm{A}t + 4eK_\mathrm{grav}\sin M_a, \qquad (32)$$

$$C_i\ddot{\gamma}_i + 2K_c\gamma_i - 2K_\mathrm{int}(\gamma_s-\gamma_i) = 4eK_c\sin M_a, \qquad (33)$$

where

$$K_s = \frac{3}{2}n^2(B_s-A_s), \qquad (34)$$

$$K_i = \frac{3}{2}n^2(B_i-A_i), \qquad (35)$$

$$K_c = \frac{3}{2}n^2[(B_i-A_i)-(B_i'-A_i')]. \qquad (36)$$

The polar moment of inertia C_j of an internal layer j is calculated from the polar flattening $\alpha = [(a+b)/2-c]/[(a+b)/2]$ by integration:

$$C_j = \frac{8\pi}{3}\int_{r_{0,b}}^{r_{0,t}} \rho_j\left[r_0'^4 + \frac{2}{15}\frac{d\,(\alpha r_0'^5)}{dr_0'}\right]dr_0', \qquad (37)$$

where $r_{0,b}$ and $r_{0,t}$ are the bottom and top mean radial coordinates of the layer. The polar flattening in the layer is determined by integrating Clairaut's Equation (2) for α with the boundary condition

$$\frac{d\alpha}{dr_0}(R) = \frac{1}{R}\left[\frac{25}{4}q - 2\alpha(R)\right]. \qquad (38)$$

This boundary condition differs from the classical Clairaut boundary condition (see, e.g., Jeffreys 1952, Moritz 1990) for polar flattening since it also includes the effect of static tides besides the rotational effect (Van Hoolst et al. 2008). We have $C_s = 2.20\,10^{34}$ kg m^2 and $C_i = 1.91\,10^{35}$ kg m^2 for model 1, and $C_s = 2.95\,10^{34}$ kg m^2 and $C_i = 2.35\,10^{35}$ kg m^2 for model 2.

The solution of Eqs. (32) and (33) can be written as a sum of two solutions, one at frequency ω_A, the second at frequency n. We here only consider the long-term LOD variations at frequency ω_A. We search for solutions of the form $\gamma_s = g_s \sin\omega_\mathrm{A} t$ and $\gamma_i = g_i \sin\omega_\mathrm{A} t$. We then have

$$\left(2K_s - \omega_\mathrm{A}^2 C_s\right) g_s + \left(2K_i - \omega_\mathrm{A}^2 C_i\right) g_i = \Gamma_\mathrm{A}, \tag{39}$$

$$-2K_\mathrm{int} g_s + \left[2\left(K_c + K_\mathrm{int}\right) - \omega_\mathrm{A}^2 C_i\right] g_i = 0. \tag{40}$$

The amplitude of the shell and interior rotation variations due to atmospheric forcing and modified by both internal couplings and external gravitational torques is then given by

$$g_s = \frac{\left[2\left(K_\mathrm{int} + K_c\right) - \omega_\mathrm{A}^2 C_i\right]\Gamma_\mathrm{A}}{\Delta}, \tag{41}$$

$$g_i = \frac{2K_\mathrm{int}\Gamma_\mathrm{A}}{\Delta}, \tag{42}$$

where

$$\begin{aligned}\Delta = {} & \omega_\mathrm{A}^2 C_i \left[\omega_\mathrm{A}^2 C_s - 2\left(K_\mathrm{int} + K_s\right)\right] \\ & -2\left\{\omega_\mathrm{A}^2 C_s \left(K_\mathrm{int} + K_c\right) - 2\left[K_\mathrm{int} K_i + \left(K_\mathrm{int} + K_c\right) K_s\right]\right\}.\end{aligned} \tag{43}$$

The shell rotation angle amplitude is $2.94\ 10^{-4}$ rad ($1.41\ 10^{-4}$ rad) for model 1 (model 2), a factor two to three larger than when Titan has no ocean. However, even when Titan has an ocean, the predicted surface rotation amplitude is about two orders of magnitude smaller than the observed value. By changing the thickness of the ice shell of our models, the rotation amplitude can be changed by at most some 10%. For example, by increasing the shell thickness by about a factor three (see model 3 in Tables 1 and 2), the amplitude reduces by about 30%.

Fig. 1 shows the variation in the small rotation angle γ_s determined from Eq. (41) in the time interval 1980 to 2020. We also numerically integrated Eqs. (30) and (31) and verified that the amplitude difference between both solutions at 14.74 yr is very small (below 0.1%). Since the rotation angle variations are predicted to be in phase with the torque (see 41), it can be seen that the rotation is predicted to be slower than synchronous during the last years (Fig. 2), as for a solid Titan but contrary to the Cassini observations (Stiles et al. 2008). Besides the long-term variations due to the atmosphere, the numerical solution also shows short-periodic variations in the rotation angle. These librations are due to the periodically changing orientation of the long axis of Titan with respect to Saturn during its orbital motion around Saturn. The largest of these librations has a period of 15.945 days, equal to the orbital period, and an amplitude of 1.49 rad.

4 Discussion

How can we explain that the predicted rotation variations are much smaller than the observed rotation and what does this difference imply for Titan? For a spherical Titan with an ocean, gravitational and pressure torques can be set equal to zero and the shell can be considered decoupled from the interior and forced only by the atmosphere. This case has been considered in previous studies (Tokano and Neubauer 2005, Lorenz et al. 2008), and the amplitude of the surface rotation variations can be expressed as

$$g_s = -\frac{\Gamma_{\mathrm{A}}}{\omega_{\mathrm{A}}^2 C_s}. \tag{44}$$

Since there are no restoring forces considered in this decoupled shell situation, the amplitude of the rotation variations is much larger than in the general case considered above. We have an amplitude of -3.99 10^{-2} rad for model 1 and -2.98 10^{-2} rad for model 2, about two to three times larger than the observed value. For model 3 with a 200km thick ice shell, the amplitude is close to the observed one. However, Titan is not spherically symmetric, as indicated by the Cassini estimates of the low-degree gravity field of Titan (Iess et al. 2007), and both external and internal torques will tend to reduce the rotation variations induced by the atmosphere.

If the torques on the ocean (Γ_o) and the interior (Γ_i) could be neglected, for example by considering the interior to be spherically symmetric, the rotation variations of the surface would be determined by the atmospheric torque and Saturn's torque on Titan's shell. This situation corresponds to that of Sect. 2, but here only the rotation of the shell instead of the whole of Titan is considered. The interior rotates synchronously with the orbital mean motion at the long time-scale considered and the amplitude of the shell rotation variations is given by

$$g_s = \frac{\Gamma_{\mathrm{A}}}{3n^2(B_s - A_s) - \omega_{\mathrm{A}}^2 C_s}. \tag{45}$$

This expression is equal to Expression (8) except for the moments of inertia, which here are for the shell instead of for the entire body of Titan. For model 1 of the interior structure of Titan, the amplitude of the rotation variations is $9.93\,10^{-4}$ rad ($g_s = 6.60\,10^{-4}$ rad for model 2, $g_s = 3.87\,10^{-4}$ rad for model 3), 4 to 10 times larger than when Titan has no ocean, but almost 50 times smaller than for a decoupled shell. Due to Saturn's large mass and proximity to Titan and due to Titan's large hydrostatic equatorial flattening, Saturn exerts a strong torque on the ice shell of Titan which largely reduces the rotation variations.

The internal coupling reduces further the rotation variations by a factor 2 to 5. Therefore, Saturn's gravitational torque on Titan is more important for the rotation variations of a hydrostatic Titan than the coupling between the internal layers. The importance of Saturn's gravitational torque is further illustrated by the following two observations. First, without Saturn's torque, internal coupling between the shell and the interior causes the rotation variations to be out of phase with the atmospheric torque (Karatekin et al. 2008). However, with Saturn's torque the rotation variations are predicted to be in phase with the atmospheric torque (see Solutions 8 and 41). Second, the dynamical effect of the interior coupling between the shell and the interior beneath the ocean is not strong enough compared to that of Saturn's torque to lock the rotation of the shell to that of the interior, whereas if Saturn's torque could be neglected, the internal coupling would force Titan to rotate almost like a rigid body (Karatekin et al. 2008). For model 1 (model 2, model 3), the ratio of the amplitude of the shell rotation to that of the interior is 0.87 (0.92, 0.88) when Saturn's torque is neglected. When Saturn's torque is included, the interior rotation amplitude is a factor 2.75 (2.00, 2.76) smaller than the shell rotation amplitude, showing that the internal torque in the presence of Saturn's torque is less efficient in locking the shell rotation to that of the interior.

The effect of ocean pressure on the rotation of Titan is surprisingly small. Pressure reduces both the external Saturn torque and the internal gravitational torque on the interior by about a factor three. Therefore, neglecting the effect of pressure corresponds to increasing the coupling constants K_{int} and K_c by about a factor three in the dynamical equation (40) for the interior. The other angular momentum equation (39) remains unaltered. Due to the long period of the LOD variations, the inertia term $-\omega_{\text{A}}^2 C_i g_i$ in Eq. (40) is two orders of magnitude smaller than the torque contributions and can be neglected in a first approximation. As a result, the dynamical equation for the interior with the pressure effect neglected is approximately equal to the equation with pressure included and the solutions are thus approximately equal. For the three models considered, pressure reduces the amplitude of rotation variations by 2%. This suggests that a more complex ocean dynamics is not likely to change our results or to explain the large difference between the observed and predicted LOD variations of Titan if its effect on the internal and external gravitational torques is equally strong.

Deviations from hydrostatic equilibrium could be important on Titan and change our results. For example, for a synchronously rotating satellite in hydrostatic equilibrium the ratio between the degree-two gravity coefficients $J_2/C_{22} = 10/3$, but estimates of this ratio from a recent analysis of radio tracking data of the Cassini spacecraft differ by some ten of percent from this value (Iess et al. 2007). If non-hydrostatic effects could strongly diminish the equatorial flattening of Titan, both the external and internal torques would be much smaller than in the hydrostatic case and the amplitude of the surface rotation variations could approach those of a decoupled shell, which is close to the observed value. However, the degree-two order-two gravity coefficient C_{22} (Iess et al. 2007) is itself an indication of the equatorial flattening and corresponds well to the values calculated for the hydrostatic models considered here. This suggests that Titan's equatorial flattening is not far from that expected for the hydrostatic case, although it is not known whether individual layers could have a much smaller flattening. Our results indicate that a very small equatorial flattening of the ice shell is required for an amplitude of rotation variations as large as the observed value. In that case, both Saturn's gravitational torque and the internal coupling almost vanish and the rotation angle amplitude is as given in Eq. (44) for a decoupled shell. The phase of the rotation variations would then also agree with the observations. Maybe convection in the ice shell could strongly reduce the equatorial flattening of the shell.

Alternatively, the effect of internal and external gravitational torques on the rotation variations could be largely reduced if a viscous ice shell can relax its shape and adjust its orientation to that of the forcing gravitational fields. The gravitational torques diminish strongly if the time scale of relaxation of the shell is much smaller than the period of the rotation variations of 14.74 yrs. In that case, the rotation variation of the shell could be large and close to that of a decoupled shell. Preliminary calculations, for a simplified case in which only internal coupling is included, show that for the lowest possible ice viscosity, the melting ice viscosity of about 10^{13} Pa s, the effective strength of the internal gravitational coupling reduces substantially (Van Hoolst and Karatekin 2008). If Saturn's gravitational torque could be strongly reduced, the rotation angle variations could also be 180° out of phase with the atmospheric torque.

Predicted amplitudes of rotation variations could also be close to the observed values if the atmospheric torque were about two orders of magnitude larger than predicted by Tokano and Neubauer (2005). Since the predicted phase of the rotation variations is in phase with the atmospheric torque, the problem of the phase difference with the observations could also be addressed by studying changes in the phase of the atmospheric torque. These possibilities require further investigation by atmospheric modelers.

5 Conclusions

Due to the gravitational torque of Saturn on a hydrostatic Titan, which is flattened by rotation and static tides, and internal gravitational and pressure coupling, calculated rotation variations induced by the atmosphere are about two orders of magnitude smaller than expected from Cassini observations. The rotation angle variations are more than 50 times smaller than the observations indicate when a subsurface ocean exists and more than 100 times smaller when Titan has no ocean. Moreover, the rotation angle variations are out of phase with the observations: the rotation is calculated to be slower than synchronous over the last few years, whereas Cassini observations indicate a faster rotation. Saturn's gravitational torque is mainly responsible for the small rotation variations and the cause of the phase difference. In the above results, the atmospheric torque of Tokano and Neubauer (2005) is used.

If Titan is entirely solid, it is unlikely that the rotation variations are not affected by a large gravitational torque from Saturn. The equatorial flattening estimated from the recently determined degree two, order two gravitational coefficient C_{22} (Iess et al. 2007) is close to that expected for hydrostatic models and the gravitational torque on a solid Titan must be close to that calculated above. Therefore, the rotation variations of a solid Titan are expected to be much smaller and out of phase with respect to the Cassini observations, which suggests that Titan is not entirely solid. Whether the actual atmospheric torque of Titan can both be sufficiently larger and bring the phase of the rotation variations into agreement with observations remains to be studied.

If Titan has an ocean, not only a larger atmospheric torque but also a viscous ice shell could increase the rotation variations. For fast viscous relaxation of the shell, Saturn's gravitational torque could possibly even be reduced to a level that it would not lead to a phase difference with observations. These possiblities require further study. Our results suggest a further solution for the difference between the predicted and observed amplitude and phase: we have shown that rotation variations with similar amplitude and phase as observed are possible if the equatorial flattening of Titan's ice shell were to be reduced strongly by non-hydrostatic effects, for example by ice convection. If this were to be the case, this study would confirm the main conclusion of Lorenz et al. (2008) that Titan's rotation reveals the existence of an ocean. Moreover, if the equatorial flattening of the shell can be neglected, the observed rotation could be used to estimate the thickness of the ice shell.

Acknowledgment

We thank T. Tokano for allowing us to use his atmospheric angular momentum series of Titan. This work was financially supported by the Belgian PRODEX program managed by the European Space Agency in collaboration with the Belgian Federal Science Policy Office. NR acknowledges the Jet Propulsion Laboratory of the California Institute of Technology visiting program scientist.

References

[25] Buffett, B.A., 1996. Gravitational oscillations in the length of day. Geophys. Res. Lett. 23(17), 2279-2282.

[25] Cayley, A., 1861. Tables of the developments of functions in the theory of elliptic motion. Mem. R. Astron. Soc. 29, 191-306.

[25] Dehant, V., Mathews, P.M., 2007, Earth rotation variations. Treatise on Geophysics, Vol. 3, 295-349

[25] Fortes, A. D., Grindrod, P.M., Trickett, S.K., Vočadlo L., 2007. Ammonium sulfate on titan: Possible origin and role in cryovolcanism. Icarus, 188, 139–153, doi:10.1016/j.icarus.2006.11.002.

[25] Hansen, P.A., 1855. Abhandlungen der mathematisch-physikalischen Classe der Königlich Sächsischen Gesellschaft der Wissenschaften. Leipzig, Vol. 2, 515-523.

[25] Hinderer, J., Legros, H., Amalvict, M., 1982. A search for Chandler and nearly diurnal wobbles using Liouville equations. Geophys. J. R. Astron. Soc. 71, 303–332

[25] Hughes, S., 1981. The computation of tables of Hansen coefficients. Cel. Mech 25, 101-107.

[25] Iess, L., Armstrong, J.W, diBenedetto, M., Graziani, A. Mackenzie, R., Racioppa, P., Rappaport, N., Tortora, P., 2007. The determination of Titan gravity field from Doppler tracking of the Cassini spacecraft. in: Proceedings of the 20th International Symposium on Space Flight Dynamics, NASA/CP-2007-214158.

[25] Jeffreys, H., 1952. The Earth. Its origin, history and physical constitution. Cambridge University Press, Cambridge, UK.

[25] Karatekin, Ö., Van Hoolst, T. Tokano, T., 2008. Effect of internal gravi-

tational coupling on Titan's non-synchronous rotation. Geophys. Res. Lett. 35, L16202, doi:10.1029/2008GL034744.

[25] Lorenz, R. D., Stiles, B.W., Kirk, R. L., Allison, M. D., Persi del Marmo, P., Iess, L., Lunine, J. I., Ostro, S. J., Hensley S., 2008. Titan's rotation reveals an internal ocean and changing zonal winds. Science 319(5870), 1649–1651, doi:10.1126/science.1151639.

[25] Mathews, P.M., Buffett, B.A., Herring, T.A., Shapiro, I.I., 1991. Forced nutations of the Earth: influence of inner core dynamics. 1. Theory. J. Geophys. Res. 96, 8219–8242

[25] Moritz, H., 1990. The figure of the Earth. Herbert Wichmann Verlag GmbH, Karlsruhe, Germany.

[25] Murray, C.D., Dermott, S.F., 1999. Solar System Dynamics. Cambridge University Press

[25] Noyelles, B., 2008. Titan's rotational state. The effects of a forced "free" resonant wobble. Cel. Mech. Dyn. Astron. 101, 13–30, doi:10.1007/s10569-008-9132-y

[25] Plummer, H.C., 1918. An introductory treatise on dynamical astronomy. Cambridge University Press, Cambridge, UK.

[25] Poincaré, H., 1910. Sur la précession des corps déformables. Bull. Astron. 27, 321-356.

[25] Sohl, F., Hussmann, H., Schwentker, B., Spohn, T., Lorenz, R. D., 2003. Interior structure models and tidal Love numbers of Titan. J. Geophys. Res. 108, 5130.

[25] Stiles, B.W., Kirk, R.L., Lorenz, R.D., Hensley, S., Lee, E., Ostro, S.J., Allison, M.D., Callahan, P.S., Gim, Y., Iess, L., Persi del Marmo, P., Hamilton, G., Johnson, W.T.K., West, R.D. 2008. Determining Titan's Spin State from Cassini Radar Images. Astron. J. 135(5), 1669–1680, 10.1088/0004-

6256/135/5/1669.

[25] Tokano, T., and Neubauer, F.M., 2005. Wind-induced seasonal angular momentum exchange at Titan's surface and its influence on Titan's length-of-day. Geophys. Res. Lett. 32, L24203, doi:10.1029/2005GL024456.

[25] Van Hoolst, T., 2007. The rotation of the terrestrial planets. Treatise on Geophysics, Vol.10: Planets and Moons, pp. 123-164, doi: 10.1007/s11214-007-9202-6

[25] Van Hoolst, T., Dehant, V. 2002. Influence of triaxiality and second-order terms in flattenings on the rotation of terrestrial planets: I. Formalism and rotational normal modes. Phys. Earth Planet. Int. 134, 17-33.

[25] Van Hoolst, T., Rambaux, N., Karatekin, Ö., Dehant, V., Rivoldini, A., 2008. The librations, shape, and icy shell of Europa. Icarus 195/1, 386–399, doi:10.1016/j.icarus.2007.12.011.

[25] Van Hoolst, T., Karatekin, Ö., 2008. The influence of internal gravitational coupling and ice viscosity on Titan's length-of-day variations. European Planetary Science Congress 2008, extended abstract.

[25] Xu, S., Crossley, D., Szeto, A.M.K, 2000. Variations in length of day and inner core differential rotation from gravitational coupling. Phys. Earth Planet. Int. 117, 95-110.

TABLES

	model 1	model 2	model 3
r_c (km)	1670.807	2124.030	1708.670
r_m (km)	2282.540	2350	2250
r_o (km)	2506.300	2495	2375
ρ_c ($\mathrm{kg\,m^{-3}}$)	3813.31	2325.1	3700.354
ρ_m ($\mathrm{kg\,m^{-3}}$)	1310	1400	1310
ρ_o ($\mathrm{kg\,m^{-3}}$)	950	1350	950
ρ_s ($\mathrm{kg\,m^{-3}}$)	917	1065	917
$I/(MR^2)$	0.304	0.360	0.304

Table 1

Size and density of the four internal layers of the Titan models. Model 1 is from Sohl et al. (2003), model 2 is based on Fortes et al. (2007), model 3 has equal densities of the ice and water layers as model 1 and almost equal mean moment of inertia as model 1 but has a thicker ice shell. The mean moment of inertia is given in the last line.

	model 1	model 2	model 3
(1) rigid, spherical	-3.23 10^{-3}	-2.73 10^{-3}	-3.24 10^{-3}
(2) rigid, flattened	9.87 10^{-5}	6.46 10^{-5}	9.88 10^{-5}
(3) ocean, spherical	-3.99 10^{-2}	-2.98 10^{-2}	-1.52 10^{-2}
(4) ocean, spherical interior	9.93 10^{-4}	6.60 10^{-4}	3.87 10^{-4}
(5) ocean, internal	-3.64 10^{-3}	-3.07 10^{-3}	-3.26 10^{-3}
(6) general	2.94 10^{-4}	1.41 10^{-4}	2.05 10^{-4}

Table 2

Amplitude of the surface rotation angle variations in radians for different interior structure models and an atmospheric torque with amplitude of 1.6 10^{17} Nm (Tokano and Neubauer 2005). In case (1), Titan is considered to behave rigidly (no ocean) and to be spherically symmetric. The modelling hypotheses of the other cases are: (2) a rigid, hydrostatically flattened Titan, (3) a spherically symmetric Titan with an ocean, (4) a hydrostatically flattened shell, spherically symmetric ocean and deeper interior, (5) Titan with an ocean but only internal coupling considered, (6) general case for a hydrostatically flattened Titan with an ocean, the gravitational torque by Saturn and internal gravitational and pressure coupling. A minus sign indicates that the rotation angle variations are 180° out of phase with respect to the atmospheric torque.

FIGURES

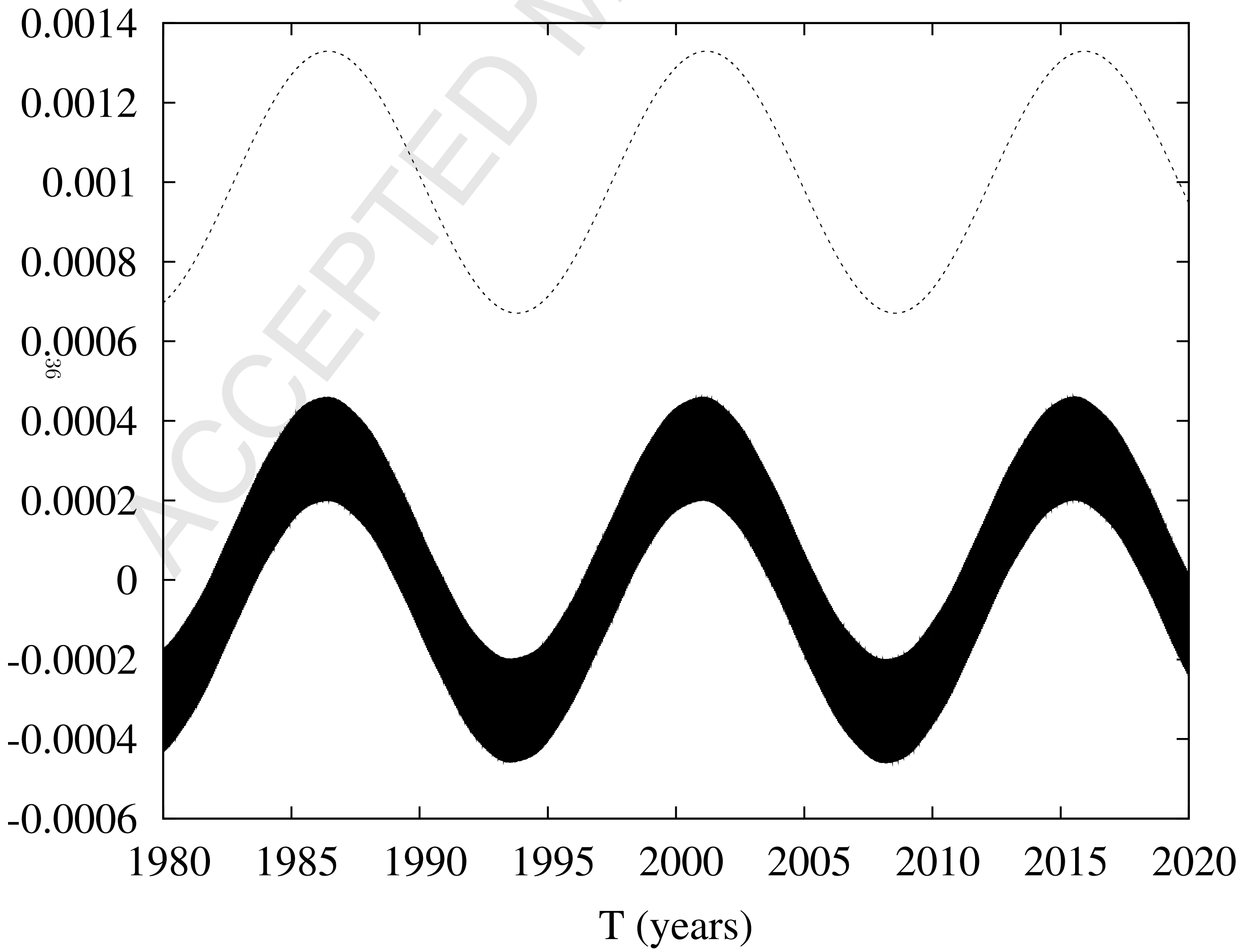


Fig. 1. Numerical (solid line) and analytical (dashed) solutions for the rotation

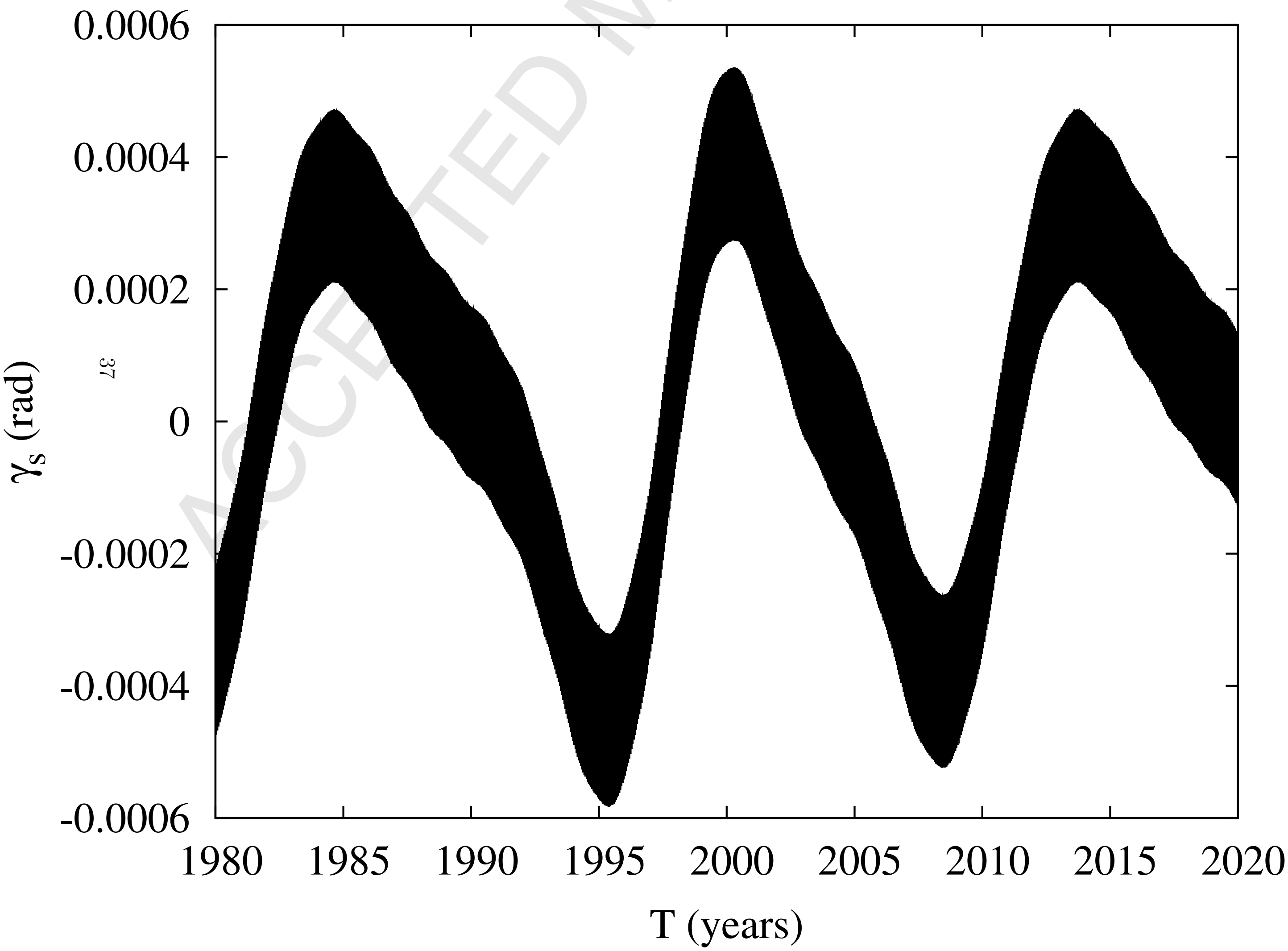


Fig. 2. Numerical solution for the rotation angle variations of model 1 when both