Accepted Manuscript

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PII: S0019-1035(07)00345-4
DOI: [10.1016/j.icarus.2007.07.018](https://doi.org/10.1016/j.icarus.2007.07.018)
Reference: YICAR 8365

To appear in: *Icarus*

Received date: 4 December 2006
Revised date: 24 April 2007
Accepted date: 27 July 2007

Please cite this article as: M. Granvik, K. Muinonen, L. Jones, B. Bhattacharya, M. Delbo, L. Saba, A. Cellino, E. Tedesco, D. Davis, V. Meadows, Linking Large-Parallax Spitzer-CFHT-VLT Astrometry of Asteroids, *Icarus* (2007), doi: 10.1016/j.icarus.2007.07.018

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Linking Large-Parallax Spitzer-CFHT-VLT¹ Astrometry of Asteroids

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Submitted to *Icarus*

Submitted 4 December 2006

Revised 18 April 2007

Manuscript pages: 59

Tables: 10

Figures: 8

¹Partially based on observations collected at the European Southern Observatory in Chile (ESO program N°271.C-5051, PI A. Cellino).

Proposed Running Head: Linking Large-Parallax Asteroid Astrometry

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Abstract

We show that the new ephemeris-space multiple-address-comparison (eMAC) method solves asteroid linking problems despite large parallaxes by applying the method to astrometric asteroid observation sets obtained nearly simultaneously with the Spitzer space telescope, the Canada-France-Hawaii Telescope (CFHT), and European Southern Observatory's Very Large Telescope (VLT). For main-belt asteroids, the parallax between Spitzer and the Earth-based telescopes is approximately one degree which is large as compared to a typical parallax for solely Earth-based telescopes in the arcseconds regime. In the eMAC method, we reduce the initially huge amount of possible linkages between observation sets by comparing samples of ephemerides that have been computed separately for all sets at, say, three common dates. If the non-zero ephemeris probability densities overlap at all common dates, we try to find an orbit solution for these so-called trial linkages. If there exists an orbit which reproduces all the astrometric observations assuming predefined observational errors, we call it a linkage. Known asteroids are independently identified among Spitzer, CFHT, and VLT astrometry, and comparing the identified observations to the linkages found shows that the method found all known correct linkages present in the data. In addition, we also found five previously unpublished linkages between Spitzer astrometry and Earth-based astrometry. Based on our simulations, we found virtually all Spitzer-related linkages between two single-night observation sets, and more than 99.4% of linkages between two single-night observation sets obtained by Earth-based observatories. Virtually all correct linkages consisting of at least three single-night sets were also detected. The results show that large-parallax discovery observations made from a spacecraft can be linked to Earth-based follow-up observations to ensure that the objects are not lost. Furthermore, we compute the heliocentric and Spitzer-centric distances as well as the corresponding solar phase angles at the dates of Spitzer observations. Based on comparisons to simulated geocentric observations, we also show that, for typical nearly-simultaneous observations, the parallax reduces the distance uncertainties by several orders of magnitude.

Key Words: ASTEROIDS, DYNAMICS, ORBITS, CELESTIAL MECHANICS, INFRARED OBSERVATIONS

1 Introduction

Observations with European Southern Observatory’s Very Large Telescope (VLT) and the Canada-France-Hawaii Telescope (CFHT) were obtained in January 2004 to provide nearly-simultaneous follow-up for the Spitzer First Look Survey Ecliptic Plane Component (FLS/EPC; see Meadows et al. (2004) for a description and early results). The goal of the observing campaign is to estimate the albedo and size distributions of main-belt asteroids (MBAs) having sizes well below the current completeness limit, by obtaining nearly-simultaneous optical and infrared observations (Tedesco et al., in preparation). To estimate asteroid sizes and albedos using the radiometric technique, one needs to know the Sun-asteroid and telescope-asteroid distances as well as the phase angles of the asteroids at the observation dates. This paper expands upon earlier works by Granvik et al. (2005) and Granvik & Muinonen (2005, hereafter GM05) that dealt solely with the linking of the VLT observations of the observing campaign.

While the deepest near-Earth-object surveys regularly go down to ~ 22 mag in the Johnson V band, objects fainter than 25 mag have been detected during the observing campaign. Some of the objects detected by Spitzer have been identified with previously known asteroids by the Minor Planet Center (MPC) and by Meadows et al. (2004). When a detection of a moving object is linked to a multi-apparition asteroid, obtaining the required distances and the phase angle for the corresponding date is reasonably straightforward. However, as many of the detected objects are previously unknown, those detections must be linked to a common object before a useful estimate for its distance can be given via an orbit solution accompanied with an uncertainty estimate. Unless linkages are found, the distance uncertainties will be very large, typically several or even tens of AUs.

There are a few circumstances that make the linking task a nontrivial one. First of all, nearly-simultaneous apparent coordinates of the same objects observed from space and from the Earth are substantially different, because of a parallax effect which, in turn, is a consequence of the comparatively large distance between the Spitzer spacecraft and the Earth-based observatories (~ 0.07 AU at the observation dates). For the observed main-belt asteroids, the observing geometry and the distance between Spitzer and an Earth-based telescope leads to a parallax of approximately one degree which can be compared to a typical order-of-magnitude estimate of arcseconds for solely Earth-based telescopes. In the latter case, the parallax is sufficiently small to allow linkages between nearly-simultaneous astrometric observations to be found by making predictions by extrapolation in Right Ascension (R.A.) and Declination (Dec.). For large parallaxes, the extrapolation in observed

coordinates is bound to fail as differences in the observation geometry lead to substantial differences in the observed R.A. and Dec.. Second, the sky-plane number density of objects grows rapidly with an increasing magnitude. Confusion in linking typically arises due to a high density of objects in a limited area. Third, the number of observations in each single-night set² (SNS) is limited (2–4)—and, finally, because of complicated apparent motion of an asteroid around the stationary point leading, for example, to orbital-element probability-density functions (p.d.f.) with two distinct maxima. To find linkages despite the obstacles, we use a method (GM05) specifically suited to solve linking problems. Alternative methods for the short-term linking task do exist, but either they suffer from the use of non-physical trajectories lacking topocentric range estimates (cf., Milani et al. 2005a, Kubica et al. 2007)—which makes it difficult to tackle large-parallax cases—or they cannot deal with the small number of observations (cf., Milani et al. 2005b). A traditional and still common method for the linking of exiguous SNSs is comparing Väisälä orbits (Väisälä 1939). But due to the assumptions, that is, perihelion time and geocentric distance, one cannot be sure to cover the entire orbital-element space of acceptable solutions, a prerequisite for finding all correct linkages. However, the data set of asteroid SNSs obtained during the observing campaign has also been independently analyzed with a Väisälä-based linking scheme by the MPC, the results of which are compared below to the ones that we have obtained.

For many of the asteroids detected by Spitzer, the distances and their uncertainty estimates can only be based on the limited amount of data obtained during the current observing campaign. However, the parallax between the Spitzer spacecraft and the Earth-based observatories helps us to constrain the distances to these previously unknown asteroids.

The paper is organized as follows. Section 2 contains descriptions of the methods and the linking-performance parameters used in the analysis. Details regarding the observations are presented in Section 3. Section 4 presents simulations that are used both to adjust the linking parameters and to estimate the correctness of our linking results lacking additional real observations. In Section 5, identifications, linkages, and distance estimates are presented and discussed. Finally, in Section 6, our conclusions are summarized.

²A single-night set is a set of sequential astrometric observations forming a detection of a moving object during one night. Typically observations from a single telescope only.

2 Methods

2.1 Inversion

Based on the number of observations and the length of the observed time interval, the inversion of astrometric observations for the orbital-element p.d.f. is performed using either the sampling methods, that is, statistical orbital ranging (Ranging; Virtanen et al. 2001, Muinonen et al. 2001) or the new Volume-of-Variation method (VoV; Muinonen et al. 2006a), or the point-estimate method of least-squares accompanied with linearized covariances (LSL; see, e.g., Danby 1992). Note that Ranging is applicable to astrometric observation sets with short observational time intervals (typically up to days) containing two or more observations, while VoV and LSL require more observations and, typically, longer observational time intervals. To find the correct solution faster than in basic Ranging, a stepwise approach is used (GM05) where the partial inversion problem is solved several times by including more observations step by step. In other words, the partial solutions are used as maps when searching for the fairly compact non-zero orbital-element p.d.f. in the six-dimensional orbital-element space.

In addition to the three orbital-inversion methods shortly described above, we also make use of the Nelder-Mead simplex method (Nelder & Mead 1965, Press et al. 1992) when a rigorous uncertainty analysis and, thus, also the occasionally badly-behaving partial derivatives, are not required. Here we use the Nelder-Mead simplex method to find an acceptable orbital fit to the astrometric data in the six-dimensional orbital-element space by starting from seven approximate orbits that can typically be satisfactorily fit only to parts of the data (Granvik & Muinonen 2007). In the context of orbit computation, the simplex method has previously also been used in collision-probability computation (Muinonen et al. 2001) and for solving the three-dimensional two-point boundary-value problem (Danby 1992) with one or more perturbing masses (Granvik & Muinonen 2007).

2.2 Identification of known objects with accurate ephemerides

To identify known objects with ephemerides uncertainties less than approximately an arcsecond among the astrometric observations, we first propagate all objects found in the Lowell asteroid orbit database (Bowell et al. 1994) to the observational mid-date of all Spitzer, CFHT, and VLT observations using an n -body dynamical model including all planets and Pluto. We use the Bulirsch-Stoer integrator and take into account the perihelion shift due to

the leading solar term in the parameterized post-Newtonian treatment of the theory of general relativity (Sitarski 1983). Then, we compute an ephemeris for each known asteroid at each date of observation using the 2-body approximation. The use of a full n -body model is unnecessary considering the lack of close approaches and relatively short time intervals. If the difference between the computed position, a point estimate of the ephemerides' p.d.f., and the observed position is smaller than a certain threshold, we perform a least-squares inversion using all available data and the n -body model. Succeeding in the effort and obtaining residuals between the observed and computed positions ($O - C$ residuals) compatible with the estimated uncertainty of the astrometric observations virtually proves that the identification is correct. The threshold we use depends on the uncertainty of each individual observation and is six times larger than the assumed astrometric uncertainty.

Note that the simplistic identification method does not take into account orbital uncertainties and may therefore not find all identifications. In addition to almost all of the approximately 137,000 numbered objects, we assume that some fraction of the unnumbered objects have ephemerides uncertainties at the arcsecond level and are thus probably detected by the identification method. However, there are, for example, some 50,000 designated objects that have been observed for less than 48 hours and will thus have ephemerides uncertainties far above the arcsecond level.

2.3 Linking observation sets

For linking SNSs of asteroid observations, we use the new ephemeris-space multiple-address-comparison (eMAC) method (GM05) which is based on Ranging and is suitable for linking problems requiring a physical propagation model, here particularly because of the substantial parallax. The key idea of the method is to compare ephemeris p.d.f.s at several different dates. For an acceptable trial linkage, we require an overlap at all comparison dates of the non-zero ephemeris p.d.f.s stemming from the two SNSs. Here we have used three different comparison dates. In other words, linkages are not directly sought between the observations as is done by, for example, Kubica et al. (2007). Instead orbital inversion is performed for all SNSs and linkages are sought between the ephemerides resulting from the orbital-element p.d.f. Thus, the eMAC linking method takes into account aspects that are treated in orbital inversion such as parallaxes, light-time corrections, and observational uncertainties. The trial linkages detected are then either accepted or discarded by computing an orbital solution based on the combined data set. We try to find an acceptable orbit by using three different methods for the orbital inversion. In order of application the methods are stepwise Ranging,

Nelder-Mead simplex, and LSL. VoV is not used here because, at this stage, we are only interested in finding a single, acceptable orbit, not in obtaining accurate uncertainty estimates. The linkage is accepted if the $O-C$ residuals are acceptable. In the current work, we require that all $O-C$ residuals should be smaller than three times the assumed uncertainty for the corresponding observations. It should be stressed that, at this stage, we do not yet know whether the linkage is correct or not. We only know that an orbit exists tying the observations together, assuming predefined observational errors.

Making the assumption that the method identifies all correct linkages comprising two SNSs (2-linkages), we locate 3-linkages by finding the three 2-linkages that can be formed from the three SNSs that constitute a single 3-linkage. Similarly, a k -linkage can be found from a set of $(k-1)$ -linkages, by requiring that the

$$\binom{k}{k-1} = k \quad (1)$$

$(k-1)$ -linkages forming the k -linkage must be present in the set of $(k-1)$ -linkages. The linkage chains can thus be detected with a stepwise chain-detection algorithm, which starts with all 2-linkages and proceeds until the step when the number of k -linkages found within a set of $(k-1)$ -linkages is zero. For SNSs, k is usually equal to or less than the number of nights of observation.

The time complexity of the eMAC method is currently $\mathcal{O}(n^2)$, where n is the number of SNSs. The chain-detection algorithm has also currently a time complexity of $\mathcal{O}(n^2)$, where n is the number of 2-linkages found. While the CPU time requirements are acceptable for current needs, it is possible to develop $\mathcal{O}(n \log n)$ algorithms in the future, if deemed necessary. Note that such a development would not change the results of the current work, but only allow the work to be done faster.

2.4 Assessing the performance of linking methods

The linking problem bears resemblance to signal detection theory (SDT; cf. Egan 1975, Spackman 1989) in the sense that we are essentially dealing with a binary classifier system. On one hand, two observation sets either belong to the same object or they do not belong to the same object. On the other hand, by solving the linking problem, we can estimate whether they can be linked or not. We therefore end up with four different outcomes when solving the linking problem:

- True Positives (TP) are linkages between observation sets that belong to the same object.

- False Positives (FP) are linkages between observation sets that do not belong to the same object.
- True Negatives (TN) are unlinked observation sets that do not belong to the same object.
- False Negatives (FN) are unlinked observation sets that belong to the same object.

Two different parameters should be used when describing the performance of a binary classifier C . In SDT, these parameters are the *sensitivity*

$$Sens(C) = \frac{TP}{TP + FN}, \quad (2)$$

and the *specificity*

$$Spec(C) = \frac{TN}{FP + TN}. \quad (3)$$

The sensitivity is a measure for how well a binary classifier correctly identifies a condition. A sensitivity of 1.0 means that a linking method finds all correct linkages. The specificity is a measure for how well a binary classifier correctly identifies negative cases. For a linking method, a specificity of 1.0 therefore means that erroneous linkages are not found, or, if found, they are not accepted.

For the linking problem, the sensitivity is given by

$$Sens(C) = \frac{l_c}{L_c}, \quad (4)$$

where l_c and L_c are the numbers of correct linkages found by the method and correct linkages that exist within the data, respectively. Specificity is a more problematic case. It is given by

$$Spec(C) = \frac{L_{\max} - (L_c + l_e)}{L_{\max} - L_c}, \quad (5)$$

where l_e is the number of erroneous linkages found and $L_{\max}$ is the total number of different and unordered pairs that can be generated from the input observation sets. For a single master set containing S internally correctly linked subsets, which have been identified with a single object, we have

$$L_{\max} = \frac{S(S-1)}{2} \quad (6)$$

(cf. Eq. 1). For two different, internally correctly linked and mutually exclusive master sets A and B consisting of S_A and S_B subsets of observations, we obtain

$$L_{\max} = S_A S_B. \quad (7)$$

Due to the S^2 and $S_A S_B$ terms, specificity comes typically very close to 1.0 in solutions to practical linking problems, rendering the discriminative power of the parameter questionable. An alternative parameter to specificity would therefore be preferable. A possibility is to use the complement of specificity

$$1 - \text{Spec}(C) = \frac{l_e}{L_{\max} - L_c}. \quad (8)$$

Another possibility is to use the *positive predictive value*, which is defined as

$$\text{PPV}(C) = \frac{l_c}{l_c + l_e}, \quad (9)$$

and reflects the probability that a detected linkage is a correct one. A positive predictive value of 1.0 means all detected linkages are correct, while 0.0 means that all detected linkages are erroneous. Note that Kubica et al. (2007) use the term “efficiency” for sensitivity and the term “accuracy” for the positive predictive value.

The performance of different classifiers can be rigorously compared to each other only when applied to the same data. Due to, for example, observational uncertainties, this is particularly true for linking methods as l_e critically depends on the observation set. Orbits corresponding to erroneous linkages can in some cases be better fit to the data than orbits corresponding to correct linkages. A statistical representation of this was given by GM05 who plotted histograms of χ^2 values computed from the $O-C$ residuals based on the orbit linking two data sets. Histograms corresponding to erroneous linkages overlapped with histograms containing solely correct linkages. Effectively, the result showed that for realistic linking problems the best-fitting erroneous linkage typically results in a lower χ^2 value than the worst-fitting correct linkage. In other words, if one wants to detect all correct linkages, a certain number of erroneous linkages are typically present in the results and the number of erroneous linkages is case-dependent. Therefore, if erroneous linkages are not detected, the specificity of the linking method is 1.0 while the sensitivity is typically less than 1.0. As shown by, e.g., GM05 and Kubica et al. (2007), the number of erroneous linkages that fit the data more or less as well as correct linkages also depends on parameters such as the timespan between the observation sets, observation geometry, and the density of objects in the observed fields. While developing linking methods,

our goal has so far not been to reduce the number of erroneous linkages, but to make sure we find the highest possible fraction of all the correct linkages present in the data, that is, to optimize the sensitivity. The development of statistical methods to reduce the number of erroneous linkages, or to increase the specificity, is left for the future.

3 Observations

The observations were made in January 2004 in the 0° and 5° fields, where the names stem from the Spitzer field centers given as ecliptic latitudes. The solar elongations ranged from $\sim 115^\circ$ for Spitzer observations to $\sim 132^\circ$ for CFHT observations. See Tables 1 and 2 for observational details. Ecliptic longitudes and latitudes for the first observation of each SNSs currently available from the observing campaign are shown in Figs. 1 and 2 for the 0° and 5° fields, respectively.

[Insert Figs. 1 and 2 approximately here]

For the infrared Spitzer FLS/EPC part, observations are presented by Meadows et al. (2004) and Tedesco et al. (in preparation). Spitzer made three observations, each consisting of several stacked images, of the same fields over a time interval of 0.09 days on January 21st. Most Spitzer SNSs, thus contain three pairs of R.A. and Dec. with an approximate uncorrelated astrometric uncertainty estimate of $\sigma_{\text{R.A.}} = \sigma_{\text{Dec.}} = 2.0''$. Tables 9 and 10 lists the re-reduced Spitzer astrometry and the heliocentric positions of the spacecraft at the observation dates. The spacecraft drifts away from the Earth on a heliocentric Earth-like orbit at a rate of $\sim 0.1 \text{ AU yr}^{-1}$. At the time of the FLS/EPC the distance to Earth was approximately 0.07 AU, resulting in a considerable parallax with respect to Earth-based observatories. Note that the $\gtrsim 1^\circ$ parallax is visible in Figs. 1 and 2 as a difference in the topocentric coordinates of the same asteroids that were observed nearly simultaneously by Spitzer and the Earth-based observatories.

Earth-based visual follow-up observations for the Spitzer FLS/EPC were obtained with VLT and CFHT. The VLT observations were only made for the 0° field on January 20th, 21st, 22nd, 23rd, and 24th. The final VLT fields are mosaics of smaller patches of the sky that can be covered by the VLT instruments in a single exposure. Each field was covered twice, thus resulting in typically two observations per SNS over a time interval of some 0.08 days. A comparison between the VLT observations and ephemerides of previously known objects suggests that we were able to detect almost all known objects that resided in the VLT fields by performing a visual inspection while blinking the images. The individual uncorrelated astrometric uncertainty for each

observation stems from the field solution and is $\sigma_{\text{R.A.}} = \sigma_{\text{Dec.}} \sim 0.3''$.

The CFHT observations were obtained on January 22nd and 30th. The 0° fields and the 5° field on the 22nd were covered at three different dates with three one-square-degree images spanning some 0.1 days. The 5° field approximately centered at $(\lambda = 178.5^\circ, \beta = 5.6^\circ)$ was not successfully obtained at CFHT in the January 30th data (Fig. 2). Moving objects were extracted with automated methods originally developed for TNOs moving slowly in the same direction (Petit et al. 2004), but here modified to allow different directions. The uncorrelated astrometric uncertainty was assumed to be $\sigma_{\text{R.A.}} = \sigma_{\text{Dec.}} = 0.5''$ for all observations.

Interestingly, when going from the 0° field to the 5° field, the sky-plane density of SNSs obtained by CFHT is reduced by $\sim 60\%$, while the sky-plane density of SNSs detected by Spitzer is reduced by only $\sim 10\%$.

4 Simulations

The parameters used in the linking procedure must allow the detection of virtually all correct linkages that exist within the data. Although we can never be sure to find all correct linkages due to the statistical nature of our approach, we can tune the method to reach a high sensitivity by using simulated MBA observations mimicking the real observations. Note that when linking real observations, the fraction of all available linkages that have been found is usually unknown, because L_c remains unknown.

Erroneous linkages should preferably be removed by additional observations. In the present case, no additional observations have been obtained and the archived observations by other telescopes do not reach the required limiting magnitudes. Thus, we need to use simulated astrometric observations to obtain an estimate for the uncertainties of linkages found. As we know a priori which linkages the method should find, the specificity and the positive predictive value can be estimated. The parameters provide simple statistical and global measures on how trustworthy the linkages found are.

The basic concept is to generate several sets of simulated MBA observations similar to those obtained in the observing campaign and run them through the linking software. We require that the sensitivity is, say, higher than 99.9%, and achieve the high sensitivity in an iterative fashion by tuning the linking parameters until the required fraction of the correct linkages are found. The specificity and the positive predictive value can then be computed based on the linking results. We use an updated version of the ASurv software (see, e.g., GM05) to generate simulated topocentric MBA observations for the three observatories at dates similar to the real observation

dates. Note that other asteroid populations were not used because the study focusses on MBAs. Uncorrelated asteroid orbits and absolute magnitudes are assigned randomly according to our current knowledge of their debiased distribution (Jedicke et al. 2002). After computing the position and a visual magnitude for a sample asteroid at an observation date, the simulated observation is accepted if its position is within the given field of view and the magnitude is below the given limiting magnitude. Because the aim of the observing campaign is to estimate the size distribution and thus the absolute magnitude distribution of the observed asteroids, the cumulative distribution function of the absolute magnitude to be used during the generation of sample asteroids is an unknown parameter. However, as we are not interested in the sizes during the linking procedure, but in the orbital distribution, we iteratively adjusted the limiting magnitudes to produce slightly higher SNS densities as compared to the observed ones (see Table 3). This is acceptable assuming that the orbital distribution of MBAs is fairly similar regardless of the size of the asteroid, that is, if the orbital elements and the absolute magnitudes of MBAs are uncorrelated. Typically, some 10% of the simulated objects detected by Spitzer are too faint or are outside of the fields of the simulated Earth-based follow-up. In other words, those Spitzer SNSs cannot be (correctly) linked to Earth-based follow-up.

By using five different seed values for the random number generator, we generated five different sets of simulated SNSs. The simulated observations mimic the distribution of the real observations which can be seen by comparing Figs. 1 (real) and 3 (simulation).

[Insert Fig. 3 approximately here]

Each simulated SNS contained three observations equally spaced in time, and the observational time interval of the simulated SNSs was 0.05 days, 0.08 days, 0.08 days, for Spitzer, VLT, and CFHT, respectively. Note that these time intervals are shorter than the real ones to make the simulated linking task more demanding. Table 3 lists the standard deviations that were used when adding uncorrelated random Gaussian noise to each simulated astrometric observation.

By using the first set of simulated SNSs, we iteratively adjusted the orbital-inversion and linking parameters until the sensitivity $Sens(eMAC) > 99.9\%$ (Table 4). By applying the linking method with the adjusted parameters to the rest of the four sets of simulated SNSs, we were able to estimate that the sensitivity $Sens(eMAC) \gtrsim 99.4\%$ (Table 5). All the correct 2-linkages missing corresponded to Earth-based observations, that is, for all sets of simulated SNSs we found all correct Spitzer 2-linkages. However, the positive predictive value is more than twelve times smaller for Spitzer-related 2-linkages ($PPV(eMAC) \sim 0.06$) as compared to the set with 2-linkages be-

tween SNSs from Earth-based observatories ($PPV(eMAC) \sim 0.75$). The large parallax thus allows approximately twelve times more erroneous 2-linkages as compared to a set of SNSs obtained solely with Earth-based telescopes. As expected, the specificity is not a good way of measuring the ability of a linking method to discard erroneous linkages, while the positive predictive value serves the purpose well.

A notable feature of the orbital inversion of the SNSs is that we did not use any regularization. Typically, the regularization does not affect the extent of the p.d.f., but only the relative weights of the sample orbits. Since the weights themselves are not used during the linking procedure, we considered their mathematically exact computation unnecessary.

As we are trying to find virtually all correct 2-linkages between the SNSs, a simulated asteroid that has been detected at all dates and with all (virtual) telescopes would result in eight SNSs and $\frac{8 \cdot (8-1)}{2} = 28$ correct 2-linkages between the SNSs. Now, if there is a problem with a single one of these SNSs, we would lack up to seven correct 2-linkages, a scenario which could explain and actually does partly explain the non-detection of $\sim 0.5\%$ of all the correct 2-linkages in the simulations. At a closer inspection, we actually found out that all the missing 2-linkages had a common reason: based on the problematic SNSs one can hardly say whether the object is moving or not. The situation is typical for observations around a stationary point—where the apparent motion of an object changes direction (for a mathematical definition of the stationary point, see, e.g., Green 1985)—and often leads to an orbital-element p.d.f. with two maxima. Since the direction of motion is not well-defined, the ephemerides are spread out symmetrically in two directions. At least in one of the problematic cases, the substantially less densely populated ephemeris cloud was the “correct” one, which made finding correct linkages impossible due to too sparse sampling in ephemeris space. However, once two different maxima have been detected in the orbital-element p.d.f., linking problems can be overcome by sampling separately and, more or less, equally densely in both regions.

5 Results and discussion

5.1 Identifications with previously known objects

By comparing each Spitzer, CFHT and VLT observation to the 356,585 orbits found in the `astorb.dat` file dated November 6, 2006, we found that 999 of the 2395 observations can be identified with 157 different asteroids. Of the 157 identified asteroids, 90 were identified with more than two SNSs

from the observing campaign, and can therefore be used when measuring the sensitivity of the eMAC method, while 67 were identified with only one SNS. Most of the 20 Spitzer asteroid identifications found (Table 7) are in agreement with the preliminary identifications by Meadows et al. (2004). Despite assuming a rather pessimistic astrometric uncertainty of $\sigma = 2''$ for the Spitzer observations, we were, however, not able to identify any Spitzer SNSs with asteroids 2004 EX₂₈, 2004 BX₁₅₉, 2004 BV₁₅₉, and 2002 TM₁₀₄ by assuming a threshold of $12''$ in contrast to the identifications suggested by Meadows et al. (2004). On one hand, the first three objects have short observational time intervals ranging from days to tens of days, and may have been excluded due to our criteria. On the other hand, the observations have been carried out around the time of the Spitzer observations and the corresponding links should have been detected. The last one has only a 2-day observational time interval making identifications over one apparition, that is, from October 2002 to January 2004, highly uncertain.

We were unable to link Spitzer objects 0j and 0m with the asteroids (27168) and (79364), respectively, as reported by Meadows et al. (2004). The $O - C$ residuals for the Spitzer source 0m with respect to the asteroid (79364) are fairly large, at least assuming a standard deviation of $2''$ for the uncertainty. There are, however, no clear trends in the $O - C$ residuals: $(0.2'', -1.0'')$, $(-10.5'', 1.0'')$, $(-10.6'', 1.8'')$, and the identification could thus be correct. The Spitzer source 0p has large $O - C$ residuals with respect to the asteroid (7987): $(-0.9'', -0.4'')$, $(-6.8'', 1.6'')$. But due to only two observations, we cannot say anything about the trends in the motion, and it could as well be a correct identification. As is customary, the $O - C$ residuals above refer to an orbit solution with the corresponding Spitzer observations included. Based on the rms values of the residuals corresponding to identified observations, we verified that the observational uncertainties assumed for Spitzer, CFHT and VLT were realistic (Fig. 4).

[Insert Fig. 4 approximately here]

5.2 Linkages between observation sets

Altogether 1,166 and 120 2-linkages between all the SNSs from Spitzer, CFHT, and VLT were found in the 0° and 5° fields, respectively. From these 2-linkages, longer linkage chains were formed by using the methodology presented in Section 2.3. To make sure that we do not miss any long linkage chains, we only used the requirement regarding the number of $(k - 1)$ -linkages to be found (Eq. 1) when searching for 3-linkages. We thus assumed that for each $(k \geq 4)$ -linkage we can find at least two $(k - 1)$ -linkages. The longest linkage chain found contained seven SNSs.

A comparison between the ($k \geq 2$)-linkages found and the identifications found, shows that the eMAC method found all 90 correct ($k \geq 2$)-linkages. The 7-linkage is one of these previously known objects. The real sensitivity of the eMAC method is thus virtually 1.0, which was expected based on the simulations. We found an additional SNS to be linked to 2004 BS₁₆₀ that was missed by the Väisälä-based linking methods. Note that 2004 BS₁₆₀ has only been observed within the current observing campaign, and the observational data is hence very limited underscoring the sensitivity of the eMAC method. In addition, we found several previously unknown linkages, five of which contain Spitzer observations and seem fairly reliable considering the $O - C$ residuals and the orbital elements (Table 6). Note that two of the linkages are mutually exclusive, which reduces the (maximum) number of new asteroids to four. According to the maximum-likelihood solution of the osculating-elements distribution weighed with the distribution of known objects, 0e₂ could belong to the Vesta family, 5f to the Nysa family, and 5k to the Themis family. A verification of the family memberships requires more astrometric observations as well as the computation of proper elements and an investigation of the objects' compositions. The orbital solutions for the two remaining linkages are MBA-like, but cannot immediately be linked to any known families. In all, the Väisälä-based linking methods found two linkages that possibly correspond to the previously unknown asteroids 2004 BS₁₆₀ and 2004 BH₁₆₀, while the eMAC method also found four additional linkages that possibly correspond to previously unknown asteroids.

Among 4-linkages including Spitzer SNSs, we found two cases that are mutually exclusive and probably both wrong because the corresponding Spitzer SNS (0l) was independently identified as asteroid (99307) by MPC, by Meadows et al. (2004), and by us. Both erroneous linkages contain nine observations over roughly two days with Earth-based and space-based observations done near simultaneously. The $O - C$ residual p.d.f.s do not show any clear trends, but systematic offsets are apparent, thus suggesting that the observations stem from several different asteroids and a chance alignment allows them to be linked. Note that this is compatible with the low positive predictive value obtained for Spitzer-related linkages in the simulations.

5.3 Distances and solar phase angles

As of November 6th, 2006, 24 of the 34 Spitzer SNSs have been either identified as previously discovered asteroids or linked to the Earth-based follow-up observations. Table 7 lists the resulting heliocentric and Spitzer-centric distances, as well as the corresponding phase angles at dates of Spitzer observations. For the identifications, an uncorrelated astrometric uncertainty

of $\sigma_{\text{R.A.}} = \sigma_{\text{Dec.}} = 0.7''$ was assumed for the observations obtained outside the current observing campaign. The fixed uncertainty is roughly equal to the estimated uncertainty (Carpino et al. 2003) of Lincoln Near Earth Asteroid Research (LINEAR) project, the largest provider of astrometric data for asteroids. By October 2004, almost 50% of all astrometric data of asteroids collected over the years had been obtained by LINEAR. The error bars correspond to 99.73% of the probability mass, which is equivalent to the $3\text{-}\sigma$ limits for a Gaussian distribution. Orbital inversion was performed with LSL for the previously known objects, and, depending on the observational data, with Ranging or VoV for all linkages and the designated asteroids 2004 BH₁₆₀ and 2004 BS₁₆₀ that were discovered and have been subsequently observed only during the current observing campaign. In Ranging and VoV, quantities stemming from the relative weights of the orbits that sample the orbital-element p.d.f. are used when estimating the uncertainties for the distances and the solar phase angle.

Ten Spitzer SNSs, that is, 0c, 0h, 0j, 5b, 5c, 5d, 5g, 5i, 5j, and 5l, remain unlinked and unidentified if we require that there are no clear trends in the $O - C$ residuals. We can extract MBA-type sample orbits with $2.1 \text{ AU} \leq a \leq 3.5 \text{ AU}$, $0 < e \leq 0.35$, $0^\circ \leq i \leq 35^\circ$, and $1.666 \dots \text{ AU} < q \leq 4.95 \text{ AU}$, where a is the semimajor axis, e is the eccentricity, i is the inclination, and q is the perihelion distance, from the orbital-element p.d.f.s of Spitzer SNSs lacking plausible linkages and compute the resulting ephemerides at the dates of the Earth-based follow-up observations. For example, in the 0° field on January 22nd, 0h might have resided in the VLT field and should have been in the CFHT field (Fig. 5).

[Insert Fig. 5 approximately here]

In the 5° field on January 22nd, all Spitzer objects that could not be linked to Earth-based follow-up reside in the CFHT field (Fig. 6).

[Insert Fig. 6 approximately here]

The total extent of the orbital-element p.d.f. stemming from unlinked and unidentified Spitzer SNSs is a lot larger, because the short observational time interval and limited amount of observations allow for a wide variety of different acceptable orbits. Hence it should be stressed that the arguments above are valid only for the assumption that the Spitzer objects are MBAs. If we would be interested in, for example, transneptunian objects (TNOs), we could proceed in a similar fashion but select TNO-like sample orbits to find out whether there are possibilities to find follow-up observations for TNOs (Granvik et al. 2003). Thus, the explanations for the hypothetical MBA observations apparently missing from the Earth-based follow-up are:

- they were outside of the fields covered by the follow-up,

- they were in the fields covered by the follow-up, but fell in CCD gaps in the mosaic array, were located in a dead CCD in the CFHT array, or appeared in front of a bright star,
- they were lost in the moving-object extraction process due to too complicated, too fast, or too slow motion (cf. problematic cases in the simulations in Sect. 4),
- they were too faint to be detected by the follow-up (cf. sky-plane densities of SNSs from different telescopes, particularly in the 5° field).

The last explanation is strengthened by preliminary asteroid size estimates from the Spitzer data (Tedesco et al. 2007, in preparation). Even when using only the highest-quality data, that is, Spitzer sources linked to known asteroids, 53% of the 0° field asteroids have diameters less than 3 km compared with 83% of the 5° field asteroids. This is significant, since most of the unlinked Spitzer sources are from the 5° field. The mean diameter and mean albedo for the five identified 5° asteroids is 1.7 km and 0.24, respectively, resulting in a mean $V = 21$. Thus, those unlinked likely have $V > 21$ at the time of the CFHT and VLT observations, and likely much greater since, e.g., a one-kilometer asteroid with geometric albedo $p_V = 0.05$ results in $V \sim 23.5$. Thus, it is not surprising that 10% to 30% of the Spitzer objects remain unlinked.

If the constraint regarding trends in the $O - C$ residuals is removed, we can find at least one linkage to all but four Spitzer SNSs, that is, 0c, 0h, 0j, and 5l. One could imagine that trends might erroneously arise due to exiguous data and comparatively large uncertainties, but since the behaviour of $O - C$ residuals stemming from scanty data has not been studied in depth, we assume that only those linkages which do not show any trends may be correct.

The importance of the parallax for reducing the distance and phase-angle uncertainties can be seen in Fig. 7.

[Insert Fig. 7 approximately here]

The distance uncertainties, the solar phase-angle uncertainties, and the observational angular arc—the angular difference between the observed coordinates of the first and the last observation—resulting from the real observations are given relative to the results obtained for simulated geocentric observations at the same dates as the real observations. Thus, the observational angular arc for the simulated observations only reflects the motions of the asteroid and the Earth, whereas the angular arc stemming from the real observations also depend on the different locations of the telescopes at a given instant, that is, the parallax. By fitting power-law functions to the

data in the least-squares sense, the following proportionalities are obtained:

$$\frac{\sigma_{r,\text{real}}}{\sigma_{r,\text{geoc}}} \propto \left(\frac{\Delta\alpha_{\text{real}}}{\Delta\alpha_{\text{geoc}}} \right)^{-1.7}, \quad \frac{\sigma_{\Delta,\text{real}}}{\sigma_{\Delta,\text{geoc}}} \propto \left(\frac{\Delta\alpha_{\text{real}}}{\Delta\alpha_{\text{geoc}}} \right)^{-1.7}, \quad \frac{\sigma_{\text{PA},\text{real}}}{\sigma_{\text{PA},\text{geoc}}} \propto \left(\frac{\Delta\alpha_{\text{real}}}{\Delta\alpha_{\text{geoc}}} \right)^{-1.5},$$

where σ is the length of the interval corresponding to 99.73% of the total probability mass, $\Delta\alpha$ is the observational angular arc, r is the heliocentric distance, Δ is the topocentric distance, PA is the solar phase angle, *real* denotes real observations, and *geoc* denotes simulated geocentric observations. Note that the ratio between the angular arcs depends on the observational time arc. The longer the time arc, the smaller the normalized angular arc. Hence, the relative reduction of the uncertainty is larger for objects having short observational time intervals than for objects having long observational time intervals (cf. Table 8). In the current case, only observational time intervals shorter than approximately two days lead to more than hundredfold reductions in the uncertainties as compared to a single geocentric observer.

Even though the $\gtrsim 1^\circ$ angular difference between Earth-based telescopes and the Spitzer spacecraft, as seen from an MBA, may seem small, the differences in the motion and relative locations of the observed MBAs is nevertheless striking when comparing the different viewing geometries. Fig. 8 shows simulated positions of nine identified Spitzer objects with well-established orbits at approximate real observation dates as seen by Spitzer and by a geocentric observer.

[Insert Fig. 8 approximately here]

The plot underscores the requirement that successful linking of large-parallax astrometry should be based on solid orbit computation methods that take into account aspects such as parallaxes, light-time corrections, and observational uncertainties. Notice that the apparent motion of most of the asteroids changes direction—the definition of the two stationary points—at different dates depending on the viewing direction.

6 Conclusions

As of November 6th, 2006, 20 of the 34 Spitzer FLS/EPC asteroid SNSs have been identified as objects previously known, or later accepted by the MPC. Of the remaining 14 SNSs, four can fairly reliably be linked with at least one SNS of follow-up observations, whereas ten SNSs (0c, 0h, 0j, 5b, 5c, 5d, 5g, 5i, 5j, and 5l) remain unidentified and unlinked. By relaxing the requirements regarding trends in the motion—a questionable requirement due to exiguous data which can easily lead to suspicious chance alignments—linkages or identifications can, nevertheless, be found to all but four Spitzer

SNSs (0c, 0h, 0j, and 5l). The results suggest that linkage chains containing at least five SNSs are required to produce unambiguous linkage configurations when linking large-parallax observations far from the direction of opposition.

Based on the simulations, we can, on one hand, be confident that virtually all, if not all, correct Spitzer linkages have been found. We have thus shown that the eMAC method is a viable and sensitive tool for solving large-parallax linking problems between exiguous SNSs of asteroid observations. On the other hand, the same simulations show that approximately 94% of the accepted Spitzer-related 2-linkages are erroneous. In the future, we therefore need to study the possibility to apply some statistical criterion to discard linkages showing, for example, trends in the $O - C$ residuals. For the eMAC method, we also need to improve the automated detection and treatment of the double maxima in the orbital-element p.d.f.. Clearly, the automated Ranging can already detect at least some of the double-maximum cases.

A future addition to our collection of linking methods is a statistical method that can link single observations to multiple-night observation sets, that is, perform the identification task by including orbital uncertainties. Here we have just used a naïve point-estimate method (Section 2.2) which may not have identified all known objects in the observational data, particularly those with short observational time intervals.

As compared to solely geocentric observations, the inclusion of parallax information reduces the distance uncertainty for the asteroids nearly simultaneously observed in the observing campaign by up to three orders of magnitude. The results are therefore particularly relevant for ESA's astrometric cornerstone mission Gaia to be launched in late 2011 according to the current plans. Gaia will operate in the vicinity of the second Lagrange point located approximately 1.5×10^6 km from the Earth in the antisolar direction. As compared to ground-based observations, a substantial parallax will be present in the observations of many solar-system objects (Mignard et al. 2007). An interesting question needing further study is whether the Gaia mission would benefit from additional Earth-based observations within, say, ± 24 hours of the detection by Gaia. The inclusion of parallax information might substantially reduce the orbital-element uncertainty of newly-discovered solar-system objects. Note that follow-up observations cannot be obtained continuously as Gaia will regularly scan regions eluding Earth-based observations such as solar elongations less than 55° . But the fact that Gaia will have a limiting magnitude of $V \approx 20$ mag would make even partial Earth-based follow-up attractive since fairly small telescopes would suffice.

Acknowledgements

The authors wish to thank the two anonymous referees whose thorough and constructive reviews helped to improve the manuscript. The research by MG and KM has been supported by the Univ. Helsinki three-year research grant, by the Academy of Finland, and by the A. Kordelin Foundation. MG is grateful to the M. Ehrnrooth Foundation and the Univ. Helsinki Chancellor for travel grants. DD's and ET's contributions were made possible by support from the National Aeronautics and Space Administration award NNG04GF40G issued through the Office of Space Science Research and Analysis Programs and Spitzer Space Telescope Archival Research grant number 14U552. The kind help provided by G. De Sanctis (Torino Observatory) in the reduction of VLT data is also acknowledged.

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Observational details for the 0° field

Day (UTC)	Telesc.	Field [deg ²]	N_{SNS}	$\frac{N_{\text{SNS}}}{\text{deg}^2}$	σ_{obs} ["]	Sol. elongation [°]
04/01/20	VLT	0.43	37	86	0.3	119.84–121.33
04/01/21	VLT	0.57	71	125	0.3	120.75–122.33
04/01/21	Spitzer	0.13	18	138	2.0	115.34–115.55
04/01/22	VLT	0.57	77	135	0.3	121.80–123.32
04/01/22	CFHT	2.5	227	91	0.5	121.40–123.94
04/01/23	VLT	0.57	45	79	0.3	122.78–124.24
04/01/24	VLT	0.1	12	120	0.3	124.69–125.31
04/01/30	CFHT	2.5	218	87	0.5	129.36–132.02

Table 1: Observational details for the 0° field. *Day* is the approximate observation date, *Telesc.* is the telescope used, *Field* is the field size without overlap, N_{SNS} is the number of SNSs, $N_{\text{SNS}} \text{ deg}^{-2}$ is the sky-plane density of SNSs, σ_{obs} is the assumed and later verified astrometric uncertainty, and *Sol. elongation* is the solar elongation interval.

Observational details for the 5° field

Day (UTC)	Telesc.	Field [deg ²]	N_{SNS}	$\frac{N_{\text{SNS}}}{\text{deg}^2}$	σ_{obs} ["]	Sol. elongation [°]
04/01/21	Spitzer	0.13	16	123	2.0	115.41–115.63
04/01/22	CFHT	2.5	89	36	0.5	121.31–123.67
04/01/30	CFHT	1.75	62	35	0.5	129.24–131.06

Table 2: Same as Table 1, but for the 5° field.

Simulation details for the 0° field

Virtual telescope	Field [deg ²]	$\bar{N}_{\text{SNS}} d$	$\frac{\bar{N}_{\text{SNS}}}{\text{deg}^2 d}$	σ_{obs} ["]	Sol. elongation [°]
Spitzer	0.19	29.0	152.6	2.0	115.30–115.54
VLT	0.96	137.5	143.2	0.4	119.66–125.40
CFHT	3.90	358.3	91.9	0.4	121.30–132.11

Table 3: Simulation details for the 0° field. *Virtual telescope* is the “virtual” telescope used, *Field* is the field size without overlap, $\bar{N}_{\text{SNS}} d^{-1}$ is the number of SNSs averaged over observation nights and the five different realizations, $\bar{N}_{\text{SNS}} \text{ deg}^{-2} d^{-1}$ is the sky-plane density of detected SNSs averaged over observation nights and five different realizations, σ_{obs} is the standard deviation of the Gaussian noise added to the astrometry, and *Sol. elongation* is the solar elongation interval.

Linking parameters	
Step 1: Phase-space address comparison	
Number of sample orbits	5,000
RA lower boundary	0°
RA upper boundary	360°
RA bin size	$\frac{1}{20}^\circ$
Dec lower boundary	-90°
Dec upper boundary	$+90^\circ$
Dec bin size	$\frac{1}{20}^\circ$
1st comparison date t_1	2004 01 24.0
2nd comparison date t_2	2004 01 26.0
3rd comparison date t_3	2004 01 28.0
Step 2a: Stepwise Ranging	
Maximum number of sample orbits	100 (at least two required)
Maximum number of trial orbits	$2 \times 10^4 \times N_{\text{obs}}$
Step 2b: Simplex optimization	
Maximum number of function evaluations	1,000
Termination criterion (χ_{max}^2)	$2 \times N_{\text{obs}} \times 2$
Step 2c: Least squares	
Maximum number of iterations	200

Table 4: Parameters used for the linking method. N_{obs} is the number of observations in an SNS, and χ_{max}^2 is the maximum χ^2 allowed for linkages.

Linking results for simulated data

Sim	SNS	Spitzer related						Not Spitzer related					
		C2L	2LD	C2LD	Sens	1 – Spec	PPV	C2L	2LD	C2LD	Sens	1 – Spec	PPV
#1	1337	79	1252	79	1.0	1.3×10^{-3}	0.063	2219	2986	2217	0.999	8.6×10^{-4}	0.742
#2	1435	100	1612	100	1.0	1.5×10^{-3}	0.062	2383	3110	2377	0.997	7.1×10^{-4}	0.764
#3	1367	65	1066	65	1.0	1.1×10^{-3}	0.061	2180	2671	2178	0.999	5.3×10^{-4}	0.815
#4	1521	90	1486	90	1.0	1.2×10^{-3}	0.061	2625	3687	2621	0.998	9.2×10^{-4}	0.711
#5	1505	82	1623	82	1.0	1.4×10^{-3}	0.051	2516	3377	2502	0.994	7.7×10^{-4}	0.741

Table 5: Linking results for the simulated 0° field data. *Sim* is the label for a realization of simulated data, and *SNS* is the number of SNSs in a realization. *Spitzer related* are linkages between SNSs one of which is a Spitzer SNSs, while *Not Spitzer related* relates to linkages between Earth-based observations. *C2L* is the number of correct 2-linkages to be detected, *2LD* is the total number of 2-linkages actually detected, that is, including both correct and erroneous links, and *C2LD* is the number of correct 2-linkages actually detected. *Sens* is the sensitivity for correct linkages, *1 – Spec* is the complement of specificity, and *PPV* is the positive predictive value.

Orbital elements for new linkages						
Spitzer SNS	a [AU]	e	i [°]	Ω [°]	ω [°]	M [°]
0e ₁	2.8 (2.4–480.6)	0.05 (0.01–0.999)	4.2 (4.0–5.9)	88.8	267.5	166.3
0e ₂	2.4 (2.3–490.5)	0.12 (0.05–0.999)	6.7 (5.0–14.2)	12.3	241.9	279.9
5e	2.4 (2.3–490.7)	0.15 (0.0004–0.999)	4.2 (4.2–6.6)	71.0	241.7	219.0
5f	2.4 (1.6–411.7)	0.24 (0.10–0.9997)	0.2 (0.1–1.0)	353.5	82.0	55.2
5k	3.1 (2.6–489.5)	0.15 (0.11–0.999)	0.9 (0.5–1.9)	160.9	0.5	0.6

Table 6: Maximum-likelihood orbits for five reasonable linkages containing Spitzer SNSs at the epoch 2004 Jan 26.0 (TT). The orbits are parameterized with Keplerian elements, where a is the semimajor axis, e is the eccentricity, i is the inclination, Ω is the longitude of the ascending node, ω is the argument of perihelion, and M is the mean anomaly. The orbital solutions were obtained with Ranging by using the two-body approximation, 10,000 sample orbits, $\Delta\chi^2 = 50$ between the best and the worst fit sample orbit corresponding to a probability mass of $1 - 4.7 \times 10^{-9}$, an upper limit of 500 AU for the semimajor axis, and an upper limit of 1.0 for the eccentricity. The resulting orbital-element p.d.f.s were multiplied with an apriori distribution derived from the astorb.dat file dated April 10, 2007. For a , e , and i the ranges of possible values are given in the parentheses. Note that the two linkages containing Spitzer SNS 0e are mutually exclusive.

Distance and solar phase-angle estimates for asteroids detected by Spitzer

Temp. des.	Date 040121.	Identified with/ linked to	Δt [days]	Distance [AU]		Solar phase angle [°]
				Spitzer	Sun	
0a	13923	2004 BH ₁₆₀	2.2	$1.81940^{+0.0012}_{-0.00067}$	$2.42111^{+0.0011}_{-0.00062}$	$21.8517^{+0.0060}_{-0.011}$
0a	18412	2004 BH ₁₆₀	2.2	$1.81956^{+0.00045}_{-0.0011}$	$2.42175^{+0.00042}_{-0.0011}$	$21.8373^{+0.010}_{-0.0041}$
0a	22889	2004 BH ₁₆₀	2.2	$1.81887^{+0.00099}_{-0.00099}$	$2.42161^{+0.00093}_{-0.00093}$	$21.8306^{+0.0089}_{-0.0089}$
0b	11203	2004 CA ₁₀₅	530.9	2.218986 ± 0.000030	2.796529 ± 0.000047	18.79077 ± 0.00078
0b	15693	2004 CA ₁₀₅	530.9	2.218434 ± 0.000030	2.796541 ± 0.000047	18.78363 ± 0.00078
0b	20170	2004 CA ₁₀₅	530.9	2.217883 ± 0.000030	2.796552 ± 0.000047	18.77650 ± 0.00079
0d	11451	2001 RP ₁₃₇	4610.9	2.36123 ± 0.00018	$2.93143505 \pm 0.00000089$	17.89738 ± 0.00034
0d	15942	2001 RP ₁₃₇	4610.9	2.36068 ± 0.00018	$2.93144418 \pm 0.00000089$	17.89071 ± 0.00034
0d	20418	2001 RP ₁₃₇	4610.9	2.36013 ± 0.00018	$2.93145327 \pm 0.00000089$	17.88405 ± 0.00034
0e ₁	11534	CFHT 22	1.5	$1.5284^{+0.0065}_{-0.0085}$	$2.1546^{+0.0059}_{-0.0077}$	$24.717^{+0.087}_{-0.080}$
0e ₁	16051	CFHT 22	1.5	$1.5271^{+0.0070}_{-0.0083}$	$2.1538^{+0.0064}_{-0.0075}$	$24.710^{+0.092}_{-0.078}$
0e ₁	20527	CFHT 22	1.5	$1.5259^{+0.0082}_{-0.0063}$	$2.1532^{+0.0075}_{-0.0069}$	$24.702^{+0.077}_{-0.084}$
0e ₂	11534	CFHT 30	9.5	$2.110^{+0.076}_{-0.064}$	$2.695^{+0.060}_{-0.066}$	$19.59^{+0.45}_{-0.50}$
0e ₂	16051	CFHT 30	9.5	$2.109^{+0.076}_{-0.064}$	$2.694^{+0.060}_{-0.066}$	$19.59^{+0.41}_{-0.59}$
0e ₂	20527	CFHT 30	9.5	$2.109^{+0.077}_{-0.070}$	$2.694^{+0.060}_{-0.066}$	$19.54^{+0.45}_{-0.54}$
0f	11675	2001 QD ₄₉	2036.9	2.191101 ± 0.000015	$2.77047213 \pm 0.00000093$	18.969916 ± 0.000095
0f	16165	2001 QD ₄₉	2036.9	2.190470 ± 0.000015	$2.77039671 \pm 0.00000093$	18.963447 ± 0.000095
0f	20642	2001 QD ₄₉	2036.9	2.189840 ± 0.000015	$2.77032149 \pm 0.00000093$	18.956983 ± 0.000095
0g	14660	2004 EU ₉₁	60.2	1.79793 ± 0.00097	2.40069 ± 0.00065	22.0554 ± 0.0035
0g	19149	2004 EU ₉₁	60.2	1.79745 ± 0.00097	2.40074 ± 0.00065	22.0468 ± 0.0035
0g	23626	2004 EU ₉₁	60.2	1.79697 ± 0.00097	2.40078 ± 0.00065	22.0383 ± 0.0034
0i	13348	2004 BS ₁₆₀	2.2	$1.8066^{+0.0042}_{-0.0046}$	$2.4086^{+0.0039}_{-0.0043}$	$21.981^{+0.041}_{-0.038}$
0i	17838	2004 BS ₁₆₀	2.2	$1.8059^{+0.0027}_{-0.0030}$	$2.4085^{+0.0025}_{-0.0028}$	$21.975^{+0.026}_{-0.024}$
0i	22314	2004 BS ₁₆₀	2.2	$1.8053^{+0.0013}_{-0.0014}$	$2.4084^{+0.0012}_{-0.0013}$	$21.967^{+0.012}_{-0.011}$
0k	11590	53273	2356.8	2.5335036 ± 0.0000014	$3.09560473 \pm 0.00000040$	16.9211871 ± 0.0000073
0k	16080	53273	2356.8	2.5329025 ± 0.0000014	$3.09557930 \pm 0.00000040$	16.9150427 ± 0.0000073

0k	20556	53273	2356.8	2.5323034 ± 0.0000014	$3.09555395 \pm 0.00000040$	16.9089063 ± 0.0000073
0l	12118	99307	2600.1	2.4548808 ± 0.0000028	3.0206168 ± 0.0000011	17.352729 ± 0.000012
0l	16634	99307	2600.1	2.4543488 ± 0.0000028	3.0206590 ± 0.0000011	17.345967 ± 0.000012
0l	21111	99307	2600.1	2.4538215 ± 0.0000028	3.0207009 ± 0.0000011	17.339252 ± 0.000012
0m*	10893	79364	2584.9	1.3585168 ± 0.0000024	$2.00081576 \pm 0.00000020$	26.766384 ± 0.000018
0m*	17486	79364	2584.9	1.3579593 ± 0.0000024	$2.00091509 \pm 0.00000020$	26.751241 ± 0.000018
0m*	21988	79364	2584.9	1.3575786 ± 0.0000024	$2.00098295 \pm 0.00000020$	26.740879 ± 0.000018
0n	13056	27293	6989.1	$1.80137149 \pm 0.00000095$	$2.40456549 \pm 0.00000013$	22.006866 ± 0.000010
0n	17983	27293	6989.1	$1.80080668 \pm 0.00000095$	$2.40456169 \pm 0.00000013$	21.998256 ± 0.000010
0n	22460	27293	6989.1	$1.80029363 \pm 0.00000095$	$2.40455823 \pm 0.00000013$	21.990417 ± 0.000010
0o	12975	136040	516.9	2.111783 ± 0.000065	2.694704 ± 0.000099	19.53664 ± 0.00035
0o	17465	136040	516.9	2.111277 ± 0.000065	2.694733 ± 0.000099	19.52937 ± 0.00035
0o	21941	136040	516.9	2.110773 ± 0.000065	2.694763 ± 0.000099	19.52211 ± 0.00035
0p*	15372	7987	9263.7	$2.53923121 \pm 0.00000095$	$3.10158503 \pm 0.00000012$	16.8818821 ± 0.0000076
0p*	19804	7987	9263.7	$2.53872186 \pm 0.00000095$	$3.10165328 \pm 0.00000012$	16.8751508 ± 0.0000076
0q	15043	9981	10010.4	$2.06739275 \pm 0.00000087$	$2.65237853 \pm 0.00000016$	19.8687768 ± 0.0000060
0q	19452	9981	10010.4	$2.06687516 \pm 0.00000087$	$2.65239077 \pm 0.00000016$	19.8615777 ± 0.0000060
0q	23955	9981	10010.4	$2.06634672 \pm 0.00000087$	$2.65240327 \pm 0.00000016$	19.8542124 ± 0.0000060
0r	13048	83004	4395.1	2.3916967 ± 0.0000019	$2.95989876 \pm 0.00000095$	17.7259524 ± 0.0000093
0r	17538	83004	4395.1	2.3911934 ± 0.0000019	$2.95996045 \pm 0.00000095$	17.7189914 ± 0.0000093
0r	22015	83004	4395.1	2.3906917 ± 0.0000019	$2.96002196 \pm 0.00000095$	17.7120392 ± 0.0000093
5a	24488	2004 EE ₅₈	866.0	2.41202 ± 0.00023	2.98109 ± 0.00022	17.57386 ± 0.00083
5a	28970	2004 EE ₅₈	866.0	2.41147 ± 0.00023	2.98110 ± 0.00022	17.56730 ± 0.00083
5a	33446	2004 EE ₅₈	866.0	2.41092 ± 0.00023	2.98111 ± 0.00022	17.56073 ± 0.00083
5e	25324	CFHT 30	9.4	$2.258^{+0.057}_{-0.10}$	$2.835^{+0.054}_{-0.096}$	$18.54^{+0.66}_{-0.37}$
5e	30999	CFHT 30	9.4	$2.257^{+0.057}_{-0.10}$	$2.835^{+0.054}_{-0.096}$	$18.53^{+0.66}_{-0.37}$
5e	35476	CFHT 30	9.4	$2.231^{+0.083}_{-0.076}$	$2.835^{+0.054}_{-0.096}$	$18.52^{+0.66}_{-0.37}$
5f	24933	CFHT 30	9.4	$1.855^{+0.059}_{-0.046}$	$2.460^{+0.051}_{-0.042}$	$21.51^{+0.35}_{-0.50}$
5f	29416	CFHT 30	9.4	$1.855^{+0.059}_{-0.046}$	$2.448^{+0.064}_{-0.030}$	$21.50^{+0.35}_{-0.50}$

5f	33892	CFHT 30	9.4	$1.854^{+0.059}_{-0.046}$	$2.460^{+0.051}_{-0.043}$	$21.50^{+0.35}_{-0.51}$
5h	25653	2004 BU ₉₉	96.7	1.54185 ± 0.00019	2.16760 ± 0.00024	24.53328 ± 0.0026
5h	30137	2004 BU ₉₉	96.7	1.54137 ± 0.00019	2.16758 ± 0.00024	24.52515 ± 0.0026
5h	34614	2004 BU ₉₉	96.7	1.54090 ± 0.00019	2.16757 ± 0.00024	24.51702 ± 0.0026
5k	26534	CFHT 22	1.4	$2.112^{+0.011}_{-0.013}$	$2.696^{+0.010}_{-0.012}$	$19.509^{+0.092}_{-0.077}$
5k	31017	CFHT 22	1.4	$2.1069^{+0.015}_{-0.0085}$	$2.695^{+0.011}_{-0.011}$	$19.511^{+0.083}_{-0.083}$
5k	35493	CFHT 22	1.4	$2.10965^{+0.011}_{-0.013}$	$2.695^{+0.010}_{-0.012}$	$19.503^{+0.091}_{-0.076}$
5m	24347	49266	5377.8	1.6070548 ± 0.0000021	$2.22719893 \pm 0.00000016$	23.833344 ± 0.000020
5m	28830	49266	5377.8	1.6065930 ± 0.0000021	$2.22721175 \pm 0.00000016$	23.824913 ± 0.000020
5m	33306	49266	5377.8	1.6061321 ± 0.0000021	$2.22722456 \pm 0.00000016$	23.816481 ± 0.000020
5n	24430	2000 CX ₁₈	1990.0	1.641886 ± 0.000054	2.2586299 ± 0.0000060	23.49021 ± 0.00033
5n	28912	2000 CX ₁₈	1990.0	1.641491 ± 0.000054	2.2587322 ± 0.0000060	23.48047 ± 0.00033
5n	33389	2000 CX ₁₈	1990.0	1.641097 ± 0.000054	2.2588344 ± 0.0000060	23.47073 ± 0.00033
5o	26666	113186	2183.7	2.1061442 ± 0.0000026	$2.68953897 \pm 0.00000096$	19.573259 ± 0.000047
5o	31148	113186	2183.7	2.1055905 ± 0.0000026	$2.68954064 \pm 0.00000096$	19.565932 ± 0.000047
5o	35654	113186	2183.7	2.1050340 ± 0.0000026	$2.68954231 \pm 0.00000096$	19.558552 ± 0.000047
5p	28636	2001 TQ ₁₃₈	3714.1	2.496226 ± 0.000043	3.0601076 ± 0.0000048	17.12140 ± 0.00039
5p	33118	2001 TQ ₁₃₈	3714.1	2.495640 ± 0.000043	3.0600933 ± 0.0000048	17.11513 ± 0.00039
5p	37594	2001 TQ ₁₃₈	3714.1	2.495055 ± 0.000043	3.0600791 ± 0.0000048	17.10887 ± 0.00039

Table 7: Distances and phase angles at the Spitzer observation dates for all identifications and selected linkages containing Spitzer sources. *Temp. des.* is the temporary designation for a moving object detected by Spitzer. Different subscripts refer to mutually exclusive linkages. Each object was observed two or three times at different dates by Spitzer, and each line refers to one of these observations. *Date* gives the observation date as a fractional part of the 2004 January 21st (UTC) when the Spitzer observations were made. *Identified with/linked to* gives either the number or the designation of a previously known object, or the telescope and dates of possible Earth-based follow-up observations in case the Spitzer object was not identified with a known object. Δt gives the length of the observational time arc. *Distance* gives the distances from the asteroid to the Spitzer spacecraft and the Sun, respectively, and *Solar phase angle* gives the phase angle with respect to Spitzer. The nominal value of the three last columns corresponds to the least-squares solution (LSL) or the maximum of a 50-column histogram of the marginal distribution (Ranging and VoV), while the error bars correspond to 99.73% of the total probability mass computed from the covariance matrix (LSL) or the histogram (Ranging and VoV). The identifications marked with an asterisk have at least one Spitzer $O - C$ residual larger than $6''$. Note that 0a and 0i have only been observed within the current observing campaign. Note also that, for 0i, we found an additional linkage which was missed by the Väisälä-based routines.

Impact of parallax on the distance and solar phase-angle uncertainties

Number / Designation	Date 040121.	SNSs	N_{obs}	Δt [days]	$\Delta \alpha$ [°]	Distance [AU]		Solar phase angle [°]
						Spitzer	Sun	
7987	15372	0p, CFHT 22	5	1.5	1.0531	$2.543^{+0.022}_{-0.020}$	$3.105^{+0.021}_{-0.019}$	$16.86^{+0.11}_{-0.12}$
7987	15372	GEOC 21 22	5	1.5	0.0291	$22.88^{+0.11}_{-0.21}$	$23.34^{+0.11}_{-0.22}$	19^{+13}_{-15}
9981	15043	0q, CFHT 22 30	9	9.5	1.2948	$2.0677^{+0.0014}_{-0.0019}$	$2.6526^{+0.0013}_{-0.0018}$	$19.867^{+0.014}_{-0.010}$
9981	15043	GEOC 21 22 30	9	9.5	0.1201	$2.077^{+0.054}_{-0.059}$	$2.662^{+0.042}_{-0.060}$	$19.79^{+0.48}_{-0.32}$
27293	13056	0n, VLT 21 22 23, CFHT 30	15	9.5	1.2989	$1.80118^{+0.0012}_{-0.00073}$	$2.40439^{+0.0011}_{-0.00068}$	$22.0086^{+0.0067}_{-0.011}$
27293	13056	GEOC 21 22 23 30	15	9.5	0.4776	$1.7998^{+0.0057}_{-0.0024}$	$2.4031^{+0.0054}_{-0.0022}$	$22.021^{+0.022}_{-0.052}$
99307	12118	0l, CFHT 30	6	9.5	1.3011	$2.389^{+0.20}_{-0.051}$	$3.095^{+0.048}_{-0.19}$	$16.92^{+1.2}_{-0.29}$
99307	12118	GEOC 21 30	6	9.5	0.2769	$2.73^{+0.26}_{-0.60}$	$3.29^{+0.20}_{-0.57}$	$15.90^{+3.6}_{-0.96}$
113186	26666	5o, CFHT 22	6	1.4	1.2904	$2.109^{+0.011}_{-0.012}$	$2.692^{+0.010}_{-0.011}$	$19.541^{+0.098}_{-0.060}$
113186	26666	GEOC 21 22	6	1.4	0.0237	$6.8^{+1.4}_{-5.4}$	$7.3^{+1.2}_{-5.6}$	$14.8^{+23}_{-8.3}$
136040	12975	0o, VLT 21, CFHT 22	11	1.5	1.1854	$2.1099^{+0.0022}_{-0.0017}$	$2.6929^{+0.0020}_{-0.0016}$	$19.550^{+0.012}_{-0.016}$
136040	12975	GEOC 21 22	11	1.5	0.0553	$2.14^{+1.5}_{-0.78}$	$2.72^{+1.4}_{-0.79}$	$19.3^{+46}_{-7.0}$
2001 QD ₄₉	11675	0f, CFHT 30	9	9.5	1.2978	$2.171^{+0.11}_{-0.063}$	$2.728^{+0.12}_{-0.036}$	$19.12^{+0.42}_{-0.71}$
2001 QD ₄₉	11675	GEOC 21 30	9	9.5	0.1141	$2.22^{+0.22}_{-0.23}$	$2.70^{+0.32}_{-0.13}$	$19.5^{+1.1}_{-2.2}$
2001 RP ₁₃₇	11451	0d, CFHT 30	6	9.5	1.2174	$2.36^{+0.14}_{-0.11}$	$2.93^{+0.13}_{-0.10}$	$17.56^{+1.1}_{-0.38}$
2001 RP ₁₃₇	11451	GEOC 21 30	6	9.5	0.1123	$2.35^{+0.47}_{-0.28}$	$2.92^{+0.40}_{-0.26}$	$18.0^{+1.9}_{-2.2}$
2001 TQ ₁₃₈	28637	5p, CFHT 30	9	9.4	1.4469	$2.49^{+0.11}_{-0.10}$	$3.056^{+0.10}_{-0.096}$	$17.18^{+0.55}_{-0.59}$
2001 TQ ₁₃₈	28637	GEOC 21 30	9	9.4	0.5040	$2.47^{+0.39}_{-0.25}$	$3.04^{+0.38}_{-0.23}$	$17.3^{+1.5}_{-2.0}$
2004 BH ₁₆₀	13923	0a, VLT 21 22 23	9	2.2	1.4221	$1.8194^{+0.0015}_{-0.0012}$	$2.4211^{+0.0014}_{-0.0011}$	$21.852^{+0.010}_{-0.014}$
2004 BH ₁₆₀	13923	GEOC 21 22 23	9	2.2	0.0474	$1.813^{+0.077}_{-0.062}$	$2.415^{+0.072}_{-0.057}$	$21.91^{+0.59}_{-0.71}$
2004 BS ₁₆₀	13348	0i, VLT 23, CFHT 22 30	14	9.5	1.3417	$1.80720^{+0.00093}_{-0.0020}$	$2.40919^{+0.00087}_{-0.0019}$	$21.9751^{+0.018}_{-0.0085}$
2004 BS ₁₆₀	13348	GEOC 21 22 23 30	14	9.5	0.1401	$1.811^{+0.012}_{-0.020}$	$2.413^{+0.011}_{-0.020}$	$21.94^{+0.19}_{-0.10}$
2004 EU ₉₁	14660	0g, CFHT 22 30	12	9.5	1.6446	$1.7983^{+0.0014}_{-0.0015}$	$2.4010^{+0.0013}_{-0.0014}$	$22.053^{+0.014}_{-0.013}$
2004 EU ₉₁	14660	GEOC 21 22 30	12	9.5	0.2926	$1.808^{+0.042}_{-0.052}$	$2.411^{+0.040}_{-0.048}$	$21.96^{+0.51}_{-0.39}$

Table 8: A comparison between the distance and phase-angle uncertainties obtained with substantial parallax information (real observations; temporary Spitzer designation, VLT, CFHT), and the uncertainties obtained without substantial parallax information (simulated geocentric observations; GEOC). *Number/Designation* refers to the observed object. *Date* gives the observation date as a fractional part of the 2004 January 21st (UTC) when the Spitzer observations were made, while the “SNSs” column gives the temporary Spitzer designation as well as telescopes and approximate observation dates in January 2004 of other SNSs. N_{obs} is the number of observations, Δt is the observational time interval, and $\Delta\alpha$ is the observational angular arc. The nominal values of the Spitzer-centric distance, the heliocentric distance, and the solar phase angle corresponds to the maximum of a 50-column histogram of the marginal distribution, while the error bars correspond to 99.73% of the total probability mass computed from the same histogram. In the simulations, all other parameters but the observer location are held constant. In the orbital inversion, the same noise that was assumed for the real observations is also assumed for the simulated observations, but no random errors were added to the simulated observations, that is, the simulated positions are the nominal ones stemming from the least-squares orbit computed for all available data.

Temp. des.	Date 040121.	R.A.			Dec.			Spitzer's heliocentric ecliptic position		
		h	m	s	°	'	''	X [AU]	Y [AU]	Z [AU]
0a	13923	12	03	28.15	-00	22	13.8	-0.44252866	0.89395611	0.00705056
0a	18412	12	03	28.66	-00	22	15.6	-0.44322297	0.89360740	0.00703611
0a	22889	12	03	29.18	-00	22	23.9	-0.44391515	0.89325910	0.00702170
0b	11203	12	02	39.34	-00	38	46.7	-0.44210784	0.89416714	0.00705931
0b	15693	12	02	39.46	-00	38	57.1	-0.44280246	0.89381868	0.00704486
0b	20170	12	02	39.53	-00	39	07.5	-0.44349480	0.89347070	0.00703046
0c	11396	12	02	49.39	-00	34	32.5	-0.44213770	0.89415217	0.00705868
0c	15886	12	02	50.16	-00	34	48.7	-0.44283231	0.89380369	0.00704424
0c	20334	12	02	50.88	-00	35	06.0	-0.44352016	0.89345794	0.00702993
0d	11451	12	02	51.67	-00	33	34.9	-0.44214621	0.89414791	0.00705851
0d	15942	12	02	52.22	-00	33	36.3	-0.44284097	0.89379934	0.00704406
0d	20418	12	02	52.68	-00	33	38.2	-0.44353314	0.89345140	0.00702966
0e	11534	12	02	47.81	-00	30	48.2	-0.44215905	0.89414147	0.00705824
0e	16051	12	02	48.67	-00	30	51.5	-0.44285783	0.89379087	0.00704371
0e	20527	12	02	49.51	-00	30	55.1	-0.44355000	0.89344292	0.00702931
0f	11675	12	02	52.27	-00	28	19.9	-0.44218087	0.89413053	0.00705779
0f	16165	12	02	52.82	-00	28	22.4	-0.44287546	0.89378202	0.00704334
0f	20642	12	02	53.35	-00	28	25.7	-0.44356778	0.89343398	0.00702894
0g	14660	12	03	11.45	-00	39	02.2	-0.44264267	0.89389890	0.00704819
0g	19149	12	03	12.24	-00	39	01.4	-0.44333693	0.89355010	0.00703374
0g	23626	12	03	13.01	-00	39	00.4	-0.44402907	0.89320171	0.00701933
0h	13212	12	04	01.22	-00	07	06.6	-0.44241867	0.89401129	0.00705284
0h	17702	12	04	01.01	-00	07	28.9	-0.44311317	0.89366259	0.00703840
0h	22179	12	04	00.84	-00	07	50.2	-0.44380539	0.89331437	0.00702399
0i	13348	12	04	00.12	-00	10	53.8	-0.44243971	0.89400074	0.00705241
0i	17838	12	04	00.96	-00	10	59.9	-0.44313420	0.89365202	0.00703796
0i	22314	12	04	01.82	-00	11	04.9	-0.44382626	0.89330386	0.00702356
0j	10809	12	02	19.45	-00	48	29.6	-0.44204687	0.89419769	0.00706057
0j	15299	12	02	19.27	-00	48	34.7	-0.44274151	0.89384928	0.00704613
0j	19776	12	02	19.15	-00	48	40.1	-0.44343388	0.89350134	0.00703172
0k	11590	12	03	00.50	-00	31	19.6	-0.44216772	0.89413713	0.00705806
0k	16080	12	03	01.08	-00	31	15.2	-0.44286231	0.89378862	0.00704362
0k	20556	12	03	01.61	-00	31	10.5	-0.44355448	0.89344067	0.00702921
0l	12118	12	03	20.71	-00	19	01.6	-0.44224941	0.89409617	0.00705636
0l	16634	12	03	21.19	-00	18	59.8	-0.44294800	0.89374558	0.00704183
0l	21111	12	03	21.62	-00	18	58.3	-0.44364029	0.89339749	0.00702743
0m	10893	12	02	35.95	-00	45	40.3	-0.44205987	0.89419118	0.00706030
0m	17486	12	02	37.46	-00	45	44.3	-0.44307977	0.89367938	0.00703909

0m	21989	12	02	38.97	-00	45	47.5	-0.44377602	0.89332916	0.00702460
0n	13056	12	03	41.93	-00	10	52.3	-0.44239454	0.89402340	0.00705335
0n	17983	12	03	42.79	-00	11	04.9	-0.44315662	0.89364075	0.00703749
0n	22460	12	03	43.51	-00	11	15.7	-0.44384883	0.89329249	0.00702309
0o	12975	12	03	09.24	-00	31	25.3	-0.44238200	0.89402968	0.00705361
0o	17465	12	03	10.01	-00	31	28.9	-0.44307652	0.89368101	0.00703916
0o	21941	12	03	10.75	-00	31	34.0	-0.44376860	0.89333289	0.00702476
0p	15372	12	02	30.40	-00	47	57.2	-0.44275281	0.89384361	0.00704589
0p	19804	12	02	30.23	-00	47	53.8	-0.44343821	0.89349916	0.00703163
0q	15043	12	02	59.16	-00	46	19.2	-0.44270192	0.89386916	0.00704695
0q	19452	12	02	59.86	-00	46	20.6	-0.44338378	0.89352654	0.00703277
0q	23955	12	03	00.62	-00	46	21.7	-0.44407992	0.89317608	0.00701827
0r	13048	12	04	02.06	-00	02	28.0	-0.44239330	0.89402402	0.00705337
0r	17538	12	04	02.62	-00	02	26.5	-0.44308781	0.89367534	0.00703893
0r	22015	12	04	03.14	-00	02	24.0	-0.44378004	0.89332713	0.00702452

Table 9: Topocentric astrometry of moving objects in the Spitzer 0° field. *Temp. des.* is the temporary designation, and *Date* is the fractional day on 2004 January 21 given in UTC. *R.A.* and *Dec.* are the observed astrometric coordinates given in the equatorial J2000 reference frame. *Spitzer's heliocentric ecliptic position* is the position of the Spitzer spacecraft at the observation date.

Temp. des.	Date 040121.	R.A.			Dec.			Spitzer's heliocentric ecliptic position		
		h	m	s	°	'	''	X [AU]	Y [AU]	Z [AU]
5a	24488	12	10	25.70	03	51	08.6	-0.44416230	0.89313456	0.00701656
5a	28970	12	10	26.11	03	51	05.0	-0.44485490	0.89278514	0.00700213
5a	33446	12	10	26.54	03	51	01.4	-0.44554630	0.89243565	0.00698771
5b	28198	12	10	56.54	03	56	47.4	-0.44473562	0.89284536	0.00700461
5b	32680	12	10	57.50	03	56	51.4	-0.44542799	0.89249550	0.00699018
5b	37156	12	10	58.42	03	56	53.9	-0.44611918	0.89214556	0.00697576
5c	28581	12	10	54.70	03	49	22.1	-0.44479480	0.89281549	0.00700338
5c	33064	12	10	55.78	03	49	26.8	-0.44548730	0.89246550	0.00698894
5c	37486	12	10	56.78	03	49	31.1	-0.44617012	0.89211974	0.00697470
5d	26431	12	11	04.32	04	04	54.1	-0.44446258	0.89298315	0.00701030
5d	30914	12	11	05.02	04	04	58.4	-0.44515522	0.89263342	0.00699587
5d	35391	12	11	05.69	04	05	02.0	-0.44584666	0.89228362	0.00698145
5e	25324	12	11	05.33	04	06	51.1	-0.44429151	0.89306943	0.00701387
5e	30999	12	11	06.00	04	06	53.6	-0.44516835	0.89262678	0.00699559

5e	35476	12	11	06.65	04	06	55.4	-0.44585978	0.89227697	0.00698117
5f	24933	12	10	51.31	04	00	57.2	-0.44423108	0.89309989	0.00701513
5f	29416	12	10	52.25	04	00	52.5	-0.44492380	0.89275034	0.00700069
5f	33892	12	10	53.18	04	00	47.5	-0.44561518	0.89240080	0.00698628
5g	27953	12	11	15.86	04	02	06.7	-0.44469777	0.89286447	0.00700540
5g	32436	12	11	16.77	04	02	09.6	-0.44539031	0.89251456	0.00699097
5g	36912	12	11	17.69	04	02	11.8	-0.44608150	0.89216465	0.00697655
5h	25654	12	11	08.09	04	17	13.2	-0.44434251	0.89304371	0.00701280
5h	30137	12	11	09.48	04	16	59.2	-0.44503519	0.89269407	0.00699837
5h	34614	12	11	10.87	04	16	45.5	-0.44572667	0.89234436	0.00698395
5i	26146	12	11	23.74	04	27	33.8	-0.44441854	0.89300537	0.00701122
5i	30654	12	11	25.32	04	27	30.2	-0.44511505	0.89265371	0.00699670
5i	35131	12	11	26.88	04	27	27.7	-0.44580651	0.89230394	0.00698228
5j	26447	12	11	39.19	04	28	03.0	-0.44446506	0.89298190	0.00701025
5j	30930	12	11	39.89	04	28	07.3	-0.44515769	0.89263217	0.00699582
5j	35406	12	11	40.56	04	28	12.4	-0.44584897	0.89228244	0.00698140
5k	26534	12	11	32.98	04	25	39.3	-0.44447850	0.89297512	0.00700997
5k	31017	12	11	33.72	04	25	40.8	-0.44517113	0.89262537	0.00699554
5k	35493	12	11	34.44	04	25	43.0	-0.44586241	0.89227564	0.00698112
5l	27145	12	11	39.26	04	19	40.4	-0.44457292	0.89292748	0.00700800
5l	31628	12	11	41.88	04	19	38.6	-0.44526551	0.89257766	0.00699357
5l	36104	12	11	44.54	04	19	35.4	-0.44595675	0.89222786	0.00697915
5m	24347	12	10	16.06	03	48	12.2	-0.44414051	0.89314555	0.00701701
5m	28830	12	10	17.54	03	48	12.6	-0.44483327	0.89279606	0.00700258
5m	33306	12	10	19.01	03	48	14.0	-0.44552468	0.89244659	0.00698816
5n	24431	12	10	29.18	03	49	34.0	-0.44415349	0.89313900	0.00701674
5n	28913	12	10	30.36	03	49	45.1	-0.44484609	0.89278959	0.00700231
5n	33390	12	10	31.54	03	49	56.6	-0.44553765	0.89244002	0.00698789
5o	26666	12	12	03.94	04	29	11.0	-0.44449890	0.89296483	0.00700955
5o	31148	12	12	04.37	04	29	12.1	-0.44519136	0.89261515	0.00699511
5o	35654	12	12	04.78	04	29	13.9	-0.44588727	0.89226305	0.00698060
5p	28637	12	10	56.28	03	46	23.9	-0.44480345	0.89281112	0.00700320
5p	33119	12	10	56.83	03	46	29.3	-0.44549580	0.89246120	0.00698877
5p	37595	12	10	57.41	03	46	35.4	-0.44618695	0.89211121	0.00697435

Table 10: As Table 9, but for the 5° field.

Figure 1. Ecliptic longitudes and latitudes for the first observation of each SNSs in the 0° field SNSs. In the legend (top left) a single symbol refer to SNSs obtained in January 2004. The designation letters for Spitzer SNSs are shown in the lower left next to the SNSs. Notice the substantial difference between the apparent coordinates of Spitzer SNSs as compared to coordinates of CFHT and VLT SNSs. The observation geometry was planned in advance to allow the same MBAs to be detected from all three locations despite the parallax.

Figure 2. As in Fig. 1, but for the 5° field.

Figure 3. An example of ecliptic longitudes and latitudes of a simulated 0° field set of SNSs (realization #4). As in Figs. 1 and 2, the coordinates refer to the first observation in each SNS. Black dots refer to SNSs that can be linked to the simulated Spitzer observations including the simulated Spitzer SNSs themselves.

Figure 4. $O - C$ residuals of observations that can be identified with known objects. The ephemerides are computed based on the orbits found in the astorb.dat file dated November 6, 2006. The rms values are $\sim 1.8''$ for the Spitzer observations, $\sim 0.52''$ for the CFHT observations, and $\sim 0.27''$ for the VLT observations. Three identified Spitzer observations result in $O - C$ residuals larger than $10''$ and were not taken into account.

Figure 5. A comparison between geocentric ephemerides for unidentified and unlinked Spitzer objects and the follow-up observations on January 22nd for the 0° field. The ephemerides are based on a subset of the orbits that are used to sample the orbital-element p.d.f. with the assumption that Spitzer detected MBAs.

Figure 6. Same as Fig. 5 but with CFHT-centric ephemerides for the 5° field.

Figure 7. The normalized heliocentric and Spitzer-centric distance uncertainties (left and center, respectively) and the solar phase-angle uncertainties

(right) as functions of the normalized observational angular arc. The distance uncertainties, the solar phase-angle uncertainties, and the observational angular arc resulting from the real observations, $\sigma_{r,\text{real}}$, $\sigma_{\Delta,\text{real}}$, $\sigma_{\text{PA},\text{real}}$, and $\Delta\alpha_{\text{real}}$, respectively, are given relative to the results obtained for simulated geocentric observations at the same dates as the real observations, $\sigma_{r,\text{geoc}}$, $\sigma_{\Delta,\text{geoc}}$, $\sigma_{\text{PA},\text{geoc}}$, and $\Delta\alpha_{\text{geoc}}$, respectively. The lines corresponds to the least-squares-fitted power-law functions $y \propto x^b$, where b is -1.7 for the distance uncertainties, and -1.5 for the phase-angle uncertainty.

Figure 8. Ephemerides for nine identified and correctly linked Spitzer objects as seen by a Spitzer-centric observer (left) and by a geocentric observer (right). The computed positions correspond to approximate real observation dates by different telescopes in January 2004 (UTC): 20.289 (VLT), 21.124 (Spitzer), 21.289 (VLT), 22.289 (VLT), 22.589 (CFHT), 23.289 (VLT), 24.289 (VLT), and 30.589 (CFTH). The $\gtrsim 1^\circ$ angular separation between the Spitzer spacecraft and Earth-bound observatories as seen from the MBAs lead to strikingly different motions and relative locations depending on the observer location. Note that the lines do not correspond to the precise paths of the objects, but merely connects simulated positions of the same object to help guide the eye.

Figure 1:

Figure 2:

Figure 3:

Figure 4:

Figure 5:

Figure 6:

Figure 7:

Figure 8: