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The long term stability of coorbital moons of the satellites of Saturn:

I. Conservative case

Apostolos A. Christou¹ Fathi Namouni² Maria Helena Moreira Morais³

¹Armagh Observatory, College Hill, Armagh BT61 9DG, Northern Ireland, United Kingdom

²CNRS, Observatoire de la Côte d'Azur, BP 4229, 06304, Nice, France

³GAUC-University of Coimbra, Largo D. Dinis, 3000, Coimbra, Portugal

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Corresponding Author:

Dr. Apostolos A. Christou

Armagh Observatory

College Hill

Armagh BT61 9DG

Northern Ireland

United Kingdom

Tel: +44 28 3751 2953

FAX: +44 28 3752 7174

E-mail: aac@star.arm.ac.uk

Abstract

The aim of this work is to understand the absence of objects along the orbits of Mimas and Enceladus in contrast to their presence at the orbits of neighbouring Tethys and Dione from the point of view of dynamical stability. Large scale numerical simulations of 360 test particles within the coorbital regions of these four saturnian satellites were carried out for 4×10^5 yr or 1.6×10^8 revolutions of the innermost moon Mimas. The tidal forcing of the satellites' orbits was not taken into account in these simulations. We have quantitatively reproduced the Mimas-Tethys 4:2 and Enceladus-Dione 2:1 mean motion resonances in the system and devised a scheme by which the parameter space of the coorbital resonance is sampled uniformly by our test particles. We observe that 6 out of the 36 integrated horseshoe particles of Enceladus escaped the coorbital region. All 54 tadpole particles remained stable. The main cause of instability for Enceladus coorbitals appears to be the overlap between the coorbital resonance and the 2:1 mean motion resonance between the particle and Dione. This leads particles with starting semimajor axes near the horseshoe-tadpole separatrix to be ejected from the resonance, as proposed by Morais (2000), over timescales of $\sim 8 \times 10^7$ revolutions of Enceladus. For Mimas we observe a larger number of coorbital escapes overall, both of tadpole (7/54) and horseshoe (29/36) librators. An analysis of the observed dynamical evolution

suggests a two-stage process at work: The semimajor axis of particles with starting conditions near the horseshoe-tadpole separatrix undergoes a slow random walk over timescales of 10^5 yr through a mechanism similar to that at Enceladus but involving the 4:2 inclination resonance with Tethys. These particles are eventually injected into a region of short-term ($\leq 10^4$ yr) instability just inside the nominal boundary of stable, symmetric horseshoe motion. The presence of the 4:2 eccentricity triplet at that location is the most likely culprit for the instability. In both the cases of Mimas and Enceladus small-amplitude tadpoles remain stable until the end of the integration. The existence of fast escapers at Mimas provides a dynamical avenue for the short-term survival of impact ejecta in horseshoe orbits within Mimas' coorbital region.

Keywords: Saturn, Satellites; Satellites, Dynamics

1 Introduction

The regular planetary satellite systems of the giant planets represent evolutionary end states of tidal evolution into or through orbital mean motion resonances. This occurred over time periods several orders of magnitude longer than the age of the solar system in terms of satellite orbital revolutions (Peale, 1976; Dermott et al, 1988). Recovering the initial state by inverting the dynamics in this case is not generally possible since the outcome of a resonant encounter under tides is probabilistic (Henrard and Lemaître, 1983). Nevertheless, an understanding of the dynamics of resonance capture has proven to be a powerful tool for constraining, not only likely initial configurations, but also the internal structure and geophysics of a planet and its satellites (Peale, 1986).

The trojan configuration, where a particle occupies the neighbourhood around either one of two stable equilibrium points in the Sun-planet-particle system leading or trailing the planet by 60 degrees in its orbit, are currently known to exist at Jupiter, Mars and Neptune (Levison et al, 1997; Marzari et al, 2003; Scholl et al, 2005). These bodies appear to be stable in their current dynamical environment, and long-term dynamical studies point to lifespans of order the age of the solar system. Consequently, their existence at some planets and not others are clues to the prevailing conditions during solar system formation and early dynamical evolution

(Morbidelli et al, 2005).

It is then reasonable to assume that the same holds for satellite systems. Of all the planets, only Saturn is known to have satellites accompanied by trojans. The satellites Telesto and Calypso occupy the leading and trailing triangular equilibrium points of Tethys; the satellites Helene and the recently discovered Polydeuces (Murray et al, 2005) play the same role at Dione. Although these bodies appear to be currently stable, the reasons why (a) only these two saturnian satellites are currently accompanied by trojans (b) there are no known trojans associated with the satellites of either Jupiter or Uranus, is still a matter of conjecture. Consequently, understanding what dynamical routes lead to long-term instability, as well as the timescale(s) over which these mechanisms operate, is necessary for explaining the current picture. In this paper we are concerned with what we call the “static” stability of saturnian satellite trojans, that is, stability over timescales within which tides do not evolve the orbits significantly. Although we expect tides to play an important role in answering the long-term stability question, it is known that sources of trojan instability exist in the conservative case (Dermott and Murray, 1981; Morais, 2000). It is important to establish which of these operate in this system and to what extent. This will form a solid basis on which to study the effect of tides in a future paper.

In previous work, Burns and Gladman (1998) followed the dynamical evolution of an ensemble of particles orbiting Saturn among the major satellites over 10^3 yr. The

main motivation for their work was the risk to the *Cassini* spacecraft due to impacts with circumplanetary material. In the course of their study, they found that objects on tadpole or horseshoe orbits with the satellite Mimas and Titan in particular are stable over this timescale. More recently, Porco et al (2005) integrated the orbits of 1000 test particles for 4800 yr in the vicinity of Mimas' orbit in the context of their discovery of two small moons, Methone (S/2004S1) and Pallene (S/2004S2), orbiting Saturn between the orbits of Mimas and Enceladus. In Figure S1 of their work, it appears that > 10 objects in horseshoe or tadpole orbits survive as such for this period of time. Mourão et al (2006) specifically addressed the stability of particles in tadpole orbits with Mimas and Enceladus and concluded that, under the present satellite configuration, these are stable over 10^4 yr.

Here we are concerned with objects in either tadpole (ie trojans) or horseshoe orbits, which we refer to collectively as coorbital. Furthermore, we limit our study to the Mimas-Enceladus-Tethys-Dione (or METD) system. The two inner satellites Mimas and Enceladus are dynamically linked to the outer, more massive Tethys and Dione respectively through mean motion resonances (Greenberg, 1984). We have carried out large scale numerical simulations of coorbital test particles perturbed by these satellites, over timescales more than an order of magnitude longer than previously attempted, to map out domains of long-term stability in phase space and their boundaries to regions where particles escape over the integration timespan.

Our results, while consistent with previous works, reveals that long-term instability does exist within the coorbital regions of Enceladus and, particularly, Mimas.

The paper is organised as follows: In Section 2, we discuss the dynamical model that we use as a background for this work. It is important to understand to what degree this set of satellites, or a subset of it, can be isolated from the whole, and still retain those dynamical features, namely the known mean motion resonances, which would enable a realistic study. In Section 3, we describe the setting up of the initial conditions of our test particles that we deem necessary for an even, unbiased, sampling of the coorbital phase space. In Section 4, we describe our results, concentrating on the dynamical sculpting of the coorbital zones of Mimas and Enceladus and the mechanisms responsible for the dynamical instability that we observe.

2 Satellite Model

Before one can proceed with large-scale numerical simulations of test particles, it is important to understand to what degree the model can represent, or reproduce, the actual dynamics. In addition, for reasons of computational expediency we aim to use the simplest model that can reproduce some main characteristics of the system. The existence of numerous resonances among the saturnian satellites presents us

with an obvious measuring stick by which to gauge how “realistic” our model is. These resonances and their corresponding critical angles, are (i) the Enceladus-Dione eccentricity-type resonance with $\phi_e = 2\lambda_{\text{Dione}} - \lambda_{\text{Enc}} - \varpi_{\text{Enc}}$ (ii) the Mimas-Tethys inclination-type resonance with $\phi_I = 4\lambda_{\text{Tethys}} - 2\lambda_{\text{Mimas}} - \Omega_{\text{Tethys}} - \Omega_{\text{Mimas}}$ and (iii) the coorbital librations of Telesto and Calypso with Tethys ($l_{r1} = \lambda_{\text{Tel}} - \lambda_{\text{Tethys}}$, $l_{r2} = \lambda_{\text{Cal}} - \lambda_{\text{Tethys}}$) and Helene with Dione ($l_{r3} = \lambda_{\text{Hel}} - \lambda_{\text{Dione}}$). Here we use the standard notation for the orbital elements where λ is the mean longitude, Ω the longitude of the ascending node and ϖ the longitude of pericentre.

For this purpose we integrated several different satellite configurations: (a) six major satellites in order of radial distance from Saturn (ie Mimas-to-Titan or METDRT) along with the three “classical” trojans mentioned above, not including the recently-discovered Polydeuces (b) the Mimas-to-Dione four-satellite subset (METD) with the three trojans (c) the Mimas-Tethys system (MT) with the two Tethys trojans Telesto and Calypso (d) the Enceladus-Dione system (ED) together with the Dione trojan Helene. We employed a hybrid integration scheme available as part of the MERCURY package (Chambers, 1999). This is an N-body symplectic code that switches to a Bulirsch-Stoer scheme during a close encounter between either a massive body and a particle or two massive bodies. The satellite masses and initial state vectors as well as the J_2 , J_4 and J_6 harmonic coefficients of Saturn’s gravity field used were taken from Jacobson (2004). We then carried a har-

monic decomposition of the numerical output using the Frequency Modified Fourier Transform (FMFT) algorithm developed by Šidlichovský and Nesvorný (1997) implemented into an IDL program written by one of us (Christou and Murray, 1999). The integration output was sampled with a period of 0.46875 days, slightly shorter than half the orbital period of Mimas to mitigate against frequency aliasing.

The results for the main satellite resonances are given in Table 1; those for the trojan librations are given in Table 2. The numerical estimates from the METDRT model give the best agreement with the observed amplitudes of libration of the Enceladus-Dione and Mimas-Tethys resonances of $1^\circ.5$ and 97° (Allan, 1969; Sinclair, 1972). The same conclusion may be reached for the respective periods of libration (Harper and Taylor, 1993) and forced actions (ie eccentricities and inclinations) corresponding to these two resonances. Note that the calculation of the libration amplitudes from Harper and Taylor’s work is not a straightforward exercise. These authors provide the amplitude of the term containing the resonant frequency in the expression for the mean longitude of the satellites, *not* the libration amplitude of the critical angle itself.

The other models differ from METDRT in various degrees. METD differs by 10% and 1% respectively in the amplitude and period of libration of the ϕ_e argument, while the differences for the ϕ_I libration are 6% and 2%. The variation in the actions forced to the smaller satellites of the pairs by the resonances is found to be 1%. Most

of the resonant parameters for the two-satellite models MT and ED exhibit marked departures from these values. In particular, the amplitude of libration for those models is a factor of $7/2$ larger for ϕ_e and smaller by a factor of $2/3$ for ϕ_I in comparison to the other models.

Finally, we see that the parameters for the Trojan librations are relatively model-insensitive in all cases. Our libration amplitudes for the Tethys trojans Telesto and Calypso are in good agreement with the values reported in Jacobson (1996) and Porco et al (2005) of $1^\circ.2$, $3^\circ.8$ and $1^\circ.3$, $3^\circ.6$ respectively. They are somewhat smaller than those reported in Table 3.2 of Dermott and Murray (1999) (2° , 4°) and by Oberti and Vienne (2003) ($2^\circ.25$, $5^\circ.25$). The trojans' libration periods as reported by the latter authors are 6% shorter than ours. Our results for the Dione trojan Helene are in excellent agreement with the libration extrema reported by Greenberg (1984) and Murray et al (2005). Our estimated libration period of 2.10 yr is bracketed by that reported by the latter authors (2.08 yr) and that in Oberti and Vienne (2003) (2.14 yr).

One can draw several conclusion from these observations. Firstly, it appears that current knowledge of the saturnian system is sufficiently close to reality that simulating more complete configurations (ie more satellites, higher order planetary zonal harmonics) results in better agreement with the observed values of sensitive indicators of the system state. Indeed, during this process we became aware of a

scarcity of self-consistent numerical models of the resonant dynamics within multi-satellite models such as METD or METDRT in the published literature; we expect the results reported here to serve as a useful reference in future work on the long-term dynamics of the system.

More pertinent to this investigation is that the 4-satellite model (METD) appears to offer the sought-after compromise between computational expediency and model realism and is thus adopted as our base model.

3 Particle setup

Our system consists of the planet or primary body (mass M), a satellite or secondary body (mass m_s) with test particles in tadpole and horseshoe orbits (hereafter referred to as H and T respectively), and possibly other perturbing satellites. The particles' osculating elements are a (semimajor axis), λ (mean longitude), e (eccentricity), ϖ (longitude of pericentre), I (inclination) and Ω (longitude of ascending node). Unsubscripted quantities refer to the test particles and quantities with subscript s refer to the secondary.

For the purposes of this work, we aim to place a parameter space grid of test particles within the coorbital regions of the satellites Mimas, Enceladus, Tethys and Dione.

We use an initial condition setup based on current knowledge of the short-term and long-term stability of the three body problem. The short-term stability is reflected by our use of the semimajor axis cutoff (outermost horseshoe). The long-term is reflected by the use of classical secular perturbation theory and what it tells us about the forced eccentricity and inclination (Morais, 1999; Morais, 2001). While unstable orbits in the three-body problem will remain unstable in the N-body problem, we test the stable orbits of the three-body problem under the perturbations of mean motion resonances (MMRs) as well as secular perturbations from additional satellites.

We choose to initially locate our test particles at the closest approach with the secondary as illustrated in Fig. 1. The expressions for the initial osculating semimajor axis and mean longitude for each particle are thus

$$a = a_s \quad (1)$$

$$\lambda = \lambda_s + 60^\circ - l(i) \quad (2)$$

where l is a parameter that characterizes the amplitude of the coorbital which we choose as

$$l(i) = \left(1 - \frac{i}{N-1}\right) \left[60^\circ - \frac{8(\mu/3)^{1/3}180^\circ}{0.74^2\pi}\right] \quad (3)$$

with $\mu = m_s/M$ and $i = 0, \dots, N-1$. A parameterisation equivalent to Eqs. 1-3 uses the relative fractional semimajor axis $a_0 = \Delta a/a_s$ ($\Delta a = a - a_s$) evaluated at

$\lambda - \lambda_s = 60^\circ$ (Morais, 1999). It is important to emphasize that a_0 and l are *constants* of the motion. The osculating quantities describing the particle's trajectory are $\Delta a/a_s$ and $\lambda - \lambda_s$.

The particles described by Eqs. 1-3 above are effectively spread at equal distances along an arc at the semimajor axis of the satellite. The factor 0.74 at $i = 0$ corresponds to the largest amplitude stable horseshoe, empirically derived from numerical experiments on the restricted circular three-body problem (Dermott and Murray, 1981). This is appropriate here as the eccentricities and inclinations of the secondaries and test particles are not large compared to the mass parameter $\epsilon = (\mu/3)^{1/3}$ (Namouni, 1999). The choice of $i = N - 1$ places a particle at the L_4 leading triangular equilibrium point. In this work we have used $N = 10$.

The corresponding expressions for the initial osculating eccentricity and inclination vectors $\mathbf{e} = e \exp i\varpi$ and $\mathbf{I} = I \exp i\Omega$ are given by Morais (2001) as

$$\mathbf{e} = \mathbf{e}_f + \mathbf{e}_p \quad (4)$$

$$\mathbf{I} = \mathbf{I}_f + \mathbf{I}_p \quad (5)$$

where

$$\mathbf{e}_f = e_s \exp [i(\varpi_s + b(l))] \quad (6)$$

$$\mathbf{I}_f = I_s \exp [i\Omega_s] \quad (7)$$

and

$$\mathbf{e}_p = e_p \exp [\mathbf{i} (\gamma(l)t + \chi)] \quad (8)$$

$$\mathbf{I}_p = I_p \exp [\mathbf{i} \Gamma(l)t + \Xi] . \quad (9)$$

The constants e_p , χ , I_p , Ξ are set by the initial conditions and the expressions for $b(l)$, $\gamma(l)$ and $\Gamma(l)$ are given in Morais (1999).

In the equations above, the “f” subscript denotes the forced component which depends on the orbital elements of the secondary and its position relative to the particle. The subscript “p” denotes the proper or free component which can be arbitrarily set to grid the initial conditions.

Fig. 2 shows a schematic representation of the eccentricity and inclination vectors defined in Eqs. 4-9, along with our chosen initial conditions. To obtain initial conditions for our simulations we decided to set the constants $\chi = b(l)$ and $\Xi = 0$. Moreover, we set $e_p = 0$, $e_s/2$, e_s and $I_p = 0$, $I_s/2$, I_s so that

$$\mathbf{e} = (1 + j/2) e_s \exp [\mathbf{i} (\varpi_s + b(l))] \quad (10)$$

$$\mathbf{I} = (1 + k/2) I_s \exp [\mathbf{i} \Omega_s] \quad (11)$$

as illustrated in Fig. 2, with $j, k = 0, 1, 2$. Hereafter, individual particles will be referred to by using the values of i , j and k applicable to them, optionally preceded by the name of the secondary eg ENCELADUS 302 or MIMAS 721. In the Figure captions we also provide the actual starting values of the orbital elements of the

particles. We refer to population subsets (ie subset $i = 3$) using the i , j and k index notation and the starting values of the orbital elements in the main text except where we deem it preferable to use only one notation to avoid repetition.

It is pertinent to the ensuing discussions to note that the critical value l_{crit} that marks the boundary between H and T motion (the H-T separatrix or HTS) is independent of μ . It is given by

$$l_{crit} = \pi/3 - 2 \arcsin \left(\sqrt{2} - 1 \right) / 2 \simeq 36^\circ.094 \quad (12)$$

and the corresponding a_{0crit} is equal to $\sqrt{8/3\mu}$.

4 Results

Our simulations were run on a CONDOR mini-cluster comprising 4-6 high-spec PCs. The integration timespan was 4×10^5 yr corresponding to 1.6×10^8 revolutions of Mimas or 5.6×10^7 revolutions of Dione. We consider that a particle escaped if its semimajor axis at the end of the integration is outside a band of half-width $a_s \epsilon$ centered at that of the satellite and $\Delta\lambda = \lambda - \lambda_s$ is circulating. In the first instance, we observe that all of the coorbitals of Tethys and Dione remained in libration. In contrast, 36 out of the 90 particles started as coorbitals of Mimas escaped, as did a small fraction of the Enceladus coorbitals (6 out of 90). In terms of libration type, all 6 Enceladus escapes were initially in horseshoe orbits, out of the 36 integrated

for that satellite. For the Mimas particles, 29 out of the 36 integrated horseshoes escaped, as did 7 out of the 54 tadpoles. In what follows we concentrate on the analysis of the cases of Mimas and Enceladus.

4.1 Enceladus

The 6 escaped Enceladus coorbitals all belong to the $i = 3$ bin, sharing an initial value of l ($37^\circ.8$) near that of the HTS (Fig. 3). Morais (2000; hereafter M00) found that the MMRs corresponding to the arguments $2\lambda_{\text{Dione}} - \lambda - \varpi$ and $2\lambda_{\text{Dione}} - \lambda - \varpi_{\text{Dione}}$ (the e and e' resonances according to the notation of Murray and Dermott (1999) which we adopt) are located deep into the coorbital zone at $a_0 \simeq 0$ and can affect those coorbitals with $a_0 \simeq a_{0\text{crit}}$ (or, equivalently, $l \simeq l_{\text{crit}}$) near the HTS. Those particles' coorbital period is sensitive to small perturbations particularly near the L3 equilibrium point at $(\Delta a/a_s = 0, \lambda - \lambda_s = 180^\circ)$. As a result, the time they spend within the MMRs varies from one coorbital libration to the next. As the e resonance also affects the eccentricity of the coorbital particle, the net result is a buildup in the eccentricity, which increases monotonically until the particle is ejected from the coorbital region by close encounters with either the secondary or nearby satellites. For the details of the mechanism we refer the reader to Fig. 6.4a and Section 6.4 of M00 (available at <http://www.astro.mat.uc.pt/~hmoais/thesis.ps.gz>) but we stress that its efficiency depends not only on the strength of a given

MMR but also its proximity to $a_0 = 0$.

The locations of these two MMRs at $a_0 \sim 0.05\epsilon$ and $a_0 \sim 0.17\epsilon$ for our model - which uses different starting conditions and parameters - agree closely with those of M00 (Fig. 4). It is also useful for completeness and the discussions to follow in the next section to note that the 4:2 MMR inclination triplet (I'^2 , II' and I^2) lies between $l = 44^\circ$ and $l = 55^\circ$, near $i = 0, 1, 2$. As the inclinations of either Enceladus or Dione are small ($\sim 0^\circ.02$) the effect of these MMRs on the particles' stability should be negligible, consistent with our results.

The types of dynamical behaviour we observe (Fig. 5) are consistent with the proposed mechanism of M00: initially, the eccentricities increase, essentially monotonically, to values around $0.06 - 0.07$, typically within 25% of the integration time; this is accompanied by moderate changes in the coorbital energy integral a_0 (Phase I) and also, something heretofore unreported in M00 and other dynamical analyses of this kind, erratic variation in I . Phase II is characterised by erratic variations in e and I due to moderately close approaches to Enceladus. Close approaches to nearby satellites require higher eccentricities and are not possible at this stage. Finally a close approach to Enceladus ejects the particle from the coorbital region (Phase III) and the particle's eccentricity can now increase sufficiently to allow close encounters with the other satellites. Notably, the first two phases (ie erratic behaviour without escape) were also observed for particles adjacent to $i = 3$. This suggests that, given

enough time, those too will share the fate of their escaped siblings. The median escape time for all particles with $i = 3$ was $\sim 2.9 \times 10^5$ yr, 72% of the integration timespan, or 8×10^7 Enceladus revolutions.

Within the limitations imposed by small numbers statistics, there appears to be a preference for instability among particles starting with high proper eccentricity e_p and low proper inclination I_p . High- e_p particles will encounter the secondary sooner than low- e_p ones while high- I_p provides a short-lived protection mechanism against close particle-secondary approaches.

4.2 Mimas

Mimas coorbitals, on the other hand, are more unstable. All escaping particles (36) have turning points within 34° of Mimas while tadpoles with values of l below that limit survive until the end of the integration. Overall, there is a trend towards faster escape as l approaches the secondary (Fig. 6). Burns and Gladman (1998) found a ratio of $< 20/3$ for H and T particles respectively after 10^3 yr. Our estimate for the same timescale is $31/54$. The difference is not surprising since the two works sample particle initial conditions in different ways.

The escape time statistics are presented in Fig 7 and show two distinct populations of particles: “fast” escapers with typical escape times less than 10^4 yr, and a slowly-escaping population similar to that seen at Enceladus with characteristic

escape times of several tens of percent of the integration timespan.

A closer, particle-by-particle inspection reveals that 9 out of the 13 particles belonging to the fast population escape within 1% of the start of the integrations or 4×10^3 yr; and that most of these particles (10 out of 13) belong to the $i = 0, 1$ set with $l > 50^\circ$. In addition, for the fast escapers only Phases II and III were observed; the small, erratic variations in a_0 and monotonic increase in e , I that characterise Phase I are absent here, probably because they occur on timescales much larger than the observed timescale for escape. In contrast, the slow population is dominant for $i = 3, 4$ particles (or $32^\circ.0 < l < 38^\circ.4$) while it is extant within the $i = 2$ set ($l = 44^\circ.8$) as well. Median escape time for the former set is 30 and 35 percent respectively of the integration timespan, $1.2 - 1.4 \times 10^5$ yr or $4.6 - 5.4 \times 10^7$ revolutions of Mimas. This is somewhat faster than for the corresponding slow escaping population of Enceladus. Within this population we see all three Phases that were found in the dynamical evolution of Enceladus coorbitals but not necessarily all for every particle. Phase I-II transition typically takes place once an eccentricity of 0.03–0.04 has been reached. For particles with $j = 1, 2$ ($44^\circ.8 < l < 51^\circ.2$) this is close to the starting eccentricity so the particles can enter directly into Phase II at the beginning of the integration. We show such an example in Fig. 8 where particle 311 undergoes erratic, zero-average-slope variation in e and I characteristic of Phase II behaviour from the onset (top panels). We contrast this with the behaviour of particle 300

(bottom panels) where e exhibits Phase I increase, passing directly into Phase III at $t = 1.8 \times 10^5$ yr.

Fig. 9 shows the locations of nearby multiples of the 2:1 MMRs with Tethys within Mimas' coorbital region where we have superimposed the dynamical evolution of two surviving particles from the $i = 0$ ($l = 57^\circ.6$) and $i = 4$ ($l = 32^\circ.0$) bins for reference. The same MMRs are also indicated in Fig. 6 where the locations are “folded over” on the l axis. These are in agreement with Fig. 6.4b of M00. The e resonance lies at 0.99ϵ , outside the coorbital zone. The e' and ee' resonances lie between $i = 0$ and $i = 1$ with the latter virtually coincident with the $i = 0$ location. In addition, the I^2 and I'^2 resonances lie between $i = 1$ ($l = 51^\circ.2$) and $i = 2$ ($l = 44^\circ.8$), within 0.005ϵ of each other. Finally, II' lies deep within the T domain, essentially coincident with Mimas' location as expected. We have checked that no other MMRs of reasonably low order with Tethys or Dione occur within this region.

Apparent in the dynamical evolution of the two particles plotted in Fig. 9 is also a modulation of the semimajor axis, but not the longitude. This is observed in all coorbitals of Mimas perturbed by the 2:1 resonant potential of Tethys and is found to be of amplitude $45 - 50 \times 10^{-5}$ in $\Delta a/a_s$. We have re-examined the corresponding plots for the Enceladus particles and identified a similar modulation there, of amplitude between 10 and 25×10^{-5} . The amplitude appears to be independent of l (with an exception at $l \simeq l_{\text{crit}}$ to be discussed shortly); it is probably due to the

differential effect of the 2:1 MMR with Tethys on Mimas and the particle.

What is particularly relevant to this work is that the amplitude of this modulation exhibits a sudden increase at $i = 3$ ($l = 38^\circ.6$) ie in the vicinity of the HTS, reaching values (in units of 10^{-5}) of 70 for Enceladus and 62 for Mimas. From the phenomenological point of view, this indicates that a mechanism similar to that responsible for the observed instability of Enceladus coorbitals is also present in Mimas' case. The effect of a mean motion resonance, namely the 4:2 II' MMR, on the semimajor axis is proportional to the time Δt that the particle spends within the MMR (Eq. 6.48 in M00) and this is maximized at $i = 3$.

The larger extent of this mechanism at Mimas, causing instability for particles with values of i adjacent to $i = 3$, may be due to the I'^2 and I^2 MMRs. In contrast to the Enceladus-Dione case, the inclination of Mimas and Tethys are not negligible ($1^\circ.5$ and $1^\circ.0$ respectively) making these resonances comparatively strong. In addition, they are located symmetrically with respect to II' at $a_0 = 0$ (Fig. 9). Hence, they are likely to affect large amplitude T or small-amplitude H motion for a good fraction of the coorbital libration period.

In parallel to what was observed at Enceladus, this mechanism affects the eccentricity as well as the inclination of the slow escapers. Regardless of the precise nature of the coupling between e and I , and whether it leads to actual growth in e (instead of zero-sum random jumps) and eventually escape from the coorbital zone,

we have identified at least one escape mechanism for these Mimas coorbitals, related to the fast escapes for $i = 0, 1$.

These latter escapes are primarily caused by the 2:1 eccentricity-type MMR triplet (Fig. 9). In Fig. 10 we show an example of particle dynamics with $i = 0$ exhibiting slow circulation near the e' resonance and corresponding slow variations in a and e . We posit that particles are transported in this way outside the region of stable H motion and escape; effectively these MMRs act to shrink the zone of coorbital stability around Mimas to just outside $i = 2$ ($l > 44^\circ.8$). The efficiency of this mechanism appears also to be dependent on the particles' starting eccentricity; the four surviving particles with $i = 0, 1$ (Fig. 6) have $j < 2$ ($e < 0.03$).

Escape for particles interior to $i = 1$ ($l < 51^\circ.2$) is facilitated by their initial random walk in a_0 caused by the interaction of motion near the HTS with the 4:2 inclination triplet as discussed earlier. In Fig. 11 we show an example of this mechanism in operation for a particle with $i = 3$.

The semimajor axis evolves until a sufficiently high value of a_0 is reached - typically 0.001 - for the e' resonance to start affecting the motion. The strength of this MMR is proportional to Tethys' eccentricity which is small (~ 0.001), but not negligible compared to that of Dione or Enceladus. The particle then essentially becomes a fast escaper. Our confidence in this two-stage mechanism is bolstered by the observations that (a) *all* particles reach this value of a_0 just before they escape

(b) it explains naturally the gradual escape time profile in Fig. 6; deeper coorbitals need to random-walk their semimajor axis for a longer period before they achieve a high enough value to be “fed” into the region of fast escape.

5 Conclusions and Discussion

In this work we have investigated the stability of coorbitals of the saturnian satellites, in particular Mimas and Enceladus, in their current orbital configuration, over timescales for which the effect of tides is not significant.

All of the 54 Enceladus tadpole particles remained stable throughout the integration. A small fraction of our Enceladus horseshoe particles (6/36) escape through the mechanism proposed by Morais (2000). This affects particles near the horseshoe-tadpole separatrix (HTS) where the interaction with a mean motion resonance (MMR) deep within the coorbital region results in eccentricity growth and escape. We thus confirm that the mechanism as studied by M00 in the Saturn-Enceladus-Dione-Particle problem is also efficient in a more complete satellite configuration with parameters closer to those of the actual Saturnian satellite system. Surprisingly, these eccentricity-type resonances were also observed to affect the inclination of Enceladus coorbitals, a matter that deserves further investigation.

For Mimas, we have found a more complicated situation. 29 out of 36 horseshoe

particles escaped as did 7 out of 54 tadpoles. Our results indicate the existence of two distinct, yet interacting, destabilising mechanisms operating on different timescales. Moderate-to-large amplitude horseshoes become quickly unstable owing to the positioning of Dione's 4:2 MMR eccentricity triplet just inside the radial boundary of Mimas' coorbital zone. The slower-acting mechanism, on the other hand, is reminiscent of the instability we observed for Enceladus coorbitals. It affects particles near the HTS and operates on a timescale comparable to the timespan of the integrations. In the absence of other plausible hypotheses, we attribute it to the symmetric positioning of Dione's 4:2 MMR inclination triplet within Mimas' coorbital region affecting near-HTS particles not just near L3 but also near the points of their maximum excursion in $\Delta a/a_s$ when $\lambda - \lambda_s \simeq \pm 60^\circ$.

This qualitative difference between the two satellites with regards to the fast escaping particles we observed at Mimas has implications for the existence of small objects in orbit-sharing configurations with the two satellites. It implies, due to the time-reversibility of the conservative dynamics, that impact ejecta originating at Mimas can survive as large-amplitude horseshoe attendants of that moon for periods of $10^3 - 10^4$ yr with eccentricities no larger than $0.03 - 0.04$. If such objects exist, they may eventually be discovered by imaging surveys at Mimas' distance from Saturn.

Finally, small to moderate libration amplitude tadpoles of both satellites ($l \leq 36^\circ$

at Enceladus and $l \leq 25^\circ$ at Mimas) survived until the end of the integration. If the absence of *known* librators is taken to represent the real picture, it implies that either such objects never existed or that they were ejected from the coorbital regions at some point in the satellites' history. The existence of tides in the Saturnian system offers fertile ground for investigating these questions further using tools such as the ones employed in this work. This will be the subject of a future paper.

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References

- Allan, R. R., 1969. The evolution of the Mimas-Tethys commensurability. *Astron. J.* 74, 497–506.
- Burns, J. A., 1986. The evolution of satellite orbits. In: Burns, J. A., Matthews, M. S. (Eds.), *Satellites*. Univ. of Arizona Press, Tucson, pp. 117–158.
- Burns, J. A., Gladman, B. J., 1998. Dynamically depleted zones for Cassini’s safe passage beyond Saturn’s rings. *Planet. Space Sci.* 46, 1401–1407.
- Chambers, J. E., 1999. A hybrid symplectic integrator that permits close encounters between massive bodies. *Mon. Not. Royal Astron. Soc.* 304, 793–799.
- Christou, A. A., Murray, C. D., 1999. The effect of near resonances on the secular precession of close orbits. *Mon. Not. Royal Astron. Soc.* 303, 806–812.
- Dermott, S. F., Murray, C. D., 1981. The dynamics of tadpole and horseshoe orbits. I - Theory. *Icarus* 48, 1–22.
- Dermott, S. F., Malhotra, R., Murray, C. D., 1988. Dynamics of the uranian and saturnian satellite systems - a chaotic route to melting Miranda? *Icarus* 76, 295–334.

- Greenberg, R., 1984. Orbital resonances among Saturn's satellites. In: Gehrels, T., Matthews, M. S. (Eds.), *Saturn*. Univ. of Arizona Press, Tucson, pp. 593–608.
- Harper, D., Taylor, D. B., 1993. The orbits of the major satellites of Saturn. *Astron. Astrophys.* 268, 326–349.
- Henrard, J., Lemaitre, A., 1983. A second fundamental model of resonance. *Cel. Mech. Dyn. Astron.* 30, 197–218.
- Jacobson, R. A., 1996. Orbits of the saturnian satellites from Earthbased and Voyager observations. *Bull. Am. Astron. Soc.* 28, 1185.
- Jacobson, R. A., 2004. The orbits of the major saturnian satellites and the gravity field of Saturn from spacecraft and Earth-based observations. *Astron. J.* 128, 492–501.
- Levison, H., Shoemaker, E. M., Shoemaker, C. S., 1997. The dispersion of the trojan asteroid swarm. *Nature* 385, 42–44.
- Marzari, F., Tricarico, P., Scholl, H., 2003. The MATROS project: Stability of Uranus and Neptune Trojans. The case of 2001 QR322. *Astron. Astrophys.* 410, 725–734.
- Morais, M. H. M., 1999. A secular theory for Trojan-type motion. *Astron. Astrophys.* 350, 318–326.

- Morais, M. H. M., 2000. The effect of secular perturbations and mean motion resonances on Trojan dynamics. PhD Thesis, Univ. of London.
- Morais, M. H. M., 2001. Hamiltonian formulation of the secular theory for Trojan-type motion. *Astron. Astrophys.* 369, 677-689.
- Morbidelli, A., Levison, H., Tsiganis, K., Gomes, R., 2005. Chaotic capture of Jupiter's Trojan asteroids in the early Solar System. *Nature* 435, 462-465.
- Mourão, D. C., Winter, O. C., Yokoyama, T., Cordeiro, R. R., 2006. On the stability of hypothetical satellites coorbital to Mimas and Enceladus. *Mon. Not. Royal Astron. Soc.* 372, 1614–1620.
- Murray, C. D., Cooper, N. J., Evans, M. W., Beurle, K., 2005. S/2004 S 5: A new co-orbital companion of Dione. *Icarus* 179, 222–234.
- Namouni, F., 1999. Secular interactions of coorbiting objects. *Icarus* 137, 293–314.
- Oberti, P., Vienne, A., 2003. An upgraded theory for Helene, Telesto and Calypso. *Astron. Astrophys.* 397, 353–359, 2003.
- Peale, S. J., 1976. Orbital resonances in the solar system. *Ann. Rev. Astron. Astrophys.* 14, 215–246.

Peale, S. J., 1986. Orbital resonances, unusual configurations and exotic rotation states among planetary satellites. In: Burns, J. A., Matthews, M. S. (Eds.), *Satellites*. University of Arizona Press, Tucson, pp. 159–223.

Porco, C. C., and 34 co-authors, 2005. Cassini imaging science: initial results on Saturn's rings and small satellites. *Science* 307, 1226–1236.

Scholl, H., Marzari, F., Tricarico, P., 2005. Dynamics of Mars Trojans. *Icarus* 175, 397–408.

Šidlichovský, M., Nesvorný, D., 1997. Frequency modified Fourier transform and its applications to asteroids. *Cel. Mech. Dyn. Astron.* 65, 137–148.

Sinclair, A. T., 1972. On the origin of commensurabilities amongst the satellites of Saturn. *Mon. Not. Royal Astron. Soc.* 160, 169–187.

Table 1: Dynamical parameters of the Mimas-Tethys and Enceladus-Dione resonances derived through FMFT decomposition of the following subsets of the Saturnian satellite system: (in order of increasing radial distance from Saturn) 6 satellites from Mimas to Titan (METDRT), 4 satellites from Mimas to Dione (METD) and the two satellite systems of Mimas-Tethys (MT) and Enceladus-Dione (ED). The four-satellite configuration was also integrated using a 15th order RADAU method (METDr) to test the robustness of the hybrid scheme used throughout the paper. The first and third lines provide the period (in years) and amplitude (in degrees) of the critical argument ϕ_{crit} equal to $\phi_e = 2\lambda_D - \lambda_E - \varpi_E$ for the Enceladus-Dione resonance and $\phi_I = 4\lambda_T - 2\lambda_M - \Omega_T - \Omega_M$ for the Mimas-Tethys resonance. The second and fourth lines give the magnitude of the eccentricity/inclination forced on the smaller member of each pair (ie Mimas in MT and Enceladus in ED). Enceladus' forced eccentricity is given in units of 10^{-5} while Mimas' forced inclination is given in degrees.

	METDRT	METD	METDr	MT	ED
ϕ_e	11.07 (1.505)	11.19 (1.663)	11.19 (1.661)		11.75 (5.37)
e_E^{forced}	474	481	481		518
ϕ_I	69.59 (96.41)	68.30 (90.66)	68.20 (90.18)	63.89 (62.42)	
I_M^{forced}	1.573	1.572	1.572	1.564	

Table 2: Dynamical parameters of the three “classical” saturnian satellite trojans derived through FMFT decomposition of numerical simulations of the METD saturnian satellite model. In this Table, l_r and a_r are the libration maxima of $\lambda - \lambda_s$ and $(a - a_s)/a_s$ respectively. The subscript 1 refers to Telesto (Tethys L_4 coorbital), 2 refers to Calypso (Tethys L_5 coorbital) and 3 refers to Helene (Dione L_4 coorbital). The first line for each object gives the libration period in yr, the second line gives the libration extrema of l_r in degrees and the third line gives the libration extrema of a_r in units of 10^{-5} . System model notations are as in Table 1.

	METDRT	METD	MT	ED
l_{r1}	1.905	1.905	1.905	
	(58.7, 61.2)	(58.8, 61.1)	(59.1, 60.8)	
a_{r1}	(-4.1, 4.3)	(-3.8, 4.0)	(-2.7, 2.8)	
l_{r2}	1.907	1.907	1.907	
	(-63.7, -56.4)	(-64.0, -56.1)	(-64.6, -55.6)	
a_{r2}	(-11.9, 12.0)	(-12.6, 12.7)	(-14.1, 14.2)	
l_{r3}	2.101	2.101		2.102
	(46.7, 76.6)	(46.7, 76.5)		(46.6, 76.7)
a_{r3}	(-62.0, 62.7)	(-61.8, 62.3)		(-62.5, 62.9)

List of Figures

- 1 Initial positioning of the test particles in (a, l) space where $l = 60^\circ - (\lambda_{\min} - \lambda_s)$. The particles are staggered at equal intervals on the $a = a_s$ circle from $l = 0$ (L_4 triangular equilibrium point) up to $l = l_{\max}$ corresponding to the horseshoe stability boundary at $a_0 = 0.74\epsilon$ derived empirically by Dermott and Murray (1981). This parameterisation completely defines the energy integral of the coorbital resonance. 38
- 2 Top row: illustration of the decomposition of a coorbital particle's eccentricity and inclination vectors $e \exp i\varpi$ and $I \exp i\Omega$ respectively into their free (or proper) and forced components. To first order, $e_{forced} \simeq e_s$ and $I_{forced} \simeq I_s$. In the case of horseshoe libration, $b(l) = 0$. Bottom row: parameterisation of the initial $\mathbf{e}_{free}$ and $\mathbf{I}_{free}$ of the test particles in this work. 39

- 3 Escape statistics for Enceladus coorbitals as a function of angular distance from the L_4 triangular equilibrium point. The red curve denotes the number of escaped coorbitals per l bin (max. is 9 on the left vertical axis) while the blue curve is the median escape time per bin, replaced by 1.0 (right vertical axis) in bins where no particles have escaped. The top and bottom dashed horizontal blue lines mark times of 10^8 and 10^7 Enceladus orbital periods respectively. The dashed vertical line marks the tadpole-horseshoe separatrix. The shorter dotted lines denote, from left to right, the locations of e and e' 2:1 MMRs with Dione. 40
- 4 The locations of the 6 different multiples of the 2:1 mean motion resonance with Dione within Enceladus' coorbital zone. These are indicated by horizontal segments on the left-hand side vertical axis. The two dashed horizontal lines indicate the outer boundary of the horseshoe region at 0.74ϵ (upper line) and the maximum amplitude tadpole (lower line). Two test particles, one in a horseshoe ($i = 0$ or $l = 56^\circ.7$) and the other in a tadpole orbit ($i = 4$ or $l = 31^\circ.5$) have been superimposed to provide a sense of scale. 41

5	Dynamical evolution of Enceladus coorbital particles over 4×10^5 yr. Particle 302 ($l = 37^\circ.8$, $e = 0.0048$, $I = 0^\circ.0066$; top) undergoes impulsive changes in a_0 (Phase I), which become more severe after the eccentricity has grown to ~ 0.06 (Phase II). This particle becomes a large-amplitude horseshoe at 1.2×10^5 yr, a state in which it remains until the end of the integration. Particle 321 ($e = 0.0096$, $I = 0^\circ.01$, bottom) undergoes a similar evolution, until it leaves the coorbital zone at 2.9×10^5 yr (Phase III).	42
6	As Fig. 3 but for Mimas coorbitals. The dashed blue vertical lines along the bottom indicate the positions of different multiples of the 2:1 mean motion MMR with Tethys. From left to right: II' , I'^2 , I^2 , e' , ee'	43
7	Escape statistics for Mimas and Enceladus coorbitals as a function of time. Bin size is 10^4 yr.	44

- 8 Dynamical evolution of Mimas coorbital particles over 3.1×10^5 yr.
Particle 311 ($l = 38^\circ.4$, $e = 0.03$, $I = 2^\circ.34$; top) undergoes changes
in a_0 causing it to alternate between tadpole and horseshoe libration
while e , already high to start with, exhibits a flat, erratic variation
characteristic of Phase II behaviour. Particle 300 on the other hand
($e = 0.02$, $I = 1^\circ.56$; bottom) starts with a smaller eccentricity which
undergoes Phase I monotonic increase until values of 0.03-0.04 are
reached. It leaves the coorbital zone at 1.8×10^5 yr (Phase III). Note
how the evolution of I parallels that of e 45
- 9 As Fig. 4 but for the 2:1 mean motion resonance with Tethys within
Mimas' coorbital zone. Two test particles, one in a horseshoe ($i = 0$
or $l = 57^\circ.6$) and the other in a tadpole orbit ($i = 4$ or $l = 32^\circ.0$)
have been superimposed to provide a sense of scale. 46

- 10 The first 300 yr (or 10^5 Mimas revolutions), from the start of the integration, of particle MIMAS 312 ($l = 38^\circ.4$, $e = 0.03$, $I = 3^\circ.12$). This is a fast escaper, leaving the coorbital zone after 6.5×10^3 yr. The two upper panels show the evolution of the semimajor axis and the eccentricity, while the bottom panel shows the critical angle of the e' resonance (“ep” in Fig. 9). This angle exhibits circulation with a ~ 10 yr period, and modulates both a and e . The longer period (~ 50 yr) apparent in e may be related to a similar period in a although there does seem to be some phase difference, particularly at the beginning of the integration. A yet slower modulation in e (several hundred years period) correlates well with a similar period in the inclination (not shown here). 47
- 11 Time history of the semimajor axis (top panel), eccentricity (middle panel) and inclination (bottom panel) of particle MIMAS 300, also shown in the bottom row of Fig. 8. The correlation in the behaviour of all three elements is apparent. When a_0 reaches a value of 0.001, e undergoes a large increase and the particle escapes. Note the monotonic increase in e between $t = 9 \times 10^4$ and $t = 1.8 \times 10^5$ yr, typical of Phase I behaviour. 48

Figure 1: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 2: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

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Figure 4: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 5: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 6: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 7: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

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Figure 9: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 10: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals

Figure 11: Christou, Namouni and Morais 2007, Saturnian satellite coorbitals