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Natural convection heat transfer coefficients in Phase Change Material (PCM) modules with external vertical fins

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Abstract

To determine the heat transfer coefficient by natural convection for specific geometries, experimental correlations are used. No correlations were found in the literature for the geometries studied in this work. These geometries consisted of a cylindrical module of 88 mm of diameter and 315 mm height with external vertical fins of 310 mm height and 20 and 40 mm length. To determine the heat transfer coefficient by natural convection, experimental work was done. This module, containing PCM (sodium acetate trihydrate), was situated in the middle upper part of a cylindrical water tank of 440 mm of diameter and 450 mm height. The calculated heat transfer coefficient changed by using external fins, as the heat transfer surface was increased. The temperature variation of the PCM and the water are presented as a function of time, and the heat transfer coefficient for different fins is presented as a function of the temperature difference. Experimental correlations were obtained, presenting the Nusselt number as a function of different dimensionless numbers. Different correlations were analysed to find which one fit better to the experimental data.

1 Corresponding author
KEYWORDS: Nusselt number, Rayleigh number, experimental correlation, heat transfer enhancement, Phase Change Material (PCM)

1. Introduction

Thermal energy storage systems are an important requirement for many applications due to the non-coincidence of heat demand and supply or availability. One of the typical examples of such mismatch is solar energy. Among the thermal energy storage concepts, latent heat thermal storage using PCM’s is regarded as a promising technology. Their use in domestic hot water (DHW) tanks would keep hot water for a longer time. In such a system, a lot of energy can be stored as latent heat, but it should be able to be transferred from the PCM to the water when needed, therefore heat transfer within the PCM and to the water is of high interest [1,2].

There are several methods to enhance the heat transfer in a latent heat thermal store. The use of fins inside the PCM has been extendedly studied. These fins can be axial or radial and are usually attached to the tubes. In this case the most important part is the formulation of phase change problems. Several theoretical techniques have been developed, such as the enthalpy method by Ismail [3,4,5], the Landau transform method associated with the finite volumes method, the interface immobilization method together with the finite volumes approach, or the integral energy method (Ismail [5]). Ismail [3, 4] presented a comprehensive review of literature on the subject as well as the results of many experimental and numerical studies on phase change heat transfer into and around simple and complex geometries.

Mehling et al., [6, 7, 8] and Py et al. [9] proposed a graphite-compound-material, where the PCM is embedded inside a graphite matrix. When using these PCM-graphite composites inside metal modules, the heat barrier is the heat transfer from the metal container to the water [8]. By using fins the heat transfer area is extended and the coefficient of heat transfer by natural convection changes, improving heat transfer form the container to the water.
Fins geometry is an important parameter when considering the addition of fins in a PCM module. In a vertical module two different fins geometries can be considered: horizontal and vertical fins. There is much more literature for horizontal fins [10-22], but this geometry interferes with the natural convection in the PCM module. On the other hand there is no literature for vertical fins around circular vertical tubes, but this geometry would improve natural convection in the water side of the PCM module. Because of this, in this work external vertical fins were used to increase the heat transfer from the PCM to the water.

There were no literature references for vertical and cylindrical modules with external and vertical fins. Heat transfer coefficient for natural convection is typically determined using experimental correlations. Lots of specific geometries have been studied and correlations are available in the literature [23, 24]. In this work an experimental set-up was used to evaluate the natural convection heat transfer coefficient for two specific geometries. Correlations of Nusselt number as a function of Rayleigh and effective Rayleigh number were obtained.

2. Experimental work

a) Experiments done

To determine the effect of adding vertical fins to the external part of the module, some experiments were conducted. The PCM-graphite composite used was sodium acetate trihydrate with graphite (90:10 vol.%). This product has a melting point of 58°C, melting enthalpy between 180 and 200 kJ/kg, density between 1.350 and 1.400 kg/m³, heat capacity of 2.5 kJ/kg·K, and thermal conductivity between 2 and 5 W/m·K.

Three different modules were used: one without fins, another one with small fins (20 mm length) and the last one with big fins (40 mm length). Each module had 8 external and vertical fins of 2 mm of thickness, providing a 28.45% and 44.28% increase of the heat transfer surface, respectively, compared to the reference module without fins. Fig. 1 shows the considered
configuration. The PCM module was 88 mm of diameter and 310 mm of height with a thickness of 2 mm.

The experimental work reproduced the behaviour of the PCM modules in a DHW tank. The dimensions of the tank used were: 440 mm of diameter and 450 mm of height. The tank was insulated using an elastomeric insulating material of 10 mm of thickness. Five K type thermocouples and a data-logger (STEP DL01-CPU) instrumented the experimental set-up. Two thermocouples were located inside the PCM module (one in the centre and the other at half distance between the first one and the metal container). The other ones were situated inside the water, outside the PCM module, one of them in contact with the external surface of it. The distance between the inside water thermocouples was 50 mm, and all thermocouples were at 135 mm distance from the top of the tank. Fig. 2 and Fig. 3 illustrate the instrumentation of the experiments.

The experimental work consisted of introducing the module containing melted PCM at 70°C into the cold water tank to evaluate the heat transfer phenomenon. The experiment was stopped when PCM and water temperatures were the same.

b) Natural convection heat transfer coefficient

Once the experimental work was done, the heat transfer coefficient by natural convection for this specific geometry could be calculated. To determine this coefficient a one-dimensional study was done in order to simplify the calculations. A three-dimensional behavior could be expected because of the stratification of the water, but since the experiments were very short it would be negligible. In future works, the stratification in the water tank when using encapsulated PCM inside (considering different distributions and geometries) will be studied. The procedure used to determine the heat transfer coefficient by natural convection was the following:
1. Calculation of the heat transfer rate.

The PCM (sodium acetate trihydrate) was mixed with graphite in a composite. Therefore, during all the experimental processes the mixture was not melted inside the PCM module, remaining in solid phase; only the sodium acetate trihydrate went through the melting/solidifying process, but not affecting the solid structure of the composite. Therefore, the heat transfer rate inside the PCM module could be determined using conduction equations (Fourier law). In this work, the transient effect was considered discretizing the data over time, therefore, the heat flux could be calculated using the following equation:

\[ q = k_{\text{PCM}} \cdot A_{\text{cond}} \cdot \left( T_{\text{PCM, \frac{1}{2}}} - T_{\text{surface}} \right) \]  

(1.a)

where:

\[ A_{\text{cond}} = \frac{2 \cdot \pi \cdot L_{\text{PCM}}}{\ln \left( \frac{r_{\text{PCM}}}{0.003} \right)} \]  

(1.b)

Note that in the equations above the thermal conductivity of the material is considered constant with time and temperature. Although this assumption might give an error to the calculation, especially during phase change, when the PCM is mixed with graphite the driving force in the conduction is the graphite, therefore the error is low enough.

2. Calculation of the natural convection heat transfer coefficient.

Once the heat transfer rate was calculated, the natural convection heat transfer coefficient could be determined using Newton law.

\[ h_{\text{PCM}} = \frac{q}{A_{\text{transfer}} \cdot (T_{\text{surface}} - T_{\text{close}})} \]  

(2.a)

where:

\[ A_{\text{transfer}} = 2 \cdot \pi \cdot r_{\text{PCM}} \cdot L_{\text{PCM}} \]  

(2.b)

for the module without fins, and
\[ A_{\text{transfer}} = 2 \cdot \pi \cdot r_{\text{PCM}} \cdot L_{\text{PCM}} + 2 \cdot N_{\text{fin}} \cdot W_{\text{fin}} \cdot L_{\text{PCM}} \]  

(2.c)

for the modules with vertical fins.

c) **Natural convection heat transfer coefficient correlations**

From the experimental data, experimental correlations were determined for each specific geometry. These equations provided the heat transfer coefficient for natural convection as a function of the temperature difference of the system. The defining dimensionless numbers of the problem, considering its geometry and boundary conditions, were:

\[ \frac{D_{\text{PCM}}}{L_{\text{PCM}}} = 0.284; \quad \frac{s}{L_{\text{PCM}}} = 0.11; \quad \frac{D_{i}}{L_{\text{PCM}}} = 1.397; \quad \frac{L_{i}}{L_{\text{PCM}}} = 1.429; \quad X = \frac{x}{L_{\text{PCM}}}; \quad Y = \frac{y}{L_{\text{PCM}}} \]

\[ \Phi = \frac{T - T_{\text{close}}}{T_{\text{surface}} - T_{\text{close}}} \]

\[ Gr = \frac{8 \cdot \beta \cdot (T_{\text{surface}} - T_{\text{close}}) \cdot L_{\text{PCM}}^3}{\nu^2}; \quad Pr = \frac{\mu \cdot C_{\text{p}}}{k} \]

and,

\[ \frac{W_{\text{fin}}}{L_{\text{PCM}}} = 0.063; \quad \text{for 20 mm fins} \]

\[ \frac{W_{\text{fin}}}{L_{\text{PCM}}} = 0.127; \quad \text{for 40 mm fins} \]

d) **Rayleigh correlations**

Different correlations for Nusselt number were studied and compared with each other to achieve the best fit with the experimental results. Three dimensionless numbers (Rayleigh, Prandtl, Grashof) and several combinations of them were used.
The most significant and used dimensionless numbers in natural convection systems and their combination were studied. The fluid properties are a function of the temperature and were evaluated at the registered temperature. The following equations were used to calculate them:

\[ Gr = \frac{g \cdot \beta \cdot (T_{\text{surface}} - T_{\text{close}}) \cdot L_{\text{PCM}}}{v^2} \]  
\( \beta = \frac{1}{T_f} \)  
\[ T_f = \frac{T_{\text{surface}} + T_{\text{close}}}{2} \]  
\[ Pr_{\text{H}_2\text{O}} = \frac{\mu_{\text{H}_2\text{O}} \cdot C_p}{k_{\text{H}_2\text{O}}} \]  
\[ Ra = Gr \cdot Pr \]  
\[ Ra_{\text{eff}} = Gr \cdot Pr \cdot \frac{W_{\text{fin}}}{L_{\text{PCM}}} \]

In equation 3f, Rayleigh number is modified by a ratio of fin width by fin height. These modifications introduced in the dimensionless number a measure of using different fins.

3. Results

The experimental results showed an increase in the heat transfer rate when using PCM modules with vertical fins. This effect can be measured by the time needed by the modules to heat the water. Fig. 4a shows the time needed to heat the water due to the PCM phase change. To cool down the PCM from 60°C to 45°C (including the solidification of the material) using a PCM module without fins the time needed was about 17 minutes.
When using 20 mm fins, to achieve the same temperature decrease as in the experiments without fins, the time necessary was about 13 minutes (a reduction of 23.53%). Fig. 4b shows the heating, and melting and cooling down processes of the water and the PCM, respectively. Finally, PCM modules with 40 mm fins reduced the cooling down time to 7 minutes for the same temperature decrease (a reduction of 58.82%). Fig. 4c shows the heating and cooling down process for these experiments.

To determine the heat transfer coefficient a one-dimensional study was done. The stratification of the water was neglected in this work. In future works, the stratification of the water tank when using PCM inside will be studied.

The heat transfer coefficient for natural convection for each PCM module geometry is compared in Fig. 5 as a function of temperature difference $\Delta T = T_{\text{surface}} - T_{\text{close}}$. When using 20 mm fins, the temperature difference $\Delta T$ necessary to achieve the maximum heat transfer coefficient ($h=179 \text{ W/m}^2\text{K}$ with no fins; $h=177 \text{ W/m}^2\text{K}$ with 20 mm fins) experienced a threefold decrease ($\Delta T=3.5 \degree\text{C}$ with no fins; $\Delta T=1.1 \degree\text{C}$ with 20 mm fins). On the other hand, using 40 mm fins the temperature difference had a fourfold decrease ($\Delta T=0.8 \degree\text{C}$), but the maximum heat transfer coefficient decreased too ($h=159 \text{ W/m}^2\text{K}$).

Fig. 6a and Fig. 6b represent the natural convection heat transfer coefficient as a function of the temperature difference $\Delta T$, for 20 mm and 40 mm fins, respectively. The experimental correlations were divided in three parts to achieve the best fit with the experimental data. The valid range of temperature differences was from 0.7 to 9.2 $\degree\text{C}$ for 20 mm fins geometry, and from 0.2 to 13.7 $\degree\text{C}$ for 40 mm fins.

For 20 mm fins, the experimental correlation is:

$$
\begin{align*}
    h_{\text{PCM}} & = -490.6 \cdot \Delta T^2 + 1249 \cdot \Delta T + 619 & \quad 0.7 \leq \Delta T \leq 1.3 \\
    h_{\text{PCM}} & = -35.7 \cdot \Delta T + 226 & \quad 1.3 < \Delta T \leq 3.7 \\
    h_{\text{PCM}} & = 1.35 \cdot \Delta T^2 - 24 \cdot \Delta T + 163.7 & \quad 3.7 \leq \Delta T \leq 9.2
\end{align*}
$$

(3.a)
For 40 mm fins, the experimental correlation is:

\[
\begin{align*}
h_{\text{PCM}} &= -232.6 \cdot \Delta T^2 + 376 \cdot \Delta T + 16.5 & \quad 0.2 \leq \Delta T \leq 0.8 \\
h_{\text{PCM}} &= 7.8 \cdot \Delta T^2 - 64.3 \cdot \Delta T + 185 & \quad 0.8 < \Delta T \leq 3 \\
h_{\text{PCM}} &= 0.33 \cdot \Delta T^2 - 9.2 \cdot \Delta T + 83 & \quad 3 < \Delta T \leq 13.7
\end{align*}
\]

(3.b)

In the first region the heat transfer coefficient increases with $\Delta T$. The maximum value for the coefficient is achieved at the intersection point between first and second region. For higher values of temperature differences the heat transfer coefficient decreased to the half of the maximum value. Finally, in the third region the coefficient described a parabolic behaviour below values the half of the maximum.

For both cases, using 20 mm or 40 mm fins, the first and third region were approached with a polynomial regression. In the second region, when using 20 mm fins a lineal regression is good enough, while for 40 mm fins a polynomial regression is necessary.

Several experimental correlations representing Nusselt number as a function of different dimensionless numbers were determined. As showed before, there were three different regions in the heat transfer coefficient behaviour, so Nusselt number should be evaluated in each region individually.

First analysis consisted in plotting Nusselt number as a function of Rayleigh number for each geometry, as showed in Fig 7a and Fig 7b. The obtained correlations were, for 20 mm fins:

\[
\begin{align*}
Nu &= -4 \cdot 10^{-22} \cdot Ra^2 + 8 \cdot 10^{-10} \cdot Ra + 308 & \quad 5 \cdot 10^{11} \leq Ra \leq 1 \cdot 10^{12} \\
Nu &= 3 \cdot 10^{-24} Ra^2 - 4 \cdot 10^{-11} \cdot Ra + 127.6 & \quad 1 \cdot 10^{12} < Ra \leq 3.6 \cdot 10^{12} \quad (4a) \\
Nu &= 4 \cdot 10^{-25} \cdot Ra^2 - 7 \cdot 10^{-12} \cdot Ra + 61.4 & \quad 3.6 \cdot 10^{12} \leq Ra \leq 6.5 \cdot 10^{12}
\end{align*}
\]

For 40 mm fins:

\[
\begin{align*}
Nu &= -2 \cdot 10^{-22} \cdot Ra^2 + 2 \cdot 10^{-10} \cdot Ra - 4.7 & \quad 7.1 \cdot 10^{10} \leq Ra \leq 6 \cdot 10^{11} \\
Nu &= 6 \cdot 10^{-24} \cdot Ra^2 - 4 \cdot 10^{-11} \cdot Ra + 91.5 & \quad 6 \cdot 10^{11} < Ra < 3 \cdot 10^{12} \quad (4b) \\
Nu &= -4 \cdot 10^{-12} \cdot Ra + 35 & \quad 3 \cdot 10^{12} \leq Ra \leq 7.5 \cdot 10^{12}
\end{align*}
\]
Nusselt number as a function of an effective Rayleigh number (Ra\textsubscript{eff}) was also studied, as showed in Fig. 8a and Fig. 8b. The experimental correlations were, for 20 mm fins:

\begin{align*}
    Nu &= 1 \cdot 10^{-11} \cdot Ra\textsubscript{eff} - 98 & 8 \cdot 10^{12} \leq Ra\textsubscript{eff} < 1.45 \cdot 10^{13} \\
    Nu &= 8 \cdot 10^{-27} (Ra\textsubscript{eff})^2 - 2 \cdot 10^{-12} \cdot Ra\textsubscript{eff} + 122 & 1.45 \cdot 10^{13} \leq Ra\textsubscript{eff} \leq 6 \cdot 10^{13} \\
    Nu &= 1 \cdot 10^{-27} (Ra\textsubscript{eff})^2 - 5 \cdot 10^{-13} \cdot Ra\textsubscript{eff} - 61.4 & 6 \cdot 10^{13} \leq Ra\textsubscript{eff} \leq 1 \cdot 10^{14}
\end{align*}

For 40 mm fins:

\begin{align*}
    Nu &= -3 \cdot 10^{-24} \cdot (Ra\textsubscript{eff})^2 + 3 \cdot 10^{-11} \cdot Ra\textsubscript{eff} - 4.7 & 5 \cdot 10^{11} \leq Ra\textsubscript{eff} \leq 5 \cdot 10^{12} \\
    Nu &= 1 \cdot 10^{-25} \cdot (Ra\textsubscript{eff})^2 - 5 \cdot 10^{-12} \cdot Ra\textsubscript{eff} + 93 & 5 \cdot 10^{12} \leq Ra\textsubscript{eff} < 2 \cdot 10^{13} \\
    Nu &= 3 \cdot 10^{-28} \cdot (Ra\textsubscript{eff})^2 - 5 \cdot 10^{-13} \cdot Ra\textsubscript{eff} + 35 & 2 \cdot 10^{13} \leq Ra\textsubscript{eff} \leq 6 \cdot 10^{13}
\end{align*}

Both correlations showed a similar behaviour. In the first region Nusselt number increased with Rayleigh and effective Rayleigh numbers respectively. The maximum value for the Nusselt number was achieved at the intersection point between first and second region. For bigger values of Rayleigh and effective Rayleigh numbers the Nusselt number decreased to values lower than half of the maximum. Finally, in the third region the Nusselt number described a parabolic behaviour obtaining values lower than a third part of the maximum.

For both cases, using 20 mm or 40 mm fins, the obtained correlations were very similar. Also comparing Rayleigh and Effective Rayleigh the results were similar. A polynomial regression was used in most cases.

4. Discussion

The increase of the heat transfer rate was a result of the increase of the heat transfer area and the lower temperature difference necessary to achieve the same heat transfer coefficient for natural convection.
The heat transfer coefficient was not increased by using vertical fins. When using small fins, a lower temperature difference was necessary to achieve the same heat transfer coefficient as with no fins. Therefore, the needed time to solidify the PCM decreased. The increase of the heat transfer area resulted in an increase of heat transfer rate.

When using big fins, the heat transfer coefficient was lower. The increase of the fins width may have interfered the natural convection. Nevertheless, the needed time to solidify the PCM was also reduced because of the increase of the heat transfer area.

The increase of the heat transfer rate obtained by using vertical fins could be very useful for applications of PCM modules inside water tanks. These PCM modules are used to store energy in a reduced volume. Using modules with vertical fins could solve the problem of slow heat transfer rate from the PCM to the water and increase the availability of the energy. The storage system would be more flexible to match the energy demand.

The behaviour of the system with vertical fins was well defined using both correlations, Rayleigh and effective Rayleigh. There were no significant differences in the quadratic mean difference or the complexity of the function describing the system, as showed in Table 1. The accuracy of the correlations was not affected by the fin width. Additional experimental work using fins with different width should be done in order to determine a unique correlation that represented all the studied cases. In this correlation, the modifying factor $\frac{W_{\text{fin}}}{L_{\text{PCM}}}$ should be important.

To simplify the calculations, the authors recommend using the Nusselt over Rayleigh correlation. The accuracy was the same as when using Nusselt over effective Rayleigh correlation, but it was easier to determine the Rayleigh number.

In conclusion, the use of external fins in PCM modules reduced the time necessary for the heat transfer to the surrounding water. The temperature difference necessary to achieve a certain
value of the heat transfer coefficient by natural convection was also reduced. The bigger the fins were, the faster was the heat transfer process, but the heat transfer coefficient was reduced. Finally, the authors recommend the use the Nusselt over Rayleigh correlation in order to simplify the calculations achieving the same precision in the results.

**Nomenclature**

- $A_{\text{cond}}$: inner conduction heat transfer area \( \text{m}^2 \)
- $A_{\text{transfer}}$: convection heat transfer area from the modules to the water \( \text{m}^2 \)
- $C_p$: specific heat \( \text{kJ/(kg-K)} \)
- $d_T$: distance between thermocouples position in the experimentation work \( \text{m} \)
- $D_{\text{PCM}}$: PCM module diameter \( \text{m} \)
- $D_t$: tank diameter \( \text{m} \)
- $g$: gravity acceleration \( \text{m/s}^2 \)
- $Gr$: Grashof number, \( \frac{g \cdot \beta \cdot (T_{\text{surface}} - T_{\text{close}}) \cdot L_{\text{PCM}}^3}{\nu^3} \)
- $h_{\text{PCM}}$: convection heat transfer coefficient \( \text{W/(m}^2\cdot\text{K)} \)
- $k_{\text{PCM}}$: PCM thermal conductivity \( \text{W/(m-K)} \)
- $L_{\text{PCM}}$: PCM module height \( \text{m} \)
- $L_t$: tank height \( \text{m} \)
- $N_{\text{fin}}$: number of fins
- $Nu$: Nusselt number, \( \frac{h \cdot L}{k} \)
- $Pr$: Prandtl number, \( \frac{\mu \cdot C_p}{k} \)
- $q$: heat transfer rate from the PCM to the water in the experimental work \( \text{W} \)
- $Ra$: Rayleigh number, \( Gr \cdot Pr \)
- $Ra_{\text{eff}}$: Effective Rayleigh number, \( Gr \cdot Pr \cdot \frac{W_{\text{fin}}}{L_{\text{PCM}}} \)
- $r_{\text{PCM}}$: PCM module radius \( \text{m} \)
\( r_{PCM, \frac{1}{4}} \) radius of the quarter of the module .......................................................... m
s distance between two adjacent fins ............................................................................. m
T temperature .............................................................................................................. °C
\( T_f \) film temperature ............................................................................................ °C
\( T_{PCM, 1/2} \) temperature in the middle of the PCM module ........................................ °C
\( T_{PCM, 1/4} \) temperature in a quarter of the PCM module ......................................... °C
\( T_{surface} \) temperature of the surface of the PCM module ....................................... °C
\( T_{close} \) temperature of the water close to the PCM module .................................... °C
\( W_{fin} \) fin width ........................................................................................................ m

Greek symbols

\( \beta \) volumetric coefficient of thermal expansion ........................................................ °C\(^{-1}\)
\( \Delta T \) temperature difference, \( \Delta T = T_{surface} - T_{close} \) ........................................ °C
\( \phi \) dimensionless number of \( \Delta T \), \( \phi = \frac{T - T_{close}}{T_{surface} - T_{close}} \)
\( \mu \) dynamic viscosity .............................................................................................. kg/(m·s)
\( \nu \) kinematic viscosity ............................................................................................ m\(^2\)/s
\( \pi \) Pi number
\( \rho \) density .............................................................................................................. kg/m\(^3\)

References


Table 1. Correlations for modules with vertical fins.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Region</th>
<th>Correlation (y vs x)</th>
<th>n(^{(1)})</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>r(^2) (^{(2)})</th>
<th>Valid range</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(h_{\text{PCM}}) vs (\Delta T)</td>
<td>14</td>
<td>-490.6</td>
<td>1249</td>
<td>619</td>
<td>0.84</td>
<td>(\geq 0.7) (\leq 1.3)</td>
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<tr>
<td></td>
<td>First region</td>
<td>Nu vs Ra</td>
<td>73</td>
<td>-4(10^{-22})</td>
<td>8(10^{-10})</td>
<td>308</td>
<td>0.79</td>
<td>(\geq 5\times10^{11}) (\leq 1\times10^{12})</td>
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<tr>
<td></td>
<td></td>
<td>Nu vs (R_{\text{eff}})</td>
<td>14</td>
<td>1(\times10^{-11})</td>
<td>-98</td>
<td></td>
<td>0.76</td>
<td>(\geq 8\times10^{12}) (&lt; 1.45\times10^{13})</td>
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<tr>
<td></td>
<td></td>
<td>(h_{\text{PCM}}) vs (\Delta T)</td>
<td>15</td>
<td>1.35</td>
<td>-24</td>
<td>163.7</td>
<td>0.98</td>
<td>(\geq 3.7) (\leq 9.2)</td>
</tr>
<tr>
<td>20 mm fin</td>
<td>Second region</td>
<td>Nu vs Ra</td>
<td>73</td>
<td>4(\times10^{-25})</td>
<td>-7(10^{-12})</td>
<td>61.4</td>
<td>0.99</td>
<td>(\geq 3.6\times10^{12}) (\leq 6.5\times10^{12})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu vs (R_{\text{eff}})</td>
<td>13</td>
<td>1(\times10^{-27})</td>
<td>-5(10^{-13})</td>
<td>61.4</td>
<td>0.99</td>
<td>(\geq 6\times10^{13}) (\leq 1\times10^{14})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h_{\text{PCM}}) vs (\Delta T)</td>
<td>30</td>
<td>0.33</td>
<td>-9.2</td>
<td>83</td>
<td>0.83</td>
<td>(&gt; 3) (\leq 13.7)</td>
</tr>
<tr>
<td></td>
<td>Third region</td>
<td>Nu vs Ra</td>
<td>73</td>
<td>3(\times10^{-28})</td>
<td>-5(10^{-13})</td>
<td>35</td>
<td>0.73</td>
<td>(\geq 3\times10^{12}) (\leq 7.5\times10^{12})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu vs (R_{\text{eff}})</td>
<td>30</td>
<td>7.8</td>
<td>-64.3</td>
<td>185</td>
<td>0.88</td>
<td>(&gt; 0.8) (\leq 3)</td>
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<tr>
<td></td>
<td>First region</td>
<td>Nu vs (R_{\text{eff}})</td>
<td>30</td>
<td>6(\times10^{-24})</td>
<td>4(\times10^{-11})</td>
<td>91.5</td>
<td>0.89</td>
<td>(&gt; 6\times10^{11}) (&lt; 3\times10^{12})</td>
</tr>
<tr>
<td>40 mm fin</td>
<td>Second region</td>
<td>Nu vs Ra</td>
<td>54</td>
<td>7.8</td>
<td>-64.3</td>
<td>185</td>
<td>0.88</td>
<td>(&gt; 5\times10^{12}) (&lt; 2\times10^{13})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu vs (R_{\text{eff}})</td>
<td>30</td>
<td>7.8</td>
<td>-64.3</td>
<td>185</td>
<td>0.88</td>
<td>(&gt; 5\times10^{12}) (&lt; 2\times10^{13})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h_{\text{PCM}}) vs (\Delta T)</td>
<td>14</td>
<td>-232.6</td>
<td>376</td>
<td>16.5</td>
<td>0.64</td>
<td>(\geq 2) (\leq 0.8)</td>
</tr>
<tr>
<td></td>
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<td>(h_{\text{PCM}}) vs (\Delta T)</td>
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<td>16.5</td>
<td>0.64</td>
<td>(\geq 2) (\leq 0.8)</td>
</tr>
<tr>
<td></td>
<td>Third region</td>
<td>Nu vs Ra</td>
<td>73</td>
<td>-2(\times10^{-22})</td>
<td>2(\times10^{-10})</td>
<td>-4.7</td>
<td>0.64</td>
<td>(\geq 7.1\times10^{10}) (\leq 6\times10^{11})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Nu vs (R_{\text{eff}})</td>
<td>30</td>
<td>-3(\times10^{-24})</td>
<td>3(\times10^{-11})</td>
<td>-4.7</td>
<td>0.64</td>
<td>(\geq 5\times10^{11}) (\leq 5\times10^{12})</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(h_{PC}) vs (T)</td>
<td>54</td>
<td>1(\times10^{-25})</td>
<td>-5(10^{-12})</td>
<td>93</td>
<td>0.88</td>
<td>(&gt; 5\times10^{12}) (\leq 2\times10^{13})</td>
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<tr>
<td></td>
<td></td>
<td>(h_{PC}) vs (T)</td>
<td>30</td>
<td>3(\times10^{-28})</td>
<td>-5(10^{-13})</td>
<td>35</td>
<td>0.73</td>
<td>(&gt; 2\times10^{13}) (\leq 6\times10^{13})</td>
</tr>
</tbody>
</table>

\(Y = ax^2 + bx + c\)

\(^{(1)}\) n → Number of experimental data

\(^{(2)}\) \(r^2\) → Quadratic mean difference

\(^{(3)}\) Low → Lower value of the range of application

\(^{(4)}\) High → Higher value of the range of application
FIGURE CAPTIONS

Fig. 1. Module with external vertical fins.
Fig. 2. Diagram of the experimental set-up.
Fig. 3. Instrumentation of the experiments.
Fig. 4a. Water and PCM temperature over time of experimental work using a PCM module without fins.
Fig. 4b. Water and PCM temperature over time of experimental work using a PCM module with vertical external fins of 20 mm length.
Fig. 4c. Water and PCM temperature over time of experimental work using a PCM module with vertical external fins of 40 mm length.
Fig. 5. Comparison of the heat transfer coefficient for natural convection for different PCM modules.
Fig. 6a. Experimental correlation for natural convection coefficient for a PCM module with 20 mm fins.
Fig. 6b. Experimental correlation for natural convection coefficient for a PCM module with 40 mm fins.
Fig. 7a Experimental correlation of Nusselt number in function of Rayleigh number for a module with 20 mm fins.
Fig. 7b Experimental correlation of Nusselt number in function of Rayleigh number for a module with 40 mm fins.
Fig. 8a Experimental correlation of Nusselt number in function of Rayleigh effective number for a module with 20 mm fins.
Fig. 8b Experimental correlation of Nusselt number in function of Rayleigh effective number for a module with 40 mm fins.
Fig. 2
Fig. 5

Natural convection heat transfer coefficient (W/m²K) vs. Temperature difference (°C)

- No Fins
- Small Fins
- Big Fins

![Graph showing natural convection heat transfer coefficient vs. temperature difference for different fin configurations.](image-url)
\[ y = -490.58x^2 + 1248.8x - 618.9 \]
\[ R^2 = 0.8409 \]

\[ y = -35.668x + 225.99 \]
\[ R^2 = 0.9772 \]

\[ y = 1.3527x^2 - 24.174x + 163.74 \]
\[ R^2 = 0.9815 \]

\[ y = -232.63x^2 + 375.92x - 16.52 \]
\[ R^2 = 0.6416 \]

\[ y = 7.7697x^2 - 64.279x + 184.88 \]
\[ R^2 = 0.8804 \]

\[ y = 0.3303x^2 - 9.1883x + 83.13 \]
\[ R^2 = 0.8269 \]
Fig. 7a

First region (20 mm fin)
Second region (20 mm fin)
Third region (20 mm fin)

\[ y = -4E-22x^2 + 8E-10x - 307.99 \]
\[ R^2 = 0.792 \]

\[ y = 3E-24x^2 - 4E-11x + 127.65 \]
\[ R^2 = 0.9831 \]

\[ y = 4E-25x^2 - 7E-12x + 61.468 \]
\[ R^2 = 0.9908 \]

Fig. 7b

First region (40 mm fin)
Second region (40 mm fin)
Third region (40 mm fin)

\[ y = -2E-22x^2 + 2E-10x - 4.6824 \]
\[ R^2 = 0.6395 \]

\[ y = 6E-24x^2 - 4E-11x + 91.521 \]
\[ R^2 = 0.8871 \]

\[ y = -4E-12x + 35.128 \]
\[ R^2 = 0.7192 \]
\[ y = 1E^{-11}x - 98,325 \]
\[ R^2 = 0.7628 \]

\[ y = 8E^{-27}x^2 - 2E^{-12}x + 121,93 \]
\[ R^2 = 0.9767 \]

\[ y = 1E^{-27}x^2 - 5E^{-13}x + 61,468 \]
\[ R^2 = 0.9908 \]

First region (20 mm fin)
Second region (20 mm fin)
Third region (20 mm fin)

Fig. 8a

\[ y = 1E^{-25}x^2 - 5E^{-12}x + 93,06 \]
\[ R^2 = 0.9767 \]

\[ y = 3E^{-28}x^2 - 5E^{-13}x + 35,244 \]
\[ R^2 = 0.7332 \]

First region (40 mm fin)
Second region (40 mm fin)
Third region (40 mm fin)

Fig. 8b