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A GENERAL REVIEW OF THE WILSON PLOT METHOD AND ITS
MODIFICATIONS TO DETERMINE CONVECTION COEFFICIENTS
IN HEAT EXCHANGE DEVICES

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Abstract
Heat exchange devices are essential components in complex engineering systems related to energy generation and energy transformation in industrial scenarios. The calculation of convection coefficients constitutes a crucial issue in designing and sizing any type of heat exchange device. The Wilson plot method and its different modifications provide an outstanding tool for the analysis and design of convection heat transfer processes in research laboratories. The Wilson plot method deals with the determination of convection coefficients based on measured experimental data and the subsequent construction of appropriate correlation equations. This paper is to present an overview of the Wilson plot method along with numerous modifications introduced by researches throughout the years to improve its accuracy and to extend its use to a multitude of convective heat transfer problems. Undoubtedly, this information will be useful to thermal design engineers.

Keywords: Wilson plot method, convection coefficients, heat exchange devices.
Nomenclature

A  heat transfer area (m$^2$)
C  constant
$C_p$ specific heat capacity at constant pressure (J/kg·K)
$d$ diameter (m)
$f$ general function
$h$ convection coefficient (W/(m$^2$·K))
k thermal conductivity (W/(m·K))
$L$ length (m)
$LMTD$ logarithmic mean temperature difference (K)
m mass flow rate (kg/s)
$Pr$ Prandtl number
$q$ heat flow (W)
$R$ thermal resistance (K/W)
$Re$ Reynolds number
$T$ temperature (K)
$U$ overall heat transfer coefficient (W/(m$^2$·K))
v velocity (m/s)
$X$ vapour quality (kg/kg)

Subscripts

A  fluid A
B  fluid B
$f$ fluid
i  inner
in  inlet
l  liquid
o  outer
out outlet
ov overall
r reduced
s surface
v vapour
w wall
Superscripts
m exponent of Prandtl number
n exponent of velocity or Reynolds number
1. INTRODUCTION

The heat transfer mechanism by convection entails energy transfer between a solid surface and a moving fluid due to a prescribed temperature difference between the solid surface and the fluid. Heat transfer by convection is indeed a combination of conduction and fluid motion. The analytical treatment of convection problems requires the solution of a system of mass, momentum, and energy conservation equations for a body geometry and fluid properties ending with the flow and temperature fields in the fluid. Although these procedures are elaborate, analytic solutions are available for simple geometries under restrictive assumptions. Most of the convective heat transfer processes inherent to heat exchangers usually involve complex geometries and intricate flows so that the analytical solutions are not possible. Therefore, an approach based on Newton's law of cooling, Eq. 1, provides a simple alternate avenue.

\[ q = A \cdot h \cdot (T_s - T_f) \]  

Newton's law of cooling established an algebraic relation between the heat flow by convection \( q \), the surface area \( A \), an average convection coefficient \( h \) and the temperature difference between the solid surface \( T_s \) and the fluid \( T_f \). Under this framework, the convection problems reduce to the estimation of the convection coefficient \( h \).

For a given flow and surface geometry, the experimental data is usually obtained by measuring the surface area and the fluid temperatures for an imposed heat condition. Then, the convection coefficient may be calculated from Eq. (1). However, the main difficulty of this methodology lies in the measurement of the surface temperature, because the surface temperature varies from point to point, and the flow pattern could be altered by the presence of the temperatures sensors. The problem is even more complicated if the heat transfer surface is not accessible, as usually happens with heat exchangers. Therefore, any alternative method conducive to the heat transfer calculation of convection coefficients in heat exchangers is attractive because of widespread use and practical applications.
The Wilson plot method constitutes a suitable technique to estimate the convection coefficients in a variety of convective heat transfer processes. The Wilson plot method avoids the direct measurement of the surface temperature and consequently the disturbance of the fluid flow and heat transfer introduced while attempting to measure those temperatures. Modifications of the Wilson plot method have been proposed by many researches continually to enhance its accuracy. This review paper focuses on the modifications published in the archival literature that led to the so-called modified Wilson plot method.

2. ORIGINAL WILSON PLOT METHOD

This Wilson plot method was developed by Wilson in 1915 [1] to evaluate the convection coefficients in shell and tube condensers for the case of a vapour condensing outside by means of a cool liquid flow inside. It is based on the separation of the overall thermal resistance into the inside convective thermal resistance and the remaining thermal resistances participating in the heat transfer process. The overall thermal resistance of the condensation process in a shell and tubes condenser \( R_{ov} \) can be expressed as the sum of three thermal resistances corresponding to 1) the internal convection \( R_i \), 2) the tube wall \( R_w \) and 3) the external convection \( R_o \), as shown in Eq. (2).

\[
R_{ov} = R_i + R_w + R_o
\]  
(2)

For the sake of simplicity, the thermal resistances due to the fluid fouling in Eq. (2) are neglected. Employing the proper expressions for the thermal resistances showing up in Eq. (2), the overall thermal resistance can be rewritten as Eq. (3).

\[
R_{ov} = \frac{1}{h_i \cdot A_i} + \frac{\ln(d_o/d_i)}{2 \cdot \pi \cdot k_w \cdot L_w} + \frac{1}{h_o \cdot A_o}
\]  
(3)

where \( h_i \) and \( h_o \) are the internal and external convection coefficients, \( d_i \) and \( d_o \) are the inner and outer tube diameters, \( k_w \) is the tube thermal conductivity, \( L_w \) is the tube length and \( A_i \) and \( A_o \) are the inner and outer tube surface areas, respectively.
On the other hand, the overall thermal resistance can be conceived as a function of the overall heat transfer coefficient referred to the inner or outer tube surfaces and the corresponding areas. In this regard, in Eq. (4) the overall thermal resistance is expressed as a function of the overall heat transfer coefficient referred to the inner or outer surface \((U_{i/o})\) and the inner or outer surface area \((A_{i/o})\).

\[
R_{ov} = \frac{1}{U_{i/o} \cdot A_{i/o}} 
\]  

(4)

Taking into account the specific conditions of a shell and tube condenser and the equations indicated above, Wilson [1] theorized that if the mass flow of the cooling liquid was modified, then the change in the overall thermal resistance would be mainly due to the variation of the in-tube convection coefficient, while the remaining thermal resistances remained nearly constant. Therefore, as indicated in Eq. (5) the thermal resistances outside of the tubes and the tube wall could be considered constant,

\[
R_w + R_o = C_t
\]

(5)

Wilson [1] determined that for the case of fully developed turbulent liquid flow inside a circular tube, the convection coefficient was proportional to a power of the reduced velocity \((v_r)\) which accounts for the property variations of the fluid and the tube diameter. Thus, the convection coefficient could be written according to Eq. (6).

\[
h_i = C_2 \cdot v_r^n
\]

(6)

where \(C_2\) is a constant, \(v_r\) is the reduced fluid velocity and \(n\) is a velocity exponent. Then, the convection thermal resistance corresponding to the inner tube flow is proportional to \(1/v_r^n\). Further, upon combining Eqs. (2), (5) and (6), the overall thermal resistance turns out to be a linear function of \(1/v_r^n\); this is represented graphically in Fig. 1. An inspection of the correlation equation (7) indicates that \(C_t\) is the intercept and \(1/(C_2A)\) is the slope of the straight line.
\[ R_{ov} = \frac{1}{C_2 \cdot A_1} \cdot \frac{1}{v_r^n} + C_1 \]  

(7)

On the other hand, the overall thermal resistance and the cooling liquid velocity can be obtained by experimentally measuring the inlet temperature \(T_{\text{in},\text{in}}\), the outlet temperature \(T_{\text{out},\text{out}}\) and the vapour condensation temperature \(T_v\) at various mass flow rates \(m_l\) of the cooling liquid under fully developed turbulent flow. Then, for each set of experimental data corresponding to each mass flow rate, the overall thermal resistance is the ratio between the logarithmic mean temperature difference of the fluids \(LMTD\) and the heat flow transferred between them, according to Eq. (8). The heat flow is determined from the enthalpy change of the cooling liquid as give in (Eq. 8).

\[ R_{ov} = \frac{LMTD}{m_l \cdot C_p l \cdot (T_{\text{out},\text{out}} - T_{\text{in},\text{in}})} \]  

(8)

Therefore, if a value of the velocity exponent \(n\) in Eq. (6) is assumed, then the experimental values of the overall thermal resistance can be represented as a linear function of the experimental values of \(1/v_r^n\). From here, the straight-line equation that fits the experimental data can be deduced by applying simple linear regression. Then, the values of the constants \(C_1\) and \(C_2\) are calculated from to Eq. (7) as indicated in Fig. 1. Once the constants \(C_1\) and \(C_2\) are determined, then the external and internal convection coefficients for a given mass flow rate can be evaluated from the combination of Eqs. (6) and (9).

\[ h_0 = \frac{1}{(C_1 - R_{ov}) \cdot A_o} \]  

(9)

The above exposition is the original Wilson plot method [1]. It relies on the fact that the overall thermal resistance can be extracted from experimental measurements in a reliable manner. As a result, the mean value of the convection coefficient outside the tubes (mean value of the outside thermal resistance) and the convection coefficient inside the tubes can be estimated as a function of the velocity of the cooling liquid. Therefore, the original Wilson plot method could be applied to convection heat transfer processes when the thermal resistance provided by one
of the fluids remains constant, while varying the mass flow of the other fluid within the adequate range of fully developed turbulent flow. Moreover, the exponent of the velocity in the general correlation equation considered for the convection coefficient of the fluid whose mass flow is changed should be assumed. Wilson [1] assumed the exponent 0.82 for the velocity. Then, the aim of the method could be, either the calculation of the convection coefficient (or thermal resistance) of the fluid that provides a constant thermal resistance, or a general correlation for the convection coefficient of the fluid whose mass flow is varied. As pointed out, the application of the Wilson plot method is based on the measurement of experimental mass flow rate and temperatures. Wójs and Tietze [2] performed an interesting analysis on the implications of the accuracy of the measured temperatures obtained by the Wilson plot method. These authors drew attention to the importance of making use of adequate experimental data to guarantee results.

An early application of the original Wilson plot method to analyse the performance of six plain and finned tube bundles forming a shell and tube heat exchanger was reported by Williams and Katz [3]. Experiments were conducted for water, lubricating oil and glycerine in the shell side and cooling water inside the tubes. For each of the fluids in the shell side, experiments were carried out at different temperature levels. The Wilson plot method was applied to all sets of experimental data and the outside convection coefficient were obtained. Afterwards, in a subsequent analysis the shell-side convection coefficients were correlated in form resembling the Sieder-Tate correlation equation [4]. New exponents for the Reynolds and Prandtl numbers and a multiplier for each one of the tubes bundles are proposed.

More applications of the original Wilson plot method are found in the archival literature. Hasim et al. [5, 6] used the original Wilson plot to investigate the heat transfer enhancement by combining ribbed tubes with wire [5] and twisted tape inserts [6]. The experimental apparatus consists of a double pipe heat exchanger with water as the cooling and heating fluids. The convection coefficients inside the enhanced tubes are assumed to be proportional to a power of the fluid velocity with known exponents, as indicated in Eq. (6). Both papers reported results of the experimental convection coefficients. Zheng et al. [7] applied the Wilson plot method to
analyze the heat transfer processes in a shell and tube flooded evaporator in an ammonia-compression refrigeration system. The study was carried out for plain tubes forming the bundle. The ammonia-lubricant mixture evaporates outside the tubes and a heated water-glycol solution flowed inside the tubes. Results of the boiling convection coefficients were reported and a suitable correlation equation was proposed.

Fernández-Seara et al. [8] described a simple experimental apparatus that allows for the measured data required for the application of the Wilson plot method. The test section consists of a transparent methacrylate enclosure, wherein water vapour generated at the bottom condenses over a test tube cooled by circulating water inside. An asset of the experimental apparatus deals with the possibility of testing different type of tubes. Once the experimental data is recorded, the original Wilson plot method or any of the Wilson plot modifications can be applied. Also, a collection of results gathered with this experimental apparatus are reported in Fernández-Seara et al. [9], this time utilizing a smooth, spirally corrugated tube made of stainless steel.

3. MODIFIED WILSON PLOT METHODS

3.1. Determining two constants in the functional form for the convection coefficient of one fluid. The other fluid has constant thermal resistance.

After the formulation of Wilson [1], general correlation equations for the analysis of internal forced convection based on the Reynolds analogy have appeared in the literature. Representative papers are those of Dittus-Boelter [10], Colburn [11], Sieder-Tate [4]. These correlation equations basically relate the Nusselt number with the Reynolds and Prandtl numbers in conformity with Eq. (10). Some of these equations are sophisticated because they incorporate the variability of the fluid properties with temperature. Therefore, early modifications of the Wilson plot method assumed a general correlation for the convection coefficient in which the mass flow is varied (fluid A) as a power of the Reynolds and Prandtl numbers instead of the fluid velocity (Eq. 10). In this format, the exponents of the Reynolds number (nA) and Prandtl number (mA) in Eq. 10) have to be assumed.
Later on, correlation equations attributable to Petukhov [12] and Gnielinski [13] contained an unknown multiplier as part of the general functional forms for turbulent forced convection. These general functional forms are applicable to laminar forced convection as well. There are also situations where the mass flow rate of the fluid undergoing a phase change process is varied. Moreover, there are also applications involving phase change where the convection coefficient is modified by changing the inlet flow quality \((X)\). In these cases, appropriate correlation equations have to be constructed for the convection coefficient as a function of an unknown multiplier and the mass flow rate or the inlet flow quality. Thereby, this simple modification of the original Wilson plot method presupposessed the existence of a general functional form for the convection coefficient of the fluid whose flow conditions can be varied in the experimental analysis (fluid A). This option responds to the general form of Eq. (11), and a constant thermal resistance for the other fluid (fluid B) obeying Eq. (12). Thereafter, the overall thermal resistance can be associated with Eq. (13), as a function of the convection coefficient of fluid A (Eq. 11) and the constant thermal resistance of fluid B in Eq. (12). The form of Eq. (13) is linear where the slope depends on the multiplier \(C_A\) (Eq. 11) and the intercept between the regression straight line and the thermal resistance axis -- the constant \(R_B\). Then, the Wilson plot permits the calculation of the unknown constant \(C_A\) (Eq. 11) and the thermal resistance \(R_B\) of fluid B, as represented graphically in Fig. 2. This sequence is synthesized as follows.

\[
\begin{align*}
    h_A &= C_A \cdot f_A[m,v,X,...] \\
    R_B &= C \quad (h_B = \text{constant}) \\
    R_{WV} &= \frac{1}{C_A} \cdot \frac{1}{f_A[m,v,X,...]} A_A + (R_w + R_B)
\end{align*}
\]

Early applications of the above modification are the topic of four publications by Young et al. [14-17]. These papers contained different ensembles such as the contact thermal resistance of bimetal tubes [14], heat transfer in clean finned tubes [15], fouling in finned tubes and coils [16].
and fouling in shell-and-tube heat exchangers operating in a refinery [17]. In all these works, the outside tube thermal resistance was taken as a constant and the in-tube convection coefficient was expressed by the general form of the Dittus-Boelter equation [10] for water as proposed by McAdams [18]. The application of the Wilson method was corroborated in terms of the corresponding quantities.

Recent papers devoted to the use of the modified Wilson plot method are reviewed in the forthcoming paragraphs. These papers are grouped according to the type of heat transfer process considered, but most of them are related with condensing gases. Chang and Hsu [19] applied the modified Wilson plot method to analyse the condensation of R-134a on two horizontal enhanced tubes with internal grooves. The authors considered the functional form of the Dittus-Boelter equation [10] for the convection coefficient of water flowing inside the tubes and a constant thermal resistance for the condensing fluid. Later, Da Silva et al. [20] used the correlation obtained by Chang and Hsu [19] for the water-side convection coefficient to study the electrohydrodynamic enhancement of R-134a condensation over the same type of enhanced tubes. Cheng et al. [21] studied the condensation of R-22 on six different horizontal enhanced tubes using water flowing inside the tubes. The Dittus-Boelter correlation equation [10] was the vehicle to obtain the water convection coefficient, whereas the condensing thermal resistance was taken as constant. Subsequently, the convection coefficients for the tested tubes were extracted by means of the Wilson plot. Kumar et al. [22] reported a similar analysis of [21] to estimate the steam condensing convection coefficient over three different types of horizontal cooper tubes. However, in this work the Sieder-Tate correlation equation [4] was modified to account for the entrance effects in short tubes. The functional form was associated with convection coefficient for the cooling water. McNeil et al. [23] examined the filmwise and dropwise condensation of steam and a steam-air mixture over a bundle of tubes with cooling water at turbine/condenser conditions. The thermal resistance of the shell-side was maintained constant and independent from the outside tube-wall temperature due to the vapour shear conditions generated by the high velocity of the steam. The Wilson plot method was implemented varying the cooling water mass flow and the convection coefficient was expressed by way of the Gnielinski equation [13] with a constant multiplier. Das et al. [24] relied on the
modified Wilson plot to determine the convection coefficient for the dropwise condensation of steam on horizontal tubes with different types of dropwise condensation promoters. It was assumed that the convection coefficient of the water inside the tubes could be accommodated into the Petukhov-Popov correlation equation [12] embracing a multiplier and also that the dropwise condensation convection coefficient was constant. Both the multiplier in the inner convection coefficient equation and the condensing convection coefficient were evaluated with the help of the Wilson plot. An iterative procedure was indispensable to account for the variability of the water properties with temperature.

Chang et al. [25] selected the Wilson plot method for the quantification of the condensing convection coefficients of R-134a and R-22 flowing inside four different extruded aluminium flat tubes and one micro fin tube. The tube testing was accomplished in a double-tube configuration set-up. In reference to the flat tubes, they were placed inside a shell with rectangular cross-section. In contrast, the micro-fin tube was located inside a shell with circular cross-section. The refrigerant condensation took place inside the tubes and coolant water flowed between the tube and the shell. The convection coefficient for the coolant water was held constant by controlling a steady flow rate of water and a small deviation of the mean water temperature was noticed. In this case, the condensing convection coefficient was varied by changing the quality of the inlet refrigerant; the quality was controlled by means of a pre-condenser. The condensing convection coefficient was considered proportional to a power of the inlet quality with an exponent of - 0.8. The paper reported experimental results of the condensing convection coefficients as a function of the quality of inlet refrigerant.

Also, this type of modification had been articulated with single-phase in-tube convection coefficients. Wang et al. [26] utilized a tube-in-tube heat exchanger to characterize the single-phase convection coefficients within micro-fin tubes. The authors invoked the Dittus-Boelter correlation equation [10] for the inner convection coefficient. The thermal resistance within the annulus was held constant by holding the average fluid temperature and the Reynolds number to constant values. These authors provided suitable correlation equations for each one of the micro-fin tubes tested. Webb et al. [27] also resorted to a tube-in-tube configuration to
recommend a correlation equation for the inner convection coefficient accounting for the dimensions of the enhancement geometry. Experiments were carried out with R-12 boiling in the annulus and varying the mass flow of the cooling water inside the tubes. The Sieder-Tate correlation equation [4] provided the framework for the turbulent heat transfer inside the tube and a constant boiling convection coefficient in the annulus (i.e., constant thermal resistance in the annulus).

Other applications of the Wilson plot method aimed at determining the thermal resistances viewed as constant, namely $R_w + R_B$ in Eq. 13. The final goal could be the calculation of the convection coefficient of the corresponding fluid or any other thermal resistance that participate in the overall thermal resistance such as a contact thermal resistance. Taylor [28] articulated the Wilson plot to the determination of the air-side convection coefficient in finned-tube heat exchangers. Despite that the air-side thermal resistance was considered constant, the cooling water mass flow was allowed to vary. The Dittus-Boelter correlation equation [10] was linked to the water-side convection coefficient and the constant multiplier was determined. The air-side convection coefficient was obtained from the thermal resistance and later correlated in the form of Colburn $j$-factor. Eight fin-and-tube heat exchangers were tested and the effects of fin spacing, number of rows and fin enhancement were reported. The work of ElSherbini et al. [29] revolved around the application of the Wilson plot method to the contact thermal resistance between fins and tubes in a plain finned heat exchanger. Using a mixture of ethylene glycol and water as coolant, the coolant flow rate was varied under constant air-side conditions. The air-side thermal resistance was idealized as constant and the Gnielinski correlation equation [13] was representative of the coolant-side convection coefficient. Experiments with unbrazed and brazed coils, both under dry and frosted conditions were conducted. The fin contact resistance was calculated from the air-side thermal resistance within the framework of the Wilson plot. Critoph et al. [30] embarked on an analysis similar to the one by ElSherbini et al. [29] to address the issue of the fin contact resistance in air-cooled plate fin air conditioning condensers. In this case, the mass flow rate of refrigerant R-22 condensing inside the tubes was varied while the air-side Reynolds number was held constant. The refrigerant-side convection coefficient was paired with the Nusselt theory [31] and the air-side thermal resistance was idealized as
constant. The contact fin-tube thermal resistance was calculated from the air-side thermal resistance with the Wilson plot method.

Some Wilson plot applications have been identified within the platform of boiling processes. Aprea et al. [32] determined the boiling convection coefficients of refrigerants R-22 and R-407C in a tube-in-tube evaporator. In this sense, the refrigerant flows inside the inner tube as opposed to a water-glycol solution that moves through the annulus. The inner thermal resistance was considered constant and the annulus-side convection coefficient was assumed to follow the Dittus-Boelter correlation equation [10]. Not only the boiling convection coefficients for the two refrigerants were obtained, but they were compared against different correlation equations.

Finally, specialized applications have been investigated such as Hariri et al. [33] who by way of Wilson plot method determined the convection coefficients in the reactor and in the jacket of a calorimeter. The overall heat transfer coefficient was expressed linearly as a function of the agitation speed. The jacket convective heat transfer was calculated and the inside convection coefficient was channeled through the Sieder-Tate correlation equation [4] having the exponent 2/3 for the Reynolds number. Lavanchy et al. [34] and Fortini et al. [35] conceived a similar analysis associated with reaction calorimeters for supercritical fluids. The convection coefficient in the reactor was also channeled through with the Sieder-Tate correlation equation [4] having a power of the stirrer rotation speed. In addition, the thermal resistance corresponding to the jacket wall and to the coolant convection were kept constant.

### 3.2. Determining two constants in the functional form for the convection coefficients of the two fluids.

Generally, if the convection coefficient of one of the fluids (fluid A) varies, the convective thermal resistance provided by the other fluid (fluid B) is altered because the surface temperatures change, and as a consequence the fluid properties will change.

A further modification of the Wilson plot method has been contemplated to avoid the assumption of a constant thermal resistance for one of the fluids. This modification consists in
formulating a functional form for the convection coefficient for the fluid (fluid B), instead of a constant thermal resistance. This general equation will absorb the variation of the surface temperature and will comprise an unknown multiplier, as indicated in Eq. (14). In this regard, the Wilson plot method envisions the calculation of the two constants embedded in the general functional forms (Eqs. 11 and 14) related to the convection coefficients of both fluids.

\[ h_B = C_B \cdot f_B(T) \]  \hfill (14)

Utilizing the general expressions given by Eqs. (11) and (14), the overall thermal resistance is supplied by Eq. (15). Despite that this equation has a nonlinear character, it can be easily rearranged into a linear functional form. This is doable in such a way that the slope depends on the constant \( C_A \) only and the intercept between the regression line and the thermal resistance axis delivers the constant \( C_B \) as shown in Eq. (16). Thereafter, the application of the Wilson plot as indicated in Fig. 3 is synonymous with for the determination of both multipliers, \( C_A \) and \( C_B \).

\[ R_{ov} = \frac{1}{C_A \cdot f_A[m(v), X, \ldots]} A_A + R_w + \frac{1}{C_B \cdot f_B(T)} A_B \]  \hfill (15)

\[ (R_{ov} - R_w) \cdot f_B(T) \cdot A_B = \frac{1}{C_A \cdot f_A[m(v), X, \ldots]} A_A + \frac{1}{C_B} \]  \hfill (16)

This systematic procedure was initially described by Young and Wall [36], who applied it to a heat exchanger owing two single-phase fluids. However, most of the references that referred to it dealt with condensation processes on tubes with cooling turbulent liquid circulating inside. The functional forms contemplated rest on the Nusselt theory [31] for the condensing side and on well-known correlation equations due to Dittus-Boelter [10], Sieder-Tate [4] or Gnielinski [13], all corresponding to turbulent flows inside the tubes. Kumar et al. [37] extended this procedure for the calculation of the condensing convection coefficient of steam and R-134a over a plane wall and different finned cooper tubes. A suitable combination of the Nusselt theory [31] and the Sieder-Tate correlation equation [4] were considered for the condensing and the inner convection coefficients. Singh et al. [38] repeated the same type of analysis to study the condensation of steam over a vertical grid of horizontal integral-fin tubes. In this case, the
authors modified the Sieder-Tate correlation equation [4] to incorporate the entrance effects in short tubes. Hwang et al. [39] also adopted the same approach to study the heat transfer characteristics of various types of enhanced titanium tubes. These authors focused on the condensation of steam over the tubes, which are internally cooled by water.

Abdullah et al. [40] and Namasivayam and Briggs [41,42] paid attention to the benefits of the modified Wilson plot for the case of condensation of pure vapour over plain tubes. The ultimate goal was the construction of a correlation equation for the coolant side. On one hand, the Nusselt theory [31] and the Shekriladze and Gomelauri equation [43] are used for the condensing vapour and on the other hand, the Sieder-Tate correlation equation [4] suffices for the cooling water. However, in the three papers the unknown constants are delimited by the minimization of the sum of squares of residuals of the overall temperature difference. Rose [44] discussed this procedure to calculate the unknown constants. Ribatski and Thome [45] also applied this modified scheme of the Wilson plot to resolve for the proper convection coefficient for nucleate boiling of R-134a on four different enhanced tubes that are commercially available. The heating medium was water flowing inside the tubes. In this case, the boiling convection coefficient was assumed proportional to a power of the heat flux, being the exponent 0.7. The inner convection coefficient was expressed in the form compatible with Gnielinski correlation equation [13]. At the end, the Wilson plot was obtained by varying the mass water flow for constant wall heat flux.

3.3. Determining more than two constants.

The most general conceptualization of the Wilson plot method involves three constants, two of them corresponding to the assumed functional form for one of the two fluids. Usually, the third unknown constant stands for the exponent of the flow parameter inherent to one of the fluids. This unknown constant does not behave linearly, and needs to be found by an iterative procedure forcibly. Initially, Briggs and Young [46-47] proposed a second linearized equation obtained by applying natural logarithms to both sides of Eq. (15) in such a way that this second equation gets linearized in the third unknown constant. Then, an iterative procedure engaging the two linear regression equations leads to the calculation of the three constants. Later, Shah
[48] envisaged a new modification of the Briggs and Young method [46] to correlate for one of the fluids when the correlation of the other fluid is unknown or is of no interest. Kedzierski and Kim [49], Yang and Chiang [50] and Fernández-Seara et al. [8-9] undertook a linearization plan of the non-linear equation by means of natural logarithms. Subsequently, several authors proposed different mathematical models for the identification of the unknown parameters in the equations derivable from the Wilson method. Khartabil and Christiensen [51] looked at a nonlinear regression model based on the resolution of three equations that emanated from the least squares method. The three equations, two of them linear and the third one nonlinear, enabled the authors to evaluate the three unknown constants. Then, the thermal resistance, along with the two constants associated with the variable thermal resistance of the other fluid, were evaluated. A drawback ascribable to this approach is its validity when the thermal resistance of one of the fluids is constant. Deviating from this format, Rose [44] proposed to compute the third unknown constant by way of an iterative scheme resulting from the minimization of the sum of squares of the residuals of the overall temperature difference. Khartabil et al. [52] scrutinized a further modification for the situation in which general forms with unknown constants are available for both fluids. This new approach ended up with the evaluation of two constants in each correlation equation together with the tube wall thermal resistance. However, this road requires the manipulation of three different sets of experimental data. Each set should be collected with a different dominant resistance (inner, outer and tube wall). Styrylska and Lechowska [53] purported a unified Wilson plot method for heat exchangers for conditions wherein a Nusselt equation is tied up to one of the fluids if the other does not suffer a phase change. This proposal is potent because it permits the calculation of any number of unknown constants, treating the unknowns as observations with a covariance matrix known a priori. Recently, Pettersen [54] chose the commercial software package DataFit (based on the Levenberg-Marquardt method) to conduct a nonlinear regression analysis of the experimental data points.

Most of the references found in the specialized literature are focalized on the modified Wilson plot method proposed by Briggs and Young [47]. Dirker and Meyer [55, 56] looked into a modification of the Wilson plot to appraise a suitable correlation equation for turbulent flow in a
smooth concentric annuli. These authors tested eight tube-in-tube heat exchangers with different annular diameter ratios using hot water in the inner tube and cold water in the annulus. The format of the Sieder-Tate correlation equation [4] was the platform for the in-tube and annular convection coefficients. The multipliers present in both correlation equations in conjunction with the exponent of the Reynolds number in the annulus were determined. Meanwhile, the Reynolds number exponent connected to the inner convection coefficient was fixed at 0.8. Results of the Reynolds number exponent and the multiplier in the annulus equation were correlated as a function of the radii ratio. The experimental-based correlation equation was also validated with a CFD (Computer Fluid Dynamics) code in Dirker et al. [57]. Further details about the CFD analysis appeared in Van der Vyver et al. [58]. An identical analysis was carried out by Coetzee et al. [59] to study the heat transfer behaviour of a tube-in-tube heat exchanger with angled spiralling tape inserts in the annulus. The analysis done in [59], was embraced by Louw and Meyer [60] with the goal at quantifying the convection coefficients. Da Veiga and Willem [61] picked out the same technique to figure out the convection coefficients in the hot and cold water sides of a semicircular heat exchanger.

A set of experiments was carried out by Rennie and Raghavan [62] on two different tube-in-tube helical heat exchangers. Cold water with constant mass flow rate was the fluid in the tube and hot water with variable mass flow rate was the fluid in the annulus. While the inner thermal resistance was taken as constant, the convective coefficient in the annulus was assumed to follow a power law of the fluid velocity (Eq. 6). The inner thermal resistance (inner convection coefficient), the constant and the velocity exponent for the annulus convection coefficient were obtained from the modified Wilson plot. Yang and Chiang [50, 63] analyzed the convection coefficients in a tube-in-tube heat exchanger with an inner curved pipe with periodically varying curvature using water as working medium. The functional forms for the inner and annulus convection coefficients were taken as proportional to the power of the Reynolds numbers. A value of 0.8 was assigned to the Reynolds number exponent in the annulus equation. Multipliers of both equations and the Reynolds number exponent for the inner convection coefficient were determined by applying the modified Wilson plot method.
Kedzierski and Kim [49] also applied the Wilson plot modified as proposed by Briggs and Young [47] to obtain the convection coefficient for an integral-spine-fin annulus in a tube-in-tube heat exchanger. However, in this case, the functional form of the annulus side includes an additional term to account for the thermally developing boundary layer. Experiments were carried out with water and 34% and 40% ethylene glycol/water mixture to determine the Prandtl number exponent of the annulus side too. Then, the modified Wilson plot was implemented in an iterative procedure that calculated the exponents of the Prandtl number and the additional term included in the equation of the annulus convection coefficient. The determination of the condensation convection coefficients of the zeotropic refrigerant mixture R-22/R-142b in a plain tube was done by Smit [64], Smit and Meyer [65] and Smit et al. [66] and in micro-fin, high-fin and twisted tape insert tubes by Smit and Meyer [67]. These studies were performed with a horizontal tube-in-tube condenser with water as the cooling medium. Butrymowicz et al. [68, 69] used the Wilson plot method to analyse the enhancement of condensation heat transfer by means of electrohydrodynamic enhancing technique. The modified Wilson plot method proposed by Briggs and Young [46] was extended to determine the annulus convection coefficient with a water-to-water configuration. Relying on the modified Wilson plot proposed by Briggs and Young [47], Yanik and Webb [70] obtained the water-side (i.e., shell-side) convection coefficient in a shell and tube evaporator with R-22 boiling inside the tubes. The refrigerant side convection coefficient was held constant by keeping the refrigerant flow rate, saturation temperature, inlet vapour quality and exit superheat constant while varying the water side mass flow rate. The functional form of the Dittus-Boelter correlation equation [10] was considered adequate for the water-side convective coefficient. Besides, the multiplier and the Reynolds number exponent were determined.

Benelmir et al. [71] examined the modified Wilson plot method appended with the improved scheme recommended by Khartabil and Christensen [51]. This combination is valid when the thermal resistance of one of the fluids is constant. Then, this thermal resistance and two constants associated with the variable thermal resistance of the other fluid are calculated from a nonlinear regression analysis. In reference [71], the influence of air humidity on the air-side convection coefficient in a plain fin and tubes heat exchanger is investigated. Here, the thermal
resistance of a glycol-water mixture flowing inside the tubes is taken as constant. The air flow and humidity are varied. The Colburn $j$-factor versus the Reynolds number is correlated with a power law model. The modified Wilson plot enabled them to quantify the multiplier and the Reynolds exponent in the Colburn $j$-factor correlation equations and the inside glycol thermal resistance. The authors reports four different correlation equations that incorporate the relative air humidity.

4. Indirect applications of the Wilson plot method.

The Wilson plot method and its variants furnish an indirect tool to generate accurate correlation equations for convective coefficients of secondary fluids in different types of heat exchange devices. Certainly, the use of general correlation equations may lead to errors since most of them do not take into account the specific operating conditions of the experimental facilities. Therefore, the application of the Wilson plot methods in a previous stage facilitates the determination of more accurate correlation equations that incorporate the particular features of the experimental equipment. In this section, a review of these indirect applications is addressed. However, it is noteworthy that, if the Wilson plot method is applied indirectly, usually the authors only quote its application and do not provide detailed information on how it was implemented. Therefore, the aim of the references to be cited lists the great variety of indirect applications that have been tackled by researchers. These references have been grouped according to the type of heat transfer process considered.

Kuo and Wang [72] analyzed the evaporation of R-22 in a smooth and in a micro-fin tube considering a tube-in-tube configuration with heating water in the annulus. By way of the modified Wilson plot method, the water convective coefficient in a water-to-water analysis was extracted. The Dittus-Boelter correlation equation [10] served as the vehicle for the convection coefficient in the annulus. The constant and the Reynolds number exponent are obtained with the modified Wilson plot method. Del Col et al. [73] focused on a similar application to determine the convective coefficient in the annulus in a water-to-water tube-in-tube heat exchanger. Kim and Shin [74] studied the evaporating heat transfer or R-22 and R-410A in horizontal smooth and microfin tubes using also a tube-in-tube configuration. These authors
framed the traditional Wilson plot with three different sets of experimental data to evaluate the convection coefficient of annulus side. Brognaux et al. [75] tested the comportment of single-phase heat transfer in micro-fin tubes using a water-to-water tube-in-tube configuration. The annulus convection coefficient was calculated by means of the modified Wilson plot method considering the Dittus-Boelter equation [10]. Yoo and France [76] experimentally observed the in-tube boiling heat transfer of R-113 using a tube-in-tube configuration. The modified Wilson plot method was used in previous experiments performed with R-113 liquid inside the tube to estimate the convective coefficient of the heating water in the annulus. In this case of the Monrad and Pelton correlation equation [77], it was taken for the annulus convection coefficient. Wang et al. [78] investigated the evaporation of R-22 inside a smooth tube in a double pipe configuration with water as the heating medium in the annulus. Within the context of the Wilson plot method, a correlation equation for the water-side convection coefficient was constructed by running separate water-to-water tests in the same experimental apparatus. Collectively, Yang and Webb [79], Webb and Zhang [80] and Kim et al. [81] summarized the use of the Wilson plot method to determine the annulus water-side convection coefficient in an experimental facility to inspect the condensation of R-12, R-134a and R-22 and R-410A in small extruded aluminium tubes with and without micro-fins.

Chang et al. [82] investigated the heat transfer performance of a heat pump filled with hydrocarbon refrigerants, in which the condenser and evaporator are concentric heat exchangers. The refrigerant flows inside the inner tube and ethyl alcohol flows through the annulus. The data was channelled through the modified Wilson plot method to obtain the convective coefficient for the secondary fluid in the annulus. Bukasa et al. [83,84] studied the condensation of R-22, R-134a and R-407C inside spiralled micro-fin tubes evaluating the effect of the spiral angle on the heat transfer rates. They used a tube-in-tube configuration in an experimental setup and with help from the Wilson plot method were able to determine a specific correlation equation that emanated from the general Sieder and Tate correlation equation [4].

Sami and Song [85], Sami and Desjardins [86-88], Sami and Grell [89,90], Sami and Maltais [91], Sami and Fontaine [92] and Sami and Comeau [93-97] designed a sequence of
experimental projects on boiling and condensation involving different refrigerants and refrigerant mixtures in an evaporator with enhanced surfaces and a condenser with coils in a vapour compression heat pump. In all the references, the data was accommodated into the Wilson plot method to eventually obtain the coils air-side convection coefficients. Sami and Desjardins [98-99], Sami and Song [100] and Sami and Poirier [101] considered the same analysis cited above but in an evaporator-condenser equipment having tube-in-tube configurations. In the first two references [98, 99], the Wilson plot was applied to isolate the water-side convection coefficient in the annulus. In contrast, in the last two references [100, 101] attention was paid to the in-tube convection coefficient since the phase change process takes place in the annulus. Yun and Lee [102] experimentally analyzed various types of fin-and-tube heat exchangers. The authors also applied the Wilson plot method to obtain the air-side convection coefficient. Cheng and Van der Geld [103] experimentally studied the heat transfer of air/water and air-steam/water in a polymer compact heat exchanger. The Wilson plot method was also used to determine the air and air-steam convection coefficients.

An experimental program was devised by Han et al. [104] to investigate the condensation heat transfer of R-134a in the annular region of a helical tube-in-tube heat exchanger. Adopting the Wilson plot method, the authors converted the data into a specific correlation equation for the water convection coefficient in the inner tube. A separate single-phase water-to-water test was performed in the same apparatus. Briggs and Young [105] and Young and Briggs [106] delineated an indirect application of the Wilson plots to evaluate the convective coefficient of water inside horizontal copper and titanium tubes of a shell and tubes condenser. Later on, the Sieder-Tate equation [4] was used to calculate the steam vapour condensing coefficient on the tube bundle. Belghazi et al. [107-110] studied the condensation heat transfer process of R-134a and of R-134a/R-23 mixture on a bundle of horizontal smooth and finned tubes. The authors applied the Wilson plot method beforehand to compute the convective coefficient of the water inside the tubes using a water-to-water counterflow configuration in a tube-in-tube heat exchanger. They considered the functional form of the Dittus-Boelter correlation equation [10] in the first reference and the Gnielinski correlation equation [13] in the last three. Gstoehl and Thome [111] examined the condensation of R-134a on tube arrays with plain and enhanced
surfaces. The Wilson plot method was coupled with a tube-in-tube configuration to transform the inside tube convection coefficient into the Gnielinski equation [13]. However, a peculiarity of this case is that nucleate pool boiling was chosen for the annulus. Zhnegguo et al. [112] experimental analysis of a shell-and-tube heat exchanger for the cooling of hot oil with water. The heat exchanger consisted of a helically baffled shell and a bundle of 32 petal-shaped finned tubes. The tube-side convection coefficient was nondimensionalized into the standard Dittus-Boelter equation [10].

Additionally, plate heat exchangers were linked with the Wilson plot method. Vlasogiannis et al. [113] analyzed a plate heat exchanger under two-phase flow conditions using air/water as the cold stream and water as the hot stream. The single-phase convection coefficient was characterized by an extension of the Wilson plot method accounting for symmetry in the two streams of the plate heat exchanger. The functional form of the Dittus-Boelter correlation equation [10] was instrumental for the convection coefficients of both streams. Both, the multiplier and the Reynolds number exponents were identified by applying this particular modification of the Wilson plot method. Longo et al. [114] investigated the vaporisation and condensation of R-22 inside herringbone-type plate heat exchangers with cross-grooved surfaces using water as heating/cooling medium. The water-side convection coefficient was obtained in a water-to-water arrangement following the footsteps of Vlasogiannis et al. [113].

The papers by Yan et al. [115] and Hsieh et al. [116] referred to the characteristics of flow condensation and flow boiling of R-134a in a vertical plate heat exchanger. Kuo et al. [117] and Hsieh and Lin [118] tackled the condensation and boiling of R-410A in a vertical plate heat exchanger also. In the four references cited above, the water convection coefficient related to a separate single-phase water-to-water test was manipulated with the Wilson plot method. Rao et al. [119] united experimentally and theoretically effects of flow maldistribution on the thermal performance of plate heat exchangers. The convective coefficients were converted into the Dittus-Boelter equation [10]. The constant multiplier and the Reynolds number exponent were treated with a modified Wilson plot method.
Finally, the reference of Kim et al. [120] was devoted to an experimental investigation on a counterflow slug flow absorber with an ammonia-water mixture. The absorber tested consisted of a vertical tube-in-tube with the mixture flowing in the inner tube and the cooling water in the annular part. The Wilson plot method was able to extract the convective on the side of the cooling water. In sum, the correlation equation expressed the Nusselt number as a power law of the Graetz number.

5. Conclusions

The general conclusions that could be drawn from the review paper is that the Wilson plot method and the different modified versions of the Wilson plot method constitute a remarkable tool for the analysis of convection heat transfer in tubes and heat exchangers for laboratory research usage. The trademark of the Wilson plot methods is that they assist in the determination of convective coefficients based on experimental data. In sum, the collection of convection coefficients are encapsulated in concise correlation equations.

The original Wilson plot method relied on two primary assumptions, a constant thermal resistance for one fluid and a known expression for the convective coefficient of the other fluid; the latter in the form of a power of the fluid velocity. The various modifications proposed by researchers all over the world have the common objective of overcoming the prevalent assumptions adhered to the original formulation of Wilson. For instance, one simple modification overcomes the assumption of constant thermal resistance and the determination of two multipliers by means of manipulating a linear regression analysis of the experimental data. Other modifications revolve around the determination of three of more constants, encompassing a second linear regression equation and iterative schemes or even advanced mathematical tools.

The range of applications of the Wilson plot methods, either directly or indirectly, includes a large variety of convective heat transfer processes and heat exchangers encountered in industry. Owing that more than one hundred publications are reviewed, it is believed that this effort will provide useful information for future users.
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Figures

Figure 1

\[ \text{Slope} = \frac{1}{C_2 \cdot A_i} \]

\{ Intercept = C_1 \}

\[ R_{ov} \]

\[ 1 / v_r^n \]

Figure 2

\[ \text{Slope} = \frac{1}{C_A} \]

\{ Intercept = (R_w + R_B) \}

\[ R_{ov} \]

\[ 1 / [f_A (m(v), X, \ldots) \cdot A_A] \]
\[ \text{Slope} = \frac{1}{C_A} \]

\[ \text{ Intercept} = \frac{1}{C_B} \]

\[ \frac{R_{ov} - R_w}{f_B(T)A_B} \]

\[ \frac{[f_B(T)A_B]}{[f_A(m(v), X,...)A_A]} \]

Figure 3
Figure captions

Fig. 1. Original Wilson plot.

Fig. 2. Wilson plot considering a functional form for the convective coefficient of one fluid and constant thermal resistance for the other fluid.

Fig. 3. Wilson plot considering functional forms for the convective coefficients of both fluids.