Piezo-ElectroMechanical (PEM) structures: passive vibration control using distributed piezoelectric transducers

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Structures Piézo-ÉlectroMécaniques (PEM) : contrôle passif de vibrations en utilisant des transducteurs piézo-électriques distribués

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Abstract

A piezo-electromechanical structural member is composed of a host member, a uniformly distributed array of piezoelectric transducers and a passive electric circuit (acting as a controller) interconnecting their electric terminals. Such a circuit has to be designed to assure the most efficient transduction of mechanical into electrical energy. The needed circuits are synthesized for bars, beams and plates and the performances of the corresponding PEM structures are determined. Once suitable dissipative elements are included in the controller, it is proven that, in PEM structures, mechanical vibrations are the most efficiently damped.

Résumé

Une structure piézo-électromécanique (PEM) se compose par un élément structurel hôte, une rangée uniformément distribuée de transducteurs piézoélectriques et un circuit électronique passif (agissant comme contrôleur) pour établir leur interconnexion électrique. Un tel circuit doit être conçu pour assurer la transduction la plus efficace de l'énergie mécanique en électrique. Les circuits nécessaires sont synthétisés pour des barres, des poutres et des plaques et les performances des structures PEM correspondantes sont déterminées. Une fois que des éléments dissipatifs appropriés sont inclus dans le contrôleur on montre que dans les structures PEM les vibrations mécaniques sont atténuées le plus efficacement possible.

Keywords: Control; Electric analog; Electric networks; Vibration damping

Mots-clés : Automatique ; Vibrations ; Analogue électrique ; Réseaux électriques ; Amortissement de vibrations

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Version française abrégée

Les développements technologiques récents dans les modes de production des transducteurs piézoélectriques et un intérêt général croissant pour l'amortissement de vibrations structurales, sont des facteurs qui ont contribué à intensifier les recherches dans le domaine de l'exploitation des transducteurs piézoélectriques dans des structures réelles (voir [1,2]). La technique d'atténuation électronique (electronic damping) attribuée à Olsen [3], et développée par Swigert et Forward [4], est l'une des premières tentatives d'exploitation de l'effet piézoélectrique dans le contrôle structural : un capteur piézoélectrique convertit la contrainte dynamique en courant électrique, ce dernier est amplifié, décalé dans la phase et appliqué à un actionneur piézoélectrique.

Cette technique, qui fut facilement améliorée et généralisée au cas à sorties et entrées multiples, doit être considérée comme active puisque elle exige nécessairement une alimentation en énergie. Si l'on veut obtenir des résultats satisfaisants en termes d'atténuation et stabilité dans cette stratégie d'amortissement active, on doit utiliser, pour le contrôle des capteurs, des amplificateurs de puissance sophistiqués et des circuits électroniques très sensibles.

Pour éviter les difficultés inhérentes au contrôle actif, Hagood et von Flotow [5] ont proposé une technique de contrôle passif : un actionneur piézoélectrique est shunté avec une résistance et un inducteur, qui sont eux mêmes accordés de façon optimale sur une fréquence modale donnée de la structure. Ce système n'est performant que dans le contrôle d'un unique mode de vibration de la structure ; en effet, le mode mécaniqueétant choisi, alors l'inductance critique, L_s^*, et la position optimale du transducteur piézoélectrique sont déterminées uniquement.

Dans le cas de ce contrôle localisé, l'inductance critique L_s^* doit être proportionnelle à $1/(C_p f_m^2)$, C_p correspondant à la capacité du transducteur piézoélectrique, f_m à la fréquence modale mécanique à contrôler. Les valeurs typiques de C_p et f_m imposent des limitations sérieuses aux inductances à employer : puisque la plage des ces inductances est de $10^2 \div 10^4$ henry, elles peuvent être obtenues soit par un circuit électronique de simulation [7] soit par des inducteurs à noyau ferroélectrique. Mais des inconvénients technologiques majeurs sont inhérents à ces deux possibilités ; il s'agit, dans le premier cas, de la nécessité d'une alimentation en énergie, dans le second, du fait que l'inducteur ajoute une fraction significative de toute la masse du système.

Chercher à réduire l'inductance critique L_s^* en ajoutant une capacité au circuit de contrôle s'avère être une solution inadéquate puisque cela implique, nécessairement, une diminution de l'efficacité du couplage piézoélectrique.

Tous les inconvénients mentionnés plus haut peuvent être évités grâce à un système de contrôle plus adapté : si l'on conçoit un contrôleur électrique passif et efficace sur plusieurs modes structuraux, on peut alors envisager d'exploiter une analogie électromécanique.

Ainsi on peut concevoir une structure PEM [8,9,17] comme :

- une rangée uniformément distribuée de transducteurs piézoélectriques placés sur la structure à contrôler (voir Fig. 2) ;
- un réseau électrique passif pour connecter l'ensemble des transducteurs, dont les propriétés spectrales sont adaptées à celles de la structure à contrôler (voir Figs. 2 et 3).

Il faut, par ailleurs, que le réseau électrique soit conçu pour être résonnant avec toutes les fréquences structurales propres mais aussi de telle sorte que ses formes modales puissent interagir piézoélectriquement avec celles de la structure à contrôler (Fig. 1).

Un système de ce type présente l'avantage de contrôler simultanément plusieurs modes sans avoir recours à des éléments actifs ou à de lourds inducteurs. De plus, grâce au comportement synergique des transducteurs piézoélectriques on obtient de très hautes performances en termes d'efficacité de transduction d'énergie mais également en termes de temps d'amortissement, voir formule (9), pour une largeur de bande de fréquence en principe infini.

La différence principale entre la stratégie exploitée par les structures PEM et la technique de shuntage passif se situe dans l'interconnexion des transducteurs piézo-électriques distribués.

Si le circuit électrique d'interconnexion est synthétisé pour être analogue à celui de la structure à contrôler, un choix simple des ses paramètres électriques permet l'amortissement de tous les modes mécaniques.

Quel que soit l' élément structural – barres, poutres et plaques – un circuit analogue totalement passif a été synthétisé. Ses performances en tant que contrôleur électrique sont comparées à celles (i) des circuits de shuntage séparés et à celles (ii) des réseaux électriques du second ordre pour toutes les structures présentées. Le contrôle multi-modal n'est possible que si l'on a recours à des circuits analogues et, alors, les inductances critiques nécessaires ont la valeur minimale.

Les structures PEM sont conçues de manière à obtenir une transduction efficace de l'énergie mécanique ; en effet, une fois que l'énergie a été transformée sous forme électrique, elle peut être dissipée rapidement.

Les temps d'amortissement des vibrations électromécaniques peuvent être optimisées en ajoutant judicieusement, dans le contrôleur électrique, quelques résistances (voyez [10] et Figs. 2 et 3). Ils dépendent des coefficients de couplage piézoélectrique et sont donnés par la formule (9).

Le développement de ces résultats doit déboucher à terme sur :

- la synthèse des analogues électriques pour des structures plus compliquées, comme coques, poutrages et treillis ;
- une analyse précise du procédé d'homogénéisation nécessaire pour obtenir les équations d'évolution pour les structures PEM ;
- la conception d'un prototype expérimental pour établir la validité de l'analyse théorique développé.

1. Introduction

1.1. State of the art

The recent technological developments in the production of piezoelectric transducers, and the relevant attention of consumers towards the suppression of structural vibrations, have increased the research efforts in the effective exploitation in actual engineering structures (see [1] and [2]). An efficient control of structural vibrations leads to other benefits such as the precision in mechanisms manoeuvres, reduced fatigue loads, reliability and durability of machineries: these were the main reasons to attract the interest of both the mechanical and aerospace industries to this topic.

Among the first attempts to exploit the piezoelectric effect in structural control, one must mention the *electronic damping technique* – attributed to Olsen [3], and developed by Swigert and Forward [4]. It consists in the following closed loop: a piezoelectric sensor converts the dynamic strain into an electric current, which is amplified, shifted in phase and applied to a piezoelectric actuator. This technique, soon improved and generalized to the multi-input multi-output case, can be labeled as *active* since it necessarily requires a power supply. Within the active control strategy, complex power amplifier and precise sensing electronics are needed to achieve satisfactory results in terms of damping and stability.

Mainly to overcome these drawbacks Hagood and von Flotow [5] proposed a *passive shunting technique*: a piezoelectric transducer is shunted with a resistor and an inductor, optimally tuned to a structural modal frequency. This control strategy electrically parallels the den Hartog theory [6] of tuned-mass damper systems. This approach is effective only for the control of a single vibration mode of the structure; indeed, once the mechanical mode is selected, the critical inductance, say L_S^*, and the optimal position of the piezoelectric patch are univocally determined. The critical inductance for this lumped control L_s^* is proportional to $1/(C_p f_m^2)$, C_p being the piezoelectric capacitance, and f_m the mechanical modal frequency to be controlled. Typical values for both C_p and f_m impose severe limitations on the inductances to be used. The subsequent inductance range ($10^2 \div 10^4$ H) can be achieved either by an electronic simulating circuit [7] or by coils wrapped around a magnetic core. In both cases serious technological drawbacks arise: in the former case power supplies are again needed, in the latter case the inductor adds a significant fraction of the total system mass. Attempting to lower the critical inductance L_s^* by adding capacitance to the shunting circuit turns out to be uneffective since it necessarily implies a decrease of the efficiency of the piezoelectric coupling.

1.2. PEM structures

All the aforementioned drawbacks can be overcome enhancing the design specifications of the control system. As one compels such an electric controller to be passive and effective on all structural modes, the idea of an electromechanical analogy arises. Thus the conceived *net-control technique* [8,9,17] results in:

- a uniformly distributed array of piezoelectric transducers positioned over the host structure;
- an electric passive network, interconnecting the transducers, whose spectral properties are adapted to those of the host structure.

The electric network is designed to be resonant at all the structural characteristic frequencies and so that its modal shapes may *piezoelectrically* interact with those of the host structure. These design specifications allow one to control simultaneously all the modes, dispensing with the use of either active elements or heavy inductors. The synergic behavior of the piezoelectric patches leads to a very high perfomance in terms of energy transduction efficiency over a frequency bandwidth in principle infinite.

2. Gallery of net-controlled structures

Within the aforementioned net-control strategy, a gallery of PEM structures is here presented. For some relevant structural members (bars, shafts, beams and plates) several circuital interconnecting schemes are exhibited; for each pair of structure and circuit, the basic dynamic properties are addressed.

In Fig. 1 both the modal frequencies and shapes are sketched for an arbitrary pair of mechanical and electrical one-dimensional sub-systems. The net-control technique introduced is based on the one-to-one correspondence between the modal frequencies and the modal shapes of the vibrating structure and the interconnecting network, assuring the complete transduction of mechanical into electrical energy.

2.1. Bars and shafts

The free vibrations of both bar and shaft are governed by the so called D'Alembert equation

$$\ddot{u} = \alpha_2 u'', \quad \alpha_2 \in \mathbb{R}^+ \tag{1}$$

where u denotes the kinematical descriptor, namely the axial displacement or the twist angle, and superposed dot and prime respectively mean the time and space derivatives. The electrical distributed circuit governed by the D'Alembert equation is the well-known second-order transmission line; a lumped circuit constituted by a set of N grounded capacitors interconnected via floating inductors represents a finite difference approximation of the aforementioned distributed circuit.

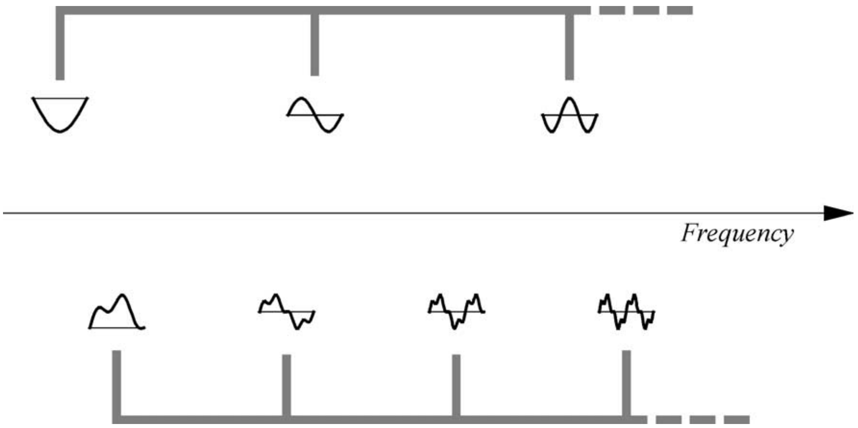

Fig. 1. Correspondence between mechanical and electrical modal properties.

Fig. 1. Correspondance entre les propriétés mécaniques et électriques du modèle.

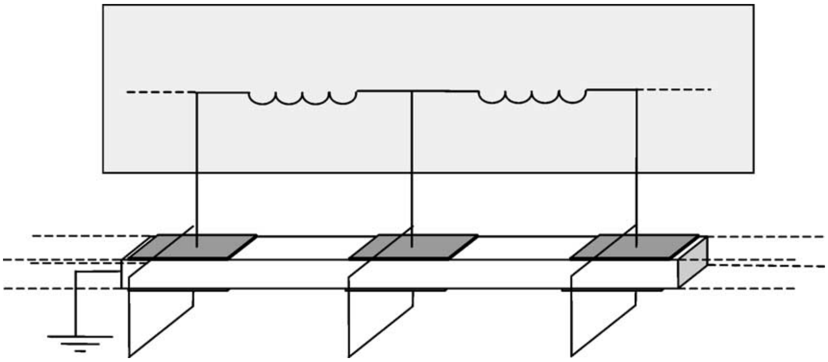

Fig. 2. Sketch of a PEM beam: the interconnection is realized by a second-order transmission line.

Fig. 2. Schéma d'un faisceau PEM : l'interconnexion a été faite d'une ligne de transmission du deuxième ordre.

Hence interconnecting the piezoelectric transducers, positioned over the structure, as the capacitors of the circuit in Fig. 2, the following set of coupled evolution equations governing the free vibrations of the net-controlled bar or shaft is derived:

$$\begin{cases} \ddot{u} = \alpha_2 u'' + \gamma_2 \dot{\psi}', \\ \ddot{\psi} = \beta_2 \psi'' - \gamma_2 \dot{u}', \end{cases} \qquad \alpha_2, \beta_2, \gamma_2 \in \mathbb{R}^+ \tag{2}$$

$\dot{\psi}$ being the electric voltage measured with respect of a common ground.

Referring to Eq. (2), in order to guarantee the one-to-one correspondence between the mechanical and electrical modal characteristics, it is sufficient to set $\beta_2 = \alpha_2$ and impose on the electric circuit boundary conditions dual to those of the vibrating structure. The parameter β_2 can be varied tuning the line inductance; let L_2^* be the critical value to get $\beta_2 = \alpha_2$. If, with the same number N of transducers, one uses the passive shunting technique (namely, shunting separately each patch with a single inductance), the multi-modal control is lost. We call $L_{s2}^*(k)$ the critical inductance when the passive shunting technique is employed to control the k-th mode. The ratio $L_{s2}^*(k)/L_2^*$ is proportional to N^2/k^2, [10]; this means that for a large number of transducers their interconnection, i.e., the basic concept of PEM systems, sensibly reduces the inductances needed.

2.2. Beams

The *elastic* equation governing the free vibrations of Euler beam:

$$\ddot{u} + \alpha_4 u^{(\mathrm{iv})} = 0, \quad \alpha_4 \in \mathbb{R}^+ \tag{3}$$

does not have a well-known electric analog. However, it is possible to couple the beam with the aforementioned *second-order transmission line*. The resulting system, discussed in [8], is governed by:

$$\begin{cases} \ddot{u} + \alpha_4 u^{(\mathrm{iv})} + \gamma_3 \dot{\psi}'' = 0, \\ \ddot{\psi} - \beta_3 \psi'' - \gamma_3 \dot{u}'' = 0, \end{cases} \quad \alpha_4, \beta_3, \gamma_3 \in \mathbb{R}^+ \tag{4}$$

With a suitable choice of the electric boundary conditions there could be similar electrical and mechanical modal shapes, but the electrical and mechanical spectra can not be superposed. The line inductance can be tuned to match only one pair of electric and mechanical eigenfrequencies, since spectral “spacings” are different (refer to Fig. 1). Nevertheless, the ratio $L^*_{s4}(k)/L^*_3(k)$, between the critical inductances required in the passive-shunting and in the second-order net-control technique (used for the k-th mode), is proportional to N^2/k^2. This circumstance represents a sensible technological advantage of the piezoelectric interconnection. In order to synthesize the *electric analog of the Euler beam*, several approaches have been investigated. The two different circuital schemes in [11] and [9] are characterized by the use of active elements. While in [11] some voltage-driven current sources are used, in [9] the circuital topology is found seeking a finite-differences symmetric scheme of the elastica, and assuming all the used electric elements to be two-terminal networks. This occurrence necessarily implies the use of negative floating inductances, which must be simulated by electronic active circuits. A step towards a feasible passive realization of the analog circuit has been made in [9] and [12], where relaxing the two-terminal network hypothesis and considering a suitable truncation of the mechanical impedance matrix of a beam element, a circuit constituted by multiport transformers is obtained. The main drawback of this scheme lies in the complexity of the circuital topology, even if low inductances are required.

A synthesis solution, technologically more convenient in terms of both required inductances and topological complexity, is given in [13] and here depicted in Fig. 3.

The proposed analog circuit has been synthesized through a finite difference approximation of the infinite dimensional Lagrangian of the Euler beam. The governing equations of the beam net-controlled via the circuits given in [9,12,13] are:

$$\begin{cases} \ddot{u} + \alpha_4 u^{(\mathrm{iv})} + \gamma_4 \dot{\psi}'' = 0, \\ \ddot{\psi} + \beta_4 \psi^{(\mathrm{iv})} - \gamma_4 \dot{u}'' = 0, \end{cases} \quad \alpha_4, \beta_4, \gamma_4 \in \mathbb{R}^+ \tag{5}$$

Obviously the parameters β_4 and γ_4 depend on the adopted electrical scheme. Eq. (5) implies that, once the parameter β_4 is set equal to α_4 and the electric boundary conditions have been chosen to be dual of the mechanical ones, a one-to-one correspondence between the electrical and mechanical modal frequencies and modal shapes is established.

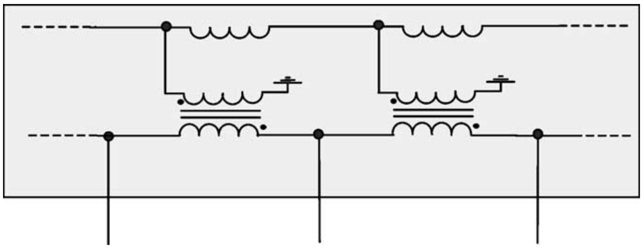

Fig. 3. Circuital scheme for a fourth-order interconnection.

Fig. 3. Schèma d’un circuit d’un interconnexion du quatriéme ordre.

Call $L^*_{s4}(k)$ and L^*_4 the critical inductances to separately shunt all the N transdurcers for controlling the k-th mode and to equate β_4 to α_4, respectively, through the circuit in Fig. 3, choosing a unitary turns ratio for the transformers (this choice is required for comparison purposes). The ratio $L^*_{s4}(k)/L^*_4$ is proportional to N^4/k^4; thus increasing the number of the piezoelectric patches results in extremely advantageous inductances with respect to the passive shunting technique. Further the net-controlled beam described by Eqs. (5) guarantees the multi-modal control of the mechanical vibrations through a unique choice of the net inductance L^*_4.

2.3. Plates

The free vibrations of the Kirchhoff plate are governed by:

$$\ddot{u} + A_4\,\Delta\Delta u = 0, \quad A_4 \in \mathbb{R}^+ \tag{6}$$

Similarly to the Euler beam, Eq. (6) has not a well known electric analog. The problems arising in the synthesis of the electric analog of the plate parallel, those addressed in the previous section, are used, since again fourth order spatial derivatives are involved.

The easiest net-controlled system is realized coupling the Kirchhoff plate with a second-order transmission network, that is the two-dimensional extension of the D'Alembert transmission line. The resulting equations are found in [14] and here reported:

$$\begin{cases} \ddot{u} + A_4\,\Delta\Delta u + \varGamma_3\,\Delta\dot{\psi} = 0, \\ \ddot{\psi} - B_3\,\Delta\psi - \varGamma_3\,\Delta\dot{u} = 0, \end{cases} \quad A_4, B_3, \varGamma_3 \in \mathbb{R}^+ \tag{7}$$

For comparison purposes, let $L^*_{sp}(h,\,k)$ be the critical inductance for the separate resonant shunting relative to the (h, k)-th plate mode. Furthermore let $L^*_{3p}(h,\,k)$ be the critical inductance needed in the net-controlled system (7) to control the same (h, k)-th mode. Similar considerations concerning Eq. (4) hold for this case: a unimodal control is guaranteed employing more advantageous inductances; indeed $L^*_{sp}(h,\,k)/L^*_{3p}(h,\,k) \propto N/(h^2 + k^2)$.

The electric analog of the Kirchhoff plate is instead synthesized in [15] and [9] by using active elements to simulate negative floating inductances. Dispensing with the use of active elements, the one-dimensional circuital topology in Fig. 3 can be extended to the two-dimensional case as done in [16]. For both the net-controlled systems the following coupled equations hold:

$$\begin{cases} \ddot{u} + A_4\,\Delta\Delta u + \varGamma_4\,\Delta\dot{\psi} = 0, \\ \ddot{\psi} + B_4\,\Delta\Delta\psi - \varGamma_4\,\Delta\dot{u} = 0, \end{cases} \quad A_4, B_4, \varGamma_4 \in \mathbb{R}^+ \tag{8}$$

Eqs. (8) guarantee the multi-modal control of the plate vibrations through a unique choice of the net inductance L^*_{4p}. Once the turns ratio of the transformers, for a fair comparison, has been chosen to be unitary, the ratio $L^*_{sp}(h,\,k)/L^*_{4p}$ becomes proportional to $N^2/(h^2 + k^2)^2$: again a large number of patches logically decreases the required inductances.

3. Conclusions and future developments

The main difference between the net-control strategy and the passive shunting technique lies in the interconnection of the distributed patches. If the interconnecting circuit is synthesized to be analog to the host structure, a single choice of the electric parameters allows for the simultaneous control of all the mechanical modes. For some relevant structural elements – bars, beams and plates – a completely passive analog circuit has been synthesized. Its performances as electric controller are compared with those of (i) separated shunted circuits and (ii) second order transmission lines for all the presented structures. The multi-modal control is obviously possible only using analog circuits and the needed critical inductances result to be lowest in this last case. PEM structures are aimed to get an effective transduction of the mechanical energy since, once the energy has been transformed in the electrical form, it can be stored or rapidly dissipated. The damping ratios for electromechanical vibrations can be optimized by suitably introducing in the electric controller some resistors (see [10,13] and Figs. 2 and 3). For instance for the system (5), introducing a suitable resistances in parallel connection with the floating ports of the transformers in Fig. 3, the modal damping ratios ζ_k turn out to be independent of the mode number k and are given by:

$$\zeta_k = \frac{\gamma_4}{2\sqrt{\alpha_4}}, \quad k = 1, 2, \ldots \tag{9}$$

Future developments of this research will lead to:

- the synthesis of electric analogs for the net-controlling of more complex structures, as frames, trusses with arbitrary shape and shells;
- an accurate analysis of the homogenization procedure lying beneath the proposed field equations for PEM structures;
- an experimental set up to validate the proposed approach.

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