Piezoelectromechanical structures: a survey of basic concepts and methodologies

Francesco Dell'Isola, E.G. Henneke, Maurizio Porfiri

Piezoelectromechanical structures: a survey of basic concepts and methodologies

F. dell'Isola*[a], E.G. Henneke[b], M. Porfiri[c]
[a]Dip. di Ingegneria Strutturale e Geotecnica, Università di Roma "La Sapienza", Roma, Italia;
[b]Dept. Engineering Sciences and Mechanics, "Virginia Tech", Blacksburg, VA, USA;
[c]Dottorato in Meccanica Teorica e Applicata, Università di Roma "La Sapienza", Roma, Italia

1-6-2003

ABSTRACT

In the literature distributed arrays of piezeoelectric patches are employed to actively control structural vibrations. In the present work in order to damp beam vibrations a completely passive electric controller is proposed, exploiting distributed piezoelectric transduction. The optimization of the distributed electric controller is performed analyzing the free wave propagation in the composite smart beam. The proposed controller allows for an optimal attenuation of wave propagation over any frequency range. A prototype of the proposed novel smart structure (Piezo-ElectroMechanical beam) is designed, allowing for appreciating its technical feasibility and effectiveness.

Keywords: Smart structures, wave propagation, piezoelectric transducers, distributed control, coupled problems.

1 Introduction

The aim of this paper is to prove that attenuation of wave propagation over any frequency band is technically feasible, when exploiting purely passive electric networks and available piezoelectric transducers. Indeed the authors prove that multimodal damping of mechanical vibrations by means of truly passive electric circuits can be obtained by uniformly distributing piezoelectric transducers on the host structure and suitably designing an optimal interconnecting electric passive network (the resulting smart structure is called PiezoElectroMechanical structure, $\mathbb{PEM}$ for brevity). This concept seems an interesting development of the method of "piezoelectric shunting" proposed by Hagood and von Flotow in[1] for single mode and extended by Hollkamp and Fleming, Behrens and Moheimani to multimodal control ([2] and[3]). Indeed the method here presented is based on the shunting of an array of distributed piezoelectric transducers with multiterminal terminals passive electric network: more precisely, instead of coupling each piezoelectric transducer to a single (eventually multifrequency) electric resonator, the whole set of transducers is coupled to a distributed electric network. It has to be remarked that the electric controller which is introduced evolves with differential equations coupled to those governing the evolution of the mechanical system to be controlled and therefore the concept introduced here differs from that studied

*francesco.dell'isola@uniroma1.it; phone (+39)0644585297.

in.[4,5] Indeed, in the quoted papers the distributed array of piezoelectric transducers is actively driven by a voltage field, which is the output of a suitable feed-back loop (in this way one gets a smart structure which is in general non-conservative). Instead, the piezoelectric controller here introduced is a completely passive controller, so that $\mathbb{PEM}$ structures are passive.

In Section 2 a refined (microscopic) model for $\mathbb{PEM}$ beams is introduced, in which the lumped nature of the electric network and the localization of piezoelectric applied couples are accounted for.

In Section 3 the homogenized model is deduced from the previously introduced refined model. Such a homogenized model:

1. is valid when the wave length is sufficiently larger than the size of each piezoelectric element,,
2. is more handleable when seeking passive optimal controllers,
3. is sufficiently detailed to suggest design criteria for truly lumped $\mathbb{PEM}$ beams.

In Section 4 the electromechanical wave propagation in homogenized $\mathbb{PEM}$ beams is addressed, without specifying completely the evolution equations of the electric controller, which is assumed to belong to a rather wide class of (local) differential controllers.

In Section 5 the pole placement technique is exploited to determine - in the class previously specified - the optimal passive electric controller.

In Section 6 the theoretical results previously established are exploited to start the design of a prototype $\mathbb{PEM}$ beam.

2 Refined model for $\mathbb{PEM}$ beams

We consider an unlimited host beam of width w and thickness h on which an array of uniformly distributed piezoelectric transducers is positioned as shown in Fig. 1 (covering both the beam faces). The length of the transducers is assumed to be equal to l_p, while the width is assumed to be equal to that of the beam; d denotes the distance between the adjacent patches.

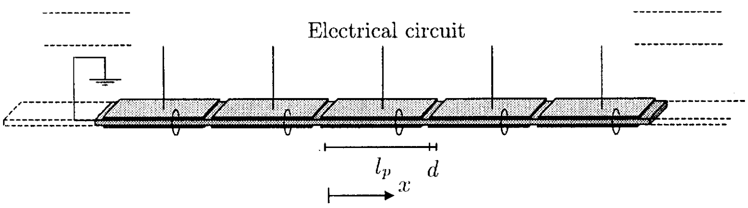

Figure 1: Geometry of the problem.

The piezotransducers are placed along the beam axis and polarized in the transverse direction, in the so called bender configuration (see e.g.[6]). These transducers will be interconnected by an electrical network which will be synthesized in order to accomplish given optimality conditions on the attenuation of propagating waves.

Introducing a set of nodes $\{x_i\}$ representing the geometrical centers of the transducers, defined by:

$$x_i = (l_p + d)\, i \tag{1}$$

the contact couple M at the section labelled by the abscissa x, over the beam span, can be expressed as the sum of mechanical and piezoelectric contributions as follows[6]:

$$M(x,t) = \left[E_b I_b + k_{mm} \sum_{i=-\infty}^{+\infty} \left(\mathrm{RECT}_{l_p}(x - x_i)\right)\right] u''(x,t) + k_{me} \sum_{i=-\infty}^{+\infty} \left(\mathrm{RECT}_{l_p}(x - x_i)\, \dot{\psi}_i(t)\right), \tag{2}$$

where:

$$\left\{ \begin{array}{c} \mathrm{RECT}_{l_p}(x - x_i) = \mathrm{H}\left(x - x_i - \frac{l_p}{2}\right) - \mathrm{H}\left(x - x_i + \frac{l_p}{2}\right) \\ k_{mm} = \dfrac{h^2 w \delta}{2 s_{11}^E}; \qquad k_{me} = \dfrac{h w d_{31}}{s_{11}^E} \end{array} \right. ; \tag{3}$$

H is the Heaviside function, ψ_i is the flux linkage[1] of the $i-$th transducer measured with respect to a common reference ground for every transducer, u is the beam deflection field, E_b is the Young modulus of the material of the beam, I_b is the beam section moment of inertia ($I_b = h^3 w/12$), δ is the transducers thickness, s_{11}^E is the piezoelectric mechanical compliance, d_{31} is the piezoelectric coupling coefficient, t denotes the time variable and superposed dot and prime respectively mean time and space derivative. The distributed inertia can be accounted for by introducing the following constitutive equation for the applied load (external mechanical forcing is excluded):

$$b_T(x,t) = -wh\left(\rho_b + 2\frac{\delta}{h} \sum_{i=-\infty}^{+\infty} \left(\mathrm{RECT}_{l_p}(x - x_i)\right) \rho_p\right) \ddot{u}(x,t), \tag{4}$$

where ρ_b and ρ_p are respectively the mass density per unit volume of the beam and transducer materials.

The balance equations for the considered electrically excited vibrating beam yield:

$$M(x,t)'' - b_T(x,t) = 0 \tag{5}$$

when $M(x,t)$ and $b_T(x,t)$ are given by Eqns. (2) and (4) respectively.

From a purely electrical point of view the $i-$th piezoelectric bender transducer can be described as a capacitor in parallel connection with a "mechanically driven" current source, which injects into the electrical circuit the current J_i driven by its mechanical time rate of deformation (see Fig. 2)

$$J_i = k_{me}\left(\dot{u}'(x_i + l_p/2, t) - \dot{u}'(x_i - l_p/2, t)\right).$$

The capacitance of each piezoelectric bender transducer can be estimated to be equal to[6]:

$$k_{ee} = 2\frac{w\left(s_{11}^E \epsilon_3^T - d_{31}^2\right)}{s_{11}^E \delta} l_p.$$

The electrical system interconnecting the electrical terminals of the bender transducers is assumed to be a multiterminal network the admittance Y_{ij}^e of which, in the Laplace domain, is given by:

$$Y_{ij}^e = D_{ij}^e + \frac{1}{s} K_{ij}^e, \tag{6}$$

[1] The flux linkage ψ_i represents the time integral of the voltage V_i of the $i-$th transducer measured with respect to the common ground, i.e. $\dot{\psi}_i \equiv V_i$.

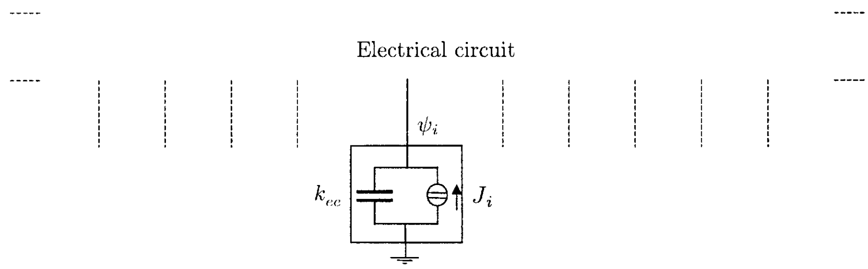

Figure 2: Sketch of the mechanically fed electric controller.

where s is the Laplace transform variable and the residues K^e_{ij} (electrical stiffness) and D^e_{ij} (electrical damping) are assumed to have finite bandwidth $2N-1$.

Thus, by Kirchhoff balance of currents at node i, the evolution equations for the variables ψ_j are given by the following second order system of ODEs:

$$k_{ee}\ddot{\psi}_i + \sum_{j=i-N}^{i+N} K^e_{ij}\psi_j + \sum_{j=i-N}^{i+N} D^e_{ij}\dot{\psi}_j - k_{me}\left(\dot{u}'\left(x_i + l_p/2, t\right) - \dot{u}'\left(x_i - l_p/2, t\right)\right) = 0 \qquad (7)$$

3 A class of Homogenized models for $\mathbb{PEM}$ beams

When it can be assumed that the number of piezoelectric transducers is sufficiently large, a continuous flux linkage field $\psi(x,t)$ can be introduced and the bending moment constitutive relation (2) becomes (for more details on the needed homogenization techniques see[4]):

$$M = \left(E_b I_b + c_f k_{mm}\right) u'' + c_f k_{me}\dot{\psi}, \qquad (8)$$

where c_f represents a covering factor defined as:

$$c_f = \frac{1}{1 + d/l_p}.$$

while the applied load (4) becomes:

$$b_T = -wh\left(\rho_b + 2c_f\frac{\delta}{h}\rho_p\right)\ddot{u}.$$

The governing equation expressed in terms of the deflection u is readily seen to be:

$$K_M u^{IV} + \rho\,\ddot{u} + c_f k_{me}\dot{\psi}'' = 0,$$

with:

$$\begin{cases} K_M = E_b I_b + c_f k_{mm} \\ \rho = wh\left(\rho_b + 2c_f\dfrac{\delta}{h}\rho_p\right) \end{cases} . \qquad (9)$$

Thus the dimensionless form of the mechanical evolution equation becomes:

$$\ddot{u} + \alpha^4 u^{IV} + \beta_{em}^2 \dot{\psi}'' = 0, \qquad \left\{ \begin{array}{l} \alpha^4 = \dfrac{K_M}{l^4 \rho \omega_0^2} \\ \beta_{em}^2 = \dfrac{c_f k_{me} \psi_0}{l^2 \rho \omega_0 u_0} \end{array} \right. , \tag{10}$$

where a characteristic frequency ω_0, a characteristic length equal to the beam length l, a characteristic deflection u_0 and a characteristic flux linkage ψ_0 have been introduced. As this cannot cause misunderstanding, we have adopted the same letters to denote both dimensional and dimensionless differential operators and kinematical descriptors.

Furthermore, when the number of transducers can be assumed to be sufficiently large, the electrical system can be described by a sole PDE which, expressed in terms of the dimensionless flux-linkage ψ, reads:

$$\ddot{\psi} + \mathbf{K}[\psi] + \mathbf{D}\left[\dot{\psi}\right] - \beta_{me}^2 \dot{u}'' = 0, \quad \beta_{me}^2 = \frac{k_{me} u_0}{l^2 \left(\dfrac{k_{ee}}{l_p}\right) \omega_0 \psi_0},$$

where it is assumed that $\mathbf{K}$ and $\mathbf{D}$ are (local) spatial differential operators of the following form:

$$\left\{ \begin{array}{l} \mathbf{K}[\psi] = \sum\limits_{i=0}^{A} (-1)^i a_{2i} \dfrac{d^{(2i)} \psi}{dx^{(2i)}} \\ \mathbf{D}\left[\dot{\psi}\right] = \sum\limits_{i=0}^{B} (-1)^i b_{2i} \dfrac{d^{(2i)} \dot{\psi}}{dx^{(2i)}} \end{array} \right. , \tag{11}$$

and a_{2i} and b_{2i} are positive real constants to assure the passivity of the interconnecting network, and the indices A and B are positive.

This hypothesis on the structure of the electrical stiffness and damping, respectively $\mathbf{K}[\cdot]$and $\mathbf{D}[\cdot]$, seems to be general enough for our design aims and leads to a simple solution of the synthesis problem of determining an optimal lumped electrical circuit to be piezoelectrically coupled to the given flexible structure.

Thus the dimensionless form of the evolution electromechanical equations is:

$$\left\{ \begin{array}{l} \ddot{u} + \alpha^4 u^{IV} + \beta^2 \dot{\psi}'' = 0 \\ \ddot{\psi} + \mathbf{K}[\psi] + \mathbf{D}\left[\dot{\psi}\right] - \beta^2 \dot{u}'' = 0 \end{array} \right. ; \qquad \left\{ \begin{array}{l} \alpha^4 = \dfrac{K_M}{l^4 \omega_0^2 \rho} \\ \beta^2 = \dfrac{k_{me}}{l^2 \omega_0} \sqrt{\dfrac{c_f l_p}{k_{ee} \rho}} \end{array} \right. , \tag{12}$$

where, in order to preserve the form of a gyroscopic coupling, the characteristic flux linkage and displacement are chosen to satisfy the following relation:

$$\sqrt{\frac{c_f k_{ee}}{l_p \rho}} = \frac{u_0}{\psi_0}.$$

Let us underline that the quantity

$$\frac{c_f k_{ee}}{l_p} =: \gamma \tag{13}$$

represents the capacitance per unit length of the PEM beam.

The coupling coefficient β^2 can be expressed in the following form:

$$\beta^2 = (c_f k_{me}) \frac{1}{l^2 \omega_0} \sqrt{\frac{1}{\gamma \rho}}.$$

The class of differential operators of the form (11) has been considered for the following reasons:

1. when boundary conditions are introduced, the electrical system piezoelectrically coupled to the flexible structure does not introduce any spillover phenomenon among the mechanical vibration modes;
2. the synthesis of an electrical circuit governed by a discrete form of these operators is very easy, exploiting the methods adopted in[6];
3. it includes at least two interesting interconnection schemes for piezoelectric arrays, developed in the vibration control literature (,[18]).

When the stiffness operator is limited at the 0 term every piezoelectric element is interconnected to a single grounded inductor and no "cooperation" exists in between the piezoelectric transducers. When only the second term is accounted for, each piezoelectric transducer is connected to the adjacent one by a floating inductor, providing a second order transmission line piezoelectrically coupled to the vibrating structure ([8]).

4 Wave propagation in homogenized $\mathbb{PEM}$ beams

In this subsection the wave propagation analysis in a $\mathbb{PEM}$ beam is performed.

Let us consider the wave solution of (12):

$$\begin{bmatrix} u(x,t) \\ \psi(x,t) \end{bmatrix} = \begin{bmatrix} \mathsf{U} \\ \Psi \end{bmatrix} e^{\left(-i\frac{2\pi}{\lambda}x+st\right)}, \tag{14}$$

in terms of the wave length $\lambda \in \mathbb{R}^+$ and the Laplace transform variable $s \in \mathbb{C}$. Substituting (14) into (12), with simple algebraic manipulations we obtain

$$\begin{cases} s^2\mathsf{U} + \alpha^4\left(\frac{2\pi}{\lambda}\right)^4\mathsf{U} - \beta^2 s\left(\frac{2\pi}{\lambda}\right)^2\Psi = 0 \\ s^2\Psi + P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right]\Psi + sP_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right]\Psi + \beta^2 s\left(\frac{2\pi}{\lambda}\right)^2\mathsf{U} = 0 \end{cases} \tag{15}$$

where the polynomials $P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right]$ and $P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right]$ are defined by the following

$$\begin{cases} P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right] = \sum_{i=0}^{A} a_{2i}\left(\frac{2\pi}{\lambda}\right)^{2i} \\ P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right] = \sum_{i=0}^{B} b_{2i}\left(\frac{2\pi}{\lambda}\right)^{2i} \end{cases}. \tag{16}$$

The algebraic equations (15) yield the following dispersion relation:

$$s^4 + P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right]s^3 + \left(\left(\alpha\frac{2\pi}{\lambda}\right)^4 + \left(\beta\frac{2\pi}{\lambda}\right)^4 + P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right]\right)s^2 + P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right]\left(\alpha\frac{2\pi}{\lambda}\right)^4 s + \left(\alpha\frac{2\pi}{\lambda}\right)^4 P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right] = 0 \tag{17}$$

5 Design of the passive optimal controller

In this section we will design the optimal passive distributed controller following the pole placement technique applied to the evolution of each pair of electrical and mechanical waves. In particular, we will determine an optimal expression for the stiffness and dissipative polynomials $P_{\mathbf{K}}$ and $P_{\mathbf{D}}$ appearing in (15); consequently, we will establish the forms of the operators $\mathbf{K}$ and $\mathbf{D}$.

The chosen optimality condition requires the determination of the values of $P_{\mathbf{K}}$ and $P_{\mathbf{D}}$ in order to maximize the exponential time decay rate τ^{λ} of the electromechanical wave propagating with the wavelength λ, defined as:

$$\tau^{\lambda} = \min_{i=1,\dots 4} \left\{ \left| \mathrm{Re}\left[s_i^{\lambda} \right] \right| \right\},$$

s_i^{λ} being the zeros of (17).

In order to maximize the exponential time decay τ^{λ}, the optimal expressions of $P_{\mathbf{K}}$ and $P_{\mathbf{D}}$ are found by requiring the four zeros s_i^{λ} to be coincident (pole placement technique, see e.g.[1] and[9]).

Hence we enforce the polynomial in the RHS of (17) to be factorized as:

$$\left(s^2 + 2\sigma\left[\lambda\right] s + \left(\sigma\left[\lambda\right]^2 + \omega\left[\lambda\right]^2 \right) \right)^2.$$

Equating the coefficients of the above mentioned polynomials the following set of conditions is obtained:

$$\left\{ \begin{array}{l} P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right] = 4\sigma\left[\lambda\right] \\ \left(\left(\alpha\frac{2\pi}{\lambda}\right)^4 + \left(\beta\frac{2\pi}{\lambda}\right)^4 + P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right] \right) = 6\sigma\left[\lambda\right]^2 + 2\omega\left[\lambda\right]^2 \\ P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right]\left(\alpha\frac{2\pi}{\lambda}\right)^4 = 4\sigma\left[\lambda\right]^3 + 4\sigma\left[\lambda\right]\omega\left[\lambda\right]^2 \\ \left(\alpha\frac{2\pi}{\lambda}\right)^4 P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right] = \left(\sigma\left[\lambda\right]^2 + \omega\left[\lambda\right]^2 \right) \end{array} \right. . \tag{18}$$

The previous set of relations imposes the following matching conditions on the electrical controller:

$$\left\{ \begin{array}{l} P_{\mathbf{K}}\left[\frac{2\pi}{\lambda}\right] = \left(\alpha\frac{2\pi}{\lambda}\right)^4 \\ P_{\mathbf{D}}\left[\frac{2\pi}{\lambda}\right] = 2\left(\beta\frac{2\pi}{\lambda}\right)^2 \end{array} \right. . \tag{19}$$

Consequently, from Eqns. (18) the values of the real and imaginary part of the coincident roots are found to be:

$$\left\{ \begin{array}{l} \sigma\left[\lambda\right] = \dfrac{\left(\beta\frac{2\pi}{\lambda}\right)^2}{2} \\ \omega\left[\lambda\right] = \sqrt{\left(\alpha\frac{2\pi}{\lambda}\right)^4 - \dfrac{\left(\beta\frac{2\pi}{\lambda}\right)^4}{4}} \end{array} \right. , \tag{20}$$

hence, the damping ratio (defined as the sine of the phase of the coincident roots measured from the imaginary axis) becomes:

$$\varsigma\left[\lambda\right] := \frac{\sigma\left[\lambda\right]}{\sqrt{\omega\left[\lambda\right]^2 + \sigma\left[\lambda\right]^2}} = \frac{\beta^2}{2\alpha^2}.$$

Taking into account the definitions (12) of the parameters α and β, we get the following expression in terms of the properties of the piezo-electromechanical beam:

$$\varsigma\left[\lambda\right] = \frac{c_f k_{me}}{2\sqrt{\gamma K_M}} \tag{21}$$

Finally, let us express the damping ratio as a function of the piezoelectric material characteristics, the beam geometry and material properties by:

$$\varsigma\left[\lambda\right] = \frac{\sqrt{c_f}\sqrt{\dfrac{\delta}{h}}}{\sqrt{\dfrac{2}{3}\left(E_b s_{11}^E\right) + 4c_f\dfrac{\delta}{h}\sqrt{\left(\dfrac{1}{k_{31}^2} - 1\right)}}}, \qquad k_{31} = d_{31}\sqrt{\frac{1}{s_{11}^E \epsilon_3^T}}; \tag{22}$$

where the electromechanical coupling coefficient k_{31} has been introduced.

Equation (22) clearly indicates that the damping ratio is independent of the mode under control and is an increasing function of the variables:

1. electromechanical coupling coefficient k_{31};
2. ratio of the patches thickness over the beam thickness $\frac{\delta}{h}$;
3. covering factor c_f;
4. ratio of the patches elastic modulus over the beam Young modulus $\left(E_b s_{11}^E\right)^{-1}$.

Conditions (16) establish that the optimal distributed passive circuit to be mechanically fed by the piezotransducers has to be governed by the following PDE:

$$\ddot{\psi} + \mathbf{K}[\psi] + \mathbf{D}\left[\dot{\psi}\right] = 0, \tag{23}$$

where the stiffness and damping operators are defined by:

$$\begin{cases} \mathbf{K}[\psi] = \alpha^4 \psi^{IV} \\ \mathbf{D}\left[\dot{\psi}\right] = -2\beta^2 \dot{\psi}'' \end{cases}, \tag{24}$$

REMARK 1. *The found electrical interconnecting network can be thought as the beam analog circuit, with an internal dissipation proportional to the curvature velocity $\dot{u}''$. This circuit guarantees the maximum exponential time rate decay τ^λ for every wavelength λ. Exploiting different circuits included in (11) it is possible to maximize τ^λ only at a given wavelength $\bar{\lambda}$.*

The synthesis problem of finding an analog circuit of the Euler beam has been addressed in[10] and.[6]

6 Design of a prototype

In order to assess the physical realizability and the efficiency of the proposed device, we will consider an aluminum beam, the geometry of which is presented in table 1:

Table 1: Properties and dimensions of the host beam.

Coefficient	Value	Units
w	2	cm
h	2	mm
E	70	GPa
ρ_b	2700	kg/m^3

Let us exploit bender transducers constituted by piezoceramic patches made of lead zirconate titanate [PZT] (for more details on the different kinds of piezoelectric actuations see[11]).

The characteristics of these piezoceramic transducers are listed in table 2.

Table 2: Properties and dimensions of the the piezoelectric transducers (Piezo System T110-H4E-602, made of PSI-5H4 piecoceramic).

Coefficient	Value	Units
ρ_p	7800	kg/m^3
$\left(s_{11}^E\right)^{-1}$	62	GPa
d_{31}	$320\,10^{-12}$	m/V
ϵ_3^T	$3800\,8.85\,10^{-12}$	F/m
δ	.267	mm
l_p	25	mm
d	2.273	mm

Thus the covering factor can be estimated to be:

$$c_f = 91.67\%.$$

While the capacitance and coupling coefficient of the bender transducers are:

$$\begin{cases} k_{ee} = 2\dfrac{w\left(s_{11}^E\epsilon_3^T - d_{31}^2\right)}{s_{11}^E\delta}l_p = 184.9\;nF \\ k_{me} = \dfrac{hwd_{31}}{s_{11}^E} = 1.436\;10^{-3}\;C \end{cases}.$$

The stiffness and mass per unit length of the PEM beam can be evaluated to be:

$$\begin{cases} K_M = E_b I_b + c_f k_{mm} = 2.032\,Nm^2 \\ \rho = wh\left(\rho_b + 2c_f\frac{\delta}{h}\rho_p\right) = 0.2462\,Kg/m \end{cases}.$$

From Eqn. (21) the damping ratio of any propagating wave is determined to be:

$$\varsigma\,[\lambda] = 17.74\%.$$

7 Conclusions

In the present paper the concept of piezoelectric shunting by means of a single transducer suitably positioned on a structural member is generalized. Indeed we consider a beam hosting an array of uniformly distributed piezoelectric transducers and look for a distributed electric network shunting them in an optimal way. If waves with wavelength reasonably greater than the size of the used transducers are considered, then we expect that the homogenized model for the controller may be suitable. In Section 3 such an infinite dimensional model is found, while in the subsequent sections its electric components are optimized to get the most efficient damping of electromechanical waves. We have proved that, in principles, a wavelength independent (i.e. a complete broadband) wave attenuation is possible by means of a completely passive electric controller. This is obtained by synthesizing a distributed electric circuit governed by the same PDE governing the beam flexural vibrations. However, the synthesis is achieved by the use of lumped circuital components, therefore the following problem arises: how efficient the finite dimensional circuit results are when compared with their homogenized counterpart. This is a crucial issue when prototypes of PEM beams must be designed, as sketched in the final section.

Future works will be devoted to the comparison between the performances of homogenized electric controllers and their lumped approximations.

Acknowledgment: The partial support of the Engineering Science and Mechanics Department of Virginia Polytechnic Institute and State University is gratefully acknowledged by the authors. This presented research has been also partially supported by MIUR, Ministero per l'Innovazione, l'Università e la Ricerca Fondi Ricerca PRIN "Sintesi di circuiti piezoelettrici e tecniche di disaccoppiamento per il controllo di vibrazioni meccaniche" (protocollo 2001097882_003) and by the Università di Roma "La Sapienza".

8 REFERENCES

[1] N. W. Hagood and A. von Flotow, "Damping of structural vibrations with piezoelectric materials and passive electrical networks", *Journal of Sound and Vibrations* **146**, 243-268, 1991.

[2] J. J. Hollkamp, "Multimodal Passive Vibration Suppression with Piezoelectric Materials and Resonant Shunts", *Journal of Intelligent Material Systems and Structures* **5**, 49-57, 1994.

[3] A. J. Fleming, S. Behrens, and S. O. R. Moheimani, "Optimization and Implementation of Multimode Piezoelectric Shunt Damping Systems", *IEEE/ASME Transactions on Mechatronics* **7**(1), 87-94, 2002.

[4] M. Kader, M. Lenczner, and Z. Mrcarica, "Distributed control based on distributed electronic circuits: application to vibration control", *Microelectronics Reliability* **41**, 1857-1866, 2001.

[5] D. Halim, and S. O. R. Moheimani, "Experimental Implementation of Spatial $\mathcal{H}_\infty$ Control on a Piezoelectric-Laminate Beam", *IEEE/ASME Transactions on Mechatronics* **7(3)**, 346-356, 2002.

[6] U. Andreaus, F. dell'Isola, and M. Porfiri, "Piezoelectric passive distributed controllers for beam flexural vibrations", to appear in *Journal of Vibration and Control.*

[7] R. W. Newcomb, *Linear Multiport Synthesis*, McGraw Hill, New York, 1966.

[8] S. Vidoli, F. dell'Isola, "Modal coupling in one-dimensional electromechanical structured continua", *Acta Mechanica* **141**, 37-50, 2000.

[9] D. W. Miller, and E. F. Crawley, "Theoretical and experimental investigation of space-realizable inertial actuation for passive and active structural control", *Journal of Guidance, Control and Dynamics* **11**(5), 449-458, 1988.

[10] F. dell'Isola, E.G. Henneke, M. Porfiri, "Synthesis of electrical networks interconnecting PZT actuators to damp mechanical vibrations", to appear in *International Journal of Applied Electromagnetics and Mechanics.*

[11] C. Niezreski, D. Brei, S. Balakrishnam, and A. Moskalik, "Piezoelectric Actuation: State of the Art", *The Shock and Vibration Digest* **33**(4), 269-280, 2001.