Localized and extended states in a disordered trap
Luca Pezzé, Laurent Sanchez-Palencia

To cite this version:

HAL Id: hal-00493626
https://hal.archives-ouvertes.fr/hal-00493626v3
Submitted on 25 Jan 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Localized and Extended States in a Disordered Trap

Luca Pezzè and Laurent Sanchez-Palencia
Laboratoire Charles Fabry de l’Institut d’Optique, CNRS and Univ. Paris-Sud,
Campus Polytechnique, RD 128, F-91127 Palaiseau cedex, France
(Dated: January 25, 2011)

PACS numbers: 03.75.-b, 03.75.Ss, 72.15.Rn

Disorder underlies many fields in physics, such as electronics, superfluid helium and optics [1–3]. It poses challenging questions, regarding quantum transport [4] and the interplay of disorder and interactions [5]. In this respect, ultracold gases offer exceptionally well controlled simulators for condensed-matter physics [6] and are particularly promising for disordered systems [7]. They recently allowed for the direct observation of one-dimensional (1D) Anderson localization of matter waves [8–11]. It should be noticed however that ultracold gases do not only mimic standard models of condensed-matter physics, but also raise new issues which require special analysis in its own right. For instance, they are most often confined in spatial traps, which has significant consequences. On the one hand, retrieving information about bulk properties requires specific algorithms [12]. On the other hand, trapping induces novel effects, such as the existence of Bose-Einstein condensates in low dimensions [13], and suppression of quantum tunneling in periodic lattices [14].

Consider Anderson localization [15]. In homogeneous disorder, linear waves can localize owing to coherent multiple scattering, with properties depending on the system dimension and the disorder strength [1]. A paradigm of Anderson localization is that localized and extended states generally do not coexist in energy. This relies on Mott’s *reductio ad absurdum* [1]: Should there exist a localized state and an extended state with infinitely close eigenenergies for a given configuration of disorder, an infinitesimal change of the configuration would hybridize them, forming two extended states. Hence, for a given energy, almost all states should be either localized or extended. Exceptions only appear for peculiar models of disorder with strong local symmetries [16]. Then, a question arises: Can inhomogeneous trapping modify this picture so that localized and extended states coexist in energy?

In this Letter, we study localization in a disordered potential combined with an inhomogeneous trap. The central result of this work is the coexistence, at intermediate energies, of two classes of eigenstates. The first class corresponds to states which spread over the full (energy-dependent) classically allowed region of the bare trap, and which we thus call “extended”. The second class corresponds to states of width much smaller than the trap size, which are localized by the disorder, and which we thus call “localized”. We give numerical evidence of the coexistence of extended and localized states for different kinds of traps. We show that while the extended states are confined by the trap and weakly affected by the disorder, the localized states correspond to eigenstates of the disordered potential, which are only affected by the trap via an inhomogeneous energy shift. These results are relevant to disordered quantum gases and we propose a realistic scheme to observe the coexistence of localized and extended states in these systems.
enough energy $E$, we find $\Delta z_n \ll L$ and the states are not significantly affected by the finite size of the box. For larger energies however, boundary effects come into the picture. The states are centered close to the box center and their extension saturates to the value obtained for a plane wave, i.e. $\Delta z^0 = L/2\sqrt{3}$. A central outcome of these results is that the curve giving $\Delta z_n$ versus $E$ displays a single branch. In particular, there is no energy window where localized and extended states coexist. This finding holds independently of the finite box size and is in agreement with Mott’s argument [1].

For inhomogeneous traps ($\alpha < \infty$), we find a completely different behavior. The curves giving $\Delta z_n$ and $z_n$ versus $E$ now display two clearly separated branches [see Figs. 1(b)-(e) and (g)-(j)]. For low energy, the states are strongly localized and, for $E > 0$, they are roughly uniformly distributed in a region bounded by the (energy-dependent) classical turning points, $z_{cl}(E)$, defined as the solutions of $V_T(z_{cl}) = E$. For higher energy, the extension of the states corresponding to the upper branch in Figs. 1(b)-(e) grows and eventually saturates to that of the eigenstates of the nondisordered trap, $\Delta z^0(E)$. The centers of mass of these states approach the trap center ($\pm z_{cl}(E)$) in panels (g)-(j). The last row shows the full DOS $\rho(E)$ (solid black line), as well as the DOS restricted to localized ($\rho_<$, solid red line) and extended ($\rho_>$, dashed blue line) states [19]. The dot-dashed green lines are the nondisordered limits.

The coexistence of localized and extended states in the same energy window for disordered traps is confirmed on more quantitative grounds in the last row of Fig. 1. It shows the full density of states (solid black line), as well as the density of localized ($\rho_<$, solid red line) and extended ($\rho_>$, dashed blue line) states [19]. The different nature of the localized and extended states is even more striking when one studies the wavefunctions. Let us focus for instance on the harmonic trap ($\alpha = 2$) and on a narrow slice of the spectrum around $E \sim 4|V_n|$, where $\rho_</\rho \simeq 14\%$ of the states are localized [20]. Figure 2(a) shows the spatial density $|\psi_n(z)|^2$ of all states found for a single realization of the disorder. We can clearly distinguish localized (thick red lines) and extended (thin blue lines) states, which shows that they coexist in the same energy window for each realization of the disorder. The localized states are very narrow and present no node (e.g. states A and E) or a few nodes (e.g. states C and H). They may be identified as bound states of the local deep wells of the disordered potential, similarly as the lowest-energy states creating the Lifshits tail in bare disorder [18]. To confirm this, let us decompose the eigenstates $|\psi_n\rangle$ of the disordered trap onto the basis of the eigenstates $|\chi_p\rangle$ of the bare disordered potential [i.e. Hamiltonian (1) with $V_T \equiv 0$], associated to the eigenenergies $\epsilon_p$. For a local-
and, due to the reduced spatial extension of $E$ states, the trap by just the energy shift $\Delta E = E \chi_p |\langle \chi_p |V'_{\epsilon}(z)|\chi_p \rangle|$. This explains that the localized states in $E$ may occupy almost-degenerate energy levels (e.g. H and I).

The states C and D are projected: (b) over the eigenstates of the disordered potential, $|\chi_p \rangle$, and (c) over those of the harmonic trap, $|\psi_n^0 \rangle$. The parameters are as in Fig. 1.

Let us now discuss a possible scheme to observe the coexistence of localized and extended states in a disordered trap. Consider a gas of noninteracting ultracold fermions prepared in a given internal state, at temperature $T$ and chemical potential $\mu$. A class of energies $E_n - E \lesssim \Delta$ [see Fig. 3(a)] deep in the Fermi sea (i.e. with $E \ll 1/n$) can be selected by applying a spin-changing radio-frequency (rf) field of frequency $\nu = E/h$ and duration $\tau \sim 1/\Delta$ (with $h$ the Planck constant) [14, 22, 23]. The rf field transfers the atoms of corresponding energies to an internal state insensitive to the disordered trap. The transferred atoms expand freely, which provides their momentum distribution:

$$D_{E,\Delta}(k) \simeq \sum_{|E_n - E| \lesssim \Delta} \psi_n(k)^2,$$

where $\psi_n(k)$ is the Fourier transform of $\psi_n(z)$ [TOF technique]. In the coexistence region, $D_{E,\Delta}(k)$ has two significantly different contributions: For localized states, $|\psi_n(k)|^2$ is centered around $k \approx 0$ with tails of width $\Delta \propto 1/\alpha$. Conversely, for extended states, $|\psi_n(k)|^2$ is peaked at $k \approx \sqrt{2mE}/h$ with long tails towards small momenta. We however found that averaging over realizations of the disorder blurs the central peak associated to the localized states in $D_{E,\Delta}(k)$. In turn, the quan-
momentum distribution which are most often created in harmonic traps [presentday experiments with disordered quantum gases, affected by the disorder. This effect is directly relevant to energy shift. Conversely, the extended states spread over exist in a given energy window. The localized states correspond to eigenstates of energy  $$E \pm \Delta$$ (shaded region) are transferred to a different internal state via rf coupling. The other parameters are as in Fig. 1.3(b). The central one is more pronounced for narrower pulses. Selecting either the localized states or the extended states [19] confirms that the central peak corresponds to the localized states and the side peak to the extended states [see Inset of Fig. 3(b)].

Finally, we have performed similar calculations as above in a 2D harmonic trap. Figure 4(a) shows the centers of mass $$r_n$$ of the eigenstates with $$E_n \approx 4\sqrt{|V_n|}$$, the color scale giving $$\Delta r_n$$. Figure 4(b) shows a density plot of $$\Delta r_n$$ versus $$|r_n|$$ for the same data. Again, the eigenstates clearly separate into two classes: Some states are extended (large $$\Delta r_n$$) and centered nearby the trap center (small $$|r_n|$$). The other states are strongly localized (small $$\Delta r_n$$) and located nearby the line of classical turning points ($$|r_n| \approx r_c(E) = \sqrt{2E/m\omega^2}$$). Hence, the two classes of states can coexist at intermediate energies also in 2D disordered traps.

In conclusion, we have shown that, in a disordered inhomogeneous trap, localized and extended states can coexist in a given energy window. The localized states correspond to eigenstates of the disordered potential which are only affected by the trap via an inhomogeneous energy shift. Conversely, the extended states spread over the classically allowed region of the trap and are weakly affected by the disorder. This effect is directly relevant to presentday experiments with disordered quantum gases, which are most often created in harmonic traps [11, 24–27]. We have proposed a realistic scheme to observe it in these systems. In the future, it would be interesting to extend our results to higher dimensions and to other kinds of inhomogeneous disordered systems.

We thank T. Giamarchi and B. van Tiggelen for discussions and ANR (Contract No. ANR-08-blan-0016-01), Triangle de la Physique, LUMAT and IFRAF for support.

Figure 3: Scheme to observe the coexistence of localized and extended states in disordered traps (solid red line). (a) Atoms occupying the eigenstates of energy $$E \pm \Delta$$ (shaded region) are transferred to a different internal state via rf coupling. The corresponding momentum distribution is then measured by TOF. (b) Correlation function $$C_{E,\Delta}(k)$$ (black solid line) and momentum distribution $$D_{E,\Delta}(k)$$ (dashed green line, arbitrary units), for $$\Delta = 2\hbar \omega$$. Inset: $$C_{E,\Delta}(k)$$ of all states (solid black line), and separating localized (dashed red line) and extended (dotted blue line) states [19], for $$\Delta = 0.01\hbar \omega$$. Here $$E = 4\sqrt{|V_n|}$$ and the other parameters are as in Fig. 1.

Figure 4: Coexistence of localized and extended states in a 2D disordered harmonic trap for $$E/|V_n| = 4 \pm 0.0003$$. The figure results from accumulation of data from 2 × 10^4 realizations of the disorder, with $$m\sigma^2|V_n|/h^2 = 0.8$$ and $$\omega = 0.05|V_n|/h$$. (a) Centers of mass $$r_n$$ of the eigenstates and corresponding values of $$\Delta r_n/\sigma_n$$ in color scale. The solid black line is the line of classical turning points, $$r_c(E) = \sqrt{2E/m\omega^2} \approx 63.2\sigma_n$$. (b) Extension $$\Delta r_n$$ versus distance from the trap center $$|r_n|$$.

[17] We obtained the same qualitative behavior as reported in the Letter using other models of disorder, such as “blue-detuned” speckle potentials and Gaussian impurities.
[19] We use the condition $$\Delta z < 10\sigma_n$$ to identify localized states, and $$\Delta z > (3/4)|\Delta z|$$ for extended states.
[20] We obtained qualitatively-similar results for different en-
ergies and different traps.

[21] For instance, we find $E_n \simeq 3.97|V_R|$, $\epsilon_p \simeq 0.24|V_R|$ and 
$\langle \chi_p|V_T|\chi_p \rangle \simeq 3.71|V_R|$ for state C in Fig. 2.


[28] Here, we use a box of length 3000$\sigma_R$ much larger than $\Delta z$ for all states considered.