ANALYTICAL COMPUTATION OF AN AC MACHINE EXTERNAL MAGNETIC FIELD

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Abstract – This paper presents an analytical approach to estimate the magnitude of a flux density component at a point located outside an AC machine. The study uses the concept of attenuation coefficients which are determined using several hypotheses. The developments consider the different media according to they are flowed by eddy currents or not. Then, the calculations are performed in each medium.

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1 Introduction

Electromagnetic emissions of electrical machines were usually considered as a disturbing phenomenon which has to be limited, particularly in special applications like military ones. But in the last few years, the interest in the electromagnetic emissions has grown because possibilities in the field of diagnosis of electrical machines have been shown. Applications were found in fault detections [1-4] but also in the estimation of the quality of the windings inter-turn insulation [5]. It has been shown that some particular components of the frequency spectrum give information about the health of the machine. Moreover, the external magnetic field can give information concerning the location of the fault [6]. The concerned frequency range depends on the kind of application: low and medium frequency (up to several kHz) for fault detection, high frequency (some MHz) for the estimation of insulation quality. The study presented in this paper used the low and medium frequency range. The analysis of external magnetic field requires to model on the one hand the sources of the electromagnetic field and on the other hand the attenuation effects due to the various ferromagnetic elements constituting the electrical machine and flown by this field.

Methods for solving the attenuation problem can be found in the literature. This kind of problem, also called “magnetic shielding” problem, is generally solved with a finite element application [7-9] and requires to take into account the eddy currents. In [10], an eddy current problem is solved analytically, not to estimate the external magnetic field, but to determine the torque delivered by a particular induction machine. Analytical methods exist [11-13] for simplified geometry and under particular hypotheses, but rarely on electrical machines.

The paper concerns the external radial magnetic field which exists in a transverse plan perpendicular to the machine axis. The radial magnetic field will be supposed to originate from the air-gap flux density. The calculation of this external magnetic field is made analytically considering the attenuation effect of the laminated stator yoke composed of thin iron sheets and attenuation produced by the external housing. The analytical approach is based on the definition of simple coefficients of attenuation and requires considering various hypotheses like the decoupling of attenuation phenomena and the use of a cartesian referential for the media where eddy currents can flow.

2 Electromagnetic modelling

2.1 Presentation of the problem

The developments are based on the analytical expression of the air-gap flux density normal component b^g which is obtained by multiplying the resulting air-gap magnetomotive force with the air-gap permeance taking the slots into account. So, considering a supply voltage with a ω angular frequency fundamental (frequency f), b^g can be written with equation (1):

$$b^g = \sum_k \sum_m b_{k,m}^g \quad (1)$$

with:

$$b_{k,m}^g = \hat{b}_{k,m}^g \cos(\omega_k t - m \alpha^s - \varphi_{k,m}) \quad (2)$$

The angular frequency ω_k , the pole pair number m and the magnitude $\hat{b}_{k,m}$ depend on the supply harmonics, the stator and rotor space harmonics, and the slotting effect. They also depend on the kind of machine (induction or synchronous machine). ω_k can be written as: $\omega_k = k\omega$. α^s is the angular position of any point relatively to a stator spatial reference axis. The electromagnetic problem consists in defining a simple coefficient K which makes it possible to determine, from the b^g components, the magnitude of the corresponding quantities at a point M outside the machine, located at a radius x from the machine axis z : $\hat{b}_{k,m}^a = K \hat{b}_{k,m}^g$. For easy exploitation in diagnosis application, K should be expressed with an analytical formulation.

The different hypotheses used for the electromagnetic modelling can be formulated as following:

- As the slotting effect appears in (1), the calculation can be made using the simplified machine representation with smooth air-gap given in figure 1 where the various geometrical parameters used for the modelling are also presented. Let us note that in the considered geometry, the gap which can possibly exist between the stator yoke and the external frame is neglected.
- The length L of the machine is supposed to be infinite, what allows one to neglect the end winding effects. Consequently, only the radial magnetic field is considered. This one is in a transverse plan of the machine axis and it is supposed to be tied only to the air-gap flux density.
- The magnetic permeability μ and the electric conductivity σ of the different materials are assumed to be constant.

Fig. 1 - simplified geometry

Let us introduce $\vec{A}$, the magnetic vector potential; in a medium of magnetic permeability μ and electric conductivity σ , the following relationship has to be solved:

$$\overrightarrow{\text{curl}} \overrightarrow{\text{curl}} \vec{A} = -\mu \sigma \partial \vec{A} / \partial t \quad (3)$$

For the media without eddy currents, the conductivity σ is zero, what leads to: $\overrightarrow{\text{curl}} \overrightarrow{\text{curl}} \vec{A} = 0$.

As only the radial field is considered, then the vector potential $\bar{A}$ is oriented along the machine z axis, and a scalar value A_{x,α^s} depending on x and α^s can be used. Moreover, if only one component at ω_k angular frequency is considered, then A_{x,α^s} varies sinusoidally and the complex notation $\bar{A}_{x,\alpha^s}$ can be introduced such as: $A_{x,\alpha^s} = \Re[\bar{A}_{x,\alpha^s} e^{j\omega_k t}]$ where “ $\Re$ ” means that only the real part of the complex quantity is considered, and “j” is the complex number such as $j^2 = -1$.

Then, equation (3) can be written in a cylindrical referential as following:

$$\frac{\partial^2 \bar{A}_{x,\alpha^s}}{\partial x^2} + \frac{1}{x} \frac{\partial \bar{A}_{x,\alpha^s}}{\partial x} + \frac{1}{x^2} \frac{\partial^2 \bar{A}_{x,\alpha^s}}{\partial \alpha^s{}^2} = j\mu\sigma\omega_k \bar{A}_{x,\alpha^s} \quad (4)$$

Equation (4) points out that if eddy currents are taken into account, then the solution will depend on the angular frequency ω_k . For the media without eddy currents, the right member of equation (4) is null and the solution does not depend on ω_k .

2.2 Decoupling of the attenuation phenomena

An experimental test has been performed on a 4-pole induction machine presented in figure 2. In that test, only one phase is supplied by a sine current with constant magnitude and a frequency f varying in a range [0-500Hz]. As the machine does not rotate, all the airgap flux density components are at ω angular frequency. Moreover, the predominant one presents $m=p=2$ pole pairs. The measurement is performed using a small flux sensor located, on the one hand, outside of the machine (figure 2.b), and on the other hand, in the same position but in the gap between the yoke and the frame (figure 2.a).

Fig. 2.a

Fig. 2.b

Fig. 2- positions of the sensor

The measurements $(u_{\text{sensor}})_{\text{in}}$ and $(u_{\text{sensor}})_{\text{out}}$ can be assimilated to sine signals. It is shown in [1] that the variable measured by the sensor (a flux) is tied to the normal component of the flux density at the centre of the sensor through a coefficient which depends on its size but also on the pole pair number of the considered flux density component. For that test, it is assumed that this coefficient is the same in the both position of the sensor (inside - outside) what enables to compare directly the both signals. Let us point out that a derivative effect is also introduced in the measurement; consequently, it is necessary to divide the magnitudes by the frequency to cancel this derivative effect and thus to obtain an image of the flux density at the centre of the sensor.

The obtained results are given in figure 3 where the outside measurement is multiplied by 10 for a better agreement of the scales.

Fig. 3- Variation of the fundamental magnitude of the external magnetic field component versus its frequency – with and without stator frame –

It can be observed that the flux density measured inside is almost constant, whatever the frequency. It also appears that the attenuation caused by the external frame strongly depends on the frequency. So, in the studied frequency range, the eddy currents have to be taken into account only in the frame. This result can be justified by the insulation between the stator laminations which limits the eddy currents flowing in the z direction, whereas the massive external frame allows the circulation of these currents.

Moreover, another property concerning the decoupling can be exploited. Actually, it appears that in the considered frequency range, the attenuation produced by the stator yoke is stronger than the one due to the external frame. Consequently, it will be assumed that the flux density at the inner surface of the external frame does not depend on the attenuation it generates, that means that a decoupling according to the different media can be introduced:

$$K = K^{sy} K^{ef} K^a \quad (5)$$

K^{sy} , K^{ef} and K^a , are the coefficient of attenuation related respectively to the stator yoke, the external frame and the external air. Figure 4 gives a representation of these different media with the associated coefficients of attenuation. These coefficients will be calculated as the ratio between the magnitude of the normal flux density at the outer radius of the considered media and this one at the inner radius. The analysis of the attenuation effect due to the whole stator will be done by considering each medium separately

Fig.4-coefficients of attenuation

2.3 Solving method

The determination of each coefficient of attenuation is done by solving equation (4) in the corresponding medium. The solution of that equation is determined using the variable separation

method [14], which enables to find solutions defined as: $\bar{A}_{x,\alpha^s} = \bar{A}_x \bar{A}_{\alpha^s}$. The method leads to solve the following independent equations:

$$\begin{cases} \bar{A}_{\alpha^s}'' + i^2 \bar{A}_{\alpha^s} = 0 \\ \bar{A}_x''(x) + \frac{1}{x} \bar{A}_x'(x) - \left(\frac{i^2}{x^2} + j\mu\sigma\omega_k \right) \bar{A}_x(x) = 0 \end{cases} \quad (6)$$

The first equation of (6) can also be expressed with the real variables, then sine functions are solutions: $A_{\alpha^s} = \gamma_1 \cos i\alpha^s + \gamma_2 \sin i\alpha^s$. Consequently, because of the constrain of periodicity: $A_{\alpha^s}(2\pi) = A_{\alpha^s}(0)$, it can be deduced that the parameter i of the variable separation method must be chosen integer. Then, the solution can be written using a serie:

$$\bar{A}_{x,\alpha^s} = \sum_i \bar{A}_i \quad (7)$$

With $\bar{A}_i = \bar{A}_{x,i} \bar{A}_{\alpha^s,i}$

The flux density is linked to the vector potential through: $\vec{B} = \text{curl} \vec{A}$. Considering the hypotheses of the study, $\vec{B}$ can be decomposed in normal and tangential components. Let us denote simply b_n and b_t , respectively the normal and tangential flux density tied to the A_i component. In a cylindrical referential the associated complex components $\bar{b}_n$ and $\bar{b}_t$ are defined as:

$$\begin{cases} \bar{b}_n = \frac{1}{x} \frac{\partial \bar{A}_i}{\partial \alpha^s} \\ \bar{b}_t = -\frac{\partial \bar{A}_i}{\partial x} \end{cases}$$

Solving the whole electromagnetic problem requires to consider the boundary conditions at the interfacing surfaces: conservation of h_t and b_n . The normal flux density at the inner surface of the stator yoke is imposed and equal to this one of the air-gap flux density given by (1). Using the complex notation, one airgap flux density component can be expressed as following:

$$\bar{b}_n = \hat{B}_{k,m}^g e^{-jm\alpha^s} \quad (8)$$

such as:

$$b_{k,m}^g = \Re \left[\bar{b}_n e^{j\omega_k t} \right]$$

Let us finally point out that the various attenuation coefficients concern the normal magnetic field in order to be in agreement with the magnetic field measurements in the initial test. Consequently, one attenuation coefficient K related to a medium of inner radius R_{int} and outer radius R_{ext} , is given by

$$K = \frac{|\bar{b}_n(x=R_{ext})|}{|\bar{b}_n(x=R_{int})|} \quad (9)$$

The symbol $||$ means that the modulus of the complex quantity must be considered.

3 Solution without eddy currents

Equation (4) has to be considered with $\sigma=0$. Then (6) becomes:

$$\begin{cases} \overline{A}_{\alpha^s}'' + i^2 \overline{A}_{\alpha^s} = 0 \\ \overline{A}_x''(x) + \frac{1}{x} \overline{A}_x'(x) - \frac{i^2}{x^2} \overline{A}_x(x) = 0 \end{cases} \quad (10)$$

The solution can be expressed with the serie given by (7) with:

$$\overline{A}_i = (\lambda_1 x^i + \lambda_2 x^{-i})(\beta_1 e^{ji\alpha^s} + \beta_2 e^{-ji\alpha^s}) \quad (11)$$

In a cylindrical referential $\overline{b}_n$ and $\overline{b}_t$ are defined as:

$$\begin{cases} \overline{b}_n = \frac{1}{x} \frac{\partial \overline{A}_i}{\partial \alpha^s} = ij(\lambda_1 x^{i-1} + \lambda_2 x^{-i-1})(\beta_1 e^{ji\alpha^s} - \beta_2 e^{-ji\alpha^s}) \\ \overline{b}_t = -\frac{\partial \overline{A}_i}{\partial x} = -i(\lambda_1 x^{i-1} - \lambda_2 x^{-i-1})(\beta_1 e^{ji\alpha^s} + \beta_2 e^{-ji\alpha^s}) \end{cases} \quad (12)$$

As the normal flux density is imposed on the inner surface of the stator through (8), the constants i , β_1 and β_2 have to be chosen in order to be in agreement with expression (8) that gives the airgap flux density. Then, it comes: $i=m$, and $\beta_1=0$, $\beta_2=j$. Now, what remain to be done is to determine the coefficients λ_1 and λ_2 for each medium. This is performed by considering the conservation of h_t and b_n at the interfacing surfaces.

3.1 Solution in the air

In the external air of the machine, b_n and b_t have to be null for $x \rightarrow \infty$. This condition imposes $\lambda_1=0$ for $m>0$, or $\lambda_2=0$ for $m<0$. Consequently, b_n and b_t have the same magnitude in the external air and they are spatially shifted by $\pi/2$. Then the coefficient related to the external air, defined as the ratio between the normal flux density at a radius x and the one the radius $x = R_{\text{ext}}^{\text{ef}}$ can be expressed as following:

$$K^a = \left(x / R_{\text{ext}}^{\text{ef}} \right)^{m-1} \quad (13)$$

3.2 Attenuation due to the stator yoke

To determine the K^{sy} coefficient tied to the stator yoke only, the external frame is supposed to be removed. By denoting μ_r^{sy} the relative magnetic permeability of the stator yoke material, the conservation of b_n and h_t , associated with the property in the air previously pointed out, leads to $\overline{b}_t = j\mu_r^{\text{sy}} \overline{b}_n$ for $x = R_{\text{ext}}^{\text{sy}}$. Then, according to (9), K^{sy} can be written as following:

$$K^{\text{sy}} = \frac{2}{(\mu_r^{\text{sy}} + 1) \left(R_{\text{int}}^{\text{sy}} / R_{\text{ext}}^{\text{sy}} \right)^{m-1} - (\mu_r^{\text{sy}} - 1) \left(R_{\text{int}}^{\text{sy}} / R_{\text{ext}}^{\text{sy}} \right)^{m-1}} \quad (14)$$

K^{sy} depends only on m , the pole pair number of the considered flux density component [15].

Figure 5 gives in log scale the variation of K^{sy} versus m for $R_{\text{int}}^{\text{sy}} = 60\text{mm}$, $R_{\text{ext}}^{\text{sy}} = 90\text{mm}$, $\mu_r^{\text{sy}} = 1000$. For these particular values, K^{sy} decreases of 4dB when m increases of 1.

Fig.5 – variation of K^{SY} versus m.

4 Solution with eddy currents

Here only the external frame is considered and equation (4), which takes into account eddy currents has to be solved. The decoupling of the attenuation phenomena allows us to consider that the flux density at the inner surface of the external frame is imposed and does not depend on the attenuation it generates. Two methods of resolution will be presented, the first one considers the real cylindrical referential and leads to solutions with Bessel functions, the second one makes a change of the structure which allows one to consider equation (3) in a cartesian referential.

4.1 Resolution in a cylindrical referential

Equation (4) and thus (6) have to be solved with $\sigma \neq 0$. Introducing x' defined as: $x' = x \sqrt{j\mu\sigma\omega_k}$, the second equation of (6) becomes:

$$\overline{A}_x''(x') + \frac{1}{x'} \overline{A}_x'(x') - \left(\frac{i^2}{x'^2} + 1\right) \overline{A}_x(x') = 0 \quad (15)$$

Then, the solution of equation (4) can be written as following:

$$\overline{A}_i = (\lambda_1 \text{Bek}_i(x') + \lambda_2 \text{Bei}_i(x')) (\beta_1 e^{j i \alpha^s} + \beta_2 e^{-j i \alpha^s})$$

Bek_i and Bei_i are respectively the first and second kind modified Bessel functions of order i . Considering again $\beta_1=0$ and $\beta_2=j$, $i=m$ to be in agreement with the inner boundary condition, the normal and tangential flux density components are defined as:

$$\begin{cases} \overline{b}_n = \frac{m}{x} (\lambda_1 \text{Bek}_m(x') + \lambda_2 \text{Bei}_m(x')) e^{-j m \alpha^s} \\ \overline{b}_t = -j \sqrt{j\mu\sigma\omega} (\lambda_1 \text{B'ek}_m(x') + \lambda_2 \text{B'ei}_m(x')) e^{-j m \alpha^s} \end{cases}$$

The determination of λ_1 and λ_2 is performed by considering the boundary conditions at the interfacing surfaces of the external frame. Then, the coefficient of attenuation K_I^{ef} associated to the external frame can be determined using (9), it is defined as following

$$K_I^{ef} = \frac{\left| \overline{b}_{n(x=R_{\text{ext}}^{\text{ef}})} \right|}{\left| \overline{b}_{n(x=R_{\text{int}}^{\text{ef}})} \right|} \quad (16)$$

The solution does not lead to define a simple analytical expression of the coefficient as it can be done in equations (13) and (14) when the eddy currents are not taken into account. To simplify, a Cartesian referential will be considered.

4.2 Solution in a cartesian referential

The structure of the machine is changed such as the stator frame is developed as shown in figure 6. The coordinates x and y are chosen such as $x=0$ corresponds to the centre of the machine and $y=0$ leads to define the same stator reference axis of the real structure ($\alpha^s=0$)

Fig. 6 – Developed machine.

Using the complex notation, the equation to be solved becomes:

$$\frac{\partial^2 \bar{A}_{x,y}}{\partial x^2} + \frac{\partial^2 \bar{A}_{x,y}}{\partial y^2} = \mu \sigma j \omega_k \bar{A}_{x,y} \quad (17)$$

The variable separation method is still used to solve this equation [16]. So, the solution can be written as a serie:

$$\bar{A}_{x,y} = \sum_M \bar{A}_M$$

where:

$$\bar{A}_{x,y} = (\lambda_1 e^{Nx} + \lambda_2 e^{-Nx}) (\gamma_1 e^{jMy} + \gamma_2 e^{-jMy})$$

M and N are defined as: $M = m / R_{int}^{ef}$, $N = \sqrt{M^2 + j \omega_k \sigma^{ef} \mu^{ef}}$

The normal and tangential components of the flux density result from the relationship:

$$\bar{b}_n = \frac{\partial \bar{A}_{x,y}}{\partial y}, \bar{b}_t = -\frac{\partial \bar{A}_{x,y}}{\partial x}$$

The solution of this electromagnetic problem leads to define the global attenuation coefficient of the external frame K_2^{ef} which can be expressed as :

$$K_2^{ef} = \left| \frac{2R_{int}^{ef} N}{(m\mu_r^{ef} + NR_{int}^{ef})e^{N(R_{ext}^{ef} - R_{int}^{ef})} - (m\mu_r^{ef} - NR_{int}^{ef})e^{-N(R_{ext}^{ef} - R_{int}^{ef})}} \right| \quad (18)$$

4.3 Comparison between the both referential

The difference between coefficient K_2^{ef} and K_I^{ef} related to the real cylindrical referential, can be computed using Matlab. Figure 7 gives the relative error (err%) defined as : $100 * (K_I^{ef} - K_2^{ef}) / K_I^{ef}$ for $\mu_r^{ef} = 150$, $\sigma^{ef} = 8,3 \cdot 10^6 \Omega^{-1} m^{-1}$ which corresponds approximately to the characteristic of the material constituting the frame of the machine used for the initial tests. The error is given versus the frequency f_k and for different quantities (width%) defined as $100 * (R_{ext}^{ef} - R_{int}^{ef}) / R_{int}^{ef}$. Various number of pole pairs are considered ($m=1,2,4,6,8,10$).

Fig. 7a. – width=50%

Fig. 7.b - width=20%

Fig 7.c - width=5%

Fig. 7 - variation of err% versus f.

It can be observed that the thinner the external frame is, the lower the error is. This result can be justified because for a thin external frame compared to the radius, its curve can be neglected. Moreover, for a thin element (figure 7.c), the error decreases when m increases. For the considered machine of the initial test in 2.2, it comes $R_{ext}^{ef} = 94\text{mm}$ and the thickness of the external frame is equal to 4mm leading to $\text{width} = 4.44\%$. The number of pole pair of the considered fundamental flux density wave is equal to 2. In these conditions, the error between the attenuation coefficients calculated in the both referential is less than 5% in the considered frequency range. So, the coefficient of attenuation defined by (18) can be used to calculate the magnitude of the radial magnetic field at a point outside of the machine.

4.4 Comparison with experimental measurements

Figure 8 gives, on the one hand the coefficient of attenuation K_{exp}^{ef} deduced from the experimental test presented in 2.2 : $K_{exp}^{ef} = (U_{\text{sensor}}/f)_{\text{out}} / (U_{\text{sensor}}/f)_{\text{in}}$, and on the other hand, the coefficient K_2^{ef} deduced from equation (18) with $m=2$, $R_{int}^{ef} = 90\text{mm}$, $R_{ext}^{ef} = 94\text{mm}$, $\mu_r^{ef} = 150$, $\sigma^{ef} = 8.3 \cdot 10^6 \Omega^{-1}\text{m}^{-1}$. One can observe that the calculated and the measured coefficients are close to each other, that validates, in the considered frequency range, the model that leads to define the coefficient of attenuation in media with eddy currents.

Fig. 8 – comparison between calculated and experimental results.

5 Conclusion

The knowledge of the attenuation of the radial flux density magnitude through the stator yoke and the housing of an AC machine provides a useful tool for analyzing the spectrum measured by small flat coil stuck on the external surface of a running motor. The method proposed in this paper deals with analytical solving of Poisson's and Laplace's equations in the various media which constitute the machine. A global approach which needs to consider the whole machine can be avoided thanks preliminary experiments which show the decoupling of the attenuation phenomena. This property leads to define attenuation coefficients related to each material considered separately. The use of a Cartesian referential to solve the electromagnetic problem in media with eddy currents allows one to affect simple coefficients to the elements which constitute the stator of the machine. With these coefficients, it is possible to perform an indirect measurement of the magnitude of each airgap flux density component what can give information about a fault (broken rotor bars, partial winding short circuits, dynamic eccentricity ...) in the machine.

6 References

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