Optimisation of an Asymmetric Three Phase-Shift Distributed Feedback Semiconductor Laser Structure Concerning the Above-Threshold Stability

Carlos A. F. Fernandes, José A. P. Morgado, and José B. M. Boavida.

Abstract

This paper shows that accurately optimised asymmetric three phase-shift (3PS)-distributed feedback (DFB) laser structures can strongly improve the stability of the single-longitudinal mode (SLM) operation, described by the mode selectivity and the flatness of the photon density profile, through an extended range of current injection, when compared to optimised symmetric 3PS-DFB structures reported elsewhere.

This study reveals its importance in modern high bit-rate optical communication systems, by enhancing the possibility of attaining high performance DFB lasers, in easily fabricated structures.

The procedure, based on matrix techniques, aims at the description of the optimal design of the laser structure and it is described step-by-step.

Above-threshold calculations have been accomplished to evaluate the performance of the optimised asymmetric 3PS-DFB structure, namely: the mode selectivity ($\mathfrak{S}$), the flatness, the lasing wavelength, the optical power, and the side-mode suppression ratio (SMSR) evolutions with the current injection. For a current injection five times bigger than the threshold current, substantially improvements in $\mathfrak{S}$ (five times bigger) and in the SMSR (about 9 dB higher) are achieved when compared to similar, but symmetric, DFB structures.

Index Terms

Distributed feedback laser, flatness, matrix-methods, mode selectivity, phase-shift, side-mode suppression ratio, threshold gain.

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I. INTRODUCTION

Nowadays, distributed-feedback (DFB) lasers play a decisive role in high bit-rate optical communication systems using direct modulation [1]–[4], where they should present single longitudinal mode (SLM) operation over the largest range of current injection. Therefore, laser structures with a very large normalised mode selectivity ($\mathfrak{S}$), combined with sufficiently flat intra-cavity fields – usually quantified by a flatness parameter ($\mathfrak{F}$) – are demanded at threshold, so that modal instabilities induced above threshold by spatial hole burning (SHB) can be avoided [5, p. 159], [6, p. 249], [7, p. 123], [8]. These constraints put increasing demands on the design of directly modulated lasers with adequate laser performance, in easily fabricated structures.

To avoid the degradation of SLM operation with the current injection, various non-conventional DFB laser structures have been proposed. Some of them point to the use of gain coupling structures with high selectivity and reduced SHB [5, p. 173], [9, p. 185]. However, they are generally associated with technological processes that are hard to implement [5, p. 173], [10], [11]. Therefore, index-coupled structures have been used more often, namely: with multiple phase-shifts (MPS-DFB) placed along the cavity [12], [13], [14], [7, pp. 124–135]; with distributed coupling coefficients (DCC-DFB) in standard quarter wavelength-shifted (QWS) structures [7, p. 136–141]; and with chirped gratings in corrugation-pitch-modulated (CPM-DFB) structures [15]. Moreover, improvements can be achieved in structures with adequate combinations of distributed phase-shift (PS) – MPS or CPM structures – and distributed coupling strength – DCC structures – along the cavity [7, p. 141–145], [5, p. 169], [16]. However, multi-section structures associated with axial variations in the pitch or/and in the coupling coefficient induce additional technological problems, making more difficult their monolithic integration [5, p. 169]. For this reason, MPS-DFB structures have been considered up to now as the most feasible solution [17]–[19]. Besides, recent studies have reported that the design of the PSs in those kind of lasers can be accurately set [20]. Unfortunately, the flatness improvement of the symmetric MPS-DFB structures is achieved at the expense of a drastic reduction in the mode selectivity [7, p. 145]. On the other hand, and as far as the authors know, the asymmetric MPS-DFB structures have not been, up to now, subjected to an accurate optimisation [13], [14].

This paper aims at to show that optimised asymmetric 3PS-DFB structures may strongly improve the laser performance

above threshold, by reducing the modal instabilities induced by SHB, when compared with currently reported symmetric DFB structures [7, p. 145].

In section II, the transfer matrix method (TMM) is reviewed, both for threshold and above-threshold regimes. Special attention is given to the details concerning the adequate strategies searching for efficient convergence procedures above threshold. The optimisation of the 3PS-DFB structure is carried out in section III at threshold regime, aiming at the calculation of the structural parameters concerning the optimised laser structure. Still in this section, the most usual laser figures of merit, like the mode selectivity, the flatness and the side mode suppression ratio (SMSR) are investigated for the optimised 3PS-DFB laser structure in the above threshold regime. A comparative analysis with other structures referred elsewhere is also performed. Finally, the main conclusions are summarised in section IV.

II. THE TRANSFER-MATRIX-METHOD

A. Threshold Analysis

To perform the TMM-based model for laser threshold analysis, the laser cavity with length L is divided into M concatenated sections, which are identified by the constancy of its structural parameters. For the m -th section with length L_m , those structural parameters are: the corrugation period Λ_m , the amount of feedback per unit length $\bar{\kappa}_m$ and the phase of the section grating with respect to the left side of the cavity Ω_m .

Each section is described by two counter propagating electrical field waves described by their complex amplitudes $\bar{E}_R(z)$ and $\bar{E}_S(z)$, which allow the internal electrical field intensity $\mathbb{E}(t, z)$, to be determined according to

$$\mathbb{E}(t, z) \propto \Re \left\{ [\bar{E}_R(z) + \bar{E}_S(z)] \cdot \exp(j\omega t) \right\}, \quad (1)$$

where t is the time, z is the z -axis coordinate, $\Re \{ \cdot \}$ is the real part operator, $j = \sqrt{-1}$ and ω is the field angular frequency. Equation (1) assumes that the longitudinal laser axis coincides with the z -axis. In the TMM the column matrices related to $\bar{E}_R(z)$ and $\bar{E}_S(z)$ components are considered for the same spatial position (see Fig. 1).

On the basis of the coupled wave equations, the transfer matrix for the m -th section of the one-dimensional DFB laser structure indicated in Fig. 1 is given by [7, p.105], [15]

$$\mathbf{T}(z_{m+1}/z_m) = \begin{bmatrix} t_{11}^{(m)} & t_{12}^{(m)} \\ t_{21}^{(m)} & t_{22}^{(m)} \end{bmatrix}, \quad (2)$$

Fig. 1 – A simplified schematic diagram for a one-dimensional DFB laser structure section, placed between $z = z_m$ and $z = z_{m+1}$.

and it links the column matrices related to the complex electric fields of the wave solutions at z_m and z_{m+1}

$$\begin{bmatrix} \overline{E}_R(z_{m+1}) \\ \overline{E}_S(z_{m+1}) \end{bmatrix} = \mathbf{T}(z_{m+1}/z_m) \cdot \begin{bmatrix} \overline{E}_R(z_m) \\ \overline{E}_S(z_m) \end{bmatrix}, \quad (3)$$

where $t_{11}^{(m)}$, $t_{12}^{(m)}$, $t_{21}^{(m)}$ and $t_{22}^{(m)}$ are given, respectively, by

$$\begin{aligned} t_{11}^{(m)} &= \frac{\xi_m - \bar{\rho}^2 \xi_m^{-1}}{(1 - \bar{\rho}^2) \zeta_m}; \quad t_{12}^{(m)} = -\frac{\bar{\rho} (\xi_m - \xi_m^{-1}) e^{-j\Omega_m}}{(1 - \bar{\rho}^2) \zeta_m} \\ t_{21}^{(m)} &= \frac{\bar{\rho} (\xi_m - \xi_m^{-1}) e^{j\Omega_m}}{(1 - \bar{\rho}^2) \zeta_m^{-1}}; \quad t_{22}^{(m)} = -\frac{\bar{\rho}^2 \xi_m - \xi_m^{-1}}{(1 - \bar{\rho}^2) \zeta_m^{-1}}, \end{aligned} \quad (4)$$

with $\xi_m = e^{\bar{\gamma}_m \cdot (z_{m+1} - z_m)}$ and $\zeta_m = e^{j\beta_m \cdot (z_{m+1} - z_m)}$. The propagation constant, β_m , and the complex propagation constant, $\bar{\gamma}_m$, are given, respectively, by

$$\beta_m = \frac{\pi}{\Lambda_m}; \quad \bar{\gamma}_m = \sqrt{(\alpha - j\delta_m)^2 + \bar{\kappa}_m^2}, \quad (5)$$

where $\bar{\rho}_m$ and δ_m are given by

$$\bar{\rho}_m = \frac{j\bar{\kappa}_m}{\alpha - j\delta_m + \bar{\gamma}_m}; \quad \delta_m = \delta + \pi \left(\frac{1}{\Lambda_1} - \frac{1}{\Lambda_m} \right), \quad (6)$$

with α and δ being, respectively, the gain and detuning for the propagation modes taking the left section as a reference. Ω_m is given by [15]

$$\Omega_m = \Omega_1 + 2 \cdot \sum_{k=1}^{m-1} \left(\frac{\pi}{\Lambda_k} \cdot L_k \right); \quad 2 \leq m \leq M. \quad (7)$$

The fields at both ends of the cavity are connected by the elementary matrix product

$$\begin{bmatrix} \overline{E}_R(L) \\ \overline{E}_S(L) \end{bmatrix} = \mathbf{T}^{\text{Total}} \cdot \begin{bmatrix} \overline{E}_R(0) \\ \overline{E}_S(0) \end{bmatrix}, \quad (8)$$

where

$$\mathbf{T}^{\text{Total}} = \prod_{m=M}^1 \mathbf{T}(z_{m+1}/z_m). \quad (9)$$

The formulation of the transfer matrices for other modified DFB laser structures is straightforward, as far as the changes are correctly translated to the matricial formalism. Namely, the

inclusion of the PS discontinuity φ , is given by the following matrix [7]

$$\begin{bmatrix} e^{j\varphi} & 0 \\ 0 & e^{-j\varphi} \end{bmatrix}, \quad (10)$$

assuming that the field discontinuity is usually small along the plane of the PS. In this case the matrix (10) should be included in the matrix product $\mathbf{T}^{\text{Total}}$ indicated in the equation (9), at the correspondent z position.

The oscillation condition corresponds to the vanishing of the incoming waves and it is determined by the following requirement

$$t_{22}^{\text{Total}}(\alpha, \delta) = 0, \quad (11)$$

where t_{22}^{Total} is the fourth element of the matrix $\mathbf{T}^{\text{Total}}$, given by (9). The solutions are the mode gain, α , and the detuning, δ , and are related to the modes that are allowed to propagate inside the cavity. For the main mode their values are, respectively, the threshold gain, α_{th} , and the threshold detuning, δ_{th} . For a grating with a first-order Bragg diffraction, the mode gain and the detuning can be expressed, respectively, as [7, p. 151]

$$\alpha(z) = \frac{\Gamma g(z) - \alpha_{\text{loss}}}{2} \quad (12)$$

and

$$\delta(z) = \frac{2\pi}{\lambda} n(z) - \frac{2\pi n_g}{\lambda \lambda_B} \cdot (\lambda - \lambda_B) - \frac{\pi}{\Lambda(z)}, \quad (13)$$

where Γ is the optical confinement factor, α_{loss} is the total loss, n is the effective index, λ_B is the Bragg wavelength, λ is the lasing mode wavelength, n_g is the group effective index and g is the material gain, given by [7, p. 151]

$$\begin{aligned} g(z) &= A_0 \cdot (N(z) - N_0) - \\ &- A_1 \cdot \left[\lambda - \left(\lambda_0 - A_2 (N(z) - N_0) \right) \right]^2. \end{aligned} \quad (14)$$

In (14), N is the carrier concentration, A_0 is the differential gain, N_0 is the carrier concentration at transparency ($g = 0$), λ_0 is the peak wavelength at transparency and A_1 and A_2 are parameters used in the parabolic model assumed for the material gain. Using the first-order approximation for the effective index n , one obtains [7, p. 151]

$$n(z) = n_0 + \Gamma \frac{\partial n}{\partial N} N(z), \quad (15)$$

where n_0 is the effective index at zero carrier injection and $\partial n / \partial N$ is the differential index. The photon concentration (S) and N are coupled together through the steady-state carrier rate equation [7, p. 152]

$$\begin{aligned} \frac{I}{q V_{\text{act}}} &= A N(z) + B N^2(z) + C N^3(z) + \\ &+ \frac{v_g g(z) S(z)}{1 + \varepsilon S(z)}, \end{aligned} \quad (16)$$

where I is the injection current, q is the modulus of the electron charge, V_{act} is the volume of the active layer, A is the spontaneous emission rate, B is the radiative spontaneous

emission coefficient, C is the Auger recombination coefficient, ε is a non-linear coefficient to take into account saturation effects and $v_g = c/n_g$ is the group velocity, with c being the free space velocity.

In an purely index-coupled DFB laser cavity, which happens to be the case considered along this paper, the mutual interaction between the coupled waves $\overline{E}_R(z)$ and $\overline{E}_S(z)$ can be neglected in the rate of total power change [7, p. 59], [10]. Therefore, the local photon density inside the cavity can be expressed as [7, p. 152]

$$S(z) \approx \frac{2 \varepsilon_0 n(z) n_g \lambda}{h c} \cdot c_0^2 \left[|\overline{E}_R(z)|^2 + |\overline{E}_S(z)|^2 \right] \quad , \quad (17)$$

where ε_0 is the free space permittivity, h is the Planck's constant and c_0 a dimensionless coefficient that allows the determination of the total electric field at the above threshold regime, taking into account that the normalization

$$|\overline{E}_R(0)|^2 + |\overline{E}_S(0)|^2 = 1 \quad (18)$$

has been imposed. The equation (18) and the boundary conditions imposed at the left facet allow the calculation of the two counter running waves, $\overline{E}_R(z)$ and $\overline{E}_S(z)$, at $z = 0$. The use of the TMM allows the calculation of the longitudinal electric field profile. The output power at the right facet can be determined as [7, p. 152]

$$P = \frac{d w}{L} \cdot v_g \cdot \frac{h c}{\lambda} \cdot S(L) \quad , \quad (19)$$

where d and w are the thickness and width of the active layer, respectively.

From the solutions of the oscillation condition (11), α_{th} and δ_{th} are determined. Using equations (12)–(15), the carrier concentration at threshold (N_{th}), the effective index at threshold (n_{th}), the threshold wavelength (λ_{th}) and λ_0 are successively evaluated. Threshold current (I_{th}) is obtained from (16), assuming that S is negligible at threshold. Within this assumption, the z dependence described in eqs. (12), (14) and (15) is also neglected. The same assumption is valid for eq. (13), except for CPM structures where a z dependence is included in $\Lambda(z)$.

B. Above-Threshold Analysis

In the above-threshold regime, $S(z)$ is high enough to induce important non-uniformities in $N(z)$ and $n(z)$. Despite the SHB effect can be minimised by an adequate design of the DFB structure, the interdependence of $S(z)$, $N(z)$ and $n(z)$ induces strong longitudinal inhomogeneities that forces the division of each section into several sub-sections, in order to ensure a correct evaluation of the above-threshold characteristics. According to Ref. [7, p. 153], for a cavity with $L = 500 \mu\text{m}$, at least a total of $M = 5000$ subsections are needed.

In this paper, the numerical procedure for the above-threshold calculations follows closely the method developed in Refs. [7, p. 149], [15]. However, in order to ensure a quick convergence in the evaluations of the laser characteristics, an adequate strategy is now proposed, which is fully described below.

1) Lasing mode analysis: For each bias current I , the numerical above-threshold calculations concerning the lasing mode are summarised as follows:

(i) Successive $(G \times G)$ grids are created in the (c_0, λ) plane. The i -th grid is centred at (c_{0c}^i, λ_c^i) and it is enclosed in the region defined by the limits $c_{0\min}^i, c_{0\max}^i, \lambda_{\min}^i$ and $\lambda_{\max}^i$. For the initial grid ($i = 1$)¹

$$\lambda_c^1 = \lambda_{th} \quad (20)$$

$$c_{0c}^1 = \frac{\sqrt{\frac{h c (I - I_{th})}{2 q V_{act} v_g g_{th} \varepsilon_0 n_{th} n_g \lambda_{th}}}}{\sqrt{|\overline{E}_R(0)|^2 + |\overline{E}_S(0)|^2}} \quad (21)$$

$$c_{0\min}^1 = c_{0c}^1 - \Delta c_0^1 \quad ; \quad c_{0\max}^1 = c_{0c}^1 + \Delta c_0^1 \quad (22)$$

$$\lambda_{\min}^1 = \lambda_c^1 - \Delta \lambda^1 \quad ; \quad \lambda_{\max}^1 = \lambda_c^1 + \Delta \lambda^1 \quad , \quad (23)$$

where $\Delta c_0^1 \triangleq c_{0c}^1/10$ and $\Delta \lambda^1 \triangleq 0.1 \text{ nm}$ seem adequate for the majority of the DFB structures, when $G \approx 10$. However, a readjustment of Δc_0^1 and $\Delta \lambda^1$ may, occasionally, be necessary in order to prevent the mode hopping. This is a critical aspect of the proposed analysis, since an inadequate choice would mislead the results, leading to solutions with no physical meaning;

(ii) For each one of the $(G \times G)$ pairs of the i -th grid (c_{0k}^i, λ_l^i) with $k, l = 1 \dots G$, the equations (14)–(17) are self-consistently solved, in order to determine the material gain, carrier density, photon density and effective index for each one of the j sub-section, respectively, g_j , N_j , S_j and n_j , with $1 \leq j \leq M$;

(iii) The equations (12) and (13) are solved in order to determine the lasing mode gain and detuning for the j -th sub-section, respectively, α_j and δ_j . The transfer matrix of the j -th sub-section, $\mathbf{T}(z_{j+1}/z_j)$, is then calculated;

(iv) Using the TMM, the two counter-running waves at the output of the j -th sub-section, $\overline{E}_{Rj}$ and $\overline{E}_{Sj}$, are obtained. For the M -th sub-section, the discrepancy found between those values and the laser right facet boundary condition is represented by ε_{kl}^i . This value is evaluated and stored for each pair (c_{0k}^i, λ_l^i) of the i -th grid. The error associated to the i -th grid is given by $\varepsilon^i = \min(\varepsilon_{kl}^i)$;

(v) Whenever $\varepsilon^i = \varepsilon^{i-1}$, the central pair remains the same $(c_{0c}^{i+1} = c_{0c}^i, \lambda_c^{i+1} = \lambda_c^i)$, but new limits are required for the next grid. c_0 and λ discretizations should be reduced, for instance: $\Delta c_0^{i+1} = \Delta c_0^i/10$ and $\Delta \lambda^{i+1} = \Delta \lambda^i/10$. Whenever $\varepsilon^i < \varepsilon^{i-1}$, the pair associated with the $\min(\varepsilon_{kl}^i)$ is chosen as next central

¹Notice that, according to (18), c_{0c}^1 is numerically equal to $\sqrt{h c (I - I_{th}) / (2 q V_{act} v_g g_{th} \varepsilon_0 n_{th} n_g \lambda_{th})}$.

pair $(c_{0_c}^{i+1}, \lambda_c^{i+1})$, while c_0 and λ discretisations remains unchangeable. For $i = 1$, ε^{i-1} is taken as the error associated with the central pair $(c_{0_c}^1, \lambda_c^1)$.

For each one of the $(G \times G)$ pairs $(c_{0_k}^{i+1}, \lambda_l^{i+1})$, the steps (i-v) are repeated until $\varepsilon^{i+1} \leq \varepsilon_{\min}$, where $\varepsilon_{\min}$ is a preset error value (less than 10^{-14} , as indicated in Ref. [7, p. 156]).

Since the gain α_j and the detuning δ_j are z -dependent, the lasing characteristics for each biasing current are associated with their mean values along the cavity, given by

$$\bar{\alpha}(I) = \frac{1}{M} \sum_{j=1}^M \alpha_j(I) \quad (24)$$

$$\bar{\delta}(I) = \frac{1}{M} \sum_{j=1}^M \delta_j(I) \quad (25)$$

Notice that the sequential analysis (i)-(v) assumes a one-mode propagation laser behaviour. This approach is itself a good assumption, since the present analysis focuses on DFB structures that must ensure SLM operation. Otherwise, different strategies must be adopted.

When studying the I influence on the laser characteristics, a considerable CPU time reduction can be achieved if, instead of using (20), $\lambda_c^1 = \lambda$ is assumed, where λ is the solution found in a previous bias current.

2) *Side mode analysis*: Now, the procedure starts by assuming the threshold solution (α, δ) , associated with the side mode under analysis. Steps (i)-(v) are carried out, with the exception of the calculations concerning $n(z)$. Since the effective index profile is settled by the main-mode distribution, $n(z)$ is the distribution obtained in subsection B.1. Therefore, while the steady-state carrier rate equation (16) is still valid in the side mode analysis, the equation (15) is no longer used. This means that the calculated $S(z)$ and $N(z)$ distributions have no physical meaning in this context, since the main mode has been suppressed in this analysis. Only the solutions $\delta(I)$, $\alpha(I)$ and $\lambda(I)$ shall be kept.

To summarise, a method of determining the laser main mode characteristics such as $N(z)$, $S(z)$, $n(z)$, λ , P , $\bar{\alpha}$ and $\bar{\delta}$ versus the biasing current has been described. Besides, the assessment of the side modes λ , $\bar{\alpha}$ and $\bar{\delta}$ is also possible. Therefore, crucial information about the mode stability, the mode selectivity and the SMSR with the current injection can be made available for the DFB structure under analysis.

III. THE ASYMMETRIC 3PS-DFB LASER

A. Structure Description and Parameters Definition

The laser structure under analysis is represented in Fig. 2. This is a multi-section anti-reflection (AR)-coated with uniform grating period ($\Lambda_m = \Lambda$) and uniform coupling coefficient ($\bar{\kappa}_m = \bar{\kappa}$). The PS discontinuities, φ_i , are located along the DFB laser structure. Their positions are represented by a normalised parameter given by

$$\text{PSP}_i = \frac{z_i}{L} \quad i = 1; 2; 3 \quad , \quad (26)$$

TABLE I – Summary of laser parameters

Laser parameter	Value
Material parameters	
Spontaneous emission rate, A	$2.5 \times 10^8 \text{ s}^{-1}$
Bimolecular recombination coefficient, B	$1.0 \times 10^{-16} \text{ m}^3 \cdot \text{s}^{-1}$
Auger recombination coefficient, C	$3.0 \times 10^{-41} \text{ m}^6 \cdot \text{s}^{-1}$
Differential gain, A_0	$2.70 \times 10^{-20} \text{ m}^2$
Gain curvature, A_1	$1.50 \times 10^{19} \text{ m}^{-3}$
Differential peak wavelength, A_2	$2.70 \times 10^{-32} \text{ m}^4$
Internal loss, α_{loss}	$4.0 \times 10^3 \text{ m}^{-1}$
Refractive index at zero injection, n_0	3.41351524
Carrier density at transparency, N_0	$1.23 \times 10^{24} \text{ m}^{-3}$
Differential index, $d n / d N$	$-1.8 \times 10^{-26} \text{ m}^3$
Group velocity, v_g	$8.33 \times 10^7 \text{ m} \cdot \text{s}^{-1}$
Nonlinear gain coefficient, ε	$1.5 \times 10^{-23} \text{ m}^3$
Structural parameters	
Active layer width, w	$1.5 \mu\text{m}$
Active layer thickness, d	$0.12 \mu\text{m}$
Cavity length, L	$500 \mu\text{m}$
Optical confinement factor, Γ	0.35
Grating period, Λ	227.039 nm

where z_i is the φ_i position. A purely index-coupled laser structure, which assures that $\bar{\kappa}$ is real [7, pp. 56–57], is assumed, with $L = 500 \mu\text{m}$, $\Lambda = 227.039 \text{ nm}$, $\Omega_1 = 0 \text{ rad}$ and $\text{PSP}_2 = 0.5$.

The normalised mode selectivity and the flatness of the electric field are given, respectively, by [7, p. 131 and p. 128]

$$\mathfrak{S} = (\alpha L) - (\alpha_{th} L) \quad ; \quad \mathfrak{F} = \frac{1}{L} \int_0^L [\mathcal{I}(z) - \bar{\mathcal{I}}]^2 dz \quad (27)$$

where $\mathcal{I}(z)$ is the normalised electric field intensity at an arbitrary position, z , given by²

$$\mathcal{I}(z) = \frac{|\bar{E}_R(z)|^2 + |\bar{E}_S(z)|^2}{|\bar{E}_R(0)|^2 + |\bar{E}_S(0)|^2} \quad , \quad (28)$$

and $\bar{\mathcal{I}}$ is its average value along the cavity. Besides, $(\alpha_{th} L)$ is the normalised threshold gain and (αL) is the normalised gain of the main side mode. The laser structural and material parameters used along the paper are summarised in Table I Ref. [7, p. 157].

B. Structure Optimisation

The objective of the 3PS-DFB laser structure optimisation is both to maximise $\mathfrak{S}$ and to minimise $\mathfrak{F}$ at threshold, by varying simultaneously and independently the following set of variables (decision variables): $(\bar{\kappa} L)$, φ_2 , PSP_1 , φ_1 , PSP_3 and φ_3 .

Initially $\mathfrak{S} \geq 0.25$ ($\mathfrak{S}_{\min} = 0.25$) and $\mathfrak{F} \leq 0.05$ ($\mathfrak{F}_{\max} = 0.050$) is required, since these limits have been generally accepted as good boundaries in order to reach a

²Notice that, according to (18), $\mathcal{I}(z)$ is numerically equal to $|\bar{E}_R(z)|^2 + |\bar{E}_S(z)|^2$.

Fig. 2 – A simplified schematic diagram for the 3PS-DFB laser structure with non-equal spaced and non-identical PSs.

stable SLM operation [7, pp. 128, 131]. After each step, the selection criteria are adjusted by fixing tighter limits, i.e., higher $\mathfrak{S}_{\min}$ and smaller $\mathfrak{F}_{\max}$. The starting point is a QWS-DFB laser structure ($\varphi_2 = 90^\circ$) with $\bar{\kappa}L = 2$. The restrictions $\text{PSP}_1 \leq \text{PSP}_2$ and $\text{PSP}_2 \leq \text{PSP}_3$ are assumed.

The step-by-step procedure can be summarised as follows:

Step 1: One PS, φ_1 , is added in the 1st half of the cavity. The optimisation of both functions $\mathfrak{S}(\text{PSP}_1, \varphi_1)$ and $\mathfrak{F}(\text{PSP}_1, \varphi_1)$ is performed by varying simultaneously and independently PSP_1 and φ_1 in their entire ranges: $0 \leq \text{PSP}_1 \leq 0.5$ and $0^\circ \leq \varphi_1 \leq 180^\circ$. The following selection criterium is used: $\mathfrak{S}(\text{PSP}_1, \varphi_1) \geq \mathfrak{S}_{\min}$ and $\mathfrak{F}(\text{PSP}_1, \varphi_1) \leq \mathfrak{F}_{\max}$. This step leads to the definition of a region in the plane $(\text{PSP}_1, \varphi_1)$, from which a solution is chosen and new boundaries ($\mathfrak{S}_{\min}, \mathfrak{F}_{\max}$) are settled;

Step 2: Assuming PSP_1 and φ_1 achieved in Step 1, another PS, φ_3 , is placed in the 2nd half of the cavity. The optimisation of both functions $\mathfrak{S}(\text{PSP}_3, \varphi_3)$ and $\mathfrak{F}(\text{PSP}_3, \varphi_3)$ is performed by varying simultaneously and independently PSP_3 and φ_3 in their entire ranges: $0.5 \leq \text{PSP}_3 \leq 1$ and $0^\circ \leq \varphi_3 \leq 180^\circ$. The following selection criterium is used: $\mathfrak{S}(\text{PSP}_3, \varphi_3) \geq \mathfrak{S}_{\min}$ and $\mathfrak{F}(\text{PSP}_3, \varphi_3) \leq \mathfrak{F}_{\max}$, where ($\mathfrak{S}_{\min}, \mathfrak{F}_{\max}$) are the new boundaries settled in Step 1;

Step 1 and Step 2 are sequentially repeated (Steps 1-2) until no improvements on $\mathfrak{S}$ and $\mathfrak{F}$ are achieved. The optima values for the structural parameters PSP_1 , φ_1 , PSP_3 , and φ_3 are found, assuming $\bar{\kappa}L = 2$ and $\varphi_2 = 90^\circ$. New optima boundaries ($\mathfrak{S}_{\min}, \mathfrak{F}_{\max}$) for the next step are settled.

Step 3 An optimisation of $(\bar{\kappa}L)$ and φ_2 is performed. Therefore, assuming the laser structure defined at the end of Steps 1-2, the optimisation of both functions $\mathfrak{S}(\bar{\kappa}L, \varphi_2)$ and $\mathfrak{F}(\bar{\kappa}L, \varphi_2)$ is performed by varying simultaneously and independently $(\bar{\kappa}L)$ and φ_2 in their entire ranges: $1 \leq \bar{\kappa}L \leq 3$ and $0^\circ \leq \varphi_2 \leq 180^\circ$. The following selection criterium is used: $\mathfrak{S}(\bar{\kappa}L, \varphi_2) \geq \mathfrak{S}_{\min}$ and $\mathfrak{F}(\bar{\kappa}L, \varphi_2) \leq \mathfrak{F}_{\max}$;

Steps 1-2 and Step 3 are repeated until no improvements on $\mathfrak{S}$ and $\mathfrak{F}$ are achieved. This means that optima values for $\mathfrak{S}$ and $\mathfrak{F}$, at which correspond the best 3PS-DFB laser structure ($\text{PSP}_1, \varphi_1, \varphi_2, \text{PSP}_3, \varphi_3$ and $(\bar{\kappa}L)$), have been obtained. In all steps, and whenever necessary, an argument based on the smallest threshold gain is used in order to decide optima solutions.

Fig. 3 shows the contour maps for $\mathfrak{S}(\text{PSP}_3, \varphi_3)$ and $\mathfrak{F}(\text{PSP}_3, \varphi_3)$, when PSP_3 and φ_3 vary along their entire range,

Fig. 3 – (a) $\mathfrak{S}(\text{PSP}_3, \varphi_3)$ and (b) $\mathfrak{F}(\text{PSP}_3, \varphi_3)$, assuming $\bar{\kappa}L = 1.7$, $\text{PSP}_2 = 0.5$, $\varphi_2 = 60^\circ$, $\text{PSP}_1 = 0.127$ and $\varphi_1 = 110.7^\circ$. Values for $\mathfrak{S} \geq \mathfrak{S}_{\min}$ or $\mathfrak{F} \leq \mathfrak{F}_{\max}$ are represented by a solid line. $\mathfrak{S}_{\min} = 0.64$ and $\mathfrak{F}_{\max} = 0.012$ are considered. (x solution: $\text{PSP}_3 = 0.64$ and $\varphi_3 = 100^\circ$, which corresponds to $\mathfrak{S} = 0.78$ and $\mathfrak{F} = 0.010$)

assuming $\bar{\kappa}L = 1.7$, $\text{PSP}_2 = 0.5$, $\varphi_2 = 60^\circ$, $\text{PSP}_1 = 0.127$ and $\varphi_1 = 110.7^\circ$. This figure corresponds to an intermediate iteration for the structure optimisation. Solid lines enclose all combinations $(\text{PSP}_3, \varphi_3)$ that ensure $\mathfrak{S}(\text{PSP}_3, \varphi_3) \geq \mathfrak{S}_{\min}$ and $\mathfrak{F}(\text{PSP}_3, \varphi_3) \leq \mathfrak{F}_{\max}$ where $\mathfrak{S}_{\min} = 0.64$ and $\mathfrak{F}_{\max} = 0.012$ have been settled in the previous iteration. The solution for the next iteration is $\text{PSP}_3 = 0.64$ and $\varphi_3 = 100^\circ$ (marked x), which corresponds to $\mathfrak{S} = 0.78$ and $\mathfrak{F} = 0.010$.

At the end of the structure optimisation process, the final solution has been found: $\bar{\kappa}L = 1.7$, $\varphi_2 = 60^\circ$, $\text{PSP}_1 = 0.127$, $\varphi_1 = 110.7^\circ$, $\text{PSP}_3 = 0.64$ and $\varphi_3 = 100^\circ$, corresponding to $\mathfrak{S} = 0.78$ and $\mathfrak{F} = 0.010$. Besides $\alpha_{th}L = 1.18$,

TABLE II – $\mathfrak{S}$, $\mathfrak{F}$ and $\alpha_{th} L$ for several laser structures. $\bar{\kappa} L = 1.7$ is assumed for the 1st structure. $\bar{\kappa} L = 2$ is assumed for the 2nd and 3rd structures.

Laser structure	$\mathfrak{S}$	$\mathfrak{F}$	$\alpha_{th} L$
Optimised 3PS-DFB	0.78	0.010	1.18
QWS-DFB	0.73	0.30	0.70
3PS-DFB PSP ₁ = 0.25 ; $\varphi_1 = 60^\circ$ PSP ₂ = 0.50 ; $\varphi_2 = 60^\circ$ PSP ₃ = 0.75 ; $\varphi_3 = 60^\circ$	0.34	0.012	0.98

corresponding to $I_{th} = 23.4$ mA. This threshold current is similar to those reported in Ref. [7, pp. 159 and 162], for the QWS-DFB and the symmetric 3PS-DFB lasers, respectively, $I_{th} = 19.8$ mA and $I_{th} = 21.8$ mA.

Table II summarises the results for $\mathfrak{S}$, $\mathfrak{F}$ and $(\alpha_{th} L)$ achieved for three different laser structures: the optimised 3PS-DFB, the QWS-DFB and the symmetric 3PS-DFB of Ref. [7, pp. 128–135] indicated as a possible optimum design for 3PS-DFB laser structures. As far as $\mathfrak{F}$ is concerned, the optimised 3PS-DFB laser structures are clearly advantageous. This is not surprising, since the inclusion of several PS along the laser cavity flattens the field distribution. However, it is worth noticing that the optimised asymmetric 3PS-DFB structure reaches higher $\mathfrak{S}$ than those obtained for both the QWS-DFB and the symmetric 3PS-DFB structures.

A threshold analysis for the 3PS-DFB structure has been presented. Nevertheless, even if a structure presents an adequate performance at threshold, an above-threshold analysis is essential in order to assess the rate at which the SHB deteriorates the laser features with the increasing current.

C. Above-Threshold Analysis of the Optimised Structure

The $S(z)$ distribution in the asymmetric 3PS-DFB laser cavity can be observed in Fig. 4, for different biasing currents. It shows the gradual increase of the photon number in the whole structure, due to the stimulated emission. The MPS-DFB lasers include local maxima other than the central one, thus leading to flatter distributions than those obtained with

Fig. 4 – Longitudinal distribution of the photon density in the optimised asymmetric 3PS-DFB laser structure under different biasing currents.

(a)

(b)

Fig. 5 – (a) Selectivity and (b) Flatness vs current injection for the optimised asymmetric 3PS-DFB, the symmetric 3PS-DFB of Ref. [7, p. 128] and the QWS-DFB laser structures, with $L = 500$ μm . In (b), the QWS-DFB laser flatness falls outside the axis limits.

QWS-DFB lasers [7, p. 159]. In the symmetric case of Ref. [7, p. 163] these local maxima coincide with the PS positions. In the optimised asymmetric 3PS-DFB laser the local maxima are displaced to the cavity facet vicinity. When compared to the QWS-DFB laser, both 3PS-DFB lasers show smaller differences between the central photon density and the escaping photon densities at the facets. This difference is especially small in the optimised asymmetric structure. This fact benefits the laser performance as far as the emitted power is concerned, as it shall be seen below.

A comparative analysis between the QWS and the 3PS-DFB lasers is available in Figs. 5–7.

Fig. 5(a) shows the laser mode selectivity versus current injection. Similar mode discriminations at threshold are shown for the QWS-DFB laser and for the asymmetric 3PS-DFB laser. Nevertheless, the mode selectivity has a severe reduction with increasing biasing currents in the QWS case, showing that this laser is strongly affected by the SHB. On the contrary, the asymmetric 3PS-DFB laser is very stable in the high power regime. For the symmetric 3PS-DFB laser, it is apparent that the stability related to flatter photon profiles was obtained at the expense of a great reduction in the mode selectivity at

Fig. 6 – (a) Lasing wavelength and (b) emitted power vs current injection for the optimised asymmetric 3PS-DFB, the symmetric 3PS-DFB of Ref. [7, p. 128] and the QWS-DFB laser structures, with $L = 500 \mu\text{m}$.

threshold (about half the value of the QWS). Concerning the mode gain discrimination, the asymmetric 3PS-DFB structure is, undoubtedly, the best option. Once more, it shall be emphasised that an adequate profile design for the 3PS-DFB structure revealed crucial for the improvement of the laser performance in the high power regime.

Fig. 5(b) focuses on the evolution of the flatness with current injection for the two 3PS-DFB lasers, showing a monotonically decreasing function for both structures. It shall be emphasised that in both cases the flatness lies within the selection criterium commonly referred for lasers with $500 \mu\text{m}$ length long: $\mathfrak{F} < 0.05$ [7, p. 131]. On the contrary, the QWS-DFB laser presents high non-uniformities in the photon profile ($\mathfrak{F} = 0.30$ at threshold, and $\mathfrak{F} = 0.079$ for $I = 5 I_{th}$).

Fig. 6(a) summarises the results obtained for the lasing wavelength versus normalised current injection for the three lasers. The QWS laser shows the larger dynamic change in the lasing wavelength. On the other hand, both 3PS-DFB lasers are associated with more stable outputs, foreseeing their potential use as optical carriers. In the current range $1 \leq I/I_{th} \leq 5$, relative variations ($\Delta\lambda/\lambda_{th}$) in the emitted wavelengths of $9.4 \times 10^{-4} \%$, $1.5 \times 10^{-3} \%$ and $5.5 \times 10^{-3} \%$ are observed

Fig. 7 – (a) Above-threshold normalised spontaneous emission spectra under two different biasing currents and (b) Side-mode suppression ratio vs current injection for the optimised asymmetric 3PS-DFB, the symmetric 3PS-DFB of Ref. [7, p. 128] and the QWS-DFB laser structures, with $L = 500 \mu\text{m}$.

for the asymmetric, the symmetric and the QWS lasers, respectively.

Fig. 6(b) compares the light-current stationary characteristic for the three lasers. Under similar normalised current injections, the asymmetric 3PS-DFB laser shows larger values for the optical power output (measured at the right facet), which establishes higher external differential efficiencies for this laser. As previously referred, this is due to the smaller difference between the central photon density and the escaping photon densities at the facets, for the asymmetric laser structure.

The measurement of the laser spectral characteristics is a way of checking its single mode stability. Fig. 7(a) shows the normalised spontaneous emission power for $I = 1.5 I_{th}$ and $I = 5 I_{th}$, for the asymmetric 3PS-DFB laser. High values of SMSR are achieved, for both bias currents. Besides, it is worth noticing that the "blue-shift" in wavelengths is negligible.

The inset of Fig. 7(a) shows the $\delta - \alpha$ plot for the modes that are allowed to propagate inside the cavity at threshold. This figure points out two positive side-modes that are very close in frequency (encircled by a dashed line), which originates the

broadening of the spectrum around 1546.4 nm. Another relevant aspect lies with the fact that, near 1547.7 nm, the spectral amplitude of the dominant mode remains at a high value with current injection, showing no severe mode competition in the high power regime. This is pin-pointed in Fig. 7(b), where the SMSR of the asymmetric DFB laser is maintained throughout the range of biasing current under analysis. Besides, with the increasing current, the other two lasers show a strong degradation of SMSR, which becomes lower than the required 30 dB for the SLM operation [5, p. 147], over the most part of the current range.

IV. CONCLUSIONS

An asymmetric 3PS-DFB laser structure especially designed to provide single-mode operation has been analysed in the above-threshold regime. A correct optimisation of the PS discontinuities profile at threshold ensures the best mode gain margin with flattened intracavity intensity profile. Based on TMM and on the carrier rate equation, above-threshold numerical simulations have shown that the optimised 3PS-DFB structure achieves very good SLM operation and maintains these characteristics up to high biasing currents. Comparative studies with a similar optimised symmetric 3PS-DFB laser, referred elsewhere in the literature, have shown that the laser proposed in this paper, presents remarkable increases in the mode selectivity (about five times for $I/I_{th} = 5$), in the SMSR (about 9 dB for $I/I_{th} = 5$) and in the lasing wavelength stability (about two times in the overall current range).

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