Sliding mode control of boost converter: Application to energy storage system via supercapacitors
Alaa Hijazi, Michaël Di Loreto, Eric Bideaux, Pascal Venet, Guy Clerc, Gérard Rojat

To cite this version:
Alaa Hijazi, Michaël Di Loreto, Eric Bideaux, Pascal Venet, Guy Clerc, et al.. Sliding mode control of boost converter: Application to energy storage system via supercapacitors. EPE, Sep 2009, Barcelone, Spain. pp.électronique. hal-00475770

HAL Id: hal-00475770
https://hal.archives-ouvertes.fr/hal-00475770
Submitted on 23 Apr 2010

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Sliding Mode Control of Boost Converter: Application to energy storage system via supercapacitors


*AMPERE UMR CNRS 5005
Bât. Omega, Université Lyon 1, Villeurbanne
Université de Lyon, F.69622, Lyon, France
Tel.: 0033 4 72 43 11 92.
Fax: 0033 4 72 43 11 93.
E-Mail: alaa.hijazi@univ-lyon1.fr
URL: http:// www.ampere-lab.fr

**AMPERE UMR CNRS 5005
Bât. Saint Exupéry, Insa de Lyon, Villeurbanne
Université de Lyon, F.69622, Lyon, France
Tel.: 0033 4 72 43 81 98.
Fax: 0033 4 72 43 85 35.
E-Mail: michael.di-loreto@insa-lyon.fr

Acknowledgements
This work was co-supported by “Direction Générale des Entreprises” and “Région Rhone-Alpes” under the contract Hysbus through the competitive pole (Lyon Urban Truck and Bus 2015).

Keywords
«Energy storage», «Robust control», «Supercapacitor».

Abstract
Sliding mode control of boost converter is studied. In order to improve dynamical performances with static and dynamic specifications, we propose a systematic procedure to compute the gains of the controller based on an optimization scheme. This method is applied to the control of an energy storage system based on supercapacitors technology in order to regulate the output voltage. Given a system with large variations of input voltage and load, it is necessary to guarantee good performance of the controller for large variations of operating point. Our study shows the great influence of the controller gains on the static and dynamic performances of the system. Hence, we point out a methodology for choosing the gains.

Introduction
In order to supply voltage, current and frequency needed for the load, and to guarantee the desired dynamics, electronics power converters must be suitably controlled. Conventionally, classical PID controllers are used for the control of power converters [1], [2], [3]. Simple models of converters are usually obtained from signals averaging and linearization techniques, these models may then be used for control design [4], [5].

On the other hand, the PID family of controllers failed to satisfactorily perform constrained specifications under large parameter variations and load disturbances [6]. Another choice for controlling power converter is to use the sliding control techniques. Sliding mode control (SMC) of variable structure systems such as power converters is particularly interesting because of its inherent robustness, its capability of system order reduction, and appropriateness for the nonlinearity aspect of power converters [5], [7], [8]. However, despite being a popular research subject, SMC is still rarely applied in practical DC-DC converters. It is mainly due to the fact that no systematic procedure is available for the design of SMC in practical applications [9]. For example, the influence of the controller gains on the closed loop system performances for a given application is not properly clarified, and most of the previous works are limited to the study of the influence of these parameters only on the existence and stability of sliding mode [10], [11]. In other cases an empirical approach is adopted for selecting these gains of
SMC, computer simulation and experiments were performed to study the effect of the various control gains on the response of the output voltage [12]. Therefore in this paper, we study the design of SMC for boost converters. After studying and analyzing different existing solutions for sliding mode control of boost converter, we propose a control mode that allows a direct control of the voltage of boost converter. The performances of the controller in terms of robustness and dynamic response will be improved. Most of literature works are concerned with the study of hitting, existence and stability conditions of the SMC. Our contribution goes beyond this direction by involving the study of the influence of control parameters on system performances. In this context, we develop an optimization algorithm in order to choose the controller parameters based on a predefined specification for a given real application.

Fig. 1: Architecture of the trolleybus supply system

The application concerned by our study is the supply of an electrical bus via supercapacitors in case of electrical microcuts. The supply system of trolleybus consists of two parallel DC-DC converters (Fig. 1). The first converter (Trolleybus converter) regulates the voltage of the bus $V_b$ to 330 Volts, the other converter (Supercapacitors converter) manages the energy transfer between supercapacitors and trolleybus. When the voltage of trolleybus is more than 350 Volts, the supercapacitors will be charged. When the voltage decreases under 310 Volts, the supercapacitors discharges to insure the continuity of the supply of the auxiliary of trolleybus (pump, air compressor, fans). The supercapacitors voltage must be always between 120 and 300 Volts. In this paper, we will focus on the discharge phase of supercapacitors. As the voltage of trolleybus is greater than the voltage of supercapacitors, our study is limited to the study of the control of boost converter.

In section II, we will present the mathematical model of boost converter. After that we will analyze the design of SMC for boost converter and then we study the influence of the choice of controller parameters on the system performances. This analysis requires an optimization algorithm to compute the parameters.

In section III, the importance of application of SMC for the alimentation of electrical bus is illustrated.

**Sliding mode control for boost converter**

**Mathematical Model of Boost Converter**

Fig. 2: Boost converter circuit
Fig. 2 shows the circuit of boost converter. It consists of a DC input voltage $E$ which represents the voltage of the supercapacitors, a smoothing inductor $L$, a controlled switch $S$, a freewheeling diode $D$, a filter capacitor $C$ and finally the load which is modeled by a resistor $R$. Assuming that the circuit is in continuous conduction mode, which means that the inductors current never falls to zero, the mathematical model of boost converter can now be easily deduced by applying Kirchhoff’s laws. The model of boost converter in continuous conduction mode is:

$$\begin{align*}
\frac{dV_b}{dt} &= (1-u)i_L - \dot{I}_{load}, \\
\frac{di_L}{dt} &= E - (1-u)V_b,
\end{align*}$$  \hspace{1cm} (1)

where $u$ is the switch state or the switch duty cycle in the case of average model, $V_b$ and $i_L$ are respectively the output voltage and the inductor current of the boost converter. In SMC, we usually determine $u$ as following:

$$u = \begin{cases} 
1 & \text{if } S < 0, \\
0 & \text{if } S > 0,
\end{cases}$$  \hspace{1cm} (2)

Where $S$ is the sliding surface.

**Design of sliding mode controller**

The objective of boost converter control is to regulate the output voltage $V_b$ to a reference voltage $V_{ref}$. The design of sliding mode controller for boost converter starts with the choice of sliding surface. As it is shown in [13], it is clear that direct surface $V_b - V_{ref}$ can be tend to zero only if the current increases continuously. Usually, a cascade control structure is applied to control boost converter, which leads to solve the control problem using two control loops [14], [15], [16]: The output voltage loop generates the reference current from voltage error and the inner current loop controls inductor current via sliding mode (Fig. 3).

This control of the output voltage of DC-DC converter meets the criteria of stability and existence of sliding mode. However, it has been shown in [17] that it is difficult to determine the gains of the voltage loop since sliding mode is a highly nonlinear method. Furthermore, since SMC is only applied to current regulation, the voltage loop will be more sensitive to high frequencies phenomena and to uncertainties on the reference current. In order to improve the performances of the controller, we propose to study a control mode based on a sliding surface which involves output voltage. Let $(V_{ref}, I_{ref} = \frac{V_{ref}^2}{R})$ be the desired equilibrium point, we take the following surface:

$$S = K_1(V_b - V_{ref}) + K_2(i_L - I_{ref}),$$  \hspace{1cm} (3)

Where $K_1$ and $K_2 \in R^+$. As reference current depends on the operating point, we will extract it, that is:

$$I_{ref} = \frac{V_{ref} \dot{I}_{load}}{E},$$  \hspace{1cm} (4)

where $\dot{I}_{load} = \frac{V_b}{R}$. 

---

**Fig. 3: Cascade control structure**
Sliding surface coefficients \((K_1, K_2)\) must be chosen to ensure that the sliding mode exists at least around the desired equilibrium point, and the dynamics of the system will reach the surface and lead toward the equilibrium point.

**Existence condition:**
The existence condition of sliding mode implies that both \(S\) and \(\dot{S}\) will tend to zero when \(t\) tend to infinity, which means that the dynamic of the system will stay into the sliding surface. The existence condition of the sliding mode is \(SS < 0\) (when \(S \to 0\)), the fulfillment of this inequality ensures the existence of sliding mode around the commutation surface.

Let us write the model of boost converter (1) in a state space where the equilibrium point is the origin. We obtain:

\[
\begin{align*}
\frac{dx_1}{dt} &= (1 - u)(x_2 + I_{ref}^e) - \left(\frac{x_1 + V_{ref}^e}{R}\right), \\
\frac{dx_2}{dt} &= E - (1 - u)(x_1 + V_{ref}^e),
\end{align*}
\]

where \(x_1 = V_b - V_{ref}^e\) and \(x_2 = i_L - I_{ref}^e\).

Replacing \(I_{ref}\) by its value from (4) in the expression of the commutation surface (3), we obtain:

\[
S = K_1(V_b - V_{ref}^e) + K_2i_L - \frac{K_2V_{ref}^eV_b}{RE},
\]

\[
= (K_1 - \frac{K_2V_{ref}^e}{RE})(V_b) + K_2i_L - K_1 V_{ref}^e. \tag{6}
\]

Writing this last equation in the new coordinate system \((x_1, x_2)\), we have:

\[
S = (K_1 - \frac{K_2V_{ref}^e}{RE})(x_1) + (K_1 - \frac{K_2V_{ref}^e}{RE})(V_{ref}^e) + K_2x_2 + K_2I_{ref}^e - K_1 V_{ref}^e,
\]

\[
= K'_1x_1 + K_2x_2, \tag{7}
\]

Where \(K'_1 = K_1 - \frac{K_2V_{ref}^e}{RE}\).

By calculating \(SS\) to be negative for \(u=0\) and \(1\), we can deduce that the sliding region is limited by the following inequalities:

\[
\begin{align*}
\begin{cases}
-x_1 \left[\frac{-K'_1}{RC} - \frac{K_2}{L}\right] + x_2 \left[\frac{K'_1}{C} + K_2 \left(\frac{E}{L} - \frac{V_{ref}^e}{L}\right)\right] - K'_1 \left[\frac{V_{ref}^e}{RC} - \frac{I_{ref}^e}{C}\right] < 0, \\
-x_1 \left[\frac{-K'_1}{RC} + \frac{K_2E}{L} - \frac{V_{ref}^eK'_1}{RC}\right] > 0.
\end{cases}
\end{align*}
\]

\[\tag{8}
\]

From these two inequalities and in order to ensure that the sliding mode exists at least around the equilibrium point \((x_1 = 0, x_2 = 0)\), the following condition must be satisfied:

\[
\frac{K'_1}{RC} < \frac{V_{ref}^e}{i_{ref}^eL}. \tag{9}
\]

**Stability condition:**
The stability of the system is guaranteed if the dynamic of the system in sliding regime is directed toward the desired equilibrium point. Our aim is to determine the dynamic of the new variable state \(x_1\) and \(x_2\) when the sliding regime is reached. Given the state space model (5), the commutation surface (6), and, from \(\dot{S}=0\), the equivalent average control that must be applied to the system in order that the system state slides along the surface is given by:
\[ u_{eq} = 1 - \frac{E}{L} - \frac{K_1'(x_1 + V_{ref}^e)}{K_2RC} \frac{x_1 + V_{ref}^e}{K_1'(x_1 + I_{ref}^e)} . \]  (10)

By replacing the equivalent control (10) in the state space model (5), and from \( S = 0 \), we deduce the dynamic of \( x_1 \) at the sliding regime:

\[ \frac{dx_1}{dt} = \frac{E \left( I_{ref}^e - \frac{x_1 K_1'}{K_2} \right) - \left( x_1 + V_{ref}^e \right)^2}{C(V_{ref}^e + x_1) - L \left( I_{ref}^e - \frac{K_1'}{K_2} x_1 \right) \frac{K_1'}{K_2}} . \]  (11)

Let \( V = \frac{x_1^2}{2} \) be a lyapunov candidate function, we have \( \dot{V} = x_1 \dot{x}_1 \) so that:

\[ \dot{V} = -x_1^2 \frac{E K_1' + V_{ref}^e + V_0}{C(V_{ref}^e + x_1) - L \left( I_{ref}^e - \frac{K_1'}{K_2} x_1 \right) \frac{K_1'}{K_2}} . \]  (12)

The condition for \( \dot{V} \) to be negative is:

\[ C(V_{ref}^e + x_1) - L \left( I_{ref}^e - \frac{K_1'}{K_2} x_1 \right) \frac{K_1'}{K_2} > 0 \Rightarrow x_1 > \frac{L V_{ref}^e}{R E} \left( \frac{K_1'}{K_2} \right) - C V_{ref}^e . \]  (13)

Taking into account the sliding region defined by (8) and the existence condition (9), we can demonstrate that the condition (13) is always satisfied along the sliding region of the commutation surface. Based on the stability theorem in the sense of Lyapunov we can say that the system is globally asymptotically stable.

**Influence of controller parameters on system performance**

Clearly, inequality (9) provides only the general information about the existence of sliding mode. On the other hand, choosing controller parameters has a significant influence on the performances of closed-loop system especially when the system presents large variations around nominal value like our system. The first criteria for choosing these parameters must be the size of sliding part of commutation surface. In fact, sliding condition is only satisfied on a subpart of the surface and not on the entire surface as shown in (8). In consequence, the controller parameters must be carefully chosen to ensure that the system dynamic will intercept the commutation surface in the sliding part. In this way we prevent undesirable behavior which is reflected by a response overshoot (Fig. 4).

![Fig. 4: Sliding mode with (a) and without overshoot (b).](image-url)
Let \( A(x_{1A}, x_{2A}) \), be the intersection of the system dynamics with the commutation surface, and let \( B(x_{1B}, x_{2B}) \) be the limit of sliding part as shown in (Fig. 4). Supposing that at \( t = 0 \) the output voltage is equal \( V_{ini} \) and the current in the inductor is zero. As at \( t = 0 \) the surface \( S \) will be negative, then \( u = 1 \) so that the state space vector will be:

\[
\begin{align*}
C \frac{dx_1}{dt} &= -\left( x_1 + \frac{V_{ref}^e}{R} \right), \\
L \frac{dx_2}{dt} &= E. 
\end{align*}
\]  
(14)

By solving these two equalities taking into account initial conditions, we obtain:

\[ x_1 = V_{ini} e^{-\frac{L(x_2 + x_{ref}^e)}{RCE}} - V_{ref}^e. \] 
(15)

This equation represents the dynamic of the system before intercepting the surface at the point A. From \( S = 0 \) and from (14) we deduce the coordinate of point A:

\[ x_{2A} = \frac{K_1'}{K_2} (-V_{ini} e^{-\frac{L(x_{2A} + x_{ref}^e)}{RCE}} + V_{ref}^e). \] 
(16)

Solving this last equation we obtain:

\[
\frac{\text{lambertw}}{\text{REL}} \left( -\frac{L V_{ini} \left( \frac{K_1'}{K_2} R E - V_{ref}^e \right) e^{-\frac{L K_1' V_{ref}^e}{RCE}}}{R^2 E^2 C} \right) CE^2 R^2 + L \frac{K_1'}{K_2} R E V_{ref}^e - L V_{ref}^e^2 \]
\[ x_{2A} = \frac{\text{lambertw}}{\text{REL}} \] 
(17)

\( \text{lambertw} \) is the lambert function defined for a complex number \( Z \) by the inverse function of \( f(Z) \ast \exp(f(Z)) = Z \).

From the equation of the commutation surface (7) and the limit of sliding zone defined by (8), we deduce the coordinate of point B, we have:

\[ x_{2B} = \frac{K_1' V_{ref}^e}{R C} - \frac{E}{L} - \frac{V_{ref}^e}{L} \left( \frac{K_1'}{K_2} R E + \frac{1}{C} \right). \] 
(18)

We can now deduce that the system intercepts the commutation surface in the right part if the controller parameters are selected in such a way that:

\[ ||OA|| < ||OB||. \]  
(19)

On the other hand, theoretical approach of sliding mode supposes that the hysteresis band is null, therefore frequency tends to infinity. It is clear that we cannot keep this assumption due to frequency limitation caused by characteristic of circuit components and losses. Usually, an hysteresis band \( \Delta \) is added around the surface in order to fix the operating frequency, looking at Fig. 5 we can calculate the rise time "\( t_1 \)" and fall time "\( t_2 \)"

\[ t_1 = \frac{2\Delta}{S^+} \quad t_2 = \frac{2\Delta}{S^-} \quad f = \frac{1}{t_1 + t_2} \] 
(20)
Fig. 5: Sliding Regime with Hysteresis Band

From equations (7) and (20), we deduce the expression of the hysteresis band function of the surface parameters, the operating point and the circuit parameters, we obtain:

\[
\Delta = \frac{1}{2f} \left( \frac{1}{K_2 E - \frac{V_{ref}^e}{RC}} + \frac{1}{K_1' \left( \frac{V_{ref}^e}{REC} - \frac{V_{ref}^e}{RC} \right) + K_2 \left( \frac{E}{L} - \frac{V_{ref}^e}{L} \right)} \right).
\]

(21)

Hence, from the expression of \( \Delta \) function of \( K_1 \) and \( K_2 \), a restriction on the choice of the controller parameters holds. In fact, values of these parameters must guarantee that the hysteresis band is greater than the perturbation generated by the converter, and in the same time, the value of the hysteresis band should be limited to guarantee the robustness.

Furthermore, as our objective is to control the output voltage and as we can see in (3) the expression of commutation surface depends on the current errors, so a major study for optimization of controller gains should be the analysis of the sensitivity of the controller facing a measurement or an estimation error of current reference. In fact, since the current reference is unknown, we are constrained to extract it from the load current (4). However, this latter can be measured or observed through an extended Luenberger observer. In both cases, an error can be occurred which affects the response of the system in terms of steady state error. So the choice of controller parameters must take into account to make the closed loop system less sensitive to an error in the current part of the surface.

Supposing that the measured reference current (\( I_{ref,mes} \)) can be expressed as the sum of the expression of reference current given by (4) and an error term (\( e I_{ref} \)), where \( e \) is defined as the error percentage, so that:

\[
I_{ref,mes} = I_{ref} - e I_{ref}
\]

(22)

By replacing this expression of \( I_{ref,mes} \) in the expression of the commutation surface (3), we have:

\[
S = K_1' (V_b - V_{ref}^e) + K_2 (i_L - I_{ref}^e) + \frac{K_2 e V_{ref}^e \text{load}}{E},
\]

\[
= K_1' (V_b - V_{ref}^e) + K_2 (i_L - I_{ref}^e) + \frac{K_2 e V_{ref}^e V_b}{ER},
\]

\[
= K_1 (V_b - V_{ref}^e) + K_2 (i_L - I_{ref}^e) + \frac{K_2 e V_{ref}^e (V_b - V_{ref}^e)}{ER} + \frac{K_2 V_{ref}^{e^2}}{RE},
\]

(23)

\[
= \left( K_1' + \frac{K_2 e V_{ref}^e}{ER} \right) (V_b - V_{ref}^e) + K_2 (i_L - I_{ref}^e) + \frac{K_2 V_{ref}^{e^2}}{RE}.
\]
The term \( \left( \frac{K_2 eV_{\text{ref}}^e}{R} \right)^2 \) is constant so we can say that:

\[
S = \left( K_1' + \frac{K_2 eV_{\text{ref}}^e}{R} \right) (V_b - (V_{\text{ref}}^e + \Delta V_b)) + K_2 (i_L - (I_{\text{ref}}^e + \Delta i_L)).
\]

Where \( \Delta V_b \) and \( \Delta i_L \) represents the voltage and the current steady state errors. We have:

\[
(K_1' + \frac{K_2 eV_{\text{ref}}^e}{R}) (\Delta V_b) + K_2 (\Delta i_L) = -\frac{K_2 eV_{\text{ref}}^e}{R}. \tag{24}
\]

The control given by (2) will tend \( S \) given by (23) to zero, so that \( V_b \to V_{\text{ref}}^e + \Delta V_b \) and \( i_L \to I_{\text{ref}}^e + \Delta i_L \). From the equality of power input-output we can say that:

\[
(V_{\text{ref}}^e + \Delta V_b) \ast i_{\text{ch}} = E \ast (I_{\text{ref}}^e + \Delta i_L). \tag{25}
\]

From equation (24) and (25), we determine the expression of \( \Delta V_b \):

\[
\Delta V_b = \frac{RE}{2} \left( \frac{K_1}{K_2} - \frac{V_{\text{ref}}^e}{RE} - \frac{eV_{\text{ref}}^e}{RE} + \left( \frac{K_1}{K_2} + \frac{V_{\text{ref}}^e}{RE} + \frac{eV_{\text{ref}}^e}{RE} \right)^2 - \frac{4eV_{\text{ref}}^e}{R^2 E^2} \right). \tag{26}
\]

**Simulations:**

Simulations were performed on a typical boost converter circuit. The input voltage of the boost E is given by 120 supercapacitors (3000 F) and can varies between 120 and 300 volts. The charge power is between 20 and 50 kW. The computer aided design software "Simplorer V7.0" is used for the simulation. The parameters of controllers are given by a genetic algorithm. The criteria are based on industrial specifications. Thus the objective of the genetic algorithm is to determine the values of parameters that ensure, regardless of the operating point, that the system will intercept the sliding part of commutation surface while respecting the following conditions:

\[
\begin{align*}
\frac{K_1}{K_2} &< \min \left( \frac{RCE}{V_{\text{ref}}^e + V_{\text{ref}}^L} \right) \quad \text{(stability and existence condition),} \\
\Delta_{\text{min}} &< \max (\Delta (K_1, K_2)) < \Delta_{\text{max}}, \\
\max (\Delta V_b) &< \Delta V_{\text{limax}} \quad \text{for } e = e_{\text{max}}.
\end{align*}
\]

Figure 6 shows that the change in supercapacitor's voltage (E) has no influence on output voltage \( (V_b) \) so the controller is absolutely robust with respect to supercapacitor’s voltage variation. On the other hand, Figure 7 shows the recovering features of the proposed controller to the imposed load. The load has been changed by various steps in the range \([2\Omega, 4\Omega]\). As expected, the sliding mode controller is robust when the load resistance is subject to a sudden variation. Figure (7.a) and (7.b) shows that dynamic response of output voltage and inductor current has an excellent behavior with non oscillation and that is independent of the operating point. The proposed controller is therefore very appropriate for systems which present large variations around its nominal operating point as our concerned system. The shape of the output voltage shows fast recovery time, which is a very important factor in application where the supercapacitors are used as an auxiliary supply system.

Figure 8 presents the startup phase plane trajectories of sliding mode control for different operating points. As mentioned before, the boost converter will regulate the output voltage to 330 volts when the voltage of the bus decreases below 310 volts in the case of electrical microcuts. As it is shown, the behavior of phase plane trajectories is similar. Looking to figure 7 and 8, it is important to note that the system is directed toward the sliding part of the commutation surface, which validates the proposed algorithm for choosing the parameters of the
controller. Unlike classical control via PID, values of parameters are not determined to satisfy desired performance only around nominal values but to cover large variations in the operating condition.

Fig. 6: Output voltage with change of supercapacitor’s voltage.

Fig. 7: Output voltage and inductor current waveforms for step load variations with different initial voltage of supercapacitors ((a) => E=300 V, (b) => E=150 V).
Fig. 8: SLMC phase plane trajectories for startup according to different levels of load and super capacitor voltages.

Conclusion

A new sliding mode control design of boost converter was developed in this paper. A control mode that improves the performances of the system was proposed. The influence of the control parameters on the performances of the system was studied. An optimization algorithm was developed in order to calculate the optimal values of parameters based on a predefined specification. The importance of the application of this type of controllers for the supply of electrical bus via supercapacitors is highlighted. Results show excellent dynamic response of controller and robustness to load and input voltage with large variations around nominal values. Further research may focus to the study of the observation of load current, and on the stability of the controller-observer closed loop.

References