Non-linear characteristics of single reed instruments : quasi-static volume flow and reed opening measurements.

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1. Introduction

Sound production in reed wind instruments is the result of self-sustained oscillations. A mechanical oscillator, the reed, acts as a valve which modulates the air flow entering into the mouthpiece. The destabilisation of the mechanical element is the result of a complex aeroelastic coupling between the reed, the air flow into the instrument driven by the mouth pressure of the musician, and the resonant acoustic field in the instrument itself. Following McIntyre et al. (1983), wind instruments can be described in terms of lumped model formed by a closed feedback loop operating as a self-sustained oscillator. In their model the loop is made up with two elements, a lumped non-linear element - the mouthpiece - and a linear passive element - the resonator, that is the instrument itself. The modelling and the measurement of the resonator has been extensively studied since Bouasse (1929) by many authors (see for example references in Nederveen, 1998, or Fletcher and Rossing, 1998). On the contrary, the non-linear element has only recently been the subject of thorough studies (see for example the review given by Hirschberg, 1995) and the knowledge of this elements action is considerably less than that of the resonator. The non-linear element can be defined by a relationship between the pressure difference across the reed and the volume flow at the inlet of the pipe of the instrument. Assuming a quasi-static response of the reed which neglects inertia and damping, the volume flow is thus an explicit function of the pressure difference. This function is called the characteristics of the reed. Most authors converge on an elementary model of the characteristic presented in section 2.1 (Wilson and Beavers, 1974 ; Fletcher, 1979 ; Fletcher 1993 ; Saneyoshi et al., 1987 ; Kergomard 1995 ; Kergomard et al.

2000 ; Ollivier et al. 2002). It is based on a quasi-stationary model of the air flow through the mouthpiece, and on a mechanical model of the reed discussed respectively in sections 2.2 and 2.3. The aim of the present paper is to obtain experimental data for the non-linear element in order to check the validity of the elementary model and to find realistic values of the parameters useful for physical modelling synthesis. The data may also give information helpful for the understanding of the physical phenomena involved.

In the present paper, a method for measuring the characteristics is proposed, the measurements being done in quasi-stationary conditions. The dynamic aspects related to the reed are not considered in the present paper. These aspects are certainly not negligible with regard to the spectrum of the instrument (Thomson, 1979). However when the playing frequency is small compared to the reed resonance frequency, it has been shown that these aspects are not essential from the strict point of view of the auto-oscillation (Wilson and Beavers, 1974 ; Dalmont et al., 1995). For the experiments an artificial blowing machine is used (section 3.1). To perform the volume flux measurements, a constriction (diaphragm) is used as a pressure reducing element for a differential pressure flowmeter (section 3.2). The diaphragm takes the place of the resonator. The major advantage of such a device is that the diaphragm also plays the role of a non-linear absorber (Ingard and Ising, 1967) which thwarts a possible standing wave in the mouthpiece. Oscillations of the reed are thus in most cases impeded and avoided if the diaphragm is well chosen. This makes it possible the measurement of the complete characteristics of real clarinet mouthpieces. The choice and dimensions of the diaphragm which depends on the reed properties are discussed section 3.3. The diaphragm does not induce additional sensors for the pressure sensors used other than those commonly present in an artificial mouth. In addition to the flow measurement, the opening of the reed is measured optically. Then the measurements of the reed opening and of the pressure difference across the reed gives information on the evolution of the reed stiffness which is important for

the understanding of the quasi-static reed mechanics. For obtaining relevant results, an accurate calibration of the sensors has to be done. Thus a specific experimental procedure has been developed which is discussed in section 3.5. Some typical experimental results are presented section 4.1. Section 4.2 and 4.3 are focused on two aspects of the mechanics of the reed, viscoelasticity and stiffness respectively. Section 4.4 is dedicated to the flow aspects. Finally in section 4.5 typical values of the parameters of the model are summarized and the accuracy of their determination is discussed.

2. State of art

2.1. Elementary model

Backus (1963) has presented the first measurements of the characteristics of a single reed instrument under steady flow conditions. The main result of Backus is a non-linear expression relating the volume flow U through the reed and two variables : the pressure difference ΔP across the reed, and the opening H between the tip of the reed and the mouthpiece. The pressure difference ΔP across the reed is equal to the mouth pressure P_m minus the pressure in the mouthpiece P_{in} . Backus fitted the experimental data by means of an expression in which U is proportional to $(\Delta P)^{4/3}$ and to $H^{2/3}$. There are good arguments to abort Backus formula (Hirschberg et al., 1990, Hirschberg, 1995) and most flow models are now based on the stationary Bernoulli equation (Wilson and Beavers, 1974 ; Fletcher, 1979 ; Saneyoshi et al., 1987 ; Fletcher 1993) which states that the volume flux U in the reed channel is given by :

$$U = wH \sqrt{\frac{2\Delta P}{\rho}}, \quad (1)$$

where w is the width of the reed channel, and ρ is the air density.

The steps leading to equation (1) are summarised by Hirschberg (1995). A reasonable prediction of the flow through the reed channel is obtained by assuming flow separation at the neck of the flow channel (figure 1a). For a uniform reed channel of height H the pressure in the reed channel is equal to the pressure in the mouthpiece. Assuming a turbulent dissipation of the kinetic energy in the jet without pressure recovery and neglecting friction in the reed channel we find, from Bernoulli equation, the velocity v_b :

$$v_b = \sqrt{\frac{2\Delta P}{\rho}} . \quad (2)$$

Multiplying v_b by the jet cross section S_j provides a prediction for the volume flow entering into the mouthpiece. If there are rounded corners at the inlet of the reed channel as in figure 1b for example), it is furthermore assumed that the jet cross section S_j is equal to wH , where w is independent of H . The motion of the reed determines the opening cross-section area and controls the volume flow entering in the mouthpiece.

FIG. 1. (a) Flow control by the clarinet reed involving free jet formation and turbulent dissipation. (b) A two dimensional model of the reed channel geometry and expected flow.

Assuming the mechanical response of the reed to be reduced to its stiffness (see section 2.3 for discussion) the variation of the reed opening $y = H_0 - H$, where H is the reed opening and H_0 the reed opening in the absence of flow, is proportional to the pressure difference ΔP across the reed :

$$y = \frac{\Delta P}{k}, \quad (3)$$

where k is a stiffness per unit area. Equation (3) is meaningful until H is equal zero (reed blocked on the lay). This corresponds to a limit value P_M of the pressure difference ΔP given by :

$$P_M = k H_0. \quad (4)$$

If the pressure difference is larger than P_M , the reed closes the opening and no flow enters into the mouthpiece. Finally the volume flow U can be written as a function of the pressure difference ΔP :

$$\begin{cases} U = w(H_o - \frac{\Delta P}{k}) \sqrt{\frac{2\Delta P}{\rho}} = wH_o(1 - \frac{\Delta P}{P_M}) \sqrt{\frac{2\Delta P}{\rho}} & \text{if } \Delta P \leq P_M \\ U = 0 & \text{if } \Delta P \geq P_M \end{cases} \quad (5)$$

This non-linear characteristics is displayed figure 2. Notice that there is a strong localised non-linearity in the characteristic model around the particular value of the pressure difference

$\Delta P = P_M$. The maximum value of the flow $U_{\max} = \frac{2}{3} wH_o \sqrt{\frac{2P_M}{3\rho}}$ is obtained for $\Delta P = P_M / 3$

which is just below the threshold of oscillation (Kergomard et al., 2000).

FIG. 2. Theoretical characteristic (equation 5) : volume flux U as a function of the pressure difference ΔP (arbitrary scales). P_M is the value of the pressure difference corresponding to the reed blocked on the mouthpiece.

2.2. Quasi-stationary model of air flow

Hirschberg et al. (1990) and Van Zon et al. (1990) have studied experimentally and theoretically the volume flow control by the motion of the reed in order to explore the limits of validity of the elementary model presented above, and to provide a better understanding of the results of Backus (1963).

Following Van Zon et al. (1990) due to the abrupt transition from the narrow reed channel of height H to the inner part of the mouthpiece of diameter D , flow separation occurs for sufficient high Reynolds numbers $Re = \rho U / (\mu_w) > 10$ (μ is the dynamic viscosity of air). A free jet is formed in the mouthpiece. For large values of $D/H > 10$, which is a typical value for single reed instruments, the pressure recovery upon deceleration of the flow in the mouthpiece is negligible. Hence the pressure P_{in} in the mouthpiece is assumed uniform and equal to the pressure at the end of the reed channel. Measurements by Van Zon et al. (1990) confirm this assumption : the pressure variations within the mouthpiece are less than 3% of the dynamic pressure in the jet.

In the limit of high Reynolds numbers and a short channel ($Re.H/L > 1000$ where L is the length of the reed channel) the volume flow can be estimated by assuming a uniform flow in the reed channel and by applying Bernoulli's equation. Ignoring the flow separation at the entrance of the channel, the volume flow is found to be given by equation 1 (elementary model). As noted in Hirschberg et al. (1990) separation occurs when the edges at the entrance are sharp, which is the case for clarinet mouthpiece and its reed. A free jet with a section S_j lower than the reed-mouthpiece opening cross-section S will be formed within in the channel. For short reed channels ($L/H < 3$) no reattachment of the flow occurs within the channel and the volume flow U will be given by :

$$U = \alpha w H \sqrt{\frac{2\Delta P}{\rho}} \text{ where } \alpha = \frac{S_j}{S} . \quad (6)$$

The coefficient α is a “contraction” coefficient which is strongly dependent on the geometry of the reed channel inlet. For typical 2D mouthpiece geometry, values in the range $0.5 < \alpha < 0.61$ are expected. Laser Doppler flow measurements and flow visualisation experiments by Van Zon (1989) confirm the typical values of α for the geometry considered. For low Reynolds number ($Re < 10$) and long reed channel ($L/H > 10$), the flow is well

approximated by a fully developed Poiseuille flow. This corresponds to the case of the reed almost closed for which the volume flow is thus given by :

$$U = \frac{wH^3\Delta P}{12\rho\nu L}. \quad (7)$$

Both Poiseuille and Bernoulli limits were also found by Gilbert (1991) and Maurin (1992). The intermediate flows between the two extreme cases mentioned above are discussed in Van Zon et al. (1990) and Hirschberg et al. (1991). For $L/H > 4$, the jet formed by the separation of the flow from the sharp edge of the reed at the entrance of the reed channel reattaches to the wall after a distance of about $2H$. If the channel is short the friction is negligible, the volume flow U approaches the value given by equation (1) then the section S_j of the jet is equal to the reed-mouthpiece opening cross-section S . The quasi-stationary models described above assume a fixed separation point at the inlet or at the exit of the reed channel and a uniform section of the reed channel. This hypothesis is questionable in the case of the clarinet mouthpiece. The transition between the “reed channel” and the mouthpiece can be smooth. In such a case the reed channel height is not uniform, and for $L/H > 4$ the separation point is not easy to determine. As a consequence the coefficient α is not so easy to predict precisely and can be equal to values larger than 1. In other words, in such a case the volume flux U can be larger than the one predicted with the elementary model (equation 1). In the case of a fully separated jet flow (short reed channels situation, $L/H < 2$), the channel geometry is not expected to be critical and the result given in equation (6) could remain valid. Notice that all the theoretical results described before have been successfully compared with experimental results obtained with a two-dimensional mouthpiece geometry. Another particularity of clarinet mouthpieces hasn't been yet mentioned. The reed channel consists of two parts : a) the front slit delimited by the edge of the mouthpiece tip and the reed and b) the lateral slits between the lay and the reed. Then the effective section S_j can be larger than the opening

cross-section S . In such a case the contraction coefficient α defined by equation 6 can be larger than one if w is always defined as the width of the tip of the mouthpiece.

2.3. Quasi-static response of the reed

The mechanics of the reed is complex. The material is orthotropic and the dimension irregular. Prediction of the deformation of the reed is difficult because reed is an essentially inhomogeneous material (Heinrich, 1991). Its mechanical properties are variable and also depend strongly on the amount of water in the material (Heinrich, 1991 ; Obataya and Norimoto, 1999). Marandas et al. (1994) suggest that a dry reed displays a viscoelastic behaviour whereas a wet reed has a viscoplastic behaviour. The reed rolls up (or not) on the lay of the mouthpiece whose geometry is said to be very critical : from the experience of craftsmen it seems that variations of some hundredths of millimeters on dimensions of the curvature of the lay lead to change of behaviour perceptible by the musician (Hirschberg et al., 1991). Finally the lips of the musician are pressed on the reed. This means that the mechanical behaviour of the reed is also dependent on the coupling with the lip, a mechanical system which is also not easy to characterise.

The reed is usually considered to be a one degree of freedom oscillator, that is the reed tip displacement y is related to the pressure difference ΔP by the following equation :

$$\ddot{y} + g\dot{y} + \omega_r^2 y = \frac{\Delta P}{\mu}, \quad (8)$$

where ω_r is the angular resonance frequency of the reed, g a viscous-damping coefficient and μ is a mass per area. The dynamic aspects related to the reed are not considered in the present paper and equation (8) is then reduced to equation (3) with

$$k = \mu\omega_r^2. \quad (9).$$

The determination of the reed stiffness k is difficult and only orders of magnitude for these parameters can be found in the literature. The difficulty lies in the fact that these parameters are generally found only in an indirect way and under experimental conditions which are not always realistic. Thus, equation (9) suggests that the stiffness can be deduced from the reed resonance frequency. In fact this is questionable because the surface per area μ is itself badly known. In addition the validity of the parameter obtained from the resonance frequency for low frequencies is also questionable. The stiffness per area was measured by Nederveen (1998) but not in a playing situation. Another solution for determining this stiffness k consists in measuring the impedance of reed (Dalmont et al., 1995 ; Boutillon and Gibiat, 1996). Unfortunately the stiffness per area is obtained by using an equivalent surface whose value is badly known. Moreover, contrary to what one could suppose, the resonance frequency of the reed itself is difficult to determine. Thus Facchinetti and Boutillon (2001) showed that the frequency of a squeak depends as much on the resonator as on the reed. A simple method to determine the stiffness consists in determining the beating pressure P_M and the opening at rest H_0 . The beating pressure can be estimated with an artificial mouth by seeking the pressure for which the reed starts to oscillate after having been plated. This method, being based on the equation (4) is used implicitly by Kergomard (1995). In the present paper another method allowing an accurate determination of the beating pressure is given.

In the elementary model presented the reed stiffness k is assumed to be constant. A priori it seems natural to think they are not. The most commonly admitted idea is that the vibrating length of the reed decreases with the opening. This would lead to an increase in the stiffness (Nederveen, 1998). To go further in the analysis various authors propose to model the reed as a bar (Stewart and Strong, 1980 ; Sommerfeld and Strong, 1988 ; Stuifmeell, 1989 ; Gazengel, 1994 ; Ducasse, 2001 ; Van Walstijn, 2002). Their results tend to show (Gazengel, 1994 ; Ducasse, 2001 ; Van Walstijn, 2002) that the reed rolls up on the table of

the mouthpiece only under certain conditions which are satisfied for only very particular reed geometries. With the dimensions of a real reed and table Ducasse (2001) showed that the reed deforms without sticking to the table until a given point near the tip touches the table. This result is confirmed by van Walstijn (2002) who shows that the stiffness is nearly constant as long as the end of the reed does not touch the table and takes a larger value afterwards. These studies would justify the "simplistic" approach of the model suggested (equation 5) at least until the reed tip touches the lay. It remain to verify whether the transverse bending of the reed which is supported only on the side by the table does not modify this two dimensional behaviour.

FIG. 3. Theoretical characteristics (equation 5) in the case of a discontinuous reed stiffness (according to Van Walstijn (2002), see text).

3. Experimental device and procedure

3.1 The artificial mouth

The artificial mouth consists of a Plexiglas box with metal reinforcement (Gazengel, 1994). The artificial lip consists of a cylindrical latex balloon of small diameter (10mm) in which a piece of foam saturated with water is inserted. The lip is fixed on a rigid support which position can be translated vertically by means of a screw. The mouthpiece is inserted in a metal barrel whose horizontal position can be adjusted. Resonators can be fixed onto the other end of the barrel. The air is supplied by a high pressure system through a sonic valve.

The pressure P_m in the mouth is measured by a static pressure sensor. A miniature differential pressure sensor mounted in the wall of the mouthpiece measures the pressure difference $\Delta P = P_m - P_{in}$ between the mouth cavity and the inside of the mouthpiece. The

reed slit opening is measured using a LASER beam and a photoelectric diode. This is the method used by Backus (1963) (see figure 4).

FIG. 4. Experimental device.

3.2 Flow measurement

To determine the static non-linear characteristics the clarinet is replaced by a diaphragm playing the role of a pressure reducing element. Using the pressure measurements, the flow through the reed is calculated by using Bernoulli's equation :

$$P_{in} = \frac{1}{2} \rho \frac{U^2}{S_{dia}^2} \quad (10)$$

where the atmospheric pressure is used as a reference, S_{dia} being the section of the opening of the diaphragm.

Compared to another flowmeter placed upstream of the cavity, the diaphragm, apart from its simplicity of implementation, has several advantages. It makes it possible, if its dimensions are well chosen, to obtain a complete characteristic since it avoid oscillations. Indeed apart from its pressure reducing effect the diaphragm plays, for acoustics, the role of a non-linear resistance thus preventing the appearance of a standing wave inside the barrel. This makes our experiment similar to the one suggested by Benade (1976, page 437).

3.3 The choice of the diaphragm

The diameter of the diaphragm is the result of a compromise. If this diameter is too large the pressure drop P_{in} created by the diaphragm is too small to be measured. If, on the contrary, it is too narrow the reed closes suddenly above a critical threshold and part of the

non-linear characteristic cannot be explored (Hirschberg, 1995). This phenomenon occurs when :

$$\frac{\partial(P_m)}{\partial U} = \frac{\partial(\Delta P + P_{in})}{\partial U} = 0 . \quad (11)$$

Considering that the pressure ΔP and P_{in} are given respectively by equations (5) and (10), it is thus necessary, to avoid phenomenon of sudden closure, that :

$$S_{dia} > wH_0 / \sqrt{3} . \quad (12)$$

In practice the section of the opening of the diaphragm is chosen to be close to the section of the reed slit at rest position. This might avoid the sudden closing of the reed while ensuring a sufficiently large pressure drop.

The cross section area of the jet formed by the diaphragm is assumed here to be equal to the section of the opening of the diaphragm. This assumption is valid only in the absence of a vena contracta. To avoid a vena contracta the diaphragms has been chamfered (conical orifice). This chamfer suppresses the vena contracta and also extends the range in which the coefficient of discharge is constant (OMEGA, 1995). In order to check this the diaphragms have been calibrated by means of a volume gas meter. The calibration of the various diaphragms proves that the vena contracta coefficient is constant and equal to unity within 1% uncertainty for all the diaphragms except for the 3.5mm diameter diaphragm for which it is estimated to be 0.97.

FIG. 5. Volume flow U as a function of the total pressure drop P_m for the clarinet mouthpiece ended with a diaphragm (arbitrary scales).

3.4 Reed opening measurement

This measurement is likely to give information on the behaviour of the jet entering the instrument. In particular the opening can be compared to the effective section of the jet entering in the mouthpiece. This measurement also makes it possible to relate the opening H and the pressure difference ΔP on both sides of the reed. This allows the determination of the evolution of the reed stiffness with the pressure difference.

The sensitivity of the optical system used to measure the opening H is calibrated by means of visual observation with a camera in macro mode. This device was used for checking the linearity of the optical system and to determine the opening at rest H_0 . A difficulty with the optical system is that the light beam can be fully stopped while the reed slit is not completely closed. To limit this problem, it is checked by visual inspection, before each experiment, that, when the reed is almost closed, the diode still detects a signal.

3.5 Experimental procedure

The two pressure signals and the optical signal are collected on a computer via a data acquisition card. The opening at rest having been measured as described above, the experiment starts without blowing pressure ($\Delta P = 0$). This state is maintained for a few seconds. This is used for the determination of the zeros of the pressure sensors and the value of the optical signal corresponding to the opening at rest. The pressure in the mouth cavity P_m is then increased gradually until the reed closes completely the opening. This state is also maintained for a few seconds. This is done for the relative calibration of the differential pressure sensor ENTRAN, as in that case $P_{in} = 0$, the sensitivity of the static pressure sensor measuring P_m being supposed to be known. This calibration is important because the pressure P_{in} is obtained by making the difference between the signals from the two pressure sensors. The pressure in the mouth is then gradually brought back to zero. A typical duration of such

an experiment is 50 to 100s with a sampling frequency of 100Hz (see figure 6). For some embouchures, when the reed is almost closed reed, oscillations occur incidently which make the corresponding part of the non-linear characteristic not exploitable.

FIG. 6. Experimental signals

_____ *static pressure sensor.*
- - - - *differential pressure sensor.*
- - - - - *optical sensor.*

4. Experimental results

4.1 A typical experimental result

The non-linear characteristic was measured for various mouthpieces, reeds and embouchures. Before each measurement of the characteristic, the embouchure is tested with a cylindrical pipe of 30cm in order to check that the instrument produces a realistic sound of good musical quality. The phenomena observed are globally reproducible even if one observes a large variation in the numerical values of the various parameters. The goal of this section is to present a typical case in order to stress the most significant results. A more detailed analysis of the experimental results is provided in the following sections.

On the figure 7.a, a non-linear characteristic $U = NL(\Delta P)$ is shown for an opening at rest of $H_0 = 0.6\text{mm}$. The reed is a Plasticover (covered with plastic) reed of force 3. This type of reed has been chosen because it is less influenced by moisture than a standard reed. The clarinet mouthpiece is a C80 by Selmer. The curve shows an hysteresis, the maximum flow being larger for increasing pressure than for decreasing pressure. This hysteresis can be attributed to the viscoplastic behaviour of the reed. This point is discussed section 4.2. The

curves are similar to the theoretical curves coming from equation (5), which are also plotted for reference in figure 7.a (dots). The parameters of these theoretical curves are chosen so that the maximum of the theoretical curves matches with those of the experiments. It appears that the closing is never total and that even when the reed can be considered as closed (beyond 60mbar) a weak flow remains. This flow decreases when the maximum pressure is maintained a few seconds. The theoretical model is thus valid until the reed is nearly closed. It does not take into account the residual flow when the reed channel is closed.

Figure 7.b plots the reed opening as a function of the pressure drop. This curve shows that the stiffness of the reed can be considered as roughly independent on the pressure. This is emphasised on figure 7.b by two straight lines of same slope, showing that the stiffness is approximately the same when the pressure is increasing as when it is decreasing. Only the rest position differs (4% difference). This is confirmed by the fact that between the two theoretical curves of figure 7.a only the values of H_0 are different in the same ratio. This result is explained on the basis of a viscoelastic model in which the return to rest position is delayed (cf. section 4.2). It appears from figure 7.b that the zero of the optical signal is reached for a pressure for which the flow is still significant. As noted section 3.4, this is due to the difficulty of adjustment of the optical set-up for which the zero is reached whereas the reed is still open.

As explained section 2.2 the jet cross section S_j can be deduced from the volume flow using equation 2. Figure 7.c plots the effective cross section as a function of the variation of the reed opening measured by the optical device. The results show systematically that the jet cross section varies linearly with the aperture. In some cases, when the optical adjustment was optimum, a line passing through zero within an uncertainty lower than 0.1mm was obtained. The assumption of a jet cross section proportional to the opening thus seems sensible. This

assumption is used for the determination, by extrapolation, of the opening corresponding to the zero value of the optical signal.

FIG. 7. Typical experimental results (continuous line experiments ; dotted lines theory)

(a) Non-linear characteristics, volume flow U versus pressure difference ΔP

(b) reed opening H versus pressure difference ΔP ,

(c) jet cross section S versus reed opening U .

4.2 Viscoelasticity of the reed

As noticed in section 4.1, some curves exhibit an hysteresis due to a change of the rest position. This result could seem contradictory with the assumption of a static measurement. Indeed the duration of the experiment and the speed of the pressure variations are such that the effects of the reed inertia are negligible during the experiment. The hysteresis can therefore only be explained as the result of a viscoelastic behaviour of the reed which only recovers its original rest position after a time delay larger than our experiments (Marandas et al., 1994). By analysing the opening as a function of time after the reed has been plated and then quickly slackened, it appears, for a given reed, that this one recovers its rest position in three steps. The reed slit opening reaches almost instantaneously 93% of its maximum value. An exponential decay of the difference between the opening value and the maximum opening with a relaxation time $\tau_1 \cong 8s$ is then observed. At the end of this second phase the reed slit opening reaches 97% of its maximum value. Rest position is finally reached at the end of a last phase for which the relaxation time is $\tau_2 \cong 900s$. In this second case, taking into account the importance of the relaxation time, it is not unreasonable to speak of a quasi-plastic deformation (Marandas et al., 1994). Taking into account the typical duration of an

experiment one can think that hysteresis observed in the experimental characteristics is due to a conjugation of these two effects. In particular the fact that when the reed is closed there remains a flow which tends to decrease if the maximum pressure is maintained a sufficiently long time. These viscoelastic effects essential for the musician (Ducasse, 2001 ; Marandas et al., 1994), are probably not relevant when considering a physical model of the autooscillation process. The actual characteristics might be found somewhere in between the two static characteristics obtained respectively upon increasing the pressure and decreasing the pressure. By chance the hysteresis being rather small the uncertainty on the relevant parameters of the model will be small. Typically it should not exceed few percent of H_0 . To limit this effect it is useful, before doing an experiment, to close the reed for a few seconds by applying a large pressure in the mouth volume in order to limit the quasi-plastic effect ($\tau_2 \cong 900s$). This is similar to some musicians practice which consists in pressing the reed with the thumb before playing. It is important to notice that the amplitude of the hysteresis can vary considerably with the reed. We observed that for some reeds, a priori not different from the others, the hysteresis did not appear. On the other hand our experiments have been done with a dry reed covered with plastic. A wet standard read would probably have emphasised a slightly different behaviour (Marandas et al., 1994).

4.3 Reed stiffness

As noted section 4.1, the reed stiffness can be regarded as a constant until the reed beats. With some mouthpieces and reeds a slightly different behaviour has been observed. Figure 8 shows a result with a clarinet mouthpiece B40 by Vandoren, a Plasticover reed n°3 and a reed opening at rest $H_0 = 0.6mm$. In figure 8.a a plot of the non-linear characteristics is compared to the simple basic model (equation 5) in which parameters P_M and wH_0 are chosen so that the maxima of the experimental and theoretical curves coincide. One can note

that the theoretical curve and the model differ for a pressure slightly lower than $\Delta P = 40\text{mbar}$. Figure 8.b plots the reed slit opening as a function of the pressure difference. One can note that the curve is linear for the pressure lower than 40mbar. The reed stiffness can be deduced from the slope of the curve. The following value for the reed stiffness is obtained: $k = 116\text{mbar/mm}$. This result is in agreement with the value $k = P_M / H_0 = 122\text{mbar/mm}$ deduced from the maximum of flow of the characteristics $P_M = 73\text{mbar}$ and the reed opening at rest $H_0 = 0.6\text{mm}$. This is not very different from the result of experiment of figure 7.b in which the reed stiffness is found to be $k = 107\text{mbar/mm}$. It is also noticed that the extrapolated line passes through zero for $\Delta P = P_M$ which validates the calibration method of the optical system based on the extrapolation of the function $S_j(H)$ (cf. section 4.1). The function $H(\Delta P)$ deviates from a linear behaviour above 40mbar. This is in agreement with the observation made on the figure 8.a that the simple model is no more valid beyond $\Delta P = 40\text{mbar}$. We expect that beyond this pressure the reed stiffness increases as a result of a reduction of the free reed length due to contact with the lay. The remarkable result here is that the change in the reed stiffness value appears only for a pressure higher than the threshold of oscillation $\Delta P \approx P_M / 3$. According to various measurements it seems that this phenomenon is general. This result is to be compared with the analysis of various authors (Gazengel, 1994 ; Ducasse, 2001 ; Van Walstjin, 2002) according to whom the curling up phenomenon is limited and has a small influence on the reed stiffness, the end of the reed touching the lay without smoothly curled up on the lay. Our measurements do not exclude this scenario but show that it should appear only for high pressures. Complementary measurements, allowing to determine directly the point of contact between the reed and the lay, are now in preparation to confirm this result.

FIG. 8. Experimental results (continuous line experiments ; dotted lines theory)

(a) non-linear characteristics, volume flow U versus pressure difference ΔP

(b) reed opening H versus pressure difference ΔP .

4.4 Air flow

To evaluate the discrepancy between the model for the volume flow and the measurement, the measured volume flux U can be divided by the theoretical one U_b calculated using equation 1 where H is measured with the optical device and $w = 14mm$ is the external width of the mouthpiece inlet. The dimensionless quantity obtained is the vena contracta coefficient α (see equation 6). It is displayed versus the reed opening in figure 9 for the same experiment as in figure 8. Parameter α remains constant along a large range of reed openings : $\alpha \approx 0.95$ for $0.2mm < H < 0.65mm$. For $H < 0.15mm$ the optical measurements are no longer relevant. From the experiment of figure 7.c the value $\alpha \approx 1.2$ is found. The constant behaviour of α has been observed for every embouchure tested, α being in the range 0.85 to 1.30. These results confirm that a volume flux calculated from the Bernoulli law with a constant vena contracta coefficient α is a reasonable approximation for sufficiently large reed opening. At this stage, the experimental device and particularly the reed opening measurement are not sufficient to draw conclusions for a small reed opening.

FIG. 9. : Contraction coefficient α versus reed opening H (same embouchure set-up as in figure 8).

It is interesting to compare the measured α values with the vena contracta coefficient obtained for simplified geometries of mouthpieces and in less realistic playing conditions. As mentioned in Section 2.2, Van Zon (1989) has obtained values in the range 0.50 to 0.61 for typical 2D mouthpiece. Furthermore Maurin (1992) observed larger values, in the range 0.60

to 0.85 for a clarinet mouthpiece mounted in an artificial mouth, the lateral parts of the reed channel being waxed to approach a 2D geometrical situation. The larger value of α could be explained by an effect of the confinement of the flow upstream of the inlet reed channel. This phenomenon would also occur in actual clarinet playing. Valkering (1993) measured volume flows through a reed channel formed by a stiff flat metal reed with sharp edges placed on an actual clarinet mouthpiece. It is shown that the flow threw lateral sides can increase the flux within 50% which lead to values for α compatible with our results.

4.5 Typical parameters range values

The model formulated in equation 5 is based on parameters of which the values have to be defined when dealing with simulations or physical modelling synthesis. Our experiments allow the determination of some of these parameters. In the present section the range of these parameters and the accuracy with which they can be determined using our setup are discussed. Results are summarised in table 1. The reed parameters, that is the opening at rest H_0 and the reed stiffness per area k , are determined using the optical device (see section 3.4 and 4.3). The reed opening is approximately $H_0 = 1mm$ when the reed is free, that is when the lips do not press the reed. When the lip presses the reed this value decreases but probably not much under $H_0 = 0.4mm$ which correspond to a rather tight embouchure. Because of the small value of the reed opening and of the geometrical irregularities of the reed this parameter H_0 can not be determined with a great accuracy. The accuracy on this parameter H_0 is estimated to be only 10%. The reed stiffness which is deduced from $k = H_0 / \Delta P$ is found to be in the range of 100mbar/mm. From our measurements it appears that the range in which these parameter varies is rather small. In all the measurement realised it did not vary more than 50%. This

parameter being deduced from H_0 , its accuracy is in the same range as H_0 , that is 10%. On the other hand the product of the two reed parameters H_0 and k is the beating pressure $P_M = kH_0$. This beating pressure P_M is found to be around 80mbar for a loose embouchure and around 60mbar for a tight embouchure. This parameter can be determined with a good accuracy from the flow measurement (see section 4.1). In some cases an accuracy of 2% can be reached (a little bit more if the hysteresis is considered, see section 4.2). From flow measurements the maximum flow can also be deduced from which the value of the effective surface of the jet at rest $S_j = \alpha w H_0$ can be deduced with a good accuracy (3%). From this surface the effective width of the reed channel αw can be deduced (section 4.4). The uncertainty on this parameter is in the same range as for the reed opening H_0 from which it is deduced. In table 1 realistic values of the parameters of the model for the clarinet are summarised. These values are in agreement with those given by other authors (Nederveen 1998 ; Stewart & Strong 1980). We guess that these values might be useful for physical modelling synthesis and simulations.

5. Conclusion and perspectives

The experiments presented in the present paper allow a better characterization of the clarinet mouthpiece behaviour. It also gives elements useful for a better comprehension of the physical phenomena involved. In the nineties a considerable effort has been carried out in parallel at the LAUM (Le Mans, France) and at the TUE (Eindhoven, Netherlands) to obtain a reliable model for the relationship between the jet section and the reed channel height characterised by the coefficient α of equation 6 (Van Zon, 1989 ; Hirschberg et al., 1990 ;

Hirschberg et al., 1994 ; Hirschberg, 1995). Using simplified 2D geometries, different stationary regimes have been identified experimentally and explained theoretically. We have now proposed an experimental procedure allowing to measure these characteristics with actual mouthpieces and reeds under conditions close to playing conditions. Our measurements with this new procedure seem to agree qualitatively with the earlier measurements in simplified geometries. The main problem in the interpretation of our results is the uncertainty in the geometry of the reed channel and of the lateral slits between the lay and the reed. Our results do confirm that a volume flux calculated from the Bernoulli law with a constant parameter vena contracta coefficient α is a reasonable first approximation for sufficiently large reed opening. For a small reed opening the effect of friction becomes significant and a correction for viscous effects should be introduced.

In the quasi-stationary basic model, the behaviour of the reed is reduced to a spring with a constant stiffness. Rather surprisingly, experimental results confirm that the latter hypothesis is reasonable, a stiffness value being associated for each embouchure adjustment. Nevertheless for some of the embouchures, the hypothesis is correct up to a critical pressure threshold above which the equivalent stiffness is increasing when the reed-lay aperture is decreasing. Some authors proposed to consider a variable stiffness $k(H)$ to take into account the curvature of the lay (see for example the time domain simulations of Ducasse, 1990). Our results show that this should be done with care. Indeed, a variation of the stiffness with the opening has been observed for some embouchures, but only when the reed is near closing. Supplementary investigations have to be done with different reeds and mouthpieces in order to check the variability of the results. A comparison with the recent theoretical works of Ducasse (2001) and Van Walstijn (2002) should be done by using a reed and a lip of which characteristics would be measurable and controllable. Incidentally, our measurements confirm the visco-elastic behaviour of reeds reported by Marandas et al. (1994). It seems however that

this behaviour is not relevant when predicting oscillations because the memory time scales involved are long compared to the oscillation period.

Finally, our study allows to conclude that the "simplistic" model described in section 2 is valid at least on the major part of the characteristics. This result confirms the practical interest of theoretical studies based on this model (Wilson and Beavers, 1974 ; Fletcher, 1979 ; Saneyoshi et al., 1987 ; Fletcher 1993 ; Kergomard, 1995 ; Kergomard et al., 2000 ; Ollivier et al., 2002). Similar studies could be done for other instruments. For instance, a saxophone alto mouthpiece has been tested too : the same behaviour as for a clarinet mouthpiece has been observed both from the point of view of the air flow and the reed mechanics in stationary regime. This suggest that, except the size, there is no major difference between a clarinet and a saxophone mouthpiece. The case of the double reed of an oboe or a bassoon could also be investigated in the same way. This could allow to check if the hypothesis by Barjau and Agullo (1989) of a cross section proportional to the opening to the power two is sensible. The conjecture that there is a significant pressure recovery inside the narrow pipes on which double reeds are mounted (Hirschberg, 1995) could also be checked.

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Figure and table legends

Table 1 : Summary of the parameters values of the parameters for the clarinet.

FIG. 1. (a) Flow control by the clarinet reed involving free jet formation and turbulent dissipation. (b) A two dimensional model of the reed channel geometry and expected flow.

FIG. 2. Theoretical characteristics (equation 5) : volume flux U as a function of the pressure difference ΔP (arbitrary scales). P_M is the value of the pressure difference corresponding to the reed blocked on the mouthpiece.

FIG. 3. Theoretical characteristics (equation 5) in the case of a discontinuous reed stiffness (according to Van Walstijn (2002), see text).

FIG. 4. Experimental device.

FIG. 5. Volume flow U as a function of the total pressure drop P_m for the clarinet mouthpiece ended with a diaphragm (arbitrary scales).

FIG. 6. Experimental signals

_____ static pressure sensor.
_ _ _ _ differential pressure sensor.
- - - - optical sensor.

FIG. 7. Typical experimental results (continuous line experiments ; doted lines theory)

(d) Non-linear characteristics, volume flow U versus pressure difference ΔP

- (e) reed opening H versus pressure difference ΔP ,
- (f) jet cross section S versus reed opening U .

FIG. 8. Experimental results (continuous line experiments ; dotted lines theory)

- (c) non-linear characteristics, volume flow U versus pressure difference ΔP
- (d) reed opening H versus pressure difference ΔP .

FIG. 9. : Contraction coefficient α versus reed opening H (same embouchure set-up as in figure 8).

CaracNL figures

Figure 1.a

Figure 1.b

Figure 2

Figure 3

Figure 4

Figure 5

Figure 6

Figure 7.a

Figure 7.b

Figure 7.c

Figure 8

Figure 9

Tableau des constantes et valeurs typiques

Quantity	Symbol	Typical values
Reed opening at rest	H_0	0.4 – 1.0mm
Beating pressure	P_M	40 – 100mbar
Reed surfacic stiffness	k	80 – 130mbar/mm
Jet effective width	αw	12 – 18mm
Maximum flow	$U_{\max}$	200 – 600cm ³ /s