Multi-criteria decision making based on DSmT-AHP

Jean Dezert, Jean-Marc Tacnet, Mireille Batton-Hubert, Florentin Smarandache

Multi-criteria decision making based on DSmT-AHP

Jean Dezert
The French Aerospace Lab.
Information Fusion Systems
29 Av. de la Division Leclerc
92320 Châtillon, France.
Email: jean.dezert@onera.fr

Jean-Marc Tacnet
CEMAGREF
Unité Etna
2 rue de la Papèterie, B.P. 76
38402 St. Martin D'Hères Cedex, France.
Email: jean-marc.tacnet@cemagref.fr

Mireille Batton-Hubert
Ecole des Mines de Saint-Etienne (EMSE)
Laboratoire SITE, Espace Fauriel
29 rue Ponchardier
42100 Saint-Etienne, France.
Email: batton@emse.fr

Florentin Smarandache
Math. & Sciences Dept.
University of New Mexico,
200 College Road,
Gallup, NM 87301, U.S.A.
Email: smarand@unm.edu

***Abstract*—In this paper, we present an extension of the multi-criteria decision making based on the Analytic Hierarchy Process (AHP) which incorporates uncertain knowledge matrices for generating basic belief assignments (bba's). The combination of priority vectors corresponding to bba's related to each (sub)-criterion is performed using the Proportional Conflict Redistribution rule no. 5 proposed in Dezert-Smarandache Theory (DSmT) of plausible and paradoxical reasoning. The method presented here, called DSmT-AHP, is illustrated on very simple examples.**

Keywords: Analytic Hierarchy Process, AHP, DSmT, Information Fusion, Decision Making, Multi-Criteria.

I. INTRODUCTION

The Multi-criteria decision-making (MCDM) problem concerns the elucidation of the level of preferences of decision alternatives through judgments made over a number of criteria [6]. At the Decision-maker (DM) level, a useful method for solving MCDM problem must take into account opinions made under uncertainty and based on distinct criteria with different importances. The difficulty of the problem increases if we consider a group decision-making (GDM) problem involving a panel of decision-makers. Several attempts have been proposed in the literature to solve the MCGDM problem. Among the interesting solutions developed, one must cite the works made by Beynon [3]–[6]. This author developed a method called DS/AHP which extended the Analytic Hierarchy Process (AHP) method of Saaty [15]–[17] with Dempster-Shafer Theory (DST) [23] of belief functions to take into account uncertainty and to manage the conflicts between experts opinions within a hierarchical model approach. In this paper, we propose to follow Beynon's approach, but instead of using DST, we investigate the possibility to use Dezert-Smarandache Theory (DSmT) of plausible and paradoxical reasoning developed since 2002 for overcoming DST limitations[1] [24]. This new approach will be referred as DSmT-AHP method in the sequel. DSmT allows to manage efficiently the fusion of quantitative (or qualitative) uncertain and possibly highly conflicting sources of evidences and proposes new methods for belief conditioning and deconditioning as well [7]. DSmT has been successfully applied in several fields of applications (in defense, medicine, satellite surveillance, biometrics, image processing, etc). In section II, we briefly introduce the principle of the AHP developed by Saaty. In section III, we recall the basis of DSmT and its main rule of combination, called PCR5 (Proportional Conflict Redistribution rule # 5). In section IV, we present the DSmT-AHP method for solving the MCDM problem. The extension of DSmT-AHP method for solving MCGDM problem is then introduced in section V. Conclusions are given in Section VI.

II. THE ANALYTIC HIERARCHY PROCESS (AHP)

The Analytic Hierarchy Process (AHP) is a structured technique developed by Saaty in [8], [15], [16] based on mathematics and psychology for dealing with complex decisions. AHP and its refinements are used around the world in many decision situations (government, industry, education, healthcare, etc.). It helps the DM to find the decision that best suits his/her needs and his/her understanding of the problem.

[1] A presentation of these limitations with a discussion is done in Chap 1 of [24], Vol. 3. It is shown clearly that the logical refinement proposed by some authors doesn't bring new insights with respect to what is done when working directly on the super-power set (i.e. on the minimal refined frame satisfying Shafer's model). There is no necessity to work with a refined frame in DSmT framework which is very attractive in some real-life problems where the elements of the refined frame do not have any (physical) sense/meaning or are just impossible to clearly determine physically (as a simple example, if Mary and Paul have possibly committed a crime alone or together, there is no way to refine these two persons into three finer exclusive physical elements satisfying Shafer's model). Aside the possibility to deal with different underlying models of the frame, it is worth to note that PCR5 or PCR6 rules provide a better ability than the other rules to deal efficiently with highly conflicting sources of evidences as shown in all fields of applications where they have been tested so far.

AHP provides a comprehensive and rational framework for structuring a decision problem, for representing and quantifying its elements, for relating those elements to overall goals, and for evaluating alternative solutions. The basic idea of AHP is to decompose the decision problem into a hierarchy of more easily comprehended sub-problems, each of which can be analyzed independently. Once the hierarchy is built, the DM evaluates the various elements of the hierarchy by comparing them to one another two at a time [21]. In making the comparisons, the DM can use both objective information about the elements as well as subjective opinions about the elements' relative meaning and importance. The AHP converts these evaluations to numerical values that are processed and compared over the entire range of the problem. A numerical weight or priority is derived for each element of the hierarchy, allowing diverse and often incommensurable elements to be compared to one another in a rational and consistent way. This is the main advantage of AHP with respect to other decision making techniques. At its final step, numerical priorities are calculated for each of the decision alternatives. These numbers represent the alternatives' relative ability to achieve the decision goal. The AHP method can be summarized as [19]:

1) Model the problem as a hierarchy containing the decision goal, the alternatives for reaching it, and the criteria for evaluating the alternatives.
2) Establish priorities among the elements of the hierarchy by making a series of judgments based on pairwise comparisons of the elements.
3) Check the consistency of the judgments and eventually revise the comparison matrices by reasking the experts when the consistency in judgments is too low.
4) Synthesize these judgments to yield a set of overall priorities for the hierarchy.
5) Come to a final decision based on the results of this process.

Example 1: According to his/her own preferences and using the Saaty's 1-9 ordinal scale, a DM wants to buy a car among four available models belonging to the set $\Theta = \{A, B, C, D\}$. To simplify the example, we assume that the objective of DM is to select one of these cars based only on three criteria (C1=Fuel economy, C2=Reliability and C3=Style). According to his/her own preferences, the DM ranks the different criteria pairwise as follows: 1 - Reliability is 3 times as important as fuel economy, 2 - Fuel economy is 4 times as important as style, 3 - Reliability is 5 times as important as style, which means that the DM thinks that Reliability criteria (C2) is the most important criteria, followed by fuel economy (C1) and style is the least important criteria[2]. The relative importance of one criterion over another can be expressed using pairwise comparison matrix (also called knowledge matrix) as follows:

$$\mathbf{M} = \begin{bmatrix} 1/1 & 1/3 & 4/1 \\ 3/1 & 1/1 & 5/1 \\ 1/4 & 1/5 & 1/1 \end{bmatrix} \approx \begin{bmatrix} 1.0000 & 0.3333 & 4.0000 \\ 3.0000 & 1.0000 & 5.0000 \\ 0.2500 & 0.2000 & 1.0000 \end{bmatrix}$$

where the element m_{ij} of the matrix $\mathbf{M}$ indicates the relative importance of criteria Ci with respect to the criteria Cj. In this example, $m_{13} = 4/1$ indicates that the criteria C1 (Fuel economy) is four times as important as the criteria C3 (Style) for the DM, etc. From this pairwise matrix, Saaty demonstrated that the ranking of the priorities of the criteria can be obtained from the normalized eigenvector[3], denoted $\mathbf{w}$, associated with the principal eigenvalue of the matrix, denoted λ. In this example, one has $\lambda = 3.0857$ and $\mathbf{w} = [0.2797\ 0.6267\ 0.0936]'$ which shows that C2 criterion (reliability) is the most important criterion with the weight 0.6267, then the fuel economy criterion C1 is the second most important criterion with weight 0.2797, and finally C3 criterion (Style) is the least important criterion with weight 0.0936 for the DM. A similar ranking procedure can be used to find the relative weights of each car A, B, C or D with respect to each criterion C1, C2 and C3 based on given DM preferences, hence one will get three new normalized eigenvectors denoted $\mathbf{w}(C1)$, $\mathbf{w}(C2)$ and $\mathbf{w}(C3)$. By example, if one has the following normalized vectors

$$[\mathbf{w}(C1)\, \mathbf{w}(C2)\, \mathbf{w}(C3)] = \begin{bmatrix} 0.2500 & 0.4733 & 0.1129 \\ 0.1304 & 0.0611 & 0.4435 \\ 0.5109 & 0.1832 & 0.0565 \\ 0.1087 & 0.2824 & 0.3871 \end{bmatrix}$$

then the solution of the MCDM problem (here the selection of the "best" car according to the DM multicriteria preferences) is finally obtained by multiplying the matrix $[\mathbf{w}(C1)\ \mathbf{w}(C2)\ \mathbf{w}(C3]$ by the criteria ranking vector $\mathbf{w}$. For this example, one will get:

$$\begin{bmatrix} 0.2500 & 0.4733 & 0.1129 \\ 0.1304 & 0.0611 & 0.4435 \\ 0.5109 & 0.1832 & 0.0565 \\ 0.1087 & 0.2824 & 0.3871 \end{bmatrix} \times \begin{bmatrix} 0.2797 \\ 0.6267 \\ 0.0936 \end{bmatrix} = \begin{bmatrix} 0.3771 \\ 0.1163 \\ 0.2630 \\ 0.2436 \end{bmatrix}$$

Based on this result, the car A which has the most important weight (0.3771) will be selected by the DM. The costs could also be included in AHP by taking into account the benefit to cost ratios which will allow to chose alternative with lowest cost and highest benefit. For example, let's suppose that the cost of car A is 21000 euros, the cost of car B is 13000 euros, the cost of car C is 12000 euros and the cost of car D is 18000 euros, then the normalized cost vector is $[0.3281\ 0.2031\ 0.1875\ 0.2812]'$, so that the benefit-cost ratios are now $[0.3771/0.3281 = 1.1492\ 0.1163/0.2031 = 0.5724\ 0.2630/0.1875 = 1.4026\ 0.2436/0.2812 = 0.8663]'$. Taking into account now the cost of vehicles, now the best solution for the DM is to choose the car C since it offers the highest benefit-cost ratio.

In this paper we do not focus on the rank reversal problem of AHP as discussed in [9], [10], [13], [18], [22], but we propose an extension of AHP using aggregation method developed in DSmT framework, able to make a difference between importance of criteria, uncertainty related to the evaluations of criteria and reliability of the different sources.

[2] The relationships between preferences given by a DM may not be transitive as shown in this example, nevertheless one has to deal with these inputs even in such situations.

[3] Note that if the relationships on the criteria is transitive, then we can easily construct the normalized vector of priorities from a system of algebraic equations, without employing Saaty's matrix approach. For example if in the previous example one assumes[4] $M_{23} = 12/1$ and $M_{32} = 1/12$ instead of 5/1 and 1/5, then the normalized weighting vector will be directly obtained as $\mathbf{w} = [4/17\ 12/17\ 1/17]'$.

III. BASICS OF DSMT

Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$ be a finite set of n elements assumed to be exhaustive. Θ corresponds to the frame of discernment of the problem under consideration. In general, we assume that elements of Θ are non exclusive in order to deal with vague/fuzzy and relative concepts [24], Vol. 2. This is the so-called free-DSm model. In DSmT, there is no need to work on a refined frame consisting in a discrete finite set of exclusive and exhaustive hypotheses[5] because DSm rules of combination work for any models of the frame. The hyper-power set D^Θ is defined as the set of all propositions built from elements of Θ with $\cup$ and $\cap$, see [24], Vol. 1 for examples. A (quantitative) basic belief assignment (bba) expressing the belief committed to the elements of D^Θ by a given source is a mapping $m(\cdot)$: $D^\Theta \to [0,1]$ such that: $m(\emptyset) = 0$ and $\sum_{A\in D^\Theta} m(A) = 1$. Elements $A \in D^\Theta$ having $m(A) > 0$ are called *focal elements* of $m(.)$. The credibility and plausibility functions are defined in almost[6] the same manner as in DST [23]. In DSmT, the Proportional Conflict Redistribution Rule no. 5 (PCR5) is used generally to combine bba's. PCR5 transfers the conflicting mass only to the elements involved in the conflict and proportionally to their individual masses, so that the specificity of the information is entirely preserved in this fusion process. For example: consider two bba's $m_1(.)$ and $m_2(.)$, $A \cap B = \emptyset$ for the model of Θ, and $m_1(A) = 0.6$ and $m_2(B) = 0.3$. With PCR5 the partial conflicting mass $m_1(A)m_2(B) = 0.6 \cdot 0.3 = 0.18$ is redistributed to A and B only with respect to the following proportions respectively: $x_A = 0.12$ and $x_B = 0.06$ because

$$\frac{x_A}{m_1(A)} = \frac{x_B}{m_2(B)} = \frac{m_1(A)m_2(B)}{m_1(A)+m_2(B)} = \frac{0.18}{0.9} = 0.2$$

In this paper, we work in the power set 2^Θ since most of readers are usually already familiar with this fusion space. Let's $m_1(.)$ and $m_2(.)$ be two independent[7] bba's, then the PCR5 rule is defined as follows (see [24], Vol. 2 for full justification and examples): $m_{PCR5}(\emptyset) = 0$ and $\forall X \in 2^\Theta \setminus \{\emptyset\}$

$$m_{PCR5}(X) = \sum_{\substack{X_1,X_2\in 2^\Theta\\ X_1\cap X_2 = X}} m_1(X_1)m_2(X_2) + \sum_{\substack{X_2\in 2^\Theta\\ X_2\cap X=\emptyset}} [\frac{m_1(X)^2 m_2(X_2)}{m_1(X)+m_2(X_2)} + \frac{m_2(X)^2 m_1(X_2)}{m_2(X)+m_1(X_2)}] \quad (1)$$

where all denominators in (1) are different from zero. If a denominator is zero, that fraction is discarded. All propositions/sets are in a canonical form. A variant of (1), called PCR6, for combining $s > 2$ sources and for working in other fusion spaces (hyper-power sets or super power-sets) is presented in [24]. Additional properties of PCR5 can be found in [7]. Extension of PCR5 for combining qualitative bba's can be found in [24], Vol. 2 & 3.

[5] referred as Shafer's model in the literature.

[6] We just replace 2^Θ by D^Θ in the definitions of credibility and plausibility functions.

[7] i.e. each source provides its bba independently of the other sources.

IV. DSMT-AHP FOR SOLVING MCDM

DSmT-AHP aimed to perform a similar purpose as AHP [15], [16], SMART [28] or DS/AHP [2], [4], etc. that is to find the preferences rankings of the decision alternatives (DA), or groups of DA. DSmT-AHP approach consists in three steps:

- Step 1: We extend the construction of the matrix for taking into account the partial uncertainty (disjunctions) between possible alternatives. If no comparison is available between elements, then the corresponding elements in the matrix is zero. Each bba related to each (sub-) criterion is the normalized eigenvector associated with the largest eigenvalue of the "uncertain" knowledge matrix (as done in standard AHP approach).
- Step 2: We use the DSmT fusion rules, typically the PCR5 rule, to combine bba's drawn from step 1 to get a final MCDM priority ranking. This fusion step must take into account the different importances (if any) of criteria as it will be explained in the sequel.
- Step 3: Decision-making can be done based either on the maximum of belief, or on the maximum of the plausibility of Decision alternatives (DA), as well as on the maximum of the approximate subjective probability of DA obtained by different probabilistic transformations.

Example 2: Let's consider now a set of three cars $\Theta = \{A, B, C\}$ and the criteria C1=Fuel Economy, C2=Reliability. Let's assume that with respect to each criterion the following "uncertain" knowledge matrices are given:

$$\mathbf{M}(C1) = \left[\begin{array}{c|ccc} & A & B\cup C & \Theta \\ \hline A & 1 & 0 & 1/3 \\ B\cup C & 0 & 1 & 2 \\ \Theta & 3 & 1/2 & 1 \end{array}\right]$$

$$\mathbf{M}(C2) = \left[\begin{array}{c|cccc} & A & B & A\cup C & B\cup C \\ \hline A & 1 & 2 & 4 & 3 \\ B & 1/2 & 1 & 1/2 & 1/5 \\ A\cup C & 1/4 & 2 & 1 & 0 \\ B\cup C & 1/3 & 5 & 0 & 1 \end{array}\right]$$

Step 1: (bba's generation) Applying AHP method, one gets the following priority vectors $\mathbf{w}(C1) \approx [0.0889\ 0.5337\ 0.3774]'$ and $\mathbf{w}(C2) \approx [0.5002\ 0.1208\ 0.1222\ 0.2568]'$ which are identified with the bba's $m_{C1}(.)$ and $m_{C2}(.)$ as follows: $m_{C1}(A) = 0.0889$, $m_{C1}(B\cup C) = 0.5337$, $m_{C1}(A\cup B\cup C) = 0.3774$ and $m_{C2}(A) = 0.5002$, $m_{C2}(B) = 0.1208$, $m_{C2}(A\cup C) = 0.1222$ and $m_{C2}(B\cup C) = 0.2568$.

Step 2: (Fusion) When the two criteria have the same full importance in the hierarchy they are fused with one of the classical fusion rules. In [4] Beynon proposed to use Dempster's rule. Here we propose to use the PCR5 fusion rule since it is known to have a better ability to deal efficiently with possibly highly conflicting sources of evidences [24], Vol. 2. With PCR5, one gets:

Elem. of 2^Θ	$m_{C1}(.)$	$m_{C2}(.)$	$m_{PCR5}(.)$
$\emptyset$	0	0	0
A	0.0889	0.5002	0.3837
B	0	0	0.1162
$A\cup B$	0	0.1208	0
C	0	0	0.0652
$A\cup C$	0	0.1222	0.0461
$B\cup C$	0.5337	0.2568	0.3887
$A\cup B\cup C$	0.3774	0	0

Step 3: (Decision-making) A final decision based on $m_{PCR5}(.)$ must be taken. Usually, the decision-maker (DM) is concerned with a single choice among the elements of Θ.

Many decision-making approaches are possible depending on the risk the DM is ready to take. A pessimistic DM will choose the singleton of Θ giving the maximum of credibility whereas an optimistic DM will choose the element having the maximum of plausibility. A fair attitude consists usually in choosing the maximum of approximate subjective probability of elements of Θ. The result however is very dependent on the probabilistic transformation (Pignistic, DSmP, Sudano's, etc) [24], Vol. 2. Below are the values of the credibility, the pignistic probability and the plausibility of A, B and C:

Elem. of Θ	$Bel(.)$	$BetP(.)$	$Pl(.)$
A	0.3837	0.4068	0.4298
B	0.1162	0.3105	0.5049
C	0.0652	0.2826	0.5000

The car A will be preferred with the pessimistic or pignistic attitudes, whereas the car B will be preferred if an optimistic attitude is adopted since one has $Pl(B) > Pl(C) > Pl(A)$.

The MCDM problem deals with several criteria having different importances and the classical fusion rules cannot be applied directly as in step 2. In AHP, the fusion is done from the product of the bba's matrix with the weighting vector of criteria. Such AHP fusion is nothing but a simple componentwise weighted average of bba's and it doesn't actually process efficiently the conflicting information between the sources. It doesn't preserve the neutrality of a full ignorant source in the fusion. To palliate these problems, we propose a solution for combining sources of different importances in the framework of DSmT and DST.

Before going further, it is essential to explain the difference between the importance and the reliability of a source of evidence. The reliability is an objective property of a source, whereas the importance of a source is a subjective characteristic expressed by the fusion system designer. The reliability of a source represents its ability to provide the correct assessment/solution of the given problem. It is characterized by a discounting reliability factor, usually denoted α in $[0,1]$, which should be estimated from statistics when available, or by other techniques [11]. The reliability can be context-dependent. By convention, we usually take $\alpha = 1$ when the source is fully reliable and $\alpha = 0$ if the source is totally unreliable. The reliability of a source is usually taken into account with Shafer's discounting method [23] defined by:

$$\begin{cases} m_\alpha(X) = \alpha \cdot m(X), \quad \text{for } X \neq \Theta \\ m_\alpha(\Theta) = \alpha \cdot m(\Theta) + (1-\alpha) \end{cases} \tag{2}$$

The importance of a source is not the same as its reliability and it can be characterized by an importance factor, denoted β in $[0,1]$ which represents somehow the weight of importance granted to the source by the fusion system designer. The choice of β is usually not related with the reliability of the source and can be chosen to any value in $[0,1]$ by the designer for his/her own reason. By convention, the fusion system designer will take $\beta = 1$ when he/she wants to grant the maximal importance of the source in the fusion process, and will take $\beta = 0$ if no importance at all is granted to this source in the fusion process. The fusion designer must be able to deal with importance factors in a different way than with reliability factors since they correspond to distinct properties associated with a source of information. The importance of a source is particularly crucial in hierarchical multi-criteria decision making problems, specially in the AHP [16], [20]. That's why it is primordial to show how the importance can be efficiently managed in evidential reasoning approaches. The main question we are concerned here is how to deal with different importances of sources in the fusion process in such a way that a clear distinction is made/preserved between reliability and importance? Our preliminary investigations for the search of the solution of this problem were based on the self/auto-combination of the sources. But such approach is very disputable and cannot be used satisfactorily in practice whatever the fusion rule is adopted because it can be easily shown that the auto-conflict tends quickly to 1 after several auto-fusions [11]. Actually a better approach can be used for taking into account the importances of the sources and can be considered as the dual of Shafer's discounting approach for reliabilities of sources. The idea was originally introduced briefly by Tacnet in [24], Vol.3, Chap. 23, p. 613. It consists to define the importance discounting with respect to the empty set rather than the total ignorance Θ (as done with Shafer's discounting). Such new discounting deals easily with sources of different importances and is very simple to use. Mathematically, we define the importance discounting of a source $m(.)$ having the importance factor β in $[0,1]$ by:

$$\begin{cases} m_\beta(X) = \beta \cdot m(X), \quad \text{for } X \neq \emptyset \\ m_\beta(\emptyset) = \beta \cdot m(\emptyset) + (1-\beta) \end{cases} \tag{3}$$

Here we allow to deal with non-normal bba since $m_\beta(\emptyset) \geq 0$ as suggested by Smets in [26]. This new discounting preserves the specificity of the primary information since all focal elements are discounted with same importance factor. Here we use the positive mass of the empty set as an intermediate/preliminary step of the fusion process. Clearly when $\beta = 1$ is chosen by the fusion designer, it will mean that the source must take its full importance in the fusion process and so the original bba $m(.)$ is kept unchanged. If the fusion designer takes $\beta = 0$, one will deal with $m_\beta(\emptyset) = 1$ which is interpreted as a fully non important source. $m(\emptyset) > 0$ is not interpreted as the mass committed to some conflicting information (classical interpretation), nor as the mass committed to unknown elements when working with the open-world assumption (Smets interpretation), but only as the mass of the discounted importance of a source in this particular context. Based on this discounting, one adapts PCR5 (or PCR6) rule for $N \geq 2$ discounted bba's $m_{\beta,i}(.)$, $i = 1, 2, \ldots N$ by considering the following extension, denoted PCR5$_\emptyset$, defined by: $\forall X \in 2^\Theta$

$$m_{PCR5_\emptyset}(X) = \sum_{\substack{X_1,X_2 \in 2^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2) + \sum_{\substack{X_2 \in 2^\Theta \\ X_2 \cap X = \emptyset}} [\frac{m_1(X)^2 m_2(X_2)}{m_1(X)+m_2(X_2)} + \frac{m_2(X)^2 m_1(X_2)}{m_2(X)+m_1(X_2)}] \tag{4}$$

A similar extension can be done for PCR5 and PCR6 formulas for $N > 2$ sources given in [24], Vol. 2. A detailed presentation of this technique with several examples will appear in [25] and thus it is not reported here. The difference between eqs. (1) and (4) is that $m_{PCR5}(\emptyset) = 0$ whereas $m_{PCR5_\emptyset}(\emptyset) \geq 0$. Since we usually work with normal bba's for decision making support, the combined bba will be normalized. In the AHP context, the importance factors correspond to the components of the normalized eigenvector $\mathbf{w}$.

Example 3: Take back example 2 assume that C2 (the reliability) is three times more important than C1 (fuel economy) so that the knowledge matrix is given by:

$$\mathbf{M} = \begin{bmatrix} 1/1 & 1/3 \\ 3/1 & 1/1 \end{bmatrix} \approx \begin{bmatrix} 1.0000 & 0.3333 \\ 3.0000 & 1.0000 \end{bmatrix}$$

Its normalized principal eigenvector is $\mathbf{w} = [0.2500\ 0.7500]'$ and indicates that C2 is three times more important than C1 as expressed in the prior DM preferences for ranking criteria. $\mathbf{w} = [w_1\ w_2]'$ can also be obtained directly by solving the algebraic system of equations $w_2 = 3w_1$ and $w_1 + w_2 = 1$ with $w_1, w_2 \in [0, 1]$. If we apply the importance discounting with $\beta_1 = w_1 = 0.25$ and $\beta_2 = w_2 = 0.75$, one gets the following discounted bba's

Elem. of 2^Θ	$m_{\beta_1,C1}(.)$	$m_{\beta_2,C2}(.)$
$\emptyset$	0.7500	0.2500
A	0.0222	0.3751
B	0	0
$A \cup B$	0	0.0906
C	0	0
$A \cup C$	0	0.0917
$B \cup C$	0.1334	0.1926
$A \cup B \cup C$	0.0944	0

With the PCR5$_\emptyset$ fusion of the sources $m_{\beta_1,C1}(.)$ and $m_{\beta_2,C2}(.)$, one gets the results in the table. For decision-making support, one prefers to work with normal bba's. Therefore $m_{PCR5_\emptyset}(.)$ is normalized by redistributing back $m_{PCR5_\emptyset}(\emptyset)$ proportionally to the masses of other focal elements as shown in the right column of the next table.

Elem. of 2^Θ	$m_{PCR5_\emptyset}(.)$	$m^{\text{normalized}}_{PCR5_\emptyset}(.)$
$\emptyset$	0.6558	0
A	0.1794	0.5213
B	0.0121	0.0351
$A \cup B$	0.0159	0.0461
C	0.0122	0.0355
$A \cup C$	0.0161	0.0469
$B \cup C$	0.1020	0.2963
$A \cup B \cup C$	0.0065	0.0188

If all sources have the same full importances (i.e. all β_i=1), then $m_{PCR5_\emptyset}(.) = m_{PCR5}(.)$ which is normal because in such case $m_{\beta_i=1,Ci}(.) = m_{Ci}(.)$. From $m^{\text{normalized}}_{PCR5_\emptyset}(.)$ one can easily compute the credibility, pignistic probability or plausibility of each element of Θ for decision-making. In this example one gets:

Elem. of Θ	$Bel(.)$	$BetP(.)$	$Pl(.)$
A	0.5213	0.5741	0.6331
B	0.0351	0.2126	0.3963
C	0.0355	0.2134	0.3974

If the classical AHP "fusion" method (i.e. weighted arithmetic mean) is used directly with bba's $m_{C1}(.)$ and $m_{C2}(.)$, one gets:

$$m_{AHP}(.) = \begin{bmatrix} 0 & 0 \\ 0.0889 & 0.5002 \\ 0 & 0 \\ 0 & 0.1208 \\ 0 & 0 \\ 0 & 0.1222 \\ 0.5337 & 0.2568 \\ 0.3774 & 0 \end{bmatrix} \times \begin{bmatrix} 0.25 \\ 0.75 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.3974 \\ 0 \\ 0.0906 \\ 0 \\ 0.0917 \\ 0.3260 \\ 0.0944 \end{bmatrix}$$

which would have provided the following result for decision-making

Elem. of Θ	$Bel(.)$	$BetP(.)$	$Pl(.)$
A	0.3974	0.5200	0.6741
B	0	0.2398	0.5110
C	0	0.2403	0.5121

In this very simple example, one sees that the importance discounting technique coupled with PCR5-based fusion rule (what we call the DSmT-AHP approach) will suggest, as with classical AHP, to choose the alternative A since the car A has a bigger credibility (as well as a bigger pignistic probability and plausibility) than cars B or C. It is however worth to note that the values of $Bel(.)$, $BetP(.)$ and $Pl(.)$ obtained by both methods are slightly different. The difference in results can have a strong impact in practice in the final result for example if the costs of vehicles have also to be included in the final decision (as explained at the end of the example 1). Note also that the uncertainties $U(X) = Pl(X) - Bel(X)$ of alternatives $X = A, B, C$ have been seriously diminished when using DSmT-AHP with respect to what we obtain with classical AHP as seen in the following table. The uncertainty reduction is a nice expected property specially important for decision-making support.

Elem. of Θ	$U(.)$ with AHP	$U(.)$ with DSmT-AHP
A	0.2767	0.1118
B	0.5110	0.3612
C	0.5121	0.3619

Important remark: If Dempster's rule is used instead of PCR5$_\emptyset$ rule, one gets the following results when comparing the fusion of $m_{C1}(.)$ with $m_{C2}(.)$ (i.e. without importance discounting) with the fusion of $m_{\beta_1=w_1=0.25,C1}(.)$ with $m_{\beta_2=w_2=0.75,C2}(.)$ (i.e. with importance discounting of criteria C1 and C2):

Elem. of 2^Θ	$m_{DS}(.)$	$m_{DS,\mathbf{w}}(.)$
$\emptyset$	0	0
A	0.3588	0.3588
B	0.0908	0.0908
$A \cup B$	0.0642	0.0642
C	0.0918	0.0918
$A \cup C$	0.0649	0.0650
$B \cup C$	0.3294	0.3294
$A \cup B \cup C$	0	0

Clearly, Dempster's rule cannot deal properly with importance discounted bba's as we have proposed in this work just because the importance discounting technique preserves the specificity of the primary information and thus Dempster's rule does not make a difference in results when combining either $m_{C1}(.)$ with $m_{C2}(.)$ or when combining $m_{\beta_1\neq 1,C1}(.)$ with $m_{\beta_2\neq 1,C2}(.)$ due to the way of processing of the total conflicting mass of belief. PRC5 deals more efficiently with importance discounted bba's as we have shown in this example. So it is not surprising that such discounting technique has never been proposed and used in DST framework and this explains why only the classical Shafer's discounting technique (the reliability discounting) is generally adopted. By using Dempster's rule, the fusion designer has no other choice but to consider importance and reliability as same notions ! The DSmT framework with PCR5 (or PCR6) rule and the importance discounting technique proposed here provides an interesting and simple solution for the fusion of sources with different importances which makes a clear distinction between importances and reliabilities of sources.

V. DSmT-AHP FOR SOLVING MCGDM

Previously, a new approach mixing AHP with DSmT solving MCDM problem has been presented. In many practical situations however, the decision must be taken by a group of $n > 1$ Decision Makers (GDM), denoted $GDM = \{DM_i, i = 1, 2, \ldots, n\}$, rather than a single DM, and from the Multi-Criteria preference rankings of the DM_i's. The importance (influence) of each member of the GDM is usually non-equivalent [1] and the importance of each DM of the GDM must be efficiently taken into account in the final decision-making process. Let's denote by $m_{DM_i}(.)$ the result of DSmT-AHP approach (see section IV) related with $DM_i \in GDM$. The MCGDM problem consists in combining all opinions/preferences rankings $m_{DM_i}(.)$, $i = 1, \ldots, n$ with their own (possibly different) importances. When all DM_i's have equal importance, the classical fusion rules[8] $\oplus$ for combining $m_{DM_i}(.)$ can be directly used to get the final result $m_{MCGDM}(.) = [m_{DM_1} \oplus m_{DM_2} \oplus \ldots \oplus m_{DM_n}](.)$; If the DM_i's have different importance weights w_i, the DSmT-AHP approach can also be used at the GDM fusion level using the importance discounting approach presented here. The result for group decision-making is given by the PCR5$_\emptyset$ fusion of $m_{\beta_i, DM_i}(.)$, with $\beta_i = w_i$ and then the result must be normalized for decision making support. In [6], Beynon used the classical discounting technique [23] to readjust $m_{DM_i}(.)$ with w_i's and he identified the importance factors with the reliability factors. In our opinions, this is disputable since importance of a DM_i is not necessarily related with its reliability but rather with the importance in the problem of the choice of his/her Multi-Criteria to establish his/her ranking, or it can come from other (political, hierarchical, etc.) reasons. In our new approach, we make a clear distinction between notions of importance and reliability and both notions can be easily taken into account [25] with DSmT-AHP for solving MCGDM problems, i.e. we can use the classical discounting technique for taking into the reliabilities of the sources, and use the importance discounting proposed here for dealing with the importances of sources.

VI. CONCLUSIONS AND PERSPECTIVES

In this paper, we have presented a new method for Multi-Criteria Decision-Making (MCDM) and Multi-Criteria Group Decision-Making (MCGDM) based on the combination of AHP method developed by Saaty and DSmT. The AHP method allows to build bba's from DM preferences of solutions which are established with respect to several criteria. The DSmT allows to aggregate efficiently the (possibly highly conflicting) bba's based on each criterion. This DSmT-AHP method allows to take into account also the different importances of the criteria and/or of the different members of the decision-makers group. The application of this DSmT-AHP approach for the prevention of natural hazards in mountains is currently under progress, see [24], Vol.3, Chap. 23, and [27].

[8] typically the PCR5 or PCR6 rules, or eventually Dempster's rule if the conflict between DM_i's is low.

REFERENCES

[1] J. Barzilai, F.A. Lootsma, *Power relations and group aggregation in the multiplicative AHP and SMART*, J. of MCDA, Vol. 6, pp. 155–165, 1997.
[2] M. Beynon, B. Curry, P.H. Morgan, *The Dempster-Shafer theory of evidence: An alternative approach to multicriteria decision modelling*, Omega, Vol. 28, No. 1, pp. 37–50, 2000.
[3] M. Beynon, D. Cosker, D. Marshall, *An expert system for multi-criteria decision making using Dempster-Shafer theory*, Expert Syst. with Appli., Vol. 20, No. 4, pp. 357–367, 2001.
[4] M. Beynon, *DS/AHP method: A mathematical analysis, including an understanding of uncertainty*, Eur. J. of Oper. Res., Vol. 140, pp. 148–164, 2002.
[5] M. Beynon, *Understanding local ignorance and non-specificity within the DS/AHP method of multi-criteria decision making*, Eur. J. of Oper. Res., Vol. 163, pp. 403–417, 2005.
[6] M. Beynon, *A method of aggregation in DS/AHP for group decision-making with non-equivalent importance of individuals in the group*, Comp. & Oper. Res., No. 32, pp. 1881-1896, 2005.
[7] J. Dezert, F. Smarandache, *Non Bayesian conditioning and deconditioning*, International Workshop on Belief Functions, Brest, France, April 2010.
[8] E.H. Forman, S.I. Gass, *The analytical hierarchy process: an exposition*, Oper. Res., Vol. 49, No. 4 pp. 46–487, 2001.
[9] R.D. Holder, *Some Comment on the Analytic Hierarchy Process*, J. of the Oper. Res. Soc., Vol. 41, No. 11, pp. 1073–1076, 1990.
[10] F.A. Lootsma, *Scale sensitivity in the multiplicative AHP and SMART*, J. of MCDA, Vol. 2, pp. 87–110, 1993.
[11] A. Martin, A.-L. Jousselme, C. Osswald, *Conflict measure for the discounting operation on belief functions*, Proc. of Fusion 2008, Cologne, Germany, July 2008.
[12] C. K. Murphy, *Combining belief functions when evidence conflicts*, Dec. Sup. Syst., vol. 29, pp. 1-9, 2000.
[13] J. Perez, *Some comments on Saaty's AHP*, Manag. Sci., Vol. 41, No. 6, pp. 1091–1095, 1995.
[14] J. Perez, J.L. Jimeno, E. Mokotoff, *Another Potential Shortcoming of AHP*, TOP: An Official J. of the Spanish Soc. of Stats and Oper. Res., Vol. 14, No. 1 , June, 2006.
[15] T.L. Saaty, *A scaling method for priorities in hierarchical structures*, J. of Math. Psych., Vol. 15, PP. 59–62, 1977.
[16] T.L. Saaty, *The Analytical Hierarchy Process*, McGraw Hill, New York, 1980.
[17] T.L. Saaty, *An exposition of the AHP in reply to the paper "Remarks on the Analytic Hierarchy Process"*, Manag. Sci., Vol. 36, No. 3, pp. 259–268, 1990.
[18] T.L. Saaty, *Response to Holder's Comments on the Analytic Hierarchy Process*, J. of the Oper. Res. Soc., Vol. 42, No. 10, pp. 909–914, October 1991.
[19] T.L. Saaty, *Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World*, RWS Publ., Pittsburgh, PA, USA, 1999.
[20] T.L. Saaty, *Fundamentals of decision making and priority theory with the analytic hierarchy process*, Vol. VI of the AHP series, RWL Publ., Pittsburgh, PA, USA.
[21] T.L. Saaty, *Relative Measurement and its Generalization in Decision Making: Why Pairwise Comparisons are Central in Mathematics for the Measurement of Intangible Factors - The Analytic Hierarchy/Network Process*, RACSAM, Vol. 102, No. 2, pp. 251–318, 2008.
[22] S. Schenkerman, *Inducement of nonexistent order by the analytic hierarchy process*, Dec. Sci., Spring 1997.
[23] G. Shafer, *A mathematical theory of evidence*, Princeton Univ. Press, 1976.
[24] F. Smarandache, J. Dezert, *Advances and applications of DSmT for information fusion (Collected works)*, Vol. 1-3, ARP, 2004–2009. http://www.gallup.unm.edu/˜smarandache/DSmT.htm
[25] F. Smarandache, J. Dezert, J.-M. Tacnet, *Fusion of sources of evidence with different importances and reliabilities*, submitted March 2010.
[26] Ph. Smets Ph., *The Combination of Evidence in the Transferable Belief Model*, IEEE Trans. PAMI 12, pp. 447–458, 1990.
[27] J.-M. Tacnet, M. Batton-Hubert, J. Dezert, *A two-step fusion process for multi-criteria decision applied to natural hazards in mountains*, Int. Workshop on Belief Functions, Brest, France, April 2010.
[28] D. Von Winterfeldt, W. Edwards *Decision analysis and behavioral research*, Cambridge Univ. Press, USA, 1986.