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A PARAMETERISATION OF VERTICAL PROFILE OF SOLAR IRRADIANCE FOR CORRECTING SOLAR FLUXES FOR CHANGES IN TERRAIN ELEVATION

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ABSTRACT

This paper deals with the modelling of vertical profile of solar irradiance for correcting solar radiation data. An approximation of the influence of the terrain elevation on all-skies radiation and a parameterisation of vertical profile on clear-skies radiation are proposed. This parameterisation, called double-z fitting function, is validated using two radiative transfer models. The main applications of this work are: computing clear-sky irradiances from satellite images, extrapolation of irradiances from one site to another of different altitude and correction of databases derived from satellite images.

1. NOMENCLATURE

\( z, z_0, z_H \) altitude (km). Altitude is defined above mean sea level
\( I_0 \) irradiance at the top of the atmosphere (W m\(^{-2}\))
\( I(z), B(z) \) global, direct irradiance at altitude \( z \) on horizontal plane (W m\(^{-2}\))
\( I_c(z), B_c(z) \) global, direct irradiance at altitude \( z \) on horizontal plane under clear sky (W m\(^{-2}\))
\( f_I, f_B \) vertical profile irradiance fitting functions for global, direct irradiance
\( A(z), A_g(z) \) attenuation coefficients of the atmosphere for global, direct irradiance (unitless)
\( \alpha, \alpha_g \) parameters of the double-z fitting function (km\(^{-1}\))
\( K_c \) clear-sky index. It is equal to the ratio of \( I \) to \( I_c \) (unitless)
\( \text{vis} \) visibility at ground level (km)

2. INTRODUCTION

The surface solar irradiance (SSI) - also called downwelling shortwave irradiance - is an important element of the surface radiation budget. It is seldom measured and several works have developed methods for the assessment of the surface solar irradiance from images taken by geostationary or polar-orbiting satellites [1, 2]. The various Heliosat methods [3 - 10], the methods of Mueller et al. [11], Perez et al. [12], Raschke et al. [13] or Stuhlmann et al. [14] are based on the principle that the irradiance at a given pixel can be computed by reducing the irradiance that should be observed under clear-sky at that pixel by a certain amount that is a function of the properties of the clouds. Such satellite-derived irradiance data is currently used in various domains, ranging from climate to materials weathering [15].

The computation of the SSI from satellite images calls upon a digital terrain model (DTM) whose cell size fits that of the pixel. For example, the ESRA atlas [16] or the HelioClim-1 database [8] exploits the DTM TerrainBase [17] whose cell size is 5° of arc angle, i.e. approximately 10 km at mid-latitude. The size of the cell is even larger for the NASA-SSE database: 1° of arc angle [18] or for the ISIS database: 280 km [19]. Generally, these sizes are too large to describe changes in altitude with a sufficient accuracy in areas of steep relief; large discrepancies can be found between the mean altitude of a cell and the altitude of a particular site within this cell. For example, in Switzerland, the altitude of the measuring station at Saentis Mountain is 2490 m while that provided by the DTM TerrainBase is 1126 m, i.e. an underestimation of 1364 m. If uncorrected, this difference leads to an error in the irradiance provided by the ESRA and HelioClim databases. Note that in very steep relief, irradiance depends upon shadows cast on the sites by surrounding obstacles and not only on change in altitude; this is not the subject of this paper.
Wahab et al. [20] established, for monthly irradiation, that when the difference in altitude reached 200 m, the error on estimated irradiation is significant and conclude that the correction for altitude should be applied in a systematic way. The main objective of this paper is to propose a parameterisation of the vertical variation of SSI with altitude z. Most models for SSI under clear-sky take into account change in SSI with z, whether they are in analytical form [16, 21, 22] or in numerical form such as libRadtran, LOWTRAN or MODTRAN. Laue [23] performed measurements of solar spectral irradiance at different altitudes. Blumthaler et al. [24], Dvorkin and Steinberger [25] focused their work on the UV part of the radiation. Hofierka and Suri [26] propose an empirical relationship between the Linke turbidity factor and the altitude; the corrected factor may then be used in a clear-sky model such as ESRA. No model describing in an explicit manner how the total irradiance changes with z in clear- or all-skies was found in the literature. For practical reasons, it would be convenient to have a simple parameterisation, preferably in analytical form. This would allow e.g., easy implementation in spreadsheet or fast calculation for demonstration and education purposes. It would help in benefiting of the availability of DTM at high spatial resolution such as SRTM: 30 or 90 m [27] to increase the accuracy of the assessment of SSI at a given site. Other applications are possible and three of them are discussed hereafter.

This paper proposes a parameterisation of the vertical profile of the global and direct irradiances. We firstly establish that, in first order, the influence of elevation on SSI in all-skies is equivalent to that in clear-skies. This parameterisation requires solar irradiance at two different altitudes as inputs that can be provided by codes simulating the radiative transfer in atmosphere (RTM: radiative transfer model) or other means. The parameterisation is validated by the results produced by the two RTM: libRadtran and 6S. These RTM are well known and serve as references. The RTM libRadtran was originally developed for modelling UV irradiance [28, 29]. Its accuracy has also been demonstrated for actinic fluxes [30, 31] and total irradiance [11, 19, 32]. The RTM 6S is largely used in processing and simulation of satellites images [33, 34]. Contrary to libRadtran, 6S deals only with clear-sky case.

3. APPROXIMATION OF INFLUENCE OF TERRAIN ELEVATION ON ALL-SKIES

The main factors influencing the SSI under clear-sky \( I_c(z) \) received at an altitude \( z \) are the ground albedo and optical path. The optical path is mainly influenced by clouds, aerosols and water vapour. Note that this paper deals with total irradiance, i.e., the irradiance received for all wavelengths less than 4 \( \mu \text{m} \). The adjective total is omitted in the following. The ground albedo has an impact on the diffuse component of the irradiance; the greater the albedo, the greater the diffuse component. This impact decreases as the altitude \( z \) increases. The optical path is a function of the altitude. As the altitude increases, the thickness of the part of the atmosphere under concern decreases, the optical path decreases leading to a decrease in the attenuation of the radiation, and the downwelling irradiance increases. Besides the purely geometrical effect induced by the reduction of the geometrical path length, the increase in irradiance with \( z \) is due to the fact that atmospheric components influencing the radiation under clear skies - water vapour, aerosols - are mostly located in the lower parts of the atmosphere [37].

On many occasions, one wants to know the SSI \( I(Q) \) at a site \( Q \) of altitude \( z(Q) \) but has only observations of SSI \( I(P) \) at a site \( P \) of altitude \( z(P) \). Practically, this is the case of a measuring site on a mountain side \( P \) and a site of interest located in the valley \( Q \) or reciprocally. This can be extended to the case where \( I(P) \) results from spatial interpolation of neighbouring measuring sites of similar altitude. In this application, the possible exploitation of the fitting functions to all-skies case is discussed. The optical state of the atmosphere, including cloud properties, is assumed the same at \( P \) and \( Q \). This means that if \( P \) and \( Q \) were having the same altitude, the SSI \( I \) at \( P \) and \( Q \) would be equal. It is believed that this assumption is better verified for daily or monthly means of irradiance than for shorter periods of integration. If the clear-sky value \( I_c(P) \) at \( P \) is known, e.g., by a clear-sky model, the clear-sky index \( K_c \) is defined as:

\[
I(P) = K_c(P) I_c(P) \tag{1}
\]

Thus, the change in \( I \) due to change in altitude \( z \) is given by

\[
\frac{\partial I}{\partial z} = K_c \frac{\partial I_c}{\partial z} + I \frac{\partial K_c}{\partial z} \tag{2}
\]

LibRadtran is exploited with many different atmospheric conditions to compute each term in Eq. 2. Figure 1 exhibits examples of the changes in SSI and \( K_c \) as a function of the cloud optical depth (COD) for
various altitudes. These curves are quasi-independent on the solar zenith angle, visibility, precipitable water, and atmospheric profile; the COD is the major influencing factor.

Figure 1. Example of the influence of the cloud optical depth on all-sky irradiance (I) and on clear-sky index (Kc), for different altitudes (z). In these simulations, the water cloud is located between 4 km and 6 km. Clear-sky inputs are solar zenith angle (30°), precipitable water (15 kg m⁻²), visibility (60 km), number of the day in the year (170), and ground albedo (0.2).

As expected, I and Kc decrease as the COD increases. For a given COD, I increases with the altitude. This is not the case for Kc; it remains quasi-constant for altitude from 0 km to 3 km. This means that ∂Kc/∂z is small. The relative contribution of the rightmost term in Eq. 2: I, ∂Kc/∂z to ∂I/∂z is now further investigated. To that purpose, derivatives are written in a discrete form:

\[ \frac{\partial I}{\partial z} \approx \frac{(I(z_2) - I(z_1))}{(z_2 - z_1)} \]  

(3)

and

\[ I, \partial Kc/\partial z \approx \frac{(I(z_2) + I(z_1))}{2} \left[ \frac{Kc(z_2) - Kc(z_1)}{(z_2 - z_1)} \right] \]  

(4)

where \( z_2 = z_1 + 1 \) km.

Figure 2 displays the comparison between these two terms as a function of the COD.

Figure 2. Comparison between ∂I/∂z (full lines and left axis) and I, ∂Kc/∂z (dashed lines, right axis) for different cloud optical depth and altitudes (z). Other inputs are similar to those in figure 1.

\[ \partial I / \partial z \] reaches its maximum for COD close to 3 and then decreases as the COD increases. \( I, \partial Kc / \partial z \) reaches its maximum for COD between 4 and 6, depending on the altitude. It then decreases as the COD increases. As a whole, the values of \( I, \partial Kc / \partial z \) are approximately 10 times lower than \( \partial I / \partial z \). This figure shows that the relative contribution of \( I, \partial Kc / \partial z \) to \( \partial I / \partial z \) decreases as the altitude increases and as the COD increases beyond values 4-6. In the studied cases, this relative contribution may reach approximately 20% but most often is less than 10%. This means that in most cases, the term \( I, \partial Kc / \partial z \) may be neglected at first order.

Eq. 2 becomes:

\[ \frac{\partial I}{\partial z} \approx Kc \frac{\partial I}{\partial z} \]  

(5)

or in a discrete form

\[ \frac{I(Q) - I(P)}{z(Q) - z(P)} \approx \frac{(Kc(Q) + Kc(P))/2}{(I(Q) + I(P) - I(P) - I(Q)) / (z(Q) - z(P))} \]  

(6)

It follows:

\[ I(Q) - I(P) \approx Kc(P) \frac{I(Q) - I(P)}{z(Q) - z(P)} \]  

(7)

Using Eq. 1 for \( I(P) \)

\[ I(Q) = Kc(Q) I(P) \]  

(8)

If an analytical function \( f(z) \), describing the vertical profile of the SSI in clear sky \( Ic(P) \), is available, then the irradiance \( I(Q) \) at site \( Q \) is easily computed as:

\[ I(Q) = Kc(Q) Ic(P) f(z, z_0, z, Ic(z_0)) \]  

(9)

Thus, the function used to model the vertical profile in clear-sky can be used in all-skies conditions at first order.

4. PARAMETERISATION OF VERTICAL PROFILE IN CLEAR-SKY

As a consequence of the previous conclusion, at follows that the search for a parameterisation of vertical profile of SSI can be restricted to clear-sky case.

Figure 3 shows an example of vertical profiles of global \( Ic \) and direct \( Bi \) irradiances. Both profiles are superimposed at high altitude where the scattering effects are small and consequently, the diffuse
irradiance is small. Close to sea level \((z = 0)\), both profiles differ because of the increasing role of the scattering effects. Most of the photons removed from the direct beam end up as diffuse irradiance. Global irradiance is therefore always larger than direct irradiance. This figure clearly shows that the direct and global components decrease with \(z\). For example, there is a decrease of 30 W m\(^{-2}\) in direct irradiance between 1 km and 0.5 km.

Figure 3. Example of vertical profiles of direct and global irradiances. These profiles have been computed with the radiative transfer model libRadtran. Main input parameters were solar zenith angle (30°), precipitable water (15 kg m\(^{-2}\)), visibility (60 km), number of the day in the year (170), and ground albedo (0.2).

We are looking for analytical functions \(f_d\) and \(f_b\) that model the vertical profile of global \(I_d(z)\) and direct \(I_b(z)\) irradiances under clear-sky at any altitude \(z\) starting from known values of \(I_d(z_0)\) and \(I_b(z_0)\) at an altitude \(z_0\):

\[
I_d(z) = I_d(z_0) f_d(z, z_0, z) \quad (13)
\]

\[
I_b(z) = I_b(z_0) f_b(z, z_0, z, B_0) \quad (14)
\]

where \(f_d\) is the quantity to be determined for the global irradiance \(I_d\), \(f_b\) that for the direct component \(I_b\), and \(I_0\) the irradiance at the top of the atmosphere. Our work is based on the assumption that within the range of terrain altitude on the earth: 0 to 8 km, the vertical profile of irradiance follows an exponential form. This assumption is supported by the law of Beer-Bouguer-Lambert and by the ESRA model [16]. Fig.3 illustrates the soundness of this assumption. Note that this assumption is more appropriate to the direct irradiance than to the global irradiance.

Knowing two values of clear-sky irradiances \(I_d(z_0)\) and \(I_b(z_0)\) at two different altitudes \(z_0\) and \(z_0\) allows to define the parameters of a fitting function in a deterministic way. This fitting function is called “double-z fitting function” because it needs as inputs the clear-sky irradiances at two altitudes. Two attenuation coefficients, \(A(z)\), for global and, \(A_d(z)\), for direct irradiances are defined:

\[
I_d(z) = I_d(0) (1 - A(z)) \quad (15)
\]

\[
I_b(z) = I_b(0) (1 - A_d(z)) \quad (16)
\]

It is assumed that \(A(z)\) and \(A_d(z)\) have the following forms:

\[
A(z) = A(z_0) \exp[-\alpha(z - z_0)] \quad (17)
\]

\[
A_d(z) = A_d(z_0) \exp[-\alpha_d(z - z_0)]
\]

Then \(A(z_0)\) and \(\alpha\) are determined from Eqs 14 and 15:

\[
A(z_0) = I_d(z_0) / I_d(0) \quad (18)
\]

\[
\alpha = -\ln[(I_d(0) - I_d(z_0)) / I_d(0)] / (z - z_0) \quad (19)
\]

where \(z_0\) has been empirically defined, in km, as:

\[
z_0 = \max(3, z_0 + 2)
\]

after several trials.

The same equations hold for \(A_d(z_0)\) and \(\alpha_d\), where \(B_d(z_0)\) and \(B_d(z_0)\) are the direct clear-sky irradiances at \(z_d\) and \(z_0\):

\[
A_d(z_0) = I_d(z_0) / I_d(0) \quad (20)
\]

\[
\alpha_d = -\ln[(I_d(0) - B_d(z_0)) / I_d(0) - B_d(z_0)] / (z_d - z_0)
\]

This parameterisation would allow e.g., easy implementation in spreadsheet or fast calculation for demonstration and education purposes. An alternative to the analytical parameterisation is a method based on look-up tables or abacus. The irradiances \(I_d(z)\) and \(I_b(z)\) are computed by invoking libRadtran for a large number of cases that are representative of all expected cases. Great attention should be brought to the description of aerosol properties as they are presently not well known and have a crucial importance on the clear sky irradiance [19, 35]. An example of these representative cases can be the following:

- every 1 km in altitude, from 0 km to 9 km,
- every 5° in solar zenith angle, from 0° to 75°,
- every 10 km in visibility, from 10 km to 110 km,
- four types of aerosol in the lower 2 km of atmosphere: rural, maritime, urban and tropospheric,
- every 10 kg m\(^{-2}\) in precipitable water, from 10 kg m\(^{-2}\) to 80 kg m\(^{-2}\),
- six atmosphere profiles: tropical, midlatitude summer, midlatitude winter, subarctic summer, subarctic winter and U.S standard,
- and every 0.2 in ground albedo, from 0 to 0.8.

The results are stored in look-up tables. For any set of inputs, the irradiances are computed by interpolation of
the known values stored in the look-up tables. Such a method would be similarly accurate and fast.

Both solutions have benefits and drawbacks. The analytical parameterisation is easy to handle and to implement. But, observations at two different altitudes of clear-sky irradiances at the same instant and same place are very rare. The look-up tables are cumbersome to compute and then to implement in the routine operations. For efficiency, the look-up tables -whose size would be approximately 200 Mbytes- should be stored in computer memory. The accuracy of the analytical parameterisation cannot be increased, except if the parameterisation is changed. The accuracy in the look-up tables can be improved by decreasing the quantification step in all inputs at the expenses of the increase of the initial computation and of the size of the look-up tables. If new descriptions of aerosol properties are available, the analytical parameterisation adapt itself to changing conditions while the look-up tables should be recomputed for the new conditions and re-implemented in the processing software.

5. COMPARISON WITH TWO RTM

The outcomes of this double-z fitting function are compared for several altitudes to the outcomes of libRadtran which serves here as a reference. LibRadtran was run for several input conditions, e.g., different solar zenith angle (0, 10, 20, 40, 50, 65, 70, 75, 80) deg, four different aerosol types, different visibilities (15, 20, 25, 30, 40, 50, 60, 80, 100) km, different amounts of precipitable water (10, 15, 30, 60, 80) kg m$^{-2}$, different standard atmospheres and different ground albedos (0.0, 0.2, 0.3, 0.6, 0.9). Similar runs were made with 6S. Similar conclusions and values were obtained, ensuring that the comparison is not biased by the use of a single RTM.

Fig. 4 illustrates the performance in the form of the relative deviation in percentage, between the vertical profiles (from 0 km to 9 km) obtained by the fitting function and those obtained by libRadtran, in the case where $z_0$ is set to mean sea level. As a whole, it is observed that the relative deviation is larger for the direct irradiance than for the global irradiance. There is a dependency of the deviation with the altitude with no clear trend. Quite often, the deviation decreases when the visibility increases but not always. The visibility and solar zenith angle are the most influencing factors on the vertical profile and therefore on the performance of a fitting function to reproduce the profile. The amount of precipitable water has little effect on the deviation.

Fig. 5 shows that there is little influence of the solar zenith angle on the deviation. Deviation is a function of the altitude. The deviation tends to increase when $z_0$ increases. But it remains very small, between –0.2 % and 0.2 % for altitudes up to 5 km, for standard atmospheric conditions and a solar zenith angle of 30°.

Generally, the relative deviation is less than 1 % (global) or 2 % (direct) for current visibilities, i.e. greater than 30 km (WMO 1981). It may amount to respectively 3 % and 5 % for very low visibilities of 15 km. For larger visibilities, the relative deviation is
almost zero for altitudes from sea level up to 6-7 km (Fig. 4). Given current visibilities and solar zenith angles less than 75°, the relative deviations compare favourably to typical deviations on ground measurements, which are 5% to 7% [36].

Since in practice, discrepancies on altitude are generally less than 1 km, it is of interest to evaluate the ability to correct the irradiance assessed incorrectly at a given altitude \( z_0 \) while the actual altitude is \( z = z_0 + 1 \) km. Given the irradiance \( I_c(z_0) \) and \( I_c(z_H) \), the double-z fitting function is applied to compute an estimate \( I^*_c(z) \). This estimate is then compared to the correct value \( I_c(z) \) given by libRadtran and the difference is computed. This comparison is performed for various atmospheric conditions and various \( z_0 \). Table 1 shows example of the deviations reached by the double-z fitting function for different \( z_0 \). One can see that the double-z fitting function exhibit small relative deviations, often less than 1%, for global and direct irradiances. It is concluded that this function may be used to correct satellite-derived SSI.

### Table 1

<table>
<thead>
<tr>
<th>( h_0 ) (km)</th>
<th>Direct irradiance</th>
<th>Global irradiance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>vis 30</td>
<td>vis 60</td>
</tr>
<tr>
<td>0</td>
<td>-0.6 (-0.1 %)</td>
<td>-1.5 (-0.4 %)</td>
</tr>
<tr>
<td>1</td>
<td>-1.2 (-0.3 %)</td>
<td>-0.8 (-0.2 %)</td>
</tr>
<tr>
<td>2</td>
<td>-2.0 (-0.4 %)</td>
<td>-0.7 (-0.1 %)</td>
</tr>
<tr>
<td>3</td>
<td>-0.7 (-0.1 %)</td>
<td>0.9 (0.2 %)</td>
</tr>
<tr>
<td>4</td>
<td>0.2 (0.0 %)</td>
<td>1.1 (0.2 %)</td>
</tr>
<tr>
<td>5</td>
<td>0.4 (0.1 %)</td>
<td>0.4 (0.1 %)</td>
</tr>
<tr>
<td>6</td>
<td>-0.2 (-0.0 %)</td>
<td>-0.2 (-0.0 %)</td>
</tr>
<tr>
<td>7</td>
<td>-0.5 (-0.1 %)</td>
<td>-0.5 (-0.1 %)</td>
</tr>
</tbody>
</table>

Table 1. Error (in W m\(^{-2}\)) between actual irradiance \( I_c(z_0+1) \) provided by libRadtran and the irradiance predicted by fitting functions. The relative deviation is indicated in brackets. Two visibilities are reported: 30 km (vis 30) and 60 km (vis 60). Solar zenith angle is 60°. Other conditions are those in Fig.3.

### 6. CONCLUSION

This study demonstrates that it is possible to find fitting functions that reproduce the vertical profile of the global and direct irradiances under clear-sky with sufficient accuracy. The possibility of using look-up-tables for the same purpose is also discussed. It is shown that in first order, the influence of altitude on all-sky irradiance is equivalent to the influence of altitude on the corresponding clear-sky irradiance. It follows that any fitting function developed for clear-sky can be used in all-sky conditions. It is also found that the double-z fitting function might be used to correct irradiance with a sufficient accuracy in the case where irradiance is incorrectly assessed because of a deviation in altitude. This result can be exploited in the case of extrapolating irradiance measuring at a station to a close site of different altitude and also in the case of design a fast method for processing satellite data based on RTM.

### REFERENCES


