On the relevance of percolation theory to the acquisition of human skills

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ABSTRACT

Knowledge can be seen as a complex system that does not reveal clearly how it works. However, some mathematical models have proved day after day their relevance to the description of complex phenomena in fields that are apparently unrelated.

This is especially the case for Percolation Theory.

After a survey of the percolation mechanism, it will be seen that Percolation Theory has proved to be relevant in a number of cases. From problems in physics or chemistry to questions in social sciences, Percolation Theory offers an unified method to approach the problems in a systematic way.

Thus, the paper examines the assumption that knowledge defines a network inside the brain, where sites are knowledge bricks that are connected (or not-connected) by p-efficient binds. This allows us to use the Percolation Theory as a model for the way our brain adopts to solve a problem (using implicitly this knowledge). It also gives a good idea of what happens when the human « insight » occurs, from the Archimede's cry « Eureka » to the "I have understood it" of our students.

As a conclusion, we illustrate the contribution of this theory, especially concerning the design of pedagogic scenarios.

Keywords: Percolation Theory, knowledge clusters, human insight, fulguracy, sciences of complexity, cognition, problems resolution.

1. INTRODUCTION

The development of technologies related to education and training systems has revealed the poorness of our knowledge about... knowledge! We don't know what « knowing » means. We do not know even less how this knowledge is used to solve a problem.

Of course, the process of human knowledge is complex; it seems occasionally chaotic and incomprehensible. And we measure nowadays how information systems, more often than not limited to collections of structured data, are removed from the knowledge process of humans.

In order to improve the efficiency of information technologies in education and training, it is relevant to try to understand (or only try to approach) how the knowledge works and is used.

Some recent studies have made considerable progress in this direction. The *Conceptual Blending* of Gilles Fauconnier and Mark Turner can be cited as one of the most famous model able to explain *The Way We Think* [1]. In another paper, I identified it as one of the processes that distinguish the human thinking from representations accessible to computers or animals [2].

The sciences of complexity and their new tools and models renew the perception of chaotic systems. Applied to cognition sciences, they allow us to approach *The Way We Use Knowledge*. This paper examines the relevance of an intuition: is the Percolation Theory suitable to understanding the acquisition of human skills?

2. PERCOLATION THEORY (PT)

Percolation Theory (PT) represents one of the simplest models of a disordered system. The first part of the paper presents a survey of the percolation mechanism.

Definition of percolation
The term « percolation » comes from the Latin word *percolatio*, meaning filtration.

Used in a wide range of situations, it evokes notions of propagation and agglutination in random environments partially interconnected.

Percolation in a coffee machine
A percolator is a coffee machine where strength of the desired coffee is adjusted by pressing the filter mechanism more or less closed. When the pressure inside the filter increases, density of coffee-grounds increases too due to the agglutination of the fine particles of coffee.

However, there is a limit density: beyond a given limit value, the water cannot cross through the coffee powder any more. This limit density is called the *percolation threshold.*

The formal problem consists in describing the agglutination of coffee particles related to the density of coffee-grounds, and characterizing the propagation of water through this random and non-homogeneous environment [3]. This phenomenon can be modelized by a network of channels between coffee particles. Each channel is then randomly open or close [4]. When the density increases, more of the channels are closed and the water crosses through with more difficulty. The percolation threshold is reached when there is no path left allowing the outflow of water through open channels.

3. EXAMPLES OF SOLVING PROBLEMS WITH PT

There are a number of applications of the PT. Some of these are cited in the second part of the paper.

Transition between archipelago of islands and continents
One application of PT refers to modelling forest fires, as a variation of the transition between archipelago of islands and continents [5].

On illustration 1 below [6], we see that a person being on one of the islands stays on this first island, unable to walk on the sea.

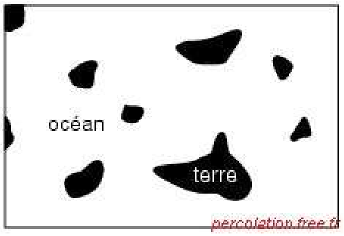

Illustration 1

When the level of the ocean decreases, the surface of islands increases and some of them join themselves, as seen on figure below.

Illustration 2

When, the level of the ocean reaches the limit value, appears a continent with a lot of lakes, but that the walker can cross without meeting any barrier of see.

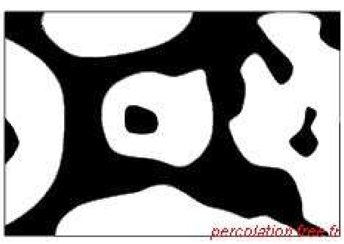

Illustration 3

Transition between insulator and conductor
PT has obviously been applied in physics or chemistry.

Let us consider a square lattice, where each site is occupied randomly with probability p or empty with probability $1-p$. Occupied and empty sites may stand for very different physical properties. For simplicity, let us assume that the occupied sites are electrical conductors, the empty sites represent insulators, and that electrical current can flow between the nearest neighbouring conductor sites. At low concentration p, the conductor sites are either isolated or form small clusters of the nearest neighbouring sites. Two conductor sites belong to the same cluster if a path from the nearest neighbouring conductor sites connects them together, and a current can flow between them. On the one hand, at low p values, the mixture is an insulator, since a conducting path connecting opposite edges of the lattice does not exist. At high p values, on the other hand, many conductive paths between opposite edges exist, where electrical current can flow, and the mixture is a conductor. Therefore, at some concentration in between, a threshold concentration p_c must exist where for the first time electrical current can percolate from one edge to the other. Below p_c, we have an insulator; above p_c we have a conductor. The threshold concentration is called the *percolation threshold*, or, since it separates two different phases, the *critical concentration*.

4. PT APPLIED TO COMMUNICATION FAILURES

Let us consider a communication network composed with n *sites*, where each site is connected with their neighbours by optical binds for instance whose efficiency is *p-random*.

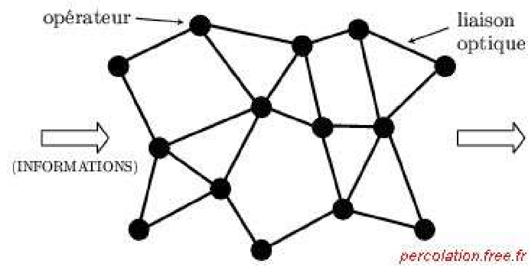

Illustration 4

Bond percolation
One can assume that a proportion *1-p* of existing binds is destroyed randomly, due to different kind of problems (poor maintenance and repair, random act of sabotage, disturbances or clamping, and so on). It becomes more and more difficult to communicate from a given point to another one in proportion as the network falls into disrepair. The path must then use relays. When the percentage amount of cut bonds increases, above a critical value p_c, there is no more possible to keep two distant sites in relation directly or indirectly.

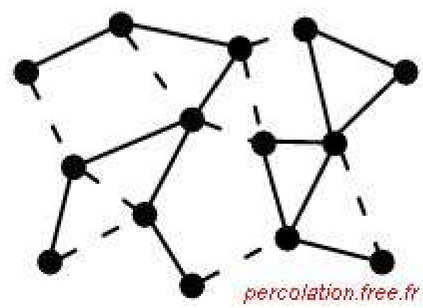

Illustration 5

Site percolation
It may be that bonds are 100% efficient, but there is a risk on the sites with a probability of p. In this case, we speak about site percolation.

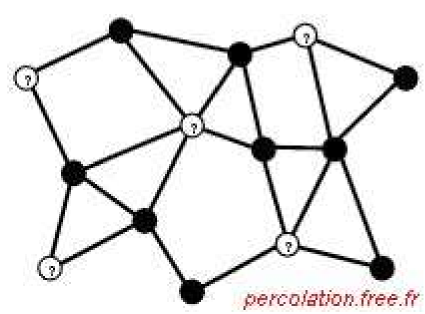

Illustration 6

Site-bond percolation
Finally, there is a site-bond percolation when sites and bonds are together randomly disrepair.

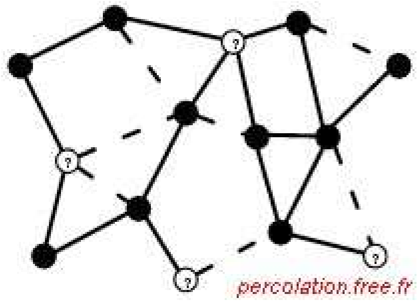

Illustration 7

Application to social sciences
In the field of “social sciences”, the interdisciplinary approach incorporating the “sciences of complexity”, which have emerged in recent years, have offered a possibility of approaching these problems using a wide range of tools that include PT.

In this case, the underlying network on which the percolation takes place is the network of social actors, while the bonds represent the interactions between them. In this way a number of significant results have been obtained such as an explanation of extreme market shares close to either 0 or 100 percent in the media industry. The same approach can be used to study the dissemination of new ideas and beliefs [7].

5. FORMAL REPRESENTATIONS OF PT

More formally, mathematicians consider a point lattice. It is an infinite array of discrete points with an arrangement and orientation that appears exactly the same, from whichever of the points the array is viewed.

One can see that the lattice in illustration 8 fulfills this condition.

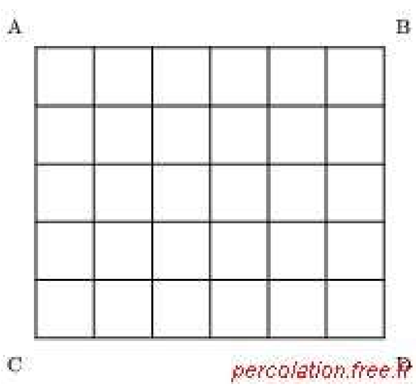

Illustration 8

Figure 9 that follows illustrates a site percolation.

Illustration 9

While illustration 10 shows a bond percolation.

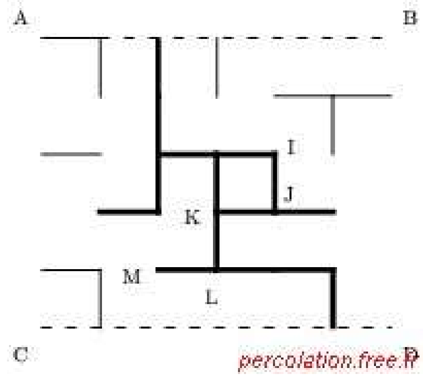

Illustration 10

6. CONTRIBUTION TO KNOWLEDGE PROCESS

We will now see how PT can be employed to describe knowledge processes.

Knowledge fragments
Here and there across the literature appears the notion of *unit*. Some authors speak about *information units* [8], others about *bricks* [9]. We prefer the term of *fragment*: it indicates a coherent and homogeneous granular parcel of knowledge that can conceptually be isolated and related to other granular fragments.

Adopting this notion is equivalent to admitting that knowledge is not continuous. So, it is possible to structure it by digitalizing its contents into a set of granular parcels and/or fragments.

At the moment, we have not said a word about the dimension of fragments, because they can be more or less small or extended, depending on the required granularity of the sharing. This aspect of the question does not influence the reasoning at this stage of it.

What is more important is to notice than fragments are not only packets of declarative data, but can also include procedural descriptions.

Global knowledge as a network between fragments
Let us now consider the assumption that global knowledge defines a network inside the brain, where sites are information or knowledge fragments that are connected (or not-connected) by *p-efficient* binds.

Then, solving any kind of problem (resolution of a mathematical problem, problem of creation, searching for a new idea, problem of management, practical problem...) can be seen as passing from the initial position (hypothesis, initial facts and assumptions) to the final position (solved problem).

Analogy with PT representations
As soon as solving a problem (finding a solution) has been formulated as finding a path between two distant sites, in relation directly or indirectly, we see an analogy with the problem of communication failures presented in part 4 and illustrated by figures from 4 to 7.

On the other hand, assuming that initial position corresponds to an edge of a square lattice and final position corresponds to the opposite edge, we can then represent the solving of a problem (implicitly using knowledge) as crossing through from one edge to the opposite edge of the lattice. So figures from 8 to 10 are relevant too.

Results obtained by applying PT
If we apply PT until its end, then PT shows that this is possible even if there are numbers of connected knowledge fragments from one edge to the opposite one, and if p is greater than or equal to p_c. Less than this value, a path cannot be found through the knowledge fragments.

Adopting this assumption shows that there are three ways to increase competency, i.e. the ability of finding a way to solve a practical problem by use of knowledge:
1)adding new fragments in the field of necessary knowledge,
2)adding new binds from one each fragment to their neighbours,
3)increasing the *p-efficiency* of the set of binds to pass above p_c.

The first important result is that knowledge is not only an amount of knowledge fragments: these fragments must be connected. Binds and connections are even more important than fragments themselves. Because binds must exist, but furthermore they must be efficient: here is the second important result.

Application to instructional contexts
Our reflections explain and make clear some difficulties in the learning process that teacher sometimes solve in an intuitive way, as tricks of the trade, and without knowing why it works.

For instance, learners can be unable to find a solution to a given problem even if they certainly have the necessary knowledge. This diagnosis means that binds are missing, or are less efficient. The pedagogical process must then increase the binds and their efficiency. In this case, it is unprofitable to add new fragments of knowledge.
In another case, when there is cognitive dissonance [10], it means that a channel between two neighbouring sites is definitely closed, and the value of *p-efficiency* is equal to 0. In this case, it is impossible to open the channel directly (because the learner thinks it is impossible: here is precisely the definition of a cognitive dissonance), and it is more profitable to proceed by finding another path between the two sites, using relays. It is the right way to make a diversion, succeed and go beyond the difficulty.

7. PT: A MODEL OF INSIGHT

Some authors have worked on the inventive process. They have proved that it is composed of four separate periods [11].

Impregnation period
In this first stage, the inventor assembles a highest number of data, observations, information, documents. The person absorbs all the elements like a sponge.

Incubation period
According to Hubert Jaoui: “The subject bears the object of his/her research, as a mother bears fecundated ovum, future embryo. This period is essentially unconscious. Inopportune interventions can disturb this stage, even prevent it. It can be favoured by a best dialog between the conscious and the unconscious. The length of this period is unpredictible”.

Bringing forth period
Always Hubert Jaoui says: “this period gives birth to the idea. One employ currently the term of insight. Most of inventors have spoken about a blinding evidence of the idea when it appears. A second before, they were in darkness and suddenly, the idea emerges perfectly accomplished”.

Validation period
After insight, it is necessary to prove that the idea is relevant. The last period changes an invention into innovation by validation. It is a more applicative phase, less creative, more modest, and sometimes longer.

PT as a model for inventive process
We can interpret those four periods in relation with Percolation Theory in the following way.

The *impregnation period* corresponds to the forging of information or knowledge units.

The *incubation period* corresponds to the construction of a structured organisation inside the network of units. It takes the form of depositing binds, with correct *p-efficiency* corresponding to each of them.

The *bringing forth period*, as described by Hubert Jaoui, is just similar to a percolation event, We have called the human “insight”, “fulguracy” (from the verb “fulgurate”, to flash like lightning, The New Collins Concise English Dictionary 1982) in other works. It is the cry of “EUREKA” of Archimedes, or the “I have understood it” of students. Finally, it is concretely supposed to occur when increasing p value starts to reach the threshold value p_c. It appears when data percolate inside the environment. It is just the discovery of a path from one site to another when p_c value is reached.

What about the computer?
As soon as we employ a network with sites and binds, we think of information systems, hypertexts and hypermedia, and the Internet.

So a question emerges: is the computer able to support PT? Information units or fragments are the basic components of hypermedia applications. The World Wide Web nowadays offers a collection of knowledge fragments, so large than it can be considered as infinite. The computer is then a good tool to help *impregnation period.*

Some of these fragments are bound together. But the informatics bind is very primitive: it exists or not (1 or 0). It would be interesting to implement binds with a p value affected to their efficiency. The question is then: is there a method to measure efficiency? What is efficiency? How can we determine a value for p?

We can observe that what occurs during the *incubation period* in the human brain is also extremely unknown. This paper reveals fertile tracks of research concerning this domain. A method like mental maps tries to approach the binding [12]. However, it is rather primitive, and represents too much summarily the human process of the "association of ideas".

In this paper, "binds" or "bonds" have been employed indifferently. The Concise Oxford Dictionary (9th edition, Oxford University Press, 1997-1999) says that a bind "fasten or hold together as a single mass, tie or fasten tightly" whereas a bond is "a uniting force, a restraint, a responsibility, a binding engagement, an agreement". In a building for instance, a bond designates the laying of bricks in one of various patterns in a wall in order to ensure strength. Let us assume that the bind between elements of a system can be seen as the means to create various types (of mental) bonds for the human user. In this context, a "bond" can be associated with the meaning that a human being gives to a particular activity. It is therefore important not to confuse the means and the ends i.e. the binds and the bonds.

As we have shown in other papers [2], the *bringing forth period* and the *validation period* are not accessible to the computer and remain specifically human.

8. CONCLUSION AND PERSPECTIVES

In this paper, we have considered the contribution of the Percolation Theory to the acquisition of human skills, especially concerning the design of instructional hypermedia scenarios.

To use this method, some fundamental assumptions need to be verified:

- The phenomenon must take place in a space containing a great number of elements.
- The relation between elements is founded on a local property, such as contiguity or another dimension of "proximity"
- This relation between elements can be characterised randomly.

Taking into account these hypotheses, Percolation Theory describes the appearance of a critical phenomenon at a global level. Below the percolation threshold, the relation is limited to the island where it has been initiated. Above this limit value, the transmission « percolates » through the whole environment.

The percolation characterises a state variation, a transition of phasis. It can explain some particular aspects of human learning or thinking as cognitive dissonance or fulguracy/insight.

As a conclusion, the advantage of the Percolation Theory is its ability to bring valuable light to the role of modern technology vis-à-vis that of knowledge seeking human beings. On the one hand, it takes into account the fact that computers cannot do everything for humans, but instead can facilitate certain tasks in the knowledge acquisition process of humans, notably in phases 1 and 2 of our insight model. On the other hand, the theory outlines what is expected of humans in terms of the various tools and resources at their disposal, namely phases 3 and 4 of the model. There is thus a clear interdependency between tools and humans as long as the role of each is defined and regularly re-evaluated. For in reality with the advances of technology and the varying needs of humans, the frontiers between the living and artefact is not always where we expect them to be. It is in this that the Percolation Theory can facilitate our understanding of what we are capable of, what we know and, thus, by extension what our tools are capable.

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