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# Defining Yield Policies in a Viability Approach

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## Abstract

Mullon *et al.* (2004) proposed a dynamical model of biomass evolution in the Southern Benguela ecosystem, including five different groups (detritus, phytoplankton, zooplankton, pelagic fish and demersal fish). They studied this model in a viability perspective, trying to assess, for a given constant yield, whether each species biomass remains inside a given interval, taking into account the uncertainty on the interaction coefficients. Instead of studying the healthy states of this marine ecosystem with a constant yield, we focus here on the yield policies which keep the system viable. Using the mathematical concept of viability kernel, we examine how yield management might guarantee viable fisheries. One of the main practical difficulties up to now with the viability theory was the lack of methods to solve the problem in large dimensions. In this paper, we use a new method based on SVMs, which gives this theory a larger practical potential. Solving the viability problem provides all yield policies (if any) which guarantee a perennial system. We illustrate our main findings with numerical simulations.

**Key words:** Viability theory, marine ecosystem, fisheries management, Support Vector Machines.

## 25Introduction

26The viability theory (Aubin, 1991) aims at controlling dynamical systems with the goal to  
27maintain them inside a given set of admissible states, called the viability constraint set. Such  
28problems are frequent in ecology or economics, when systems die or badly deteriorate if they  
29leave some regions of the state space. For instance Béné *et al.* (2001) studied the management  
30of a renewable resource as a viability problem. They pointed out irreversible overexploitation  
31related to the resource extinction. Bonneuil (2003) studied the conditions the prey-predator  
32dynamics must satisfy to avoid extinction of one or the other species as a viability problem.  
33Cury *et al.* (2005) consider viability theory to advise fisheries.

34Mullon *et al.*, (2004) proposed a dynamical model of biomass evolution of the Southern  
35Benguela ecosystem, involving five different groups (detritus, phytoplankton, zooplankton,  
36pelagic fish and demersal fish). They studied this model in a viability perspective (Aubin,  
371991), trying to assess, for a given constant yield, whether each species biomass remains  
38inside a given interval, taking into account the uncertainty on the interaction coefficients. The  
39aim was to identify constant yield values that allow persistence of the ecosystem. We extend  
40the problem and we focus here on the yield policies which keep the system viable, instead of  
41considering a constant yield.

42Using the mathematical concept of *viability kernel*, we examine how yield management might  
43guarantee viable fisheries. The *viability kernel* designates the set of all viable states, i.e. for  
44which there exists a control policy maintaining them within the set of constraints. Outside the  
45viability kernel, there is no evolution which prevents the system from collapsing. Aubin  
46(1991) proved the viability theorems which enable to determine the viability kernel, without  
47considering the combinatorial exploration of control actions series. These theorems also  
48provide the control functions that maintain viability.

49This general approach shows several interesting specific aspects:

- 50 - It can take into account the uncertainties on the parameters which are generally high in  
51 ecosystem modelling. Here, we manage the uncertainties like in (Mullon *et al.* 2004).
- 52 - The viability kernel can define a variety of different policies, which respect the  
53 viability constraints. Therefore, it offers more possibilities for negotiations and  
54 discussions among the concerned stakeholders than techniques which propose a single  
55 optimal policy.

56The main limitation of the viability approach is its computational complexity. The existing  
57algorithm for viability kernel approximation (Saint-Pierre, 1994) supposes an exhaustive  
58search in the control space at each time step. This makes the method impossible to use when  
59the control space is of a 51 dimensions like in our problem. Mullon *et al.* (2004) solved this  
60problem with a method which is only adapted to linear equations of evolution. Here, we use a  
61new method, based on support vector machines, which can be applied to non-linear models as  
62well (Deffuant *et al.* 2007).

63We present the viability model of the Southern Benguela ecosystem and we recall the main  
64concepts of the viability theory. Then, we describe our main numerical results. We show the  
65shape of the found viability kernel, and the corresponding possible yield policies. Finally, we  
66discuss the results and draw some perspectives.

### 67**The viability model of the Southern Benguela ecosystem**

68Following a classical approach (Walters and Pauly, 1997), we suppose that the variation of  
69the biomass of species  $i$  due to its predation by other species  $j$  depends linearly on the  
70recipient and donor biomasses ( $B_j$  and  $B_i$ ), with respective coefficients  $r_{ji}$  and  $d_{ji}$ . The biomass  
71lost by species  $i$  due to the predation by the other species is expressed by equation (1):

$$\frac{dB_i(i \rightarrow)}{dt} = - \sum_j (r_{ji} B_j + d_{ji} B_i). \quad (1)$$

The variation of the donor biomass  $B_i$  due to this interaction takes into account the assimilation of the biomass of other species  $j$ , multiplied by a growth efficiency coefficient (denoted below by  $g_i$ ). Therefore, the biomass gained by species  $i$ , because of its consumption of other species, is expressed by:

$$\frac{dB_i(i \leftarrow)}{dt} = g_i \sum_j (r_{ij} B_i + d_{ij} B_j). \quad (2)$$

For the detritus, the variation of the biomass follows the same principle, but it also integrates the non-assimilated biomass of the other species, except phytoplankton, which is added to the detritus biomass  $B_1$  (multiplied by its growth efficiency  $g_1$ ):

$$\frac{dB_1(non - assimilated)}{dt} = \sum_{j>1} \sum_k g_1 (1 - g_j) (r_{jk} B_j + d_{jk} B_k). \quad (3)$$

The model of the Southern Benguela ecosystem considers trophic interactions (predation, consumption and catch) among 5 components: detritus ( $i = 1$ ), phytoplankton ( $i = 2$ ), zooplankton ( $i = 3$ ), pelagic fish ( $i = 4$ ), demersal fish ( $i = 5$ ). In total, the biomass evolution can be written as follows:

$$\frac{dB_i}{dt} = \frac{dB_i(i \leftarrow)}{dt} - \frac{dB_i(i \rightarrow)}{dt} + \frac{dB_1(non - assimilated)}{dt} - Y_i \quad (4)$$

$$\left\{ \begin{array}{l} \frac{dB_i}{dt} = \frac{dB_i(i \leftarrow)}{dt} - \frac{dB_i(i \rightarrow)}{dt} - Y_i. \end{array} \right.$$

where  $g_i$  is the growth efficiency of species  $i$ ,  $Y_i$  is the yield of species  $i$ . Figure 1 shows the structure of the ecosystem.

85

Mullon *et al.* (2004) take into account the uncertainty on parameters  $r_{ij}$  and  $d_{ij}$ , which is expressed by:

$$r_{ij} \in [\bar{r}_{ij} - \delta r_{ij}, \bar{r}_{ij} + \delta r_{ij}] , d_{ij} \in [\bar{d}_{ij} - \delta d_{ij}, \bar{d}_{ij} + \delta d_{ij}] \quad (5)$$

They consider this model in a viability perspective, in order to study the persistence of the ecosystem and to define the impact of the fisheries. Given a constant yield, they define scenarios which result in a “healthy” system.

Extending the work of (Mullon *et al.*, 2004), we incorporate the fisheries in this study as a control variable of the system, in order to find the yield policies which allow keeping the system viable. To guarantee a perennial system, the viability constraints are defined by:

$$\begin{cases} m_i \leq B_i \leq M_i, \\ y_{\min} \leq Y_i \leq y_{\max}, Y'_i \in [-\delta y, +\delta y], i = 4, 5, \end{cases} \quad (6)$$

where  $m_i$  is the minimum level for the resource,  $M_i$  the maximal biomass that can be contained in the ecosystem,  $y_{\min}$  is the minimum level for yield for demersal and pelagic fish, and  $y_{\max}$  the maximum level. The parameter  $\delta y$  limits the evolution of the fisheries between two time steps. We suppose that the levels of yields of pelagic fish and demersal fish are the same. These constraints, which attain critical values of a “healthy” system allow one to link yield objectives with the principle of ecosystem persistence.

#### The viability analysis control problem and viability kernel approximation

In the viability problem, the controls are the yields on pelagic fish ( $Y_4$ ), demersal fish ( $Y_5$ ), and the uncertainty on coefficients  $r_{ij}$  and  $d_{ij}$ . This means that for any state of the system located in the viability kernel, there exist values of these parameters for which the system remains in the viability kernel at the next time step. Adding the constraints on the derivatives of  $Y_4$  and  $Y_5$  implies to add two dimensions to the state space, which would then be 7. This reaches the current computational limits, therefore, we supposed that  $Y_4 = Y_5 = Y$ . This hypothesis is of course not realistic, but we thought it would nevertheless be an interesting first step.

The viability control problem is to determine a control function:

$$Y_i \in [-\delta y, +\delta y] \quad \text{with } i, j = 1, 2, 3, 4, 5 \quad (7)$$

109which enables to keep the viability constraints (6) satisfied indefinitely. Solving this problem  
110requires to determine the viability kernel, which is the set of states for which such a control  
111function exists.

112Saint Pierre (1994) proposed an algorithm to approximate the viability kernel from the  
113problem defined on a grid but the result is a set of points that is viable and it requires an  
114exhaustive search in the control space, which is not possible in our case because the control is  
115in the dimension 51.

116To approximate the viability kernel of the Southern Benguela ecosystem, we use a new  
117algorithm (Deffuant *et al.*, 2007) (see Appendix 1) which is built on previous work from  
118Saint-Pierre (1994), using a discrete approximation of the viability constraint set  $K$  by a grid.  
119Its main characteristic is to use an explicit analytical expression of the viability kernel  
120approximation, in order to make it possible to use standard optimization methods to compute  
121the control. This analytical expression is provided by a classification procedure, the support  
122vector machines (SVMs) (Vapnik, 1998, Cristianini and Shaw-Taylor, 2000). This algorithm  
123is interesting in the case we study, because the analytical expression of the viability kernel  
124allows to use optimization techniques in order to find the best evolution in high dimensional  
125control spaces.

## 126Numerical simulations

127The donor and recipient control coefficients are derived from a mass-balanced Ecopath model  
128for the ecosystem (Shannon, 2003). We use the evaluation of the parameters provided in  
129(Mullon *et al.*, 2004). Table 1 gives the values of the viability constraint set and we put  
130  $y_{\min} = 0$  tons/km<sup>2</sup> (no catches at all),  $y_{\max} = 5$  tons/km<sup>2</sup> (the minimal level of the biomass of  
131pelagic and demersal fish, corresponding on the maximum constant value tested by Mullon *et*  
132*al.* (2004)),  $\delta y = 0.5$  (which represents a variation of 10% of the maximal yield). The yield for

133others species has been set to 0, except for detritus ( $Y_d = 1000$  tons/km<sup>2</sup>, which correspond of  
134an import of detritus).

135The following figures present some results for given values of biomasses of each species. The  
136boundaries of the axes are the constraints defined on the species represented. The  
137approximation of the viability kernel is represented in grey. Inside the viability kernel, there is  
138at least one viable path which allows keeping a healthy system and outside, there is no  
139evolution which prevents the system from collapsing. We focus here on values of detritus  
140biomass = 2000 tons/km<sup>2</sup> because this ensures the existence of a viability kernel for almost all  
141the values of the others compartments. For a level of detritus biomass = 1600 tons/km<sup>2</sup>, given  
142values of zooplankton and phytoplankton are necessary to guarantee a viable path. For lower  
143detritus biomass, there is no viable path: a threshold of detritus biomass is necessary for  
144ensuring a perennial system.

145In the algorithm used to approximate the viability kernel (Deffuant *et al.*, 2007), we used a  
146grid with 6 points per dimension (46000 points in total) and 1642 support vectors are  
147necessary to define the boundary of the kernel.

148We focus on the effects of fisheries on demersal and pelagic fish.

#### 149Effects of fisheries on demersal fish

150Figure 2 presents a 2D slices of the viability kernel where detritus biomass = 2000 tons/km<sup>2</sup>,  
151phytoplankton = 100 tons/km<sup>2</sup>, zooplankton = 90 tons/km<sup>2</sup> and for different values of pelagic  
152fish biomass. Horizontal axis represents demersal fish and vertical one the fisheries.

153

154The levels of pelagic fish, demersal fish and yield have an influence on the boundary of the  
155viability kernel:



- 156 - For low values of pelagic fish biomass, the demersal fish biomass must not be too high  
 157 and consequently intensive fishery must be avoid (see the circle at the top left of  
 158 Figure 2, Pelagic 5);
- 159 - In the same way, when the biomass of pelagic fish is high, the value of demersal fish  
 160 biomass must not be too low to guarantee a perennial system and some low levels of  
 161 catch must be avoided (see the circle at the bottom of Figure 2, Pelagic 60);
- 162 - For mean values of pelagic fish biomass, there is no restriction about the fisheries.

163 Figure 3 presents a 2D slice of the viability kernel, when detritus biomass = 2000 tons/km<sup>2</sup>,  
 164 phytoplankton = 400 tons/km<sup>2</sup> and zooplankton = 130 tons/km<sup>2</sup>. We note that the viability  
 165 kernel is smaller: a high level of pelagic fish represents a non-viable situation. Again, some  
 166 high and low levels of fisheries must be avoided. In general, when the value of zooplankton is  
 167 higher, the viability kernel is smaller and there is no viable path starting from pelagic fish  
 168 biomass = 60 tons/km<sup>2</sup>.

#### 169 Effects of fisheries on pelagic fish

170 We explore now the impact of fisheries on pelagic fish, keeping the same values for others  
 171 species.

172 Figure 4 presents the viability kernel where detritus biomass = 2000 tons/km<sup>2</sup>, phytoplankton  
 173 = 90 tons/km<sup>2</sup>, zooplankton = 100 tons/km<sup>2</sup> and for demersal fish = 5, 15, 30 tons/km<sup>2</sup>.

174 We notice that fisheries affect the boundary of the viability kernel only when the demersal  
 175 fish biomass is too low: the more the pelagic biomass is, the more the catch can be important.  
 176 However, whatever the level of demersal fish, the level of fisheries must be controlled to  
 177 guarantee a healthy system. For mean values of demersal fish, the system is not viable for low  
 178 and high values of pelagic fish. For high values of demersal fish, the pelagic biomass must not  
 179 be too low to guarantee the persistence of the ecosystem.

180For high values of zooplankton (see Figure 5), the viability kernel is smaller: some values of  
181demersal fish and fisheries are necessary to ensure a viable path:

- 182 - For low values of demersal fish, the level of fisheries must be carefully set; lower and  
183 higher values of catch represent non-viable situation;
- 184 - For mean value of demersal fish, the system is not viable for high values of pelagic  
185 fish;
- 186 - Fisheries have an influence for high biomass of demersal and pelagic fish: a minimum  
187 level of yields is necessary to ensure ecosystem persistence (area surrounded in Figure  
188 5).

#### 189Main results

190Our study illustrates the potential utility of the viability kernel to help the definition of viable  
191fishery policies: given values of the biomass of the five species, the viability kernel provides  
192the levels of catch to avoid. In addition, the viability kernel defines some conditions in which  
193the fisheries can be increased without compromising the viability. We notice that the  
194maximum thresholds for fisheries used by Mullon *et al.* (2004) can also be increased.

#### 195**Discussion and conclusion**

196Solving the viability problem provides all yield policies (if any) which guarantee a perennial  
197system. This study shows that it is possible and interesting to integrate fisheries as a control  
198parameter of a viability problem. We made strong simplifications: we supposed the same  
199yield for the two species, and we should obviously take other parameters into account, like  
200social and economics issues (Mullon *et al.*, 2004). Nevertheless, we think that this work  
201illustrates the potential of the viability approach to help the definition of fishery policies.

202One of the main practical difficulties up to now with the viability theory was the lack of  
203methods to solve the problem in a large number of dimensions. The use of learning

procedures such as SVMs gives this theory a larger practical potential. However, to deal with a problem of six dimensions with the current algorithm can only be done with a very rough precision and several improvements are necessary to get more reliable and accurate results.

Moreover, it will be interesting to define yield strategies which allow the system to come from a non-viable state back to a viable state in minimum time, or minimizing some cost. This relates to the definition of the resilience proposed in (Martin, 2004).

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## 235 Appendix 1: Algorithm of SVM viability kernel approximation

236 We consider a given time interval  $dt$  and we define the set-value map  $G : X \rightarrow X$

$$(8) \quad G(\mathbf{x}) = \{\mathbf{x} + \varphi(\mathbf{x}, \mathbf{u})dt \text{ for } \mathbf{u} \in U(\mathbf{x})\}$$

237 Considering the compact viability constraint set  $K$ , the viability kernel of  $K$  under  $G$  is the  
238 largest set included in  $K$  such that, for any  $\mathbf{x}$  in  $Viab(K)$ :

$$(9) \quad G(\mathbf{x}) \cap Viab(K) \neq \emptyset$$

239 We define a grid  $K_h$  as a finite set of  $K$  such that:

$$(10) \quad \forall \mathbf{x} \in K, \exists \mathbf{x}_h \in K_h \text{ such as } \|\mathbf{x} - \mathbf{x}_h\| < \beta(h)$$

240 At each step  $n$ , we define a discrete set  $K_h^n \subset K_h^{n-1} \subset K_h$  and a continuous set  $L(K_h^n)$  which  
241 is a generalization of the discrete set and which constitutes the current approximation of the  
242 viability kernel. The boundary of this set is define thanks to a particularly procedure, the  
243 support vector machines (SVM), which is a method for data classification. Given a set of  
244 examples  $\{(\mathbf{x}_i, y_i)\}_{i=1}^n$ , where  $\mathbf{x}_i$  is a real vector and  $y_i \in \{-1, 1\}$ , SVM define a function  $f$   
245 which separates examples of each labels:

$$(11) \quad f(\mathbf{x}) = \sum_{i=1}^n \alpha_i y_i k(\mathbf{x}_i, \mathbf{x}) + b$$

246 with  $\alpha_i \geq 0$  and  $k(\mathbf{x}_i, \mathbf{x}) = \exp\left(\frac{-\|\mathbf{x}_i - \mathbf{x}\|^2}{2\sigma^2}\right)$ .

247 In (Deffuant *et al.*, 2007), we show that it is possible to find an optimal control vector  $\mathbf{u}^*$ ,  
248 which defines the position the most inside the current approximation of the kernel among all  
249 possibilities in  $G(\mathbf{x})$  (we use a gradient algorithm).

250 The steps of the algorithm are the following:

- Initialize the sets  $K_h^1 = K_h$  and  $L(K_h^1) = K$ .
- Iterate:

- Define the discrete set  $K_h^{n+1}$  from  $K_h^n$  and  $f_n$  as follows:

$$K_h^{n+1} = \left\{ \mathbf{x}_h \in K_h^n \text{ such that } f_n(\mathbf{x}_h + \varphi(\mathbf{x}_h, \mathbf{u}^*)dt) \geq -\gamma \right. \\ \left. \text{and } (\mathbf{x}_h + \varphi(\mathbf{x}_h, \mathbf{u}^*)dt) \in K \right\}$$

- If  $K_h^{n+1} \neq K_h^n$  then run the SVM on the learning sample obtained with the points

$\mathbf{x}_h$  of the grid  $K_h$ , associated with the labels  $+\gamma$  if  $\mathbf{x}_h \in K_h^{n+1}$ , and with labels

$-\gamma$  otherwise. Let  $f_{n+1}$  be the obtained classification function.  $L(K_h^{n+1})$  is

defined as follows:

$$L(K_h^{n+1}) = \left\{ \mathbf{x} \in K \text{ such that } f_{n+1}(\mathbf{x}) = +\gamma \right\}$$

- Else stop and return  $L(K_h^n)$ .

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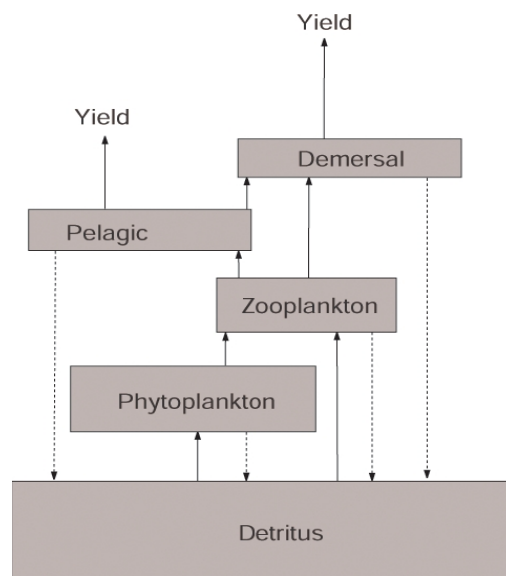
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<i>Compartment</i>	$m_i$ (tons/km <sup>2</sup> )	$M_i$ (tons/km <sup>2</sup> )
Detritus	100	2000
Phytoplankton	30	400
Zooplankton	20	200
Pelagic fish	5	60
Demersal fish	5	30

**Tab 1 - Estimation of the minimal and maximal biomasses (Bi) for the five species.**



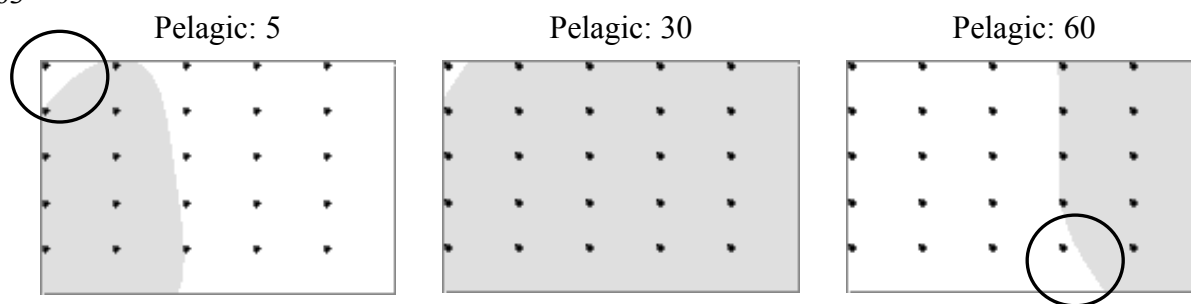


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261 **between compartments (from Mullon *et al.*, 2004).**

262

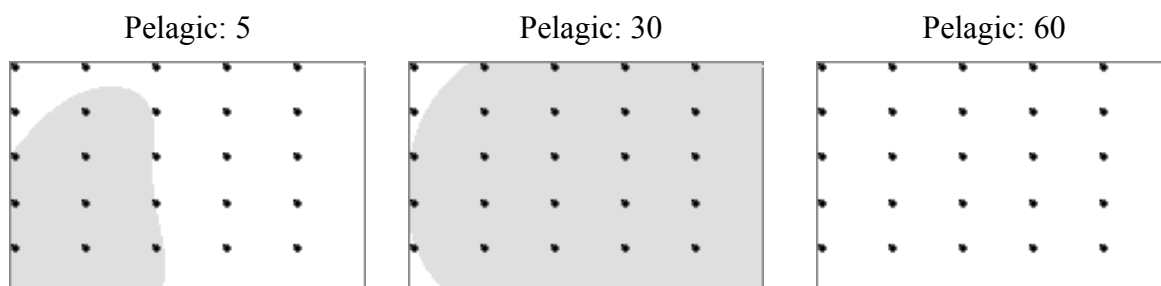
263



264 **Figure 2 - Approximation of viability kernel. The horizontal axis represents demersal fish, vertical axis**  
265 **fisheries, detritus = 2000 tons/km<sup>2</sup>, zooplankton = 90 tons/km<sup>2</sup> and phytoplankton = 100 tons/km<sup>2</sup>.**

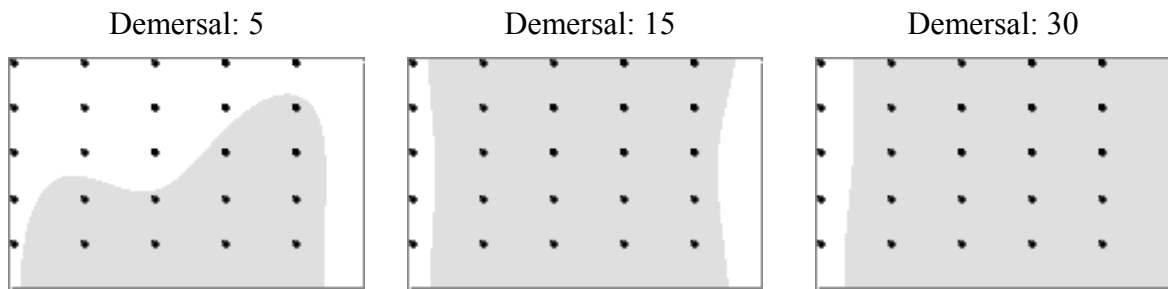
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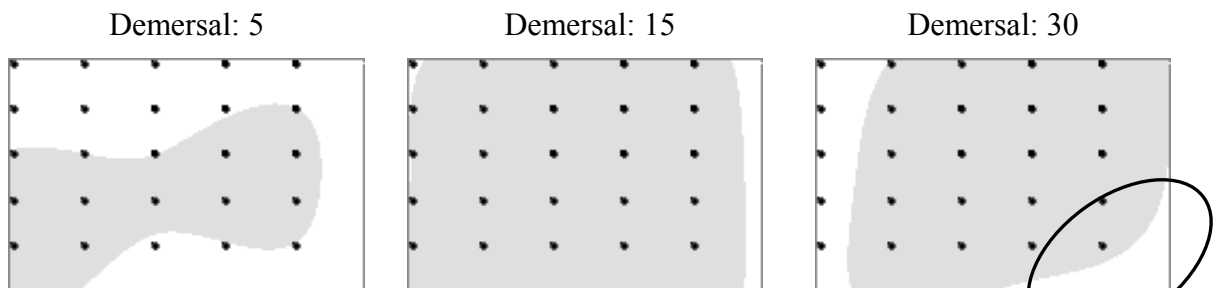


268 **Figure 3 - Approximation of viability kernel. The horizontal axis represents demersal fish, vertical axis**  
269 **fisheries, detritus = 2000 tons/km<sup>2</sup>, zooplankton = 130 tons/km<sup>2</sup> and phytoplankton = 400 tons/km<sup>2</sup>.**

270



**Figure 4 - Approximation of viability kernel. The horizontal axis represents pelagic fish, vertical axis fisheries, detritus = 2000 tons/km<sup>2</sup>, zooplankton = 90 tons/km<sup>2</sup> and phytoplankton = 100 tons/km<sup>2</sup>.**



**Figure 5 - Approximation of viability kernel. The horizontal axis represents pelagic fish, vertical axis fisheries, detritus = 2000 tons/km<sup>2</sup>, zooplankton = 130 tons/km<sup>2</sup> and phytoplankton = 400 tons/km<sup>2</sup>.**