Optical Parabolic Pulse Generation and Applications
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Abstract — Parabolic pulses in optical fibers have stimulated an increasing number of applications. We review here the physics underlying the generation of such pulses as well as the results obtained in a wide-range of experimental configurations.

Index Terms — Fiber amplifiers, Nonlinear dynamics, Optical pulse processing.

I. INTRODUCTION

For more than three decades, optical fibers have been recognized as a versatile testbed with which to investigate a very wide range of nonlinear concepts. In 1980, Mollenauer et al. used optical single mode fibers to demonstrate the propagation of optical solitons [1] and the balance between linear anomalous dispersion and the Kerr nonlinearity of silica. More recently, parabolic pulse generation and evolution have motivated many studies [2]. Specifically, it has been shown that any pulse propagating in a normally dispersive fiber amplifier progressively reshapes into a pulse having a parabolic intensity profile combined with a perfectly linear chirp [3]. This new class of optical pulse is often referred to as a similariton pulse, and exhibits several fundamental features that are clearly attractive for applications ranging from high-power ultrashort pulse generation to optical nonlinear processing of telecommunication signals.

Although much of the background of this field is now well-known, a number of important applications are becoming apparent that take particular advantage of the particular properties and features of this class of pulse. In this paper, we review a number of features of similariton pulse formation, as well as the wide range of experimental techniques that have been proposed to generate a parabolic intensity profile in both amplifying fibers as well as passive devices. Our intention is to provide a self-contained up-to-date overview of several specific aspects of this field that have not been reviewed in detail in previous studies. We also focus on some new applications that can benefit from the intrinsic properties of these pulses.

II. SELF-SIMILAR PARABOLIC PULSES

A. Ultrashort pulse dynamics in a fiber amplifier

During propagation in a fiber amplifier, gain, dispersion and nonlinearity interact together to strongly modify the properties of the initial ultrashort pulses. The resulting longitudinal evolution of the complex electric field envelope \( \psi(z,T) \) can be quantitatively modeled in terms of the non-linear Schrödinger equation (NLSE) with a constant gain:

\[
i \frac{\partial \psi}{\partial z} + \beta_2 \frac{\partial^2 \psi}{\partial T^2} - \gamma |\psi|^2 \psi + i \frac{g}{2} \psi = 0,
\]

with \( \beta_2 \), \( \gamma \) and \( g \) the second order dispersion, the Kerr nonlinear coefficient and the gain of the fiber, respectively.

Depending on the dispersion regime of the optical fiber, very different behaviors will then be observed. In the case of anomalous dispersion (\( \beta_2 < 0 \)), the pulses will undergo soliton-like compression [1] before splitting as a result of perturbed higher-order soliton dynamics [2]. On the other hand, in the case of normal dispersion (\( \beta_2 > 0 \)), the pulses will progressively broaden both in the temporal and spectral domains. More specifically, techniques based on the asymptotic self-similar analysis of this equation have outlined that, in the normal regime of dispersion, the evolution of any pulse can be accurately described in the limit \( z \to \infty \) by the following asymptotic complex solution \( \psi_f(z,T) \) [3]:

\[
\psi_f(z,T) = A_f(z) \sqrt{1 - \frac{T^2}{T_f^2(z)}} \exp \left( -i \frac{C_f}{2} T^2 + i \phi_f \right) \quad \text{if} \quad |T| \leq T_f \tag{2}
\]

This solution corresponds to a compactly-supported pulse with a parabolic intensity profile whose zero-crossing points are given by \( T_f(z) \). \( A_f(z) \), \( C_f \) and \( \phi_f \) are the amplitude of the pulse, the linear chirp slope and a constant phase offset, respectively. Their longitudinal evolution are explicitly given by the following set of equations first obtained by V.I. Kruglov [3-5]:

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\[
\begin{align*}
A_r(z) &= \frac{1}{2} U_{in}^{1/3} \left( \frac{g}{\sqrt{\beta_2} \gamma / 2} \right)^{1/3} \exp \left( \frac{g}{3} z \right) \\
T_r(z) &= 3 U_{in}^{1/3} \left( \frac{2 \beta_2}{\gamma / 2 g^{3/2}} \right)^{1/3} \exp \left( \frac{g}{3} z \right) \\
C_r &= \frac{g}{3 \beta_2} \\
\phi_r &= \frac{3 \gamma A_r^2}{2 g} \exp \left( -2 g z / 3 \right)
\end{align*}
\]

with \( U_{in} \) being the initial pulse energy.

In the frequency domain, the corresponding asymptotic power spectrum is also parabolic with zero-crossing points of \( \pm \nu_p(z) \) where \( \nu_p(z) = C_r T_r(z) \).

In order to illustrate those analytical predictions, we have simulated the propagation of a Gaussian pulse having an initial duration of 40 ps and an initial energy of 8 pJ. The pulse propagates in a 15-km Raman amplifier based on a normally dispersive fiber with \( \beta_2 = 16.6 \times 10^{-3} \text{ ps}^2/\text{m} \) and \( \gamma = 4 \text{ W}^{-1} \text{ km}^{-1} \). The integrated gain of the amplifier is 27 dB. Results of the numerical integration of the NLSE are plotted on Fig. 1. Figure 1(a) highlights the significant reshaping undergone by the initial Gaussian pulse which exhibits after propagation the expected parabolic shape. The temporal width and amplitude as well as the chirp of the pulse have been found to be in excellent agreement with the analytical results. When plotted on a semi-logarithmic scale, the intensity profile clearly reveals rapidly decreasing wings typical of the compact nature of the parabolic pulse. Some slowly decreasing wings can however be noticed below \( 10^{-15} \) and are residues of the pre-asymptotic evolution [6-8].

![Fig. 1. (a) Temporal chirp and intensity profiles. (b) Temporal intensity profile plotted on a logarithmic scale. (c) Spectral intensity profile. The results of the numerical simulations (solid black lines) are compared with the analytical predictions (grey circles) and the initial pulse properties (dotted grey line)](image)

Of course it is important to stress that the parabolic shape is efficiently generated only so long as the NLSE is a valid model to describe the pulse evolution in the fiber amplifier. Additional higher-order linear or nonlinear effects may indeed occur and ultimately limit the propagation length of the pulses. Among these deleterious effects, the Raman response of silica or the finite amplification bandwidth of the device have been shown to have relevant impact on the parabolic dynamics [9-12]. We also note that the constant gain assumption is not mandatory and parabolic pulses can also develop in the presence of longitudinal gain fluctuations [4, 13].

**B. A novel class of pulses: optical similaritons**

From the analytical expressions given by (2) and (3), we can identify several important features. First, once acquired, the parabolic shape is maintained as the pulse experiences exponential temporal and spectral broadening as well as an amplitude increase (see Fig. 2). In other words, the parabolic pulse is able to resist the deleterious effects of the wave-breaking [2, 8, 14, 15], a phenomenon which usually seriously degrades the evolution of any pulse evolving in a nonlinear normally dispersive medium [16-18]. As a consequence of this ability to propagate self-similarly, parabolic pulses are often referred to as optical similariton pulses [19].

![Fig. 2. Self-similar longitudinal evolution of the pulse in the normally dispersive Raman amplifier.](image)

More remarkably, the output characteristics only depend on the energy of the initial pulse and are independent of the details of the initial intensity profile: the asymptotic state actually behaves as an attractor of the system [2, 20, 21]. In other words, initial pulses with significantly different temporal widths but with the same energy will converge after a sufficient propagation length in the amplifier towards the analytical state predicted by (2) and (3). Figure 3 illustrates this typical behavior: the longitudinal temporal evolutions of several initial pulses with identical energy but with temporal widths varying by a factor of more than ten are plotted on a logarithmic scale. As it can be seen, after a given propagation length which depends on the initial pulse properties, all the pulses undergo the exponential temporal broadening predicted by Eq. (3). The problem of choosing the optimum initial pulse property in order to shorten the propagation length required to enter into the asymptotic parabolic regime has been the subject of several studies [7, 18, 21-24] that have stressed the interactions between dispersion and nonlinearity in the initial stages of propagation.
The dynamics of a pair of optical similaritons with the same or different central wavelengths has also been theoretically and experimentally investigated [25]: similaritons are robust against collisions [26], whereas the interaction of similaritons leads to the generation of high-repetition rate dark soliton trains [27]. Evolution of dark structures embedded on compact parabolic background has also been the subject of intense recent theoretical studies [28-31].

C. Extension of optical similaritons to other pulse shapes and other nonlinear media

The interest in parabolic self-similar pulses has not been restricted to the configuration of a single lumped amplifier, and the potential of similariton pulse shaping in fiber cavities has been demonstrated numerically as early as 2001 [25]. Additional experiments and theoretical studies have lead to a brand new class of high power femtosecond mode-locked oscillators [32, 33]. Compared to soliton lasers or dispersion-managed cavities, similariton pulses have been shown to yield significant improvements in terms of pulse energy [34, 35]. Although there are many competing technologies based on alternative fiber designs, similariton lasers (like similariton amplifiers) are particularly attractive for the simplicity of their experimental realization.

Although parabolic pulses have been the first kind of optical pulses to be clearly identified as having a self-similar dynamics, other pulse shapes can also maintain their typical shape unchanged during propagation in an optical nonlinear fiber. More specifically, it has been shown that under certain conditions, optical fibers with longitudinally varying parameters can sustain the propagation of exact self-similar pulses having chirped hyperbolic secant or hyperbolic tangent profiles for example [36-40]. Other recent theoretical studies have also successfully investigated the possibility to extend the concept of self-similar bright and dark optical structures to birefringent fibers [41, 42], to optical non linear waveguides with a graded index profile [28, 30, 31, 43-46] as well as to two- or three- dimensional optical beam propagation [47, 48].

III. Experimental Generation

A. Generation in active media

The first clear experimental demonstration of the theoretical predictions of Eq. (2) and (3) have been carried out in the framework of ytterbium-doped fiber amplifiers [3]. A precise intensity and phase measurement based on the frequency resolved optical gating (FROG) definitively confirmed the parabolic intensity profile of the emerging picosecond pulses, as well as the high linearity of the chirp. Since then, several other experiments have benefited of the parabolic features in ytterbium doped fibers with increasing output power levels and various initial femtosecond seeds from passively modelocked fiber lasers with repetition rates of a few tens of MHz [3] to VECSELS with GHz repetition rates [49]. Since the pulse reshaping process is not restricted to a specific rare-earth dopant, generation in erbium-doped amplifiers at telecommunication wavelengths has also been successfully tested [8, 50, 51]. Indeed, it should be noted that as soon as 1996, experiments in erbium doped fiber done by Tamura et al. provided indirect evidence of the chirp linearity of pulses amplified in a normally dispersive fiber [52]. At this time, however, the self-similar nature of the parabolic pulses and their underlying fundamental properties were not clearly pointed out.

![Fig. 3. Evolution of the pulse parameters towards the asymptotic parabolic profile.](image)

![Fig. 4. Generation of parabolic pulses through Raman amplification at telecommunication wavelength.](image)
profile is recorded in real-time by a high-speed photodetector and oscilloscope. The emerging 200-ps pulses clearly exhibit the expected parabolic intensity profile combined with a highly linear chirp. The output optical spectrum also highlights the clear reshaping of the pulse, with a low ripple in the central part and a rapid decrease of the wings. This then confirms that parabolic pulses can be efficiently generated over a wide range of initial pulse durations (from 100 fs to 10 ps) and fiber lengths (from a few meters to several kilometers).

B. Generation in passive fibers relying on a nonlinear dynamics

Active similariton amplifiers have found great success, but it can be also of interest to study alternative generation methods, especially in the context of optical telecommunications or for wavelengths for which adequate rare earth dopants or pumps are not available. Several specific techniques have been recently experimentally demonstrated. Based on the observation that a longitudinal decrease of the normal dispersion is formally equivalent to optical gain, it has for example been proposed to use continuously tapered dispersion decreasing with a dispersion profile exponentially decreasing down to 0 [55]. However, third order dispersion and optical attenuation have been found to ultimately limit the device performance [56, 57]. Using tapering down to nonzero (normal) dispersion values with possibly additional Raman pumping can overcome those limitations [58]. A potential drawback of the method based on dispersion decreasing fibers is that such tapered fibers are not to date commercially available. Fortunately, the use of comb-like structures made of several segments with fixed dispersion has been found to efficiently mimic a continuous dispersion decrease [59].

Recent observations have led to another approach taking advantage of the progressive nonlinear reshaping of a pulse propagating in a nonlinear fiber with fixed dispersion [61]. Indeed, it has been shown that for a given initial pulse shape with precise input parameters, the temporal profile obtained after a carefully chosen propagation distance is in close agreement with a parabolic intensity profile combined with a nearly linear chirp [24, 62]. There are however some fundamental differences when compared to the previously described parabolic pulses in the sense that the pulse generated by this latter technique are not able to maintain their characteristic shape during further propagation. In order to stabilize their parabolic features, one has to use a second stage with higher nonlinearity and less dispersion [24, 63]. Figure 5 illustrates such an approach that has successfully been used to reshape an initial Gaussian pulse train at a repetition rate of 40 GHz into a train of parabolic pulses [60].

C. Generation based on linear pulse shaping techniques

All the previously described generation methods are based on the intrinsic nonlinear reshaping of an optical pulse propagating in an active or passive normally dispersive fiber. It is of course also possible to benefit from the progress achieved in the field of linear pulse shaping. This removes the need for a high initial peak power, which is especially beneficial in the context of telecommunication applications at high repetition rates where the required average power could be prohibitive. We note, however, that whereas the nonlinear passive reshaping does not induce losses, the linear techniques are intrinsically lossy.

Three approaches have been to date reported. The first approach relies on the use of a superstructured fiber Bragg grating to tailor the initial spectrum of picosecond pulses [64-66]. Contrary to the nonlinear methods, it is here possible to record chirp-free parabolic intensity profiles. A second flexible approach takes advantage of the line by line shaping in an arrayed waveguide grating [67, 68] to produce bright or dark parabolic pulse trains. A third shaping scheme that has been recently proposed is based on the use of long period fiber grating filter [69].

Fig. 5. Nonlinear pulse reshaping observed in a two segment configuration. Initial Gaussian pulse train at a repetition rate of 40 GHz (a) is reshaped into parabolic pulse train (b) after propagation in a normal dispersion shifted fiber followed by a highly nonlinear fiber. Experimental results obtained by FROG characterization (empty circles) are compared with the numerical integration of the NLSE (solid line). (c) Details of parabolic pulse after linear compression (results recorded for a repetition rate of 20 GHz). Experimental results (circles) are compared with a first order Bessel function of the first kind. Results adapted from reference [60].

Fig. 6. Various active and passive approaches for parabolic pulse generation.
IV. APPLICATIONS

Optical similarities are not only of theoretical interest but they have very wide practical potential. The remarkable features of parabolic pulses discussed above have indeed stimulated numerous applications both in ultrashort high-power pulse generation and in optical pulse processing.

A. High Power pulse amplification and ultrashort pulse generation

The field of ultrashort high-power pulse generation has strongly benefited from the highly linear chirp of the amplified pulses [3, 49]. Indeed, the direct nonlinear amplification of pulses avoids the necessity of a prechirping stage as required in the linear chirped pulse amplification approach [70] where the ultimate goal is to avoid any nonlinearity by stretching the initial pulse and therefore reducing the initial peak power. The approach based on parabolic amplification is intrinsically different, as it takes advantage of the nonlinear optical spectrum expansion of the pulse. Indeed, a strong frequency chirp develops during amplification. Thanks to the high level of linearity of this chirp, it is possible to efficiently compensate for the positive chirp slope by using a pair of diffraction gratings so that high quality recompressed pulses can be obtained with final temporal durations much shorter than the initial seed pulse. Indeed, the theoretical pulse shape obtained after compression of a perfectly parabolic pulse is a first order Bessel function of the first kind that exhibits noticeably low sidelobes [23] (see Fig. 5(c) for experimentally compressed parabolic pulses compared to a first order Bessel function fit; the sidelobe level is more than 18 dB below the peak power). Therefore, several experimental works have reported high performance results [8, 71-73], with temporal widths below 100 fs and peak-powers exceeding 4 MW [74] or pulse energy at the microjoule level [71]. Improvements in the technique have led to the use of cubicon pulses where the effect of residual third order dispersion can be overcome by an initial pulse shaping [75, 76]. Parabolic amplification can also sustain GHz repetition rates [49, 51] and can be integrated in an all-fiber set-up with the judicious use of hollow-core photonic bandgap fibers employed as compression stages [5]. Additional nonlinear higher-order soliton compression has enabled the generation of pulses with temporal durations as short as 20 fs (i.e. four optical cycles at 1550 nm) [77]. If the primary goal is not to reach high peak powers, a length of anomalously dispersive fiber can be used [60, 62, 64, 69].

B. Highly coherent continuums for optical telecommunications

Highly coherent pulse generation has also taken advantage of the parabolic pulses ability to resist the effects of optical wave-breaking. Continuum with low spectral ripple and high energy spectral density in the central part can indeed be achieved [64] and the temporal coherence of the continuum obtained in the normal dispersion regime has been found superior to the case of continuum generated in the anomalous regime in the presence of solitonic effects or modulational instability. Such features can be of great interest for applications such as pulse shaping [78] or multiwavelength picosecond sources. Examples of low-noise multichannel sources running at 10 GHz and covering the whole C-band of the optical telecommunications have therefore been demonstrated based on km-long erbium doped fiber amplifier [51] or on the passive propagation of a parabolic pulse in a highly non-linear fiber [64, 69].

C. Ultrafast all-optical signal processing

In the high-bit rate telecommunication context [19], using parabolic pulses has also enabled noticeable improvements of existing optical pulse processing techniques based on the quasi-instantaneous Kerr response of highly nonlinear optical fibers. Various applications targeting the optical regeneration of an optical signal impaired by amplitude or timing jitter have been proposed.

In an extension of the technique initially proposed by P.V. Mamyshev [79], distributed Raman amplification in a normally dispersive fiber followed by offset spectral filtering has been shown to improve the ability of a 2R regenerator to eliminate spurious noise pulses as well as to simultaneously reduce any fluctuations in the signal ‘1’ bit level [80]. Given their typical parabolic intensity profiles, the cross-phase modulation (XPM) induced by a parabolic pulse onto another pulse is also strictly parabolic. This feature is of great interest for several optical high-speed applications where electro-optic phase modulators are not available, or are unable to provide a sufficient phase shift. Therefore, taking advantage of the perfectly linear chirp that can be generated through XPM induced by a parabolic pulse, it is possible to increase the level of tolerable temporal jitter that an optical retiming system can accommodate [65]. More recently, parabolic phase profiles induced by XPM have also been shown to be beneficial for the restoration of the intensity profile of pulses degraded by linear dispersive effects. Indeed, the time-domain optical Fourier transformation approach is based on imparting
a highly linear chirp to the distorted data bits [81] and the use of parabolic pulses represents an attractive solution to generate the required chirp for picosecond pulse signals [66, 68]. We also anticipate that in the near future other applications, such as those based on time-lens effects, will benefit from such an approach.

V. CONCLUSION

In conclusion, parabolic self-similar pulses represent a new class of ultrashort optical pulses that have opened new avenues of fundamental and applied photonics research. A wide set of active or passive techniques has progressively become available to generate these pulses for applications such as ultrashort high-power pulse generation, highly coherent continuum sources and optical signal processing in high-bit rate telecommunications, to name just a few.

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