Power allocation games in wireless networks of multi-antenna terminals

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Abstract

We consider wireless networks that can be modeled by multiple access channels in which all the terminals are equipped with multiple antennas. The propagation model used to account for the effects of transmit and receive antenna correlations is the unitary-invariant-unitary model, which is one of the most general models available in the literature. In this context, we introduce and analyze two resource allocation games. In both games, the mobile stations selfishly choose their power allocation policies in order to maximize their individual uplink transmission rates; in particular they can ignore some specified centralized policies. In the first game considered, the base station implements successive interference cancellation (SIC) and each mobile station chooses his best space-time power allocation scheme; here, a coordination mechanism is used to indicate to the users the order in which the receiver applies SIC. In the second framework, the base station is assumed to implement single-user decoding. For these two games a thorough analysis of the Nash equilibrium is provided: the existence and uniqueness issues are addressed; the corresponding power allocation policies are determined by exploiting random matrix theory; the sum-rate efficiency of the equilibrium is studied analytically in the low and high signal-to-noise ratio regimes and by simulations in more typical scenarios. Simulations show that, in particular, the sum-rate efficiency is high for the type of systems investigated and the performance loss due to the use of the proposed suboptimum coordination mechanism is very small.

Index Terms

MIMO, MAC, non-cooperative games, Nash equilibrium, power allocation, price of anarchy, random matrix theory.

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I. INTRODUCTION

In this paper, we consider the uplink of a decentralized network of several mobile stations (MS) and one base station (BS). This type of network is commonly referred to as the decentralized multiple access channel (MAC). The network is said to be decentralized in the sense that each user can freely choose his power allocation (PA) policy in order to selfishly maximize a certain individual performance criterion, which is called utility or payoff. This means that, even if the the BS broadcasts some specified policies, every user is free to ignore the policy intended for him if the latter does not maximize his performance criterion.

To the best of the authors' knowledge, the problem of decentralized PA in wireless networks has been properly formalized for the first time in [1], [2]. Interestingly, this problem can be formulated quite naturally as a non-cooperative game with different performance criteria (utilities) such as the carrier-to-interference ratio [3], aggregate throughput [4] or energy efficiency [5], [6]. In this paper, we assume that the users want to maximize information-theoretic utilities and more precisely their Shannon transmission rates. Indeed, the point of view adopted here is close to the one proposed by the authors of [7] for DSL (digital subscriber lines) systems, which are modeled as a parallel interference channel; [8] for the single input single output (SISO) and single input multiple output (SIMO) fast fading MACs with global CSIR and global CSIT (Channel State Information at the Receiver/Transmitters); [9] for MIMO (Multiple Input Multiple Output) MACs with global CSIR, channel distribution information at the transmitters (global CDIT) and single-user decoding (SUD) at the receivers; [10], [11] for Gaussian MIMO interference channels with global CSIR and local CSIT and, by definition of the conventional interference channel [12], SUD at the receivers. Note that reference [13] where the authors considered Gaussian MIMO MACs with neither CSIT nor CDIT differs from our approach and that of [7], [8], [9], [10], [11] because in [13] the MIMO MAC is seen as a two-player zero-sum game where the first player is the group of transmitters and the second player is the set of MIMO sub-channels. The closest works to the work presented here are [9] and [14]. Although this paper is in part based on these works, it still provides significant contributions w.r.t. to them, as explained below.

In [9], the authors consider MIMO multiple access channels and assume SUD at the BS; the authors formulate the PA problem into a team game in which each user chooses his PA to maximize the network sum-rate. In [14], the same type of decentralized networks is considered but SIC is assumed at the BS. As each user needs to know his decoding rank in order to adapt his PA policy to maximize his individual transmission rate, a coordination mechanism has to be introduced: the coordination signal precisely indicates to all the users the decoding order used by the receiver. The present paper differs from these two contributions on at least four important technical points: (i) when SUD is assumed, the PA game is not formulated as a team game but as a non-cooperative one; (ii) we exploit several proof techniques that are different from [9]; (iii) while [9] and [14] assume a Kronecker propagation model with *common* receive correlation we assume here a more general model, the unitary-invariant-unitary (UIU) propagation model introduced by [21], for which the users can have different receive antenna correlation profiles. This is useful in practice since, for instance, it allows one to study propagation scenarios where some users can be in line of sight with the BS (the receive antenna are strongly correlated) whereas other users can be surrounded by many obstacles, which can strongly decorrelate the receive antennas for these users; (iv) while the authors of [14] restricted their attention to either a purely spatial PA problem or a purely temporal PA problem, we tackle here the general space-time PA problem.

In this context, our main objective is to study the equilibrium of two power allocation games associated with the two types of decoding schemes aforementioned (namely SIC and SUD). The motivation for this is that the existence of an equilibrium allows network designers to predict, with a certain degree of stability, the effective operating state(s) of the network. Clearly, in our context, uniqueness is a desirable feature of the equilibrium. As it will be seen, it is possible to prove the existence in both games under investigation. Uniqueness is proven in the case of SUD while it is conjectured for the case of SIC. In order to establish the corresponding results, the paper is structured as follows. After presenting the general system model in Sec. II, we analyze in detail the space-time PA game when SIC and a corresponding coordination mechanism are assumed (Sec. III). For this game, the existence and uniqueness of the NE are proven and the equilibrium is determined by exploiting random matrix theory when the numbers of antennas are sufficiently large. Its sum-rate efficiency is also analyzed. In Sec. IV, we analyze the case of SUD since this decoding scheme, although suboptimal in terms of performance (even in the case of a network with single-antenna terminals), has some features that can be found desirable in some contexts: the receiver complexity is low, there is no need for a coordination signal, there is no propagation error since the data flows are decoded in parallel and not successively and also it is intrinsically fair. To analyze the case of the SUD-based PA game, we will follow the same steps as in Sec. III and we will see that, the equilibrium analysis can be deduced, to a large extent, from the SIC case. Numerical results are provided in Sec. V to illustrate our theoretical analysis and to better assess the sum-rate efficiency of the considered games. Sec. VI corresponds to the conclusion.

II. System Model

We assume a MAC with arbitrary number of users, $K \geq 2$. Regarding the original definition of the MAC by [15] and [16], the system under consideration has two common features: all transmitters send at once and at different rates over the entire bandwidth, and the transmitters are using good codes in the sense of the Shannon rate. Our system differs from [15][16] in the sense that multiple antennas are considered at the terminal nodes, channels vary over time and the BS does not dictate the PA policies to the MSs. Also, we assume the existence of coordination signal which is perfectly known to all the terminals. If the coordination signal is generated by the BS itself, this induces a certain cost in terms of downlink signaling but the distribution of the coordination signal can then be optimized. On the other hand, if the coordination signal comes from an external source, e.g., an FM transmitter, the MSs can acquire their coordination signal for free in terms of downlink signaling. However this generally involves a certain sub-optimality in terms of uplink rate. In both cases, the coordination signal will be represented by a random variable denoted by $S \in \mathcal{S}$. Since we study the $K-$user MAC, $\mathcal{S} = \{0, 1, ..., K!\}$ is a $K! + 1$-element alphabet. When the realization is in $\{1, ..., K!\}$, the BS applies SIC with a certain decoding order (game 1). When $S = 0$ the BS always applies SUD (game 2), where all users are decoded simultaneously (no interference cancellation). In a real wireless system the frequency at which the realizations would be drawn would be roughly proportional to the reciprocal of the channel coherence time (i.e., $1/T_{\text{coh}}$). Note that the proposed coordination mechanism is suboptimal because it does not depend on the realizations of the channel matrices. We will see that the corresponding performance loss is in fact very small.

We will further consider that each mobile station is equipped with n_t antennas whereas the base station has n_r antennas (thus we assume the same number of transmitting antennas for all the users). In our analysis, the flat fading channel matrices of the different links vary from symbol vector (or space-time codeword) to symbol vector. We assume that the receiver knows all the channel matrices (CSIR) whereas each transmitter has only access to the statistics of the different channels (CDIT). The equivalent baseband signal received by the base station can be written as:

$$\underline{Y}^{(s)}(\tau) = \sum_{k=1}^{K} \boldsymbol{H}_k(\tau)\underline{X}_k^{(s)}(\tau) + \underline{Z}^{(s)}(\tau), \tag{1}$$

where $\underline{X}_k^{(s)}(\tau)$ is the n_t-dimensional column vector of symbols transmitted by user k at time τ for the realization $s \in \mathcal{S}$ of the coordination signal, $\mathbf{H}_k(\tau) \in \mathbb{C}^{n_r \times n_t}$ is the channel matrix (stationary and ergodic process) of user k and $\underline{Z}^{(s)}(\tau)$ is a n_r-dimensional complex white Gaussian noise distributed as $\mathcal{N}(\underline{0}, \sigma^2 \mathbf{I}_{n_r})$. For the sake of clarity we will omit the time index τ from our notations.

In order to take into account the antenna correlation effects at the transmitters and receiver, we will assume the different channel matrices to be structured according to the unitary-independent-unitary model introduced in [21]:

$$\forall k \in \{1, ..., K\},\ \mathbf{H}_k = \mathbf{V}_k \tilde{\mathbf{H}}_k \mathbf{W}_k, \tag{2}$$

where $\mathbf{V}_k$ and $\mathbf{W}_k$ are deterministic unitary matrices that allow one to take into consideration the correlation effects at the receiver and transmitter. Also $\tilde{\mathbf{H}}_k$ is an $n_r \times n_t$ matrix whose entries are zero-mean independent complex Gaussian random variables with an arbitrary profile of variances, such that $\mathbb{E}|\tilde{H}_k(i,j)|^2 = \frac{\sigma_k(i,j)}{n_t}$. The Kronecker propagation model for which the channel transfer matrices factorizes as $\mathbf{H}_k = \mathbf{R}_k^{1/2}\tilde{\mathbf{\Theta}}_k\mathbf{T}_k^{1/2}$ is a special case of the UIU model where the profile of variances is separable i.e., $\mathbb{E}|\tilde{H}_k(i,j)|^2 = \frac{d_k^{(\mathrm{R})}(i) d_k^{(\mathrm{T})}(j)}{n_t}$, with for each k: $\mathbf{\Theta}_k$ is a random matrix with zero-mean i.i.d. entries, $\mathbf{T}_k$ is the transmit antenna correlation matrix, $\mathbf{R}_k$ is the receive antenna correlation matrix, $\{d_k^{(\mathrm{T})}(j)\}_{j\in\{1,\dots,n_t\}}$ and $\{d_k^{(\mathrm{R})}(i)\}_{i\in\{1,\dots,n_r\}}$ are their associated eigenvalues. In this paper we will consider that $\mathbf{V}_k = \mathbf{V}$ for all users. The reason for assuming this will be made clearer a little further. In spite of this simplification, we will still be able to deal with some useful scenarios where the users see different propagation conditions in terms of receive antenna correlation.

III. Successive Interference Cancellation

When SIC is assumed at the BS, the strategy of user $k \in \{1, 2, ..., K\}$, consists in choosing the best vector of precoding matrices $\mathbf{Q}_k = \left(\mathbf{Q}_k^{(1)}, \mathbf{Q}_k^{(2)}, ..., \mathbf{Q}_k^{(K!)}\right)$ where $\mathbf{Q}_k^{(s)} = \mathbb{E}\left[\underline{X}_k^{(s)}\underline{X}_k^{(s),H}\right]$, for $s \in \mathcal{S}$, in the sense of his utility function. For clarity sake, we will introduce another notation which will be used in the remaining of this section to replace the realization s of the coordination signal. We denote by $\mathcal{P}_K$ the set of all possible permutations of K elements, such that $\pi \in \mathcal{P}_k$ denotes a certain decoding order for the K users and $\pi(k)$ denotes the rank of user $k \in \mathcal{K}$ and $\pi^{-1} \in \mathcal{P}_K$ denotes the inverse permutation (i.e. $\pi^{-1}(\pi(k)) = k$) such that $\pi^{-1}(r)$ denotes the index of the user that is decoded with rank $r \in \mathcal{K}$. We denote by $p_\pi \in [0,1]$ the probability that the receiver implements the decoding order $\pi \in \mathcal{P}_K$, which means that $\sum\limits_{\pi\in\mathcal{P}_K} p_\pi = 1$. At last note that there is a one-to-one mapping between the set of realizations of the coordination signal $\mathcal{S}$ and the set of permutations $\mathcal{P}_K$, i.e. $\xi : \mathcal{S} \rightarrow \mathcal{P}_k$ such that $\xi(\cdot)$ is a bijective function. This is the reason why the index s can be replaced with the index π without introducing any ambiguity or loss of generality. The vector of precoding matrices can be denoted by $\mathbf{Q} = \left(\mathbf{Q}_k^{(\pi)}\right)_{\pi\in\mathcal{P}_K}$ and the utility function can be written as:

$$u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k}) = \sum_{\pi\in\mathcal{P}_K} p_\pi R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)}) \tag{3}$$

where

$$R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)}) = \mathbb{E}\log_2\left|\mathbf{I} + \rho\mathbf{H}_k\mathbf{Q}_k^{(\pi)}\mathbf{H}_k^H + \rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\mathbf{H}_\ell\mathbf{Q}_\ell^{(\pi)}\mathbf{H}_\ell^H\right| - \mathbb{E}\log_2\left|\mathbf{I} + \rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\mathbf{H}_\ell\mathbf{Q}_\ell^{(\pi)}\mathbf{H}_\ell^H\right| \tag{4}$$

with $\rho = \frac{1}{\sigma^2}$ and $\mathcal{K}_k^{(\pi)} = \{\ell \in \mathcal{K} | \pi(\ell) \geq \pi(k)\}$ represents, for a given decoding order π, the subset of users that will be decoded after user k. Also, we use the standard notation $-k$, which stands for the other players than k. An important point to mention here is the power constraint under which the utilities are maximized. Indeed for user $k \in \{1, ..., K\}$, the strategy set is defined as follows:

$$\mathcal{A}_k^{\mathrm{SIC}} = \left\{ \mathbf{Q}_k = \left(\mathbf{Q}_k^{(\pi)} \right)_{\pi \in \mathcal{P}_K} \quad | \quad \forall \pi \in \mathcal{P}_K, \mathbf{Q}_k^{(\pi)} \succeq 0, \sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\mathbf{Q}_k^{(\pi)}) \leq n_t \overline{P}_k \right\}. \tag{5}$$

In order to tackle the existence and uniqueness issues for Nash equilibria in the general space-time PA game, we exploit and extend the results from Rosen [17], which we will briefly state here below in order to make this paper sufficiently self-contained.

Theorem 1: [17] *Let $\mathcal{G} = (\mathcal{K}, \{\mathcal{A}_k\}_{k \in \mathcal{K}}, \{u_k\}_{k \in \mathcal{K}})$ be a game where $\mathcal{K} = \{1, ..., K\}$ is the set of players, $\mathcal{A}_1, ..., \mathcal{A}_K$ the corresponding sets of strategies and $u_1, ..., u_k$ the utilities of the different players. If the following three conditions are satisfied: (i) each u_k is continuous in the all the strategies $\underline{a}_j \in \mathcal{A}_j, \forall j \in \mathcal{K}$; (ii) each u_k is concave in $\underline{a}_k \in \mathcal{A}_k$; (iii) $\mathcal{A}_1, ..., \mathcal{A}_K$ are compact and convex sets; then $\mathcal{G}$ has at least one NE.*

Theorem 2: [17] *Consider the K-player concave game of Theorem 1. If the following (diagonally strict concavity) condition is met: for all $k \in \mathcal{K}$ and for all $(\underline{a}_k', \underline{a}_k'') \in \mathcal{A}_k^2$ such that there exists at least one index $j \in \mathcal{K}$ for which $\underline{a}_j' \neq \underline{a}_j''$, $\sum_{k=1}^{K} (\underline{a}_k'' - \underline{a}_k')^T \left[\nabla_{\underline{a}_k} u_k(\underline{a}_k', \underline{a}_{-k}') - \nabla_{\underline{a}_k} u_k(\underline{a}_k'', \underline{a}_{-k}'') \right] > 0$; then the uniqueness of the NE is insured.*

In the space-time power allocation game under investigation, the obtained results are stated in the following theorem.

Theorem 3: [Existence of an NE] The joint space-time power allocation game described by: the set of players $k \in \{1, 2\}$; the sets of actions $\mathcal{A}_k^{\mathrm{SIC}}$ and the utility functions $u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k})$ given in (3), has a Nash equilibrium.

Proof: It is quite easy to prove that the strategy sets $\mathcal{A}_k^{\mathrm{SIC}}$ are convex and compact sets and that the utility functions $u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k})$ are concave w.r.t. $\mathbf{Q}_k$ and continuous w.r.t. to $(\mathbf{Q}_k, \mathbf{Q}_{-k})$ and by Theorem 1 at least one Nash equilibrium exists. For more details, the reader is referred to Appendix A. ■

Theorem 4: [Sufficient condition for uniqueness] If the following condition is met

$$\sum_{\pi \in \mathcal{P}_K} \sum_{k=1}^{K} \mathrm{Tr} \left\{ (\mathbf{Q}_k^{(\pi)''} - \mathbf{Q}_k^{(\pi)'}) \left(\nabla_{Q_k^{(\pi)}} u_k^{\mathrm{SIC}}(\mathbf{Q}_k', \mathbf{Q}_{-k}') - \nabla_{Q_k^{(\pi)}} u_k^{\mathrm{SIC}}(\mathbf{Q}_k'', \mathbf{Q}_{-k}'') \right) \right\} > 0 \tag{6}$$

for all $\mathbf{Q}_k' = \left(\mathbf{Q}_k^{(\pi)'} \right)_{\pi \in \mathcal{P}_K}, \mathbf{Q}_k'' = \left(\mathbf{Q}_k^{(\pi)''} \right)_{\pi \in \mathcal{P}_K} \in \mathcal{A}_k^{\mathrm{SIC}}$ such that $(\mathbf{Q}_1', \dots, \mathbf{Q}_K') \neq (\mathbf{Q}_1'', \dots, \mathbf{Q}_K'')$, then the Nash equilibrium in the power allocation game of Theorem 3 is unique.

This theorem corresponds to the matrix generalization of the diagonally strict concavity (DSC) condition of [17] and is proven in Appendix B. To know whether this condition is verified or not in the MIMO MAC one needs to re-write it in a more exploitable manner. It can be checked that $\mathcal{C}$ expresses as $\mathcal{C} = \sum_{\pi \in \mathcal{P}_K} p_\pi \mathcal{T}_\pi$ where for each $\pi \in \mathcal{P}_K$, $\mathcal{T}_\pi$ is given by:

$$
\begin{aligned}
\mathcal{T}_\pi &= \sum_{k=1}^{K} \mathrm{Tr}\left\{ (\mathbf{Q}_k^{(\pi)''} - \mathbf{Q}_k^{(\pi)'}) \left[\nabla_{Q_k^{(\pi)}} R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)'}, \mathbf{Q}_{-k}^{(\pi)'}) - \nabla_{Q_k^{(\pi)}} R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)''}, \mathbf{Q}_{-k}^{(\pi)''}) \right] \right\} \\
&= \mathbb{E} \sum_{r=1}^{K} \mathrm{Tr}\Big\{ \rho \mathbf{H}_{\pi^{-1}(r)} (\mathbf{Q}_{\pi^{-1}(r)}^{(\pi)''} - \mathbf{Q}_{\pi^{-1}(r)}^{(\pi)'}) \mathbf{H}_{\pi^{-1}(r)}^H \\
&\quad \Bigg[\left(\mathbf{I} + \rho \mathbf{H}_{\pi^{-1}(r)} \mathbf{Q}_{\pi^{-1}(r)}^{(\pi)'} \mathbf{H}_{\pi^{-1}(r)}^H + \rho \sum_{s=r+1}^{K} \mathbf{H}_{\pi^{-1}(s)} \mathbf{Q}_{\pi^{-1}(s)}^{(\pi)'} \mathbf{H}_{\pi^{-1}(s)}^H \right)^{-1} - \\
&\quad \left(\mathbf{I} + \rho \mathbf{H}_{\pi^{-1}(r)} \mathbf{Q}_{\pi^{-1}(r)}^{(\pi)''} \mathbf{H}_{\pi^{-1}(r)}^H + \rho \sum_{s=r+1}^{K} \mathbf{H}_{\pi^{-1}(s)} \mathbf{Q}_{\pi^{-1}(s)}^{(\pi)''} \mathbf{H}_{\pi^{-1}(s)}^H \right)^{-1} \Bigg] \Big\} \\
&= \mathbb{E} \sum_{r=1}^{K} \mathrm{Tr}(\mathbf{A}_r^{(\pi)''} - \mathbf{A}_r^{(\pi)'}) \left[\left(\mathbf{I} + \sum_{s=r}^{K} \mathbf{A}_r^{(\pi)'} \right)^{-1} - \left(\mathbf{I} + \sum_{s=r}^{K} \mathbf{A}_r^{(\pi)''} \right)^{-1} \right]
\end{aligned} \tag{7}
$$

where $\mathbf{A}_r^{(\pi)'} = \rho \mathbf{H}_{\pi^{-1}(r)} \mathbf{Q}_{\pi^{-1}(r)}^{(\pi)'} \mathbf{H}_{\pi^{-1}(r)}^H$, $\mathbf{A}_r^{(\pi)''} = \rho \mathbf{H}_{\pi^{-1}(r)} \mathbf{Q}_{\pi^{-1}(r)}^{(\pi)''} \mathbf{H}_{\pi^{-1}(r)}^H$ and the users have been ordered using their decoding rank rather than their index.

Theorem 5: [A sufficient condition for DSC] If for any positive definite matrices $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{A}_i \neq \mathbf{B}_i$, $i \in \{1, ..., K\}$ we have that

$$
\sum_{i=1}^{K} \mathrm{Tr}\left\{ (\mathbf{A}_i - \mathbf{B}_i) \left[\left(\sum_{j=1}^{i} \mathbf{B}_j \right)^{-1} - \left(\sum_{j=1}^{i} \mathbf{A}_j \right)^{-1} \right] \right\} > 0, \tag{8}
$$

then the DSC condition is met: $\mathcal{C} > 0$.

It turns out that the trace inequality (9) always holds for any K et for any positive matrices.

Lemma 1: [Trace inequality] For any positive definite matrices $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{A}_i \neq \mathbf{B}_i$, $i \in \{1, ..., K\}$ we have that

$$
\sum_{i=1}^{K} \mathrm{Tr}\left\{ (\mathbf{A}_i - \mathbf{B}_i) \left[\left(\sum_{j=1}^{i} \mathbf{B}_j \right)^{-1} - \left(\sum_{j=1}^{i} \mathbf{A}_j \right)^{-1} \right] \right\} > 0. \tag{9}
$$

The proof can be found in [27], for $K = 2$, and in [28] for arbitrary $K \geq 2$.

Determination of the Nash equilibrium. In order to find the optimal covariance matrices, we proceed in the same way as described in [9]. First we will focus on the optimal eigenvectors and then we will determine the optimal eigenvalues by approximating the utility functions under the large system assumption.

Theorem 6: [Optimal eigenvectors] For all $k \in \mathcal{K}$, $\mathbf{Q}_k \in \mathcal{A}_k^{\mathrm{SIC}}$ there is no loss of optimality by imposing the structure $\mathbf{Q}_k = ({\mathbf{Q}_k}^{(\pi)})_{\pi \in \mathcal{P}_K}$, $\mathbf{Q}_k^{(\pi)} = \mathbf{W}_k {\mathbf{P}_k}^{(\pi)} {\mathbf{W}_k}^H$, in the sense that:

$$\max_{\mathbf{Q}_k \in \mathcal{A}_k^{\mathrm{SIC}}} u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k}) = \max_{\mathbf{Q}_k \in \mathcal{S}_k^{\mathrm{SIC}}} u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k}),$$

where $\mathcal{S}_k^{\mathrm{SIC}} = \left\{ \mathbf{Q}_k = ({\mathbf{Q}_k}^{(\pi)})_{\pi \in \mathcal{P}_k} \in \mathcal{A}_k^{\mathrm{SIC}} | \mathbf{Q}_k^{(\pi)} = \mathbf{W}_k \mathbf{P}_k^{(\pi)} \mathbf{W}_k^H \right\}$, $s \in \mathcal{S}$, model from (2) and $\mathbf{P}_k^{(s)} = \mathrm{Diag}(P_k^{(\pi)}(1), \ldots, P_k^{(\pi)}(n_t))$.

The detailed proof of this result is given in Appendix C. This result, although easy to obtain, it is instrumental in our context for two reasons. First, the search of the optimum precoding matrices boils down to the search of the eigenvalues of these matrices. Second, as the optimum eigenvectors are known, available results in random matrix theory can be exploited to find an accurate approximation of these eigenvalues. Indeed, the eigenvalues are not easy to find in the finite setting. They might be found using numerical techniques based on extensive search. Here, our approach consists in approximating the utilities in order to obtain expressions which are not only easier to interpret but also easier to be optimized w.r.t. the eigenvalues of the precoding matrices. The key idea is to approximate the different transmission rates by their large-system equivalent in the regime of large number of antennas. The corresponding approximates can be found to be accurate even for relatively small number of antennas (see e.g., [18][19] for more details).

Since we have assumed $\mathbf{V}_k = \mathbf{V}$, we can exploit the results in [20][21] for single-user MIMO channels, assuming the asymptotic regime in terms of the number of antennas: $n_r \to \infty$, $n_t \to \infty$, $\frac{n_r}{n_t} \to \beta$. The corresponding approximated utility for user k is:

$$\tilde{u}_k^{\mathrm{SIC}}(\{\mathbf{P}_k^{(\pi)}\}_{k \in \mathcal{K}, \pi \in \mathcal{P}_K}) = \sum_{\pi \in \mathcal{P}_K} p_\pi \tilde{R}_k^{(\pi)}(\mathbf{P}_k^{(\pi)}, \mathbf{P}_{-k}^{(\pi)}) \tag{10}$$

where

$$\begin{aligned}\tilde{R}_k^{(\pi)}(\mathbf{P}_k^{(\pi)}, \mathbf{P}_{-k}^{(\pi)}) &= \frac{1}{n_r} \sum_{\ell \in \mathcal{K}_k^{(\pi)} \cup \{k\}} \sum_{j=1}^{n_t} \log_2 \left(1 + (N_k^{(\pi)}+1)\rho P_\ell^{(\pi)}(j) \gamma_\ell^{(\pi)}(j)\right) + \\ & \frac{1}{n_r} \sum_{i=1}^{n_r} \log_2 \left(1 + \frac{1}{(N_k^{(\pi)}+1)n_t} \sum_{\ell \in \mathcal{K}_k^{(\pi)} \cup \{k\}} \sum_{j=1}^{n_t} \sigma_\ell(i,j) \delta_\ell^{(\pi)}(j)\right) - \\ & \frac{1}{n_r} \sum_{\ell \in \mathcal{K}_k^{(\pi)} \cup \{k\}} \sum_{j=1}^{n_t} \gamma_\ell^{(\pi)}(j) \delta_\ell^{(\pi)}(j) \log_2 e - \\ & \frac{1}{n_r} \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \sum_{j=1}^{n_t} \log_2 \left(1 + N_k^{(\pi)} \rho P_\ell^{(\pi)}(j) \phi_\ell^{(\pi)}(j)\right) - \\ & \frac{1}{n_r} \sum_{i=1}^{n_r} \log_2 \left(1 + \frac{1}{N_k^{(\pi)} n_t} \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \sum_{j=1}^{n_t} \sigma_\ell(i,j) \psi_\ell^{(\pi)}(j)\right) + \\ & \tfrac{1}{n_r} \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \sum_{j=1}^{n_r} \phi_\ell^{(\pi)}(j) \psi_\ell^{(\pi)}(j) \log_2 e \end{aligned} \tag{11}$$

where $N_k^{(\pi)} = |\mathcal{K}_k^{(\pi)}|$ and the parameters $\gamma_k^{(\pi)}(j)$ and $\delta_k^{(\pi)}(j)$ $\forall j \in \{1, \ldots, n_t\}$, $k \in \mathcal{K}$, $\pi \in \mathcal{P}_K$ are the solutions of:

$$\left\{ \begin{aligned} & \qquad \forall j \in \{1, \ldots, n_t\}, \ell \in \mathcal{K}_k^{(\pi)} \cup \{k\} : \\ \gamma_\ell^{(\pi)}(j) &= \frac{1}{(N_k^{(\pi)}+1)n_t} \sum_{i=1}^{n_r} \frac{\sigma_\ell(i,j)}{1 + \frac{1}{(N_k^{(\pi)}+1)n_t} \sum_{r \in \mathcal{K}_k^{(\pi)} \cup \{k\}} \sum_{m=1}^{n_t} \sigma_r(i,m) \delta_r^{(\pi)}(m)} \\ \delta_\ell^{(\pi)}(j) &= \frac{(N_k^{(\pi)}+1)\rho P_\ell^{(\pi)}(j)}{1 + (N_k^{(\pi)}+1)\rho P_\ell^{(\pi)}(j) \gamma_\ell^{(\pi)}(j)}, \end{aligned} \right. \tag{12}$$

and $\phi_\ell^{(\pi)}(j)$, $\psi_\ell^{(\pi)}(j)$, $\forall j \in \{1, \ldots, n_t\}$ and $\pi \in \mathcal{P}_K$ are the unique solutions of the following system:

$$\left\{ \begin{aligned} & \qquad \forall j \in \{1, \ldots, n_t\}, \ell \in \mathcal{K}_k^{(\pi)} : \\ \phi_\ell^{(\pi)}(j) &= \frac{1}{N_k^{(\pi)} n_t} \sum_{i=1}^{n_r} \frac{\sigma_\ell(i,j)}{1 + \frac{1}{N_k^{(\pi)} n_t} \sum_{r \in \mathcal{K}_k^{(\pi)}} \sum_{m=1}^{n_t} \sigma_r(i,m) \psi_r^{(\pi)}(m)} \\ \psi_\ell^{(\pi)}(j) &= \frac{N_k^{(\pi)} \rho P_\ell^{\pi)}(j)}{1 + N_k^{(\pi)} \rho P_\ell^{(\pi)}(j) \phi_\ell^{(\pi)}(j)}. \end{aligned} \right. \tag{13}$$

The corresponding water-filling solution is:

$$P_k^{(\pi),\mathrm{NE}}(j) = \left[\frac{1}{\ln 2 n_r \lambda_k} - \frac{1}{N_k^{(\pi)} \rho \gamma_k^{(\pi)}(j)}\right]^+, \tag{14}$$

where $\lambda_k \geq 0$ is the Lagrangian multiplier tuned in order to meet the power constraint:

$$\sum_{\pi \in \mathcal{P}_K} \sum_{j=1}^{n_t} p_\pi \left[\frac{1}{\ln 2 n_r \lambda_k} - \frac{1}{N_k^{(\pi)} \rho \gamma_k^{(\pi)}(j)} \right]^+ = n_t \overline{P}_k.$$

Note that to solve the system of equations given above, we can use the same iterative power allocation algorithm as the one described in [9].

At this point, an important point has to be mentioned. The existence and uniqueness issues have be analyzed in the finite setting (exact game) whereas the determination of the NE is performed in the asymptotic regime (approximated game). It turns out that large system approximates of ergodic transmission rates have the same properties as their exact counterparts, as shown recently by [23], which therefore ensures the existence and uniqueness of the NE in the approximated game.

Nash Equilibrium efficiency. In order to measure the efficiency of the decentralized network w.r.t. its centralized counterpart we introduce the following quantity:

$$\mathrm{SRE} = \frac{R_{\mathrm{sum}}^{\mathrm{NE}}}{C_{\mathrm{sum}}} \leq 1, \tag{15}$$

where SRE stands for sum-rate efficiency; the quantity $R_{\mathrm{sum}}^{\mathrm{NE}}$ represents the sum-rate of the decentralized network at the Nash equilibrium, which is achieved for certain choices of coding and decoding strategies; the quantity C_{sum} corresponds to the sum-capacity of the centralized network, which is reached only if the optimum coding and decoding schemes are known. Note that this is the case for the MAC but not for other channels like the interference channel. Obviously, the efficiency measure we introduce here is strongly connected to the price of anarchy [24] (POA). The difference between SRE and POA is subtle. In our context, information theory provides us with fundamental physical limits on the social welfare (network sum-capacity) while in general no such upper bound is available. In our case, the sum-capacity is given by:

$$C_{\mathrm{sum}} = \max_{(\mathbf{\Omega}_1,\ldots,\mathbf{\Omega}_K) \in \mathcal{A}^{(C)}} \mathbb{E} \log \left| \mathbf{I} + \rho \sum_{k=1}^{K} \mathbf{H}_k \mathbf{\Omega}_k \mathbf{H}_k^H \right|, \tag{16}$$

with

$$\mathcal{A}^{(C)} = \left\{ (\mathbf{\Omega}_1, ..., \mathbf{\Omega}_K) | \forall k \in \mathcal{K}, \mathbf{\Omega}_k \succeq 0, \mathbf{\Omega}_k = \mathbf{\Omega}_k^H, \mathrm{Tr}(\mathbf{\Omega}_k) \leq n_t \overline{P}_k \right\}. \tag{17}$$

In general, it is not easy to find a closed-form expression of the SRE. This is why we will respectively analyze the SRE in the regimes of high and low signal-to-noise ratio (SNR), and for intermediate regimes simulations will complete our analysis. It turns out that the SRE tends to 1 in the two mentioned extreme regimes, which is the purpose of what follows.

In the *high SNR regime*, where $\rho \to \infty$, we observe from (12) that $\delta_\ell^{(\pi)}(j) \to \frac{1}{\gamma_\ell^{(\pi)}(j)}$. Under this condition, it is easy to check that by setting the derivatives of $\mathcal{L}_k$ w.r.t. $P_k^{(s)}(j)$ to zero, we obtain that the power allocation policy at the NE is the uniform power allocation $\mathbf{P}_k^{(\pi),\mathrm{NE}} = \overline{P}_k\mathbf{I}$, regardless the realization of the coordination signal S. Furthermore, in the high SNR regime, the sum-capacity is achieved by the uniform power allocation. Thus, we obtain that the gap between the NE achievable sum-rate and the sum-capacity is optimal, $\mathrm{SRE} = 1$ for any distribution of S.

In the *low SNR regime*, where $\rho \to 0$, from (12) we obtain that $\delta_\ell^{(\pi)}(j) \to 0$ and that $\gamma_\ell^{(\pi)}(j) = \frac{1}{(N_k^{(\pi)}+1)n_t}\sum_{i=1}^{n_r}\sigma_\ell(i,j)$. By approximating $\ln(1+x) \approx x$ when $x << 1$, the power allocations policies at the NE are the solutions of the following linear programs:

$$\begin{array}{c} \max\limits_{\{P_k^{(\pi)}(j)\}_{1\leq j\leq n_t}} \displaystyle\sum_{j=1}^{n_t}\left\{\sum_{p_\pi\in\mathcal{P}_K}\pi P_k^{(\pi)}(j)\sum_{i=1}^{n_r}\sigma_1(i,j)\right\} \\ \text{s.t.} \displaystyle\sum_{j=1}^{n_t}\sum_{\pi\in\mathcal{P}_K}P_k^{(\pi)}(j) \leq \overline{P}_k n_t \end{array}, \tag{18}$$

given by:

$$\sum_{\pi\in\mathcal{P}_K} p_\pi P_k^{(\pi),\mathrm{NE}}(j) = \left| \begin{array}{ll} n_t\overline{P}_k & \text{if } j = \arg\max\limits_{1\leq m\leq n_t}\displaystyle\sum_{i=1}^{n_r}\sigma_k(i,m) \\ 0 & \text{otherwise} \end{array} \right. . \tag{19}$$

The optimal power allocation that achieves the sum-capacity is equal to the equilibrium power allocation, $\mathbf{P}_k^* = \sum_{\pi\in\mathcal{P}_K} p_\pi \mathbf{P}_k^{(\pi),\mathrm{NE}}(j)$ Thus, the achievable sum-rate at the NE is equal to the centralized upper bound and thus $\mathrm{SRE} = 1$ for any distribution of S. In conclusion, when either the low or high SNR regime is assumed, the sum-capacity of the fast fading MAC is achieved at the NE although a sub-optimum coordination mechanism is assumed and also regardless of the distribution of the coordination channel.

IV. Single User Decoding

In this section the coordination signal is deterministic (namely $\Pr[S = s] = \delta(s)$, δ being the Kronecker symbol) and therefore the amount of downlink signalling the BS needs in order to indicate to the MSs that it is using SUD can be made arbitrary small (by letting the frequency at which the realizations of the coordination signal are drawn tend to zero). In this framework, each user has to optimize only one precoding matrix. Indeed, the strategy of user $k \in \mathcal{K}$, consists in choosing the best precoding matrix $\mathbf{Q}_k^{(0)} = \mathbb{E}\left[\underline{X}_k^{(0)}\underline{X}_k^{(0)H}\right]$, in the sense of his utility function obtained with SUD:

$$u_k^{\mathrm{SUD}}(\mathbf{Q}_k^{(0)}, \mathbf{Q}_{-k}^{(0)}) = \mathbb{E}\log\left|\mathbf{I} + \rho\mathbf{H}_k\mathbf{Q}_k^{(0)}\mathbf{H}_k^H + \rho\sum_{\ell\neq k}\mathbf{H}_\ell\mathbf{Q}_\ell^{(0)}\mathbf{H}_\ell^H\right| - \mathbb{E}\log\left|\mathbf{I} + \rho\sum_{\ell\neq k}\mathbf{H}_\ell\mathbf{Q}_\ell^{(0)}\mathbf{H}_\ell^H\right| \tag{20}$$

. The strategy set of user k becomes

$$\mathcal{A}_k^{\mathrm{SUD}} = \left\{ \mathbf{Q}_k^{(0)} \succeq 0, \mathbf{Q}_k^{(0)} = \mathbf{Q}_k^{(0),H}, \mathrm{Tr}(\mathbf{Q}_k^{(0)}) \leq n_t \overline{P}_k \right\}. \tag{21}$$

It turns out that the equilibrium analysis in the game with SUD can be, to a large extent, deduced from the game with SIC. For this reason, we will not detail the corresponding proofs. The existence and uniqueness issues are given in the following theorem.

Theorem 7: [Existence and uniqueness of an NE] The space power allocation game described by: the set of players $k \in \mathcal{K}$; the sets of actions $\mathcal{A}_k^{\mathrm{SUD}}$ and the payoff functions $u_k^{\mathrm{SUD}}(\mathbf{Q}_k^{(0)}, \mathbf{Q}_{-k}^{(0)})$ given in (20), has a unique Nash equilibrium.

To prove the *existence* of a Nash equilibrium we also exploit Theorem 1 and the four necessary conditions on the utility functions and strategy sets can be verified using the same tools as described in Appendix A.

Uniqueness of the Nash equilibrium. Here we can specialize Theorem 4, which is the matrix extension of Theorem 2. When the strategies sets are not sets of pairs of matrices but only sets of matrices, the diagonally strict concavity condition in (6) can be written as follows. For all $\mathbf{Q}_k^{(0)'}, \mathbf{Q}_k^{(0)''} \in \mathcal{A}_k^{\mathrm{SUD}}$ such that $(\mathbf{Q}_1^{(0)'}, \ldots, \mathbf{Q}_K^{(0)'}) \neq (\mathbf{Q}_1^{(0)''}, \ldots, \mathbf{Q}_K^{(0)''})$:

$$\mathcal{C} \;=\; \sum_{k=1}^{K} \mathrm{Tr}\left\{ (\mathbf{Q}_k^{(0)''} - \mathbf{Q}_k^{(0)'}) \left[\nabla_{Q_k^{(0)}} u_1(\mathbf{Q}_k^{(0)'}, \mathbf{Q}_k^{(0)'}) - \nabla_{Q_k^{(0)}} u_1(\mathbf{Q}_k^{(0)''}, \mathbf{Q}_k^{(0)''}) \right] \right\}. \tag{22}$$

Now we can evaluate $\mathcal{C}$ and obtain that:

$$\begin{aligned} \mathcal{C} &= \sum_{k=1}^{K} \mathrm{Tr}\left\{ \left[\rho \mathbf{H}_k (\mathbf{Q}_k^{(0)'} - \mathbf{Q}_k^{(0)''}) \mathbf{H}_k^H \right] \left[\left(\mathbf{I} + \rho \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell^{(0)''} \mathbf{H}_\ell^H \right)^{-1} - \left(\mathbf{I} + \rho \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell^{(0)'} \mathbf{H}_\ell^H \right)^{-1} \right] \right\} \\ &= \mathrm{Tr}\{(\mathbf{A}' - \mathbf{A}'')[(\mathbf{A}'')^{-1} - (\mathbf{A}')^{-1}]\}, \end{aligned} \tag{23}$$

which is strictly positive for all $\mathbf{A}' \neq \mathbf{A}''$, $\mathbf{A}' \succ \mathbf{0}$, $\mathbf{A}'' \succ \mathbf{0}$ after [27] applied when $K = 1$. This result can be applied here since we have

$$\mathbf{A}' = \mathbf{I} + \rho \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell^{(0)'} \mathbf{H}_\ell^H$$

$$\mathbf{A}'' = \mathbf{I} + \rho \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell^{(0)''} \mathbf{H}_\ell^H.$$

Determination of the Nash equilibrium. As for the optimal eigenvectors of the covariance matrices, we follow the same lines as in Appendix C. In this case also there is no loss of optimality by choosing the covariance matrices $\mathbf{Q}_k^{(0)} = \mathbf{W}_k \mathbf{P}_k^{(0)} \mathbf{W}_k^H$, where $\mathbf{W}_k$ is the same unitary matrix as in (2) and $\mathbf{P}_k$ is the diagonal matrix containing the eigenvalues of $\mathbf{Q}_k^{(0)}$.

Here also we further exploit the asymptotic results for the MIMO channel given in [20] [21]. The approximated utility for user k is:

$$\begin{array}{rcl} \tilde{u}_k^{\mathrm{SUD}}(\mathbf{P}_k^{(0)}, \mathbf{P}_{-k}^{(0)}) & = & \displaystyle\frac{1}{n_r}\sum_{k=1}^{K}\sum_{j=1}^{n_t}\log_2(1+K\rho P_k^{(0)}(j)\gamma_k(j))+ \\ & & \displaystyle\frac{1}{n_r}\sum_{i=1}^{n_r}\log_2\left(1+\frac{1}{Kn_t}\sum_{k=1}^{K}\sum_{j=1}^{n_t}\sigma_k(i,j)\delta_k(j)\right)- \\ & & \displaystyle\frac{1}{n_r}\sum_{k=1}^{K}\sum_{j=1}^{n_t}\gamma_k(j)\delta_k(j)\log_2 e- \\ & & \displaystyle\frac{1}{n_r}\sum_{\ell\neq k}\sum_{j=1}^{n_t}\log_2(1+(K-1)\rho P_\ell^{(0)}(j)\phi_\ell(j))- \\ & & \displaystyle\frac{1}{n_r}\sum_{i=1}^{n_r}\log_2\left(1+\frac{1}{(K-1)n_t}\sum_{\ell\neq k}\sum_{j=1}^{n_t}\sigma_\ell(i,j)\psi_\ell(j)\right)+ \\ & & \frac{1}{n_r}\displaystyle\sum_{\ell\neq k}\sum_{j=1}^{n_r}\phi_\ell(j)\psi_\ell(j)\log_2 e \end{array} \tag{24}$$

where the parameters $\gamma_k(j)$ and $\delta_k(j)$ $\forall j \in \{1,\ldots,n_t\}$, $k \in \{1,2\}$ are solution of:

$$\left\{\begin{array}{rcl} & & \forall j \in \{1,\ldots,n_t\}, k \in \mathcal{K}: \\ \gamma_k(j) & = & \displaystyle\frac{1}{Kn_t}\sum_{i=1}^{n_r}\frac{\sigma_k(i,j)}{1+\frac{1}{Kn_t}\displaystyle\sum_{\ell=1}^{K}\sum_{m=1}^{n_t}\sigma_\ell(i,m)\delta_\ell(m)} \\ \delta_k(j) & = & \displaystyle\frac{K\rho P_k^{(0)}(j)}{1+K\rho P_k^{(0)}(j)\gamma_k(j)}. \end{array}\right. \tag{25}$$

and $\phi_\ell(j)$, $\psi_\ell(j)$, $\forall j \in \{1,\ldots,n_t\}$ are the unique solutions of the following system:

$$\left\{\begin{array}{rcl} & & \forall j \in \{1,\ldots,n_t\}, \ell \in \mathcal{K}\setminus\{k\}: \\ \phi_\ell(j) & = & \displaystyle\frac{1}{(K-1)n_t}\sum_{i=1}^{n_r}\frac{\sigma_\ell(i,j)}{1+\frac{1}{(K-1)n_t}\displaystyle\sum_{r\neq k}\sum_{m=1}^{n_t}\sigma_r(i,m)\psi_r(m)} \\ \psi_\ell(j) & = & \displaystyle\frac{(K-1)\rho P_\ell^{(0)}(j)}{1+(K-1)\rho P_\ell^{(0)}(j)\phi_\ell(j)}. \end{array}\right. \tag{26}$$

The corresponding water-filling solution is:

$$P_k^{(0),\mathrm{NE}}(j) = \left[\frac{1}{\ln 2 n_r \lambda_k} - \frac{1}{K\rho\gamma_k(j)}\right]^+, \tag{27}$$

where $\lambda_k \geq 0$ is the Lagrangian multiplier tuned in order to meet the power constraint: $\sum_{j=1}^{n_t}\left[\frac{1}{\ln 2 n_r \lambda_k} - \frac{1}{K\rho\gamma_k(j)}\right]^+ = n_t\overline{P}_k$.

In what the efficiency of the NE point is concerned, we already know that the SUD decoding technique is sub-optimal in the centralized case (SUD does allow the network to operate at an arbitrary point of the centralized MAC capacity region) and it is impossible to reach the sum-capacity C_{sum} even if the high and low SNR regime are assumed.

V. SIMULATION RESULTS

In what follows, we assume the regime of large numbers of antennas. From [9], [20], [21], we know that the approximates of the ergodic achievable rates in the asymptotic regime are accurate even for relatively small number of antennas. For the channel matrices, we assume the Kronecker model $\mathbf{H}_k = \mathbf{R}_k^{1/2}\mathbf{\Theta}_k\mathbf{T}_k^{1/2}$ mentioned in Sec. II, where the receive and transmit correlation matrices $\mathbf{R}_k$, $\mathbf{T}_k$ follow an exponential profile characterized by the correlation coefficients (see e.g., [25], [26]) $r = [r_1, r_2]$ and $t = [t_1, t_2]$ such that $\mathbf{R}_k(i,j) = r_k^{|i-j|}$, $\mathbf{T}_k(i,j) = t_k^{|i-j|}$. By assuming that the receive antenna is a uniform linear array (ULA) and knowing that, when the dimensions of Toeplitz matrices increase they can be approximated by circular matrices we obtain that all the receive correlation matrices $\mathbf{R}_k$ can be diagonalized in the same vector basis (i.e., the Fourier basis). Thus the considered model is included in the UIU model that we studied where $\mathbf{V}_k = \mathbf{V}$.

Fair SIC decoding versus SUD decoding. First we compare the results of the general space-time PA game considered in Sec. III, where SIC decoding is used at the receiver, and the game described in Sec. IV, where SUD decoding is used. Fig. 1 depicts the achievable sum-rate at the equilibrium as a function of the transmit power $P_1 = P_2 = P$, for the scenario $n_r = n_t = 10$, $r = [0.5, 0.2]$, $t = [0.5, 0.2]$, $\rho = 3dB$. In order to have a fair comparison we assume that $p = \frac{1}{2}$ (on average each user is decoded second half of the time when SIC is assumed). We observe that, even in this scenario, which was thought to be a bad one in terms of sub-optimality, the sum-rate obtained with the first game is very close to the sum-capacity upper bound. Also, the sum-rate reached when the BS uses SUD is clearly much lower than the sum-rate obtained by using SIC.

SIC decoding, comparison between the joint space-time PA and the special cases of spatial PA and temporal PA. Now we want to compare the results of the general space-time PA with the two particular cases that were studied in [14]: the spatial PA, where the users are forced to allocate their power uniformly over time (regardless of their decoding rank) but are free to allocate their power over the transmit antennas; the temporal PA, where the users are forced to allocate their power uniformly over their antennas but they can adjust their power as a function of the decoding rank at the receiver. Fig. 2 represents the sum-rate efficiency as a function of the coordination signal distribution parameter $p \in [0,1]$ when $n_r = n_t = 10$, $r = [0.3, 0]$, $t = [0.5, 0.2]$, $\rho = 4dB$, $P_1 = 5$, $P_2 = 50$. We observe that the three types of power allocation policies perform very close to the upper bound. What is most interesting is the fact that the performance of the network at the equilibrium is better by using a purely spatial PA instead of the most general space-time PA. This has been confirmed by many other simulations and illustrates a Braess paradox: although the sets of strategies for the space-time case include those of the purely spatial case, the performance obtained at the NE are not better in the space-time case.

SIC decoding, spatial PA, achievable rate region. In Fig. 3, we observe that the rate region achieved at the NE of the space PA as a function of the distribution of the coordination signal p for the scenario $n_r = n_t = 10$, $r = [0.4, 0.2]$, $t = [0.6, 0.3]$, $\rho = 3dB$, $P_1 = 5$, $P_2 = 50$. It is quite remarkable that in large MIMO MACs, the capacity region comprises a full cooperation segment just like the SISO MACs. The coordination signal precisely allows one to move along the corresponding line. This shows the relevance of large systems in decentralized networks since they allow to determine the capacity region of certain systems whereas it is unknown in the finite setting. Furthermore, they induce an averaging effect, which makes the users' behavior predictable.

VI. CONCLUSIONS

Interestingly, the existence and uniqueness of the Nash equilibrium can be proven in multiple access channels with multi-antenna terminals for a general propagation channel model (namely the unitary-invariant-unitary model) and the most general case of space-time power allocation schemes. In particular, the uniqueness proof requires a matrix generalization of the second theorem of Rosen [17] and proving a trace inequality [28]. For all the types of power allocation policies (purely temporal PA, purely spatial PA, space-time PA), the sum-rate efficiency of the decentralized network is close to one when SIC is assumed and the network is coordinated by the proposed suboptimum coordination mechanism. Quite surprisingly, the space-time power allocation performs a little worse than its purely spatial counterpart, which puts in evidence a Braess paradox in the types of wireless networks under consideration. One of the interesting extensions of this work would be to analyze the impact of a non-perfect SIC on the PA problem. Indeed, the effect of propagation errors could then be assessed (which does not exist with SUD).

APPENDIX A

A. Concavity of the utility functions u_k^{SIC}

Let us focus on user $k \in \mathcal{K}$. We want to prove that $u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k})$ is concave w.r.t. $\mathbf{Q}_k \in \mathcal{A}_1^{\mathrm{SIC}}$. We observe that the term $R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)})$ in (3) depends only on $\mathbf{Q}_k^{(\pi)}$ and $\mathbf{Q}_{-k}^{(\pi)}$ and not on the covariance matrices $\mathbf{Q}_k^{(\tau)}$, $\mathbf{Q}_{-k}^{(\tau)}$ for any other possible decoding rule $\tau \in \mathcal{P}_K \setminus \{\pi\}$. Thus, in order to prove that $u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_k)$ is strictly concave w.r.t. to $\mathbf{Q}_k = (\mathbf{Q}_k^{(\pi)})_{\pi \in \mathcal{P}_K}$, it suffices to prove that $R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)})$ is concave w.r.t. $\mathbf{Q}_k^{(\pi)}$ for all $\pi \in \mathcal{P}_K$.

To this end, we study the concavity of the function $f(\lambda) = R_k^{(\pi)}(\lambda \mathbf{Q}_k^{(\pi)'} + (1-\lambda)\mathbf{Q}_k^{(\pi)''})$ over the interval $[0, 1]$ for any pair of matrices $(\mathbf{Q}_k^{(\pi)'}, \mathbf{Q}_k^{(\pi)''})$. The second derivative of f is equal to:

$$
\begin{aligned}
\frac{\partial^2 f}{\partial \lambda^2}(\lambda) &= -\mathbb{E}\mathrm{Tr}\left[\rho^2 \mathbf{H}_k^H \left(\mathbf{I} + \rho \mathbf{H}_k \mathbf{Q}_k^{(\pi)''} \mathbf{H}_k^H + \rho\lambda \mathbf{H}_k \Delta\mathbf{Q}_k^{(\pi)} \mathbf{H}_k^H + \rho \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \mathbf{H}_\ell \mathbf{Q}_\ell^{(\pi)} \mathbf{H}_\ell^H\right)^{-1} \mathbf{H}_k \Delta\mathbf{Q}_k^{(\pi)}\right. \\
&\quad \left. \times \mathbf{H}_k^H \left(\mathbf{I} + \rho \mathbf{H}_k \mathbf{Q}_k^{(\pi)''} \mathbf{H}_k^H + \rho\lambda \mathbf{H}_k \Delta\mathbf{Q}_k^{(\pi)} \mathbf{H}_k^H + \rho \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \mathbf{H}_\ell \mathbf{Q}_\ell^{(\pi)} \mathbf{H}_\ell^H\right)^{-1} \mathbf{H}_k \Delta\mathbf{Q}_k^{(\pi)}\right], \\
&= -\mathbb{E}\mathrm{Tr}[\mathbf{A}\Delta\mathbf{Q}_k^{(\pi)} \mathbf{A}\Delta\mathbf{Q}_k^{(\pi)}]
\end{aligned}
$$

with $\mathbf{A} = \rho^2 \mathbf{H}_k^H \left(\mathbf{I} + \rho \mathbf{H}_k \mathbf{Q}_k^{(\pi)''} \mathbf{H}_k^H + \rho\lambda \mathbf{H}_k \Delta\mathbf{Q}_k^{(\pi)} \mathbf{H}_k^H + \rho \sum_{\ell \in \mathcal{K}_k^{(\pi)}} \mathbf{H}_\ell \mathbf{Q}_\ell^{(\pi)} \mathbf{H}_\ell^H\right)^{-1} \mathbf{H}_k$, which can be proven to be a Hermitian positive definite matrix, $\Delta\mathbf{Q}_k^{(\pi)} = \mathbf{Q}_k^{(\pi)'} - \mathbf{Q}_k^{(\pi)''}$ also a Hermitian matrix, and $\rho = \frac{1}{\sigma^2}$.

$$
\begin{aligned}
\frac{\partial^2 f}{\partial \lambda^2}(\lambda) &= -\mathbb{E}\mathrm{Tr}[\mathbf{A}^{1/2}\Delta\mathbf{Q}_k^{(\pi)}\mathbf{A}^{1/2}\mathbf{A}^{1/2}\Delta\mathbf{Q}_k^{(\pi)}\mathbf{A}^{1/2}] \\
&= -\mathbb{E}\mathrm{Tr}[\mathbf{B}\mathbf{B}^H] < 0
\end{aligned},
$$

with $\mathbf{B} = \mathbf{A}^{1/2}\Delta\mathbf{Q}_k^{(\pi)}\mathbf{A}^{1/2}$.

B. Continuity of the utility functions u_k^{SIC}

Considering the Leibniz formula, the determinant of a matrix can be expressed as a weighted sum of products of its entries. Knowing that the product and the sum of continuous functions are continuous, we conclude that the determinant function is continuous. Also, it is well known that the logarithmic function is a continuous function. Thus, for any $\pi \in \mathcal{P}_K$, the function $R_k^{(\pi)}(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)})$ is nothing else but the composition of two continuous functions which is also continuous w.r.t. $(\mathbf{Q}_k^{(\pi)}, \mathbf{Q}_{-k}^{(\pi)})$. This suffices to prove that $u_k^{\mathrm{SIC}}(\mathbf{Q}_k, \mathbf{Q}_{-k})$ is continuous w.r.t. $(\mathbf{Q}_k, \mathbf{Q}_{-k})$.

C. Convexity of the strategy sets $\mathcal{A}_k^{\text{SIC}}$

In order to prove that the set $\mathcal{A}_k^{\text{SIC}}$ is convex, we need to verify that, for any two matrices $(\mathbf{Q}_k^{'}, \mathbf{Q}_k^{''}) \in \mathcal{A}_k^{\text{SIC}} \times \mathcal{A}_k^{\text{SIC}}$, we have:

$$\alpha \mathbf{Q}_1^{'} + (1-\alpha)\mathbf{Q}_1^{''} \in \mathcal{A}_k^{\text{SIC}},$$

for all $\alpha \geq 0$.

For any $\mathbf{Q}_k^{'}, \mathbf{Q}_k^{''} \in \mathcal{A}_k^{(\text{SIC})}$, the matrices $\mathbf{Q}_k^{(\pi)}$ are Hermitian which implies that $\alpha \mathbf{Q}_k^{(\pi)'} + (1-\alpha)\mathbf{Q}_k^{(\pi)''}$ are also Hermitian matrices, for all $\pi \in \mathcal{P}_K$.

Furthermore, for any $\mathbf{Q}_k^{'}, \mathbf{Q}_k^{''} \in \mathcal{A}_k^{\text{SIC}}$, we have that $\mathbf{Q}_k^{(\pi)'}$, $\mathbf{Q}_k^{(\pi)''}$ are non-negative matrices which implies that $\alpha \mathbf{Q}_k^{(\pi)'} + (1-\alpha)\mathbf{Q}_k^{(\pi)''}$ are also non-negative matrices, for all $\pi \in \mathcal{P}_K$.

Finally, knowing that the trace is a linear application we have that:

$$\begin{aligned}
&\sum_{\pi \in \mathcal{P}_k} p_\pi \mathrm{Tr}\left(\alpha \mathbf{Q}_k^{(\pi)'} + (1-\alpha)\mathbf{Q}_k^{(\pi)''}\right) = \\
&= \alpha \sum_{\pi \in \mathcal{P}_k} p_\pi \mathrm{Tr}(\mathbf{Q}_k^{(\pi)'}) + (1-\alpha) \sum_{\pi \in \mathcal{P}_k} p_\pi \mathrm{Tr}(\mathbf{Q}_k^{(\pi)'}) \\
&\leq \alpha n_t \overline{P}_k + (1-\alpha) n_t \overline{P}_k \\
&= n_t \overline{P}_k.
\end{aligned}$$

Thus $\alpha \mathbf{Q}_k^{'} + (1-\alpha)\mathbf{Q}_k^{''} \in \mathcal{A}_k^{\text{SIC}}$ and the set is convex.

D. Compactness of the strategy sets $\mathcal{A}_k^{\text{SIC}}$

To prove that the strategy sets are compact sets we use the fact that, in finite dimension spaces, a closed and bounded set is compact.

First let us prove that $\mathcal{A}_k^{\text{SIC}}$ is a closed set. We define the function $g : \mathcal{A}_k^{\text{SIC}} \longrightarrow [0, n_t \overline{P}_k]$, with

$$f(\mathbf{Q}_k) = \sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\mathbf{Q}_k^{(\pi)}).$$

We see that $g(\cdot)$ is a continuous function and that its image is a compact and thus closed set. Knowing that the continuous inverse image of a closed set is closed, we conclude that $\mathcal{A}_k^{\text{SIC}}$ is closed.

Now we want to prove that the set $\mathcal{A}_k^{\text{SIC}}$ is a bounded set. We associate to the tuple of matrices $(\mathbf{Q}_k^{(\pi)})_{\pi \in \mathcal{P}_K}$ the following norm $||\mathbf{Q}_k|| = \sqrt{\sum_{\pi \in \mathcal{P}_K} ||\mathbf{Q}_k^{(\pi)}||_2^2}$ where $||.||_2$ is is the spectral norm of a matrix.

$$||\mathbf{Q}_k^{(\pi)}||_2 = \sqrt{\max\{\lambda_{\mathbf{Q}_k^{(\pi)H}\mathbf{Q}_k^{(\pi)}}(i)\}_{i=1}^{n}}.$$

Since for all $\mathbf{Q}_k \in \mathcal{A}_k^{\text{SIC}}$, $\mathbf{Q}_k^{(\pi)}$ is a non-negative, Hermitian matrix we have that:

$$\max\{\lambda_{\mathbf{Q}_k^{(\pi)}}(i)\}_{i=1}^n \leq Tr(\mathbf{Q}_k^{(\pi)}) \leq \infty,$$

and thus:

$$||\mathbf{Q}_k^{(\pi)}||_2 = \sqrt{\max\{\lambda_{\mathbf{Q}_k^{(\pi)2}}(i)\}_{i=1}^n} = \sqrt{\max\{\lambda_{\mathbf{Q}_k^{(\pi)}}(i)^2\}_{i=1}^n} \leq \infty.$$

In conclusion the associated norm $||\mathbf{Q}_k|| \leq \infty$.

APPENDIX B

We suppose that there exist two different equilibrium strategy profiles: $(\widetilde{\mathbf{Q}}_k, \widetilde{\mathbf{Q}}_{-k}) \in \mathcal{A}_k^{\text{SIC}} \times \mathcal{A}_{-k}^{\text{SIC}}$ and $(\widehat{\mathbf{Q}}_k, \widehat{\mathbf{Q}}_{-k}) \in \mathcal{A}_k^{\text{SIC}} \times \mathcal{A}_{-k}^{\text{SIC}}$, such that $(\widetilde{\mathbf{Q}}_k, \widetilde{\mathbf{Q}}_{-k}) \neq (\widehat{\mathbf{Q}}_k, \widehat{\mathbf{Q}}_{-k})$. Then the condition given in the theorem, $\mathcal{C} > 0$ is met for the particular choice of $(\mathbf{Q}'_k, \mathbf{Q}'_{-k}) = (\widetilde{\mathbf{Q}}_k, \widetilde{\mathbf{Q}}_{-k})$ and $(\mathbf{Q}''_k, \mathbf{Q}''_{-k}) = (\widehat{\mathbf{Q}}_k, \widehat{\mathbf{Q}}_{-k})$.

By the definition of the Nash Equilibrium, the strategies $\widetilde{\mathbf{Q}}_k$, $k \in \mathcal{K}$, are the solutions of the following maximization problems:

$$\max_{\mathbf{Q}_k \in \mathcal{A}_k^{\text{SIC}}} u_k(\mathbf{Q}_k, \widetilde{\mathbf{Q}}_{-k}).$$

Thus, $\widetilde{\mathbf{Q}}_k$ satisfy the following Kuhn-Tucker optimality conditions:

1) $\widetilde{\mathbf{Q}}_k \in \mathcal{A}_k^{\text{SIC}}$, which means that:

$$\left\{ \begin{array}{lll} \widetilde{\mathbf{Q}}_k^{(\pi)} = (\widetilde{\mathbf{Q}}_k^{(\pi)})^H & \succeq & 0 \qquad ,\forall \pi \in \mathcal{P}_K \\ \displaystyle\sum_{\pi \in \mathcal{P}_K} p_\pi \text{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)}) & \leq & n_t \overline{P}_k, \end{array} \right.$$

2) There exist $\widetilde{\lambda}_k \geq 0$, and the following Hermitian non-negative matrices of rank 1, $\widetilde{\Phi}_k^{(\pi)}$, for all $\pi \in \mathcal{P}_K$, such that:

$$\left\{ \begin{array}{lll} \widetilde{\lambda}_k \left[\displaystyle\sum_{\pi \in \mathcal{P}_K} p_\pi \text{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)}) - n_t \overline{P}_k \right] & = & 0 \\ \text{Tr}(\widetilde{\Phi}_k^{(\pi)} \widetilde{\mathbf{Q}}_k^{(\pi)}) & = & 0 \quad ,\forall \pi \in \mathcal{P}_K, \end{array} \right.$$

3)

$$\left\{ \begin{array}{lll} \forall \pi \in \mathcal{P}_K : & & \\ \nabla_{\mathbf{Q}_k^{(\pi)}} u_k(\widetilde{\mathbf{Q}}_k, \widetilde{\mathbf{Q}}_{-k}) & = & p_\pi \widetilde{\lambda}_k \mathbf{I} - \widetilde{\Phi}_k^{(\pi)} \end{array} \right. ,$$

Having assumed that $(\widehat{\mathbf{Q}}_k, \widehat{\mathbf{Q}}_{-k})$ is also a Nash Equilibrium, $\widehat{\mathbf{Q}}_k$, with $k \in \mathcal{K}$ are the solution of:

$$\max_{\mathbf{Q}_k \in \mathcal{A}_k^{\mathrm{SIC}}} u_k(\mathbf{Q}_k, \widehat{\mathbf{Q}}_{-k}),$$

and thus $\widehat{\mathbf{Q}}_k$ satisfy the following Kuhn-Tucker optimality conditions:

4) $\widehat{\mathbf{Q}}_k \in \mathcal{A}_k^{\mathrm{SIC}}$, which means that:

$$\begin{cases} \widehat{\mathbf{Q}}_k^{(\pi)} = (\widehat{\mathbf{Q}}_k^{(\pi)})^H & \succeq & 0 & ,\forall \pi \in \mathcal{P}_K \\ \displaystyle\sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)}) & \leq & n_t \overline{P}_k, & \end{cases}$$

5) There exist $\widehat{\lambda}_k \geq 0$, $k \in \mathcal{K}$ and the following non-negative, Hermitian matrices of rank 1, $\widehat{\Phi}_k^{(\pi)}$, for all $\pi \in \mathcal{P}_K$ such that:

$$\begin{cases} \widehat{\lambda}_k \left[\displaystyle\sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)}) - n_t \overline{P}_k \right] & = & 0 & \\ \mathrm{Tr}(\widehat{\Phi}_k^{(\pi)} \widehat{\mathbf{Q}}_k^{(\pi)}) & = & 0 & ,\forall \pi \in \mathcal{P}_K, \end{cases}$$

6)

$$\begin{cases} \forall \pi \in \mathcal{P}_K : & & \\ \nabla_{\mathbf{Q}_k^{(\pi)}} u_k(\widehat{\mathbf{Q}}_k, \widehat{\mathbf{Q}}_{-k}) & = & p_\pi \widehat{\lambda}_k \mathbf{I} - \widehat{\Phi}_k^{(\pi)} \end{cases}.$$

Using the third and the sixth optimality conditions, the condition given in (6) becomes:

$$\begin{aligned} \mathcal{C} &= \sum_{\pi \in \mathcal{P}_K} \sum_{k=1}^{K} \Big\{ p_\pi \tilde{\lambda}_k \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)}) + p_\pi \hat{\lambda}_k \mathrm{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)}) - p_\pi \tilde{\lambda}_k \mathrm{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)}) - p_\pi \hat{\lambda}_k \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)}) - \\ &\quad \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)} \widetilde{\mathbf{\Phi}}_k^{(\pi)}) - \mathrm{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)} \widehat{\mathbf{\Phi}}_k^{(\pi)}) + \mathrm{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)} \widetilde{\mathbf{\Phi}}_k^{(\pi)}) + \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)} \widehat{\mathbf{\Phi}}_k^{(\pi)}) \Big\} \\ &\leq \sum_{k=1}^{K} \left\{ \tilde{\lambda}_k \left[\sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\widehat{\mathbf{Q}}_k^{(\pi)}) - n_t \overline{P}_k \right] + \hat{\lambda}_k \left[\sum_{\pi \in \mathcal{P}_K} p_\pi \mathrm{Tr}(\widetilde{\mathbf{Q}}_k^{(\pi)}) - n_t \overline{P}_k \right] \right\} \\ &\leq 0. \end{aligned}$$

From the other four K-T conditions, we obtain that all the terms on the right are negative and thus $\mathcal{C} \leq 0$. But this contradicts the diagonally strict concavity condition and so the Nash Equilibrium is unique.

APPENDIX C

We want to prove that there is no optimality loss when restricting the search for the optimal covariance matrices to $\mathbf{Q}_k \in \mathcal{A}_k^{\mathrm{SIC}}$ such that $\mathbf{Q}_k^{(\pi)} = \mathbf{W}_k \mathbf{P}_k^{(\pi)} \mathbf{W}_k^H$, for all $\pi \in \mathcal{P}_K$. Let us consider user $k \in \mathcal{K}$. We have that:

$$\begin{aligned}
&\arg\max_{\mathbf{Q}_k\in\mathcal{A}_k^{\mathrm{SIC}}} u_k(\mathbf{Q}_k,\mathbf{Q}_{-k})\\
=\ &\arg\max_{\mathbf{Q}_k\in\mathcal{A}_k^{\mathrm{SIC}}}\left\{\sum_{\pi\in\mathcal{P}_K} p_\pi \mathbb{E}\log_2\left|\mathbf{I}+\rho\mathbf{H}_k\mathbf{Q}_k^{(\pi)}\mathbf{H}_k^H+\rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\mathbf{H}_\ell\mathbf{Q}_\ell^{(\pi)}\mathbf{H}_\ell^H\right|\right\}\\
=\ &\arg\max_{\mathbf{Q}_k\in\mathcal{A}_k^{\mathrm{SIC}}}\left\{\sum_{\pi\in\mathcal{P}_K} p_\pi \mathbb{E}\log_2\left|\mathbf{I}+\rho\mathbf{V}\tilde{\mathbf{H}}_k\mathbf{W}_k^H\mathbf{Q}_k^{(\pi)}\mathbf{W}_k\tilde{\mathbf{H}}_k^H\mathbf{V}^H+\rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\mathbf{V}\tilde{\mathbf{H}}_\ell\mathbf{W}_\ell^H\mathbf{Q}_\ell^{(\pi)}\mathbf{W}_\ell\tilde{\mathbf{H}}_\ell^H\mathbf{V}^H\right|\right\},\\
=\ &\arg\max_{\mathbf{Q}_k\in\mathcal{A}_k^{\mathrm{SIC}}}\left\{\sum_{\pi\in\mathcal{P}_K} p_\pi \mathbb{E}\log_2\left|\mathbf{I}+\rho\tilde{\mathbf{H}}_k\mathbf{W}_k^H\mathbf{Q}_k^{(\pi)}\mathbf{W}_k\tilde{\mathbf{H}}_k^H+\rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\tilde{\mathbf{H}}_\ell\mathbf{W}_\ell^H\mathbf{Q}_\ell^{(\pi)}\mathbf{W}_\ell\tilde{\mathbf{H}}_\ell^H\right|\right\}\\
=\ &\arg\max_{\mathbf{Q}_k\in\mathcal{A}_k^{\mathrm{SIC}}}\left\{\sum_{\pi\in\mathcal{K}_k^{(\pi)}} \mathbb{E}\log_2\left|\mathbf{I}+\rho\tilde{\mathbf{H}}_k\mathbf{X}_k^{(\pi)}\tilde{\mathbf{H}}_k^H+\rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\tilde{\mathbf{H}}_\ell\mathbf{W}_\ell^H\mathbf{Q}_\ell^{(\pi)}\mathbf{W}_\ell\tilde{\mathbf{H}}_\ell^H\right|\right\}
\end{aligned} \tag{28}$$

where we denoted with $\mathbf{X}_k^{(\pi)} \triangleq \mathbf{W}_k^H\mathbf{Q}_k^{(\pi)}\mathbf{W}_k$. Knowing that the utility function is concave w.r.t. the new defined matrices $\mathbf{X}_k^{(\pi)}$, and the channel matrix $\mathbf{H}_k$ has independent entries, we can directly apply the results given in [22] to prove that annulling the non-diagonal entries of $\mathbf{X}_k^{(\pi)}$ can only increase the values of the functions $\mathbb{E}\log_2\left|\mathbf{I}+\rho\tilde{\mathbf{H}}_k\mathbf{X}_k^{(\pi)}\tilde{\mathbf{H}}_k^H+\rho\sum_{\ell\in\mathcal{K}_k^{(\pi)}}\tilde{\mathbf{H}}_\ell\mathbf{W}_\ell^H\mathbf{Q}_\ell^{(\pi)}\mathbf{W}_\ell\tilde{\mathbf{H}}_\ell^H\right|$. In conclusion the optimal matrices $\mathbf{X}_k^{(\pi)}$ are diagonal, that we will denote with $\mathbf{P}_k^{(\pi)}$. The spectral decomposition of the optimal covariance matrices are: $\mathbf{Q}_k^{(\pi)} = \mathbf{W}_k\mathbf{P}_k^{(\pi)}\mathbf{W}_k^H$.

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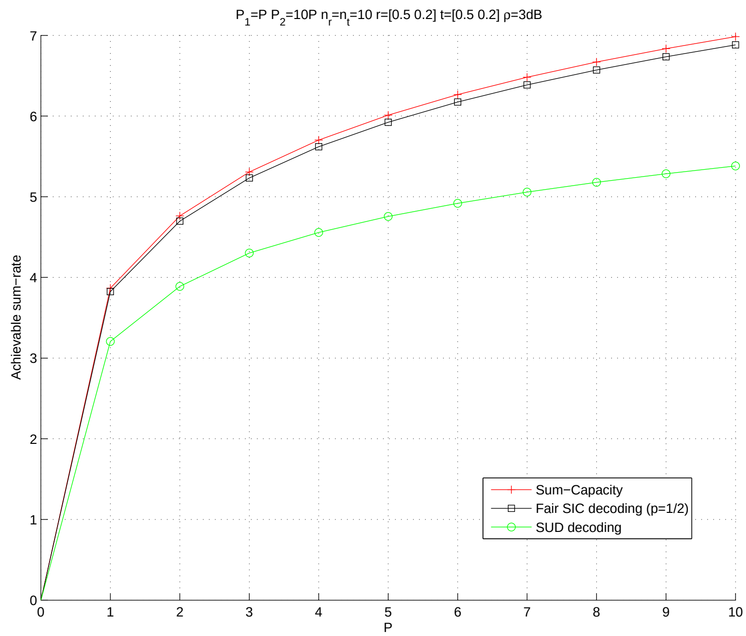

Fig. 1. Fair SIC (joint space-time power allocation) vs. SUD decoding. Achievable network sum-rate versus the available transmit power P for $p = \frac{1}{2}$, $n_r = n_t = 10$, $r = [0.5, 0.2]$, $t = [0.5, 0.2]$, $\rho = 3dB$. The fair SIC performs much closer to the sum-capacity upper bound than SUD.

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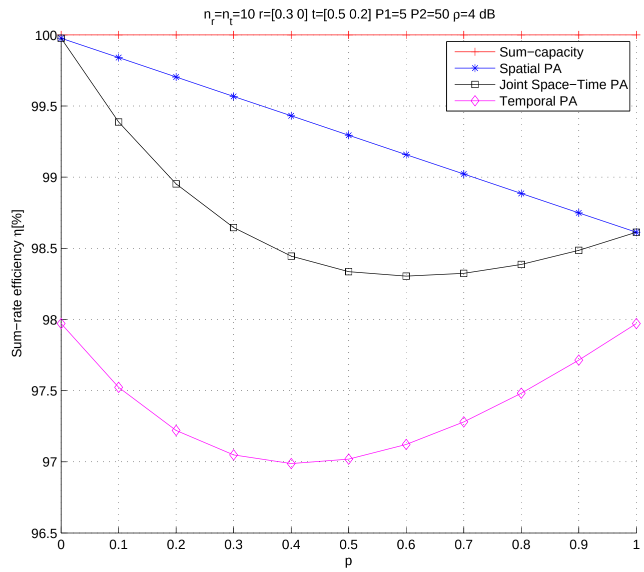

Fig. 2. SIC decoding, comparaison between the joint space-time PA and the two special cases: the space PA and temporal PA. Sum-rate efficiency versus the distribution of the coordination signal $p \in [0, 1]$ for $n_r = n_t = 10$, $r = [0.3, 0]$, $t = [0.5, 0.2]$, $\rho = 4dB$, $P_1 = 5$, $P_2 = 50$. The spatial PA outperforms the joint space-time PA (Braess paradox).

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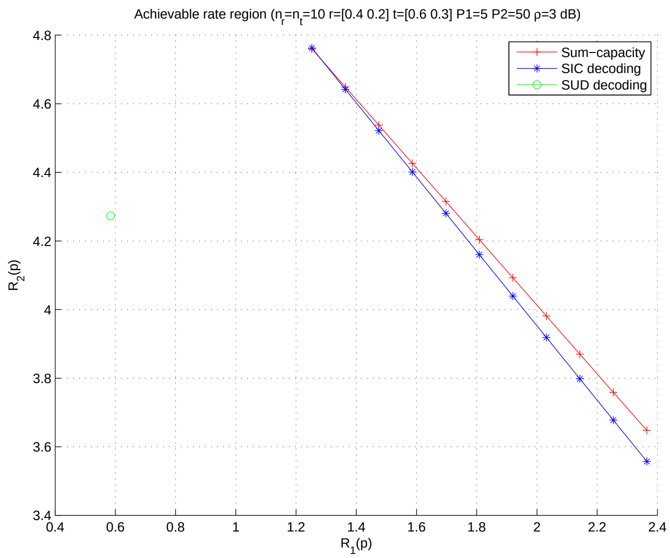

Fig. 3. SIC decoding, space PA. The achievable rate region at the NE versus the distribution of the coordination signal $p \in [0, 1]$ for $n_r = n_t = 10$, $r = [0.4, 0.2]$, $t = [0.6, 0.3]$, $\rho = 3dB$, $P_1 = 5$, $P_2 = 50$. Varying p allows to move along a segment close to the sum-capacity, similar to SISO MAC.

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