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Abstract: This paper deals with the quantitative and qualitative ultrasonic imaging of media contrasting strongly with the background (bones, for example). The wave propagation in these structures, which involves on several phenomena, generates ultrasound signals forming packages with different time and frequency signatures. To correlate the complex physical processes involved with the various packets forming the signals, it is advisable to separate the useful information (about the various packages, for example) from the parasite information (electronic noise). After testing several possible methods (filtering, deconvolution) with variable levels of success, it was proposed here to increase the resolution of the ultrasound signals, using a method of processing which was optimized by simultaneously analyzing the time and frequency parameters. For this purpose, algorithms based on the wavelet decomposition of the signals were used and suitable transmitted signals correlated with the parameters of the experimental device and the incident wave, were selected. The mathematical properties of the transmitted signal could be adapted themselves well to this time and frequency based approach. This method, which we have called the “wavelet based deconvolution method”, makes it possible to determine the transfer function of the scatterer, which we attempted to reconstruct.

Key words: Ultrasonic tomography, hard tissues, deconvolution, wavelet transform.

A. Introduction

Medical imaging is the main field in which ultrasonic computed tomography (UCT) is applied. UCT, which involves the linearisation of the inverse problem of acoustic wave propagation, can be used to detect small perturbations occurring against a reference background. In the case of soft tissues, the mean medium (the background) serves as a homogeneous reference medium, and the waves cross an almost homogeneous and isotropic medium with similar acoustical properties to that of water. When the contrasts are too strong (in the case of bone structures for example), linear theories no longer reasonably apply, and the presence of some physical phenomena associated with the wave propagation makes it necessary to change the methods used for the acquisition of the ultrasonic signals. The more the frequency of the wave increases, the more its propagation will obviously be perturbed. To overcome these problems, the use of low ultrasound frequencies (< 3 MHz) provides an effective possible alternative [1]: when the frequency of the transmitted wave decreases, certain zones turn out to be more homogeneous and the attenuation is also reduced. However, if the depth of the field increases, the resolution of the signals and hence that of the reconstructed images is bound to decrease. Even with low frequencies, the complexity of the wave propagation process generates extremely complex acoustic signals consisting of several packets with different signatures, which it is often difficult to analyze (in terms of the wavepaths, volume, guided or surface waves, and attenuation) and to interpret.

To improve the quality of the signals used either to reconstruct a tomographic image or to characterize the physical processes involved, or both, we focused on the signal processing. In the particular case where ultrasonic imaging is applied to structures, which contrast strongly with the background, optimizing the signal processing used seemed to be a useful approach because tissue characterization requires quantified acoustical information extracted from signals. Several approaches to this problem have been used so far, such as filtering, spectrum analysis and a method involving the deconvolution of the signals using a characteristic transfer function of the experimental device [2]. The latter operation improves the information in the high and low frequency ranges. The alternative method presented in the present study, which is based on a multi-scale decomposition procedure of the signals, makes it possible to process all the information available in terms of frequency and time. But this analysis alone does not suffice to optimize the signal processing because the acoustic signature of the transmitted wave, i.e. the incident wave reaching the scatterer, also has to match some or all of the wavelets’ mathematical properties. The main aim of this study is to show the usefulness of this method based on the optimized shaping of the electronically transmitted signal associated with the incident wave propagating through the media, in comparison with the conventional signal processing approach.

B. Acoustical statements

The frequencies used here (1 MHz) where performing UCT on strongly contrasted materials are therefore lower than those formerly used in practice (for clinical and other purposes, > 3 MHz). This increases the wavelength of the transmitted wave (1.5 mm in water) and decreases the quantity of energy lost during the propagation of the wave. It also increases the depth of penetration of the wave into the medium. The scatterer analyzed will be then taken to consist of a quasi-homogeneous medium. But this reduces the accuracy of the separating power and finally

- 1 -
the resolution decreases. It is then more difficult on the
tomogram to identify details such as boundaries (see Fig.1).

Fig.1 Effects of the frequency on the resolution of ultrasonic
computed tomography performed on adult human thighbone
(left) 500 kHz, (right) 2.25 MHz.

The propagation of the waves will result in signals
with specific characteristics, which will depend on the
pathways taken and on the mode of data acquisition
(reflection, transmission or diffraction) used. The
following examples (see Fig.2) show how difficult it is to
analyse the acoustical signatures. In this example,
working in the transmission mode (but similar complexity
occurs in the reflection and diffraction modes), a different
signature and different packets can be detected,
depending on whether the wave is propagating through a
solid region (2), a medium solid region (1) comparable to
a multi-layer (solid/fluid/solid) material, or the region
near the border contrasting with the background (3).

Fig.2. Ultrasonic signals depending on the bone region
involved: (1) multi-layer solid/fluid/solid (2) solid (3) limit
contrasting with the background

In the case of objects showing a strong contrast with
the background, a large proportion of the incident energy
is scattered (or simply reflected) and the transmitted
proportion is refracted (deviated) after modal conversion
at the interface between the water/scatterer/central cavity,
at various angles. The diffraction will not be isotropic.
The various wave packets constituting the signals depend
decisively on the pathway taken and on the nature of the
waves (compressional or shear waves, surface or guided
waves). The idea of using a specific signal processing
method is to extract as much information as possible
about these various packets by analyzing the
time and frequency signatures of the signals. It is therefore
necessary to decompose these signals so as to identify
their constituents.

Wavelet transformation makes it possible to process a
signal and to analyze the very local, and much more
global, frequency and time parameters. Based on this
transformation, a time versus scale diagram can be
obtained, giving the evolution of the frequencies with
time. Like time-scale processing, wavelet decomposition
lends itself very well to detecting and discriminating
between signals during the data pre-processing phase as
well as the filtering step carried out during the image
reconstruction phase.

C. Material and methods

The test bench

The test bench we have developed consists in a main
symmetrical arm carrying two secondary transversal arms.
Each arm supports an aligned transducer, which can be
moved linearly or turned.

Fig.3. Ultrasonic control of a high-contrast scatterer

The electro-acoustic device and the transducers
therefore serve as a continuous, linear stationary causal
filter. The input and output signals are connected by
convolution:

\[ s(t) = (x \otimes h_M)(t) = (h_T \otimes e \otimes h_M)(t), \]

where \( e(t) \) is the electric signal conveyed to the
transmitter, \( h_T(t) \) is the pulse response of the two
transducers (which is assumed to be known and to be
identical), and \( h_M \) is the pulse response of the scatterer.

When the input \( x(t) \) cannot be reasonably
approximated by a Dirac function, the function \( h_M(t) \) has
to be deduced from the output measurement \( s(t) \) and then
we must deal with deconvolution.

C.1. Wavelet based deconvolution

Since \( e(t) \) is generally comparable to a unit impulse
function (i.e. a Dirac delta function), the input will be
equal to the transducer’s response \( x(t) = h_T(t) \). Let us
now generate a wave corresponding to a transmitted
signal \( x(t) \) having suitable properties for specific
processing method, such as wavelet analysis.

If \( x(t) \) is a wavelet denoted \( \varphi_j(t) \) which is centered
on the (dilation) scale denoted \( J (J \in \mathbb{Z}) \), we have:

\[ s(t) = \varphi_j(t) \otimes h_M(t), \]

Time-scale decomposition of the signal \( s(t) \) on a
dyadic grid [4-5] makes it possible to calculate the
coefficients \( X_j \),

\[ X_j(t) = s(t) \otimes \phi_{j0}(t) = h_M \otimes \varphi_j \otimes \varphi_{j0}(t) \]

where \( \phi_{j0}(t) \) is a wavelet centered on the scale j.
The properties of the wavelet decomposition (an orthogonal decomposition in this case) are such that the coefficients, $X_j$, nullify everywhere except for $j = J$, which makes it possible to directly determine the response of the scatterer without encountering any further filtering effects.

$$X_j(t) = \begin{cases} (h_M \otimes \delta)(t) = h_M(t) & \text{if } j = J \\ 0 & \text{if } j \neq J \end{cases}, \quad (4)$$

If it is possible to process the transfer function of the experimental device in such a way that it is identical to a wavelet function, this algorithm directly yields the response of the scatterer.

**D. Results, simulation and experiments**

Let us take the case of a multi-layer plane interface (water/Plexiglas/water) with thickness $D$ and compressional wave velocity $c_M = 2700 \text{ m/s}$.

The case of the interface at a depth of 2 mm is interesting because the wavelength in the water at 1 MHz (equal to 1.5 mm for a compression wave velocity of 1500 m/s) is close to the depth, and the signals have therefore merged, which affects the resolution (see Fig.4.).

![Simulated ultrasonic signals](image1)

**Fig.4.** Simulated ultrasonic signals transmitted in plane interfaces at various depths $D$ (left) 5 mm, (right) 2 mm; (height), pulse echo (bottom), wavelet at $J = -4$, 512 samples, frequency rate 20 MHz, velocity of compression waves $= 2700 \text{ m/s}$

These signals were then decomposed into wavelet functions (several scales $j$). The maximum coefficients of the dyadic grid were used to identify the scale $j$, where the echoes are separated.

![Dyadic grid](image2)

**Fig.5.** Dyadic grid corresponding to the scale $J$ of non-zero coefficients (height) pulse echo (bottom), wavelet $J = -4$ (left) 5 mm, (right) 2 mm, 512 samples, frequency rate 20 MHz, velocity of compression waves $= 2700 \text{ m/s}$

Based on the time at which the maximum coefficient occurred in the 1st group and the time at which the...
maximum coefficient was reached in the 2nd group, we obtained the following thickness values (Table 1).

<table>
<thead>
<tr>
<th>Real thickness (mm)</th>
<th>Calculated thickness (mm) (error %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pulse echo</td>
</tr>
<tr>
<td>5</td>
<td>5.4 (8 %)</td>
</tr>
<tr>
<td>2</td>
<td>3.24 (62 %)</td>
</tr>
</tbody>
</table>

When the echoes are fairly well separated, it is also possible to discriminate between the -input and output- interfaces, and the error was about 8%. With the wavelet-based deconvolution method, the error was reduced to 0.099%. At a thickness of 2 mm, the discrimination based on the pulse echo was more difficult (the error was about 62%), whereas using the wavelet-based deconvolution method, the error at the interface was only about 5.5%.

It is not easy to test this method experimentally because approaching the numerical transmitted signal \( x(t) \) in the form of wavelets makes it necessary to process an initial function, which has to be filtered by the transducer to obtain a specific wavelet. Several approaches were previously tested with various levels of success, such as those based on direct deconvolution and simulated annealing, and especially manual determination, which consists in iteratively producing the signal by correcting the quadratic error at every iteration.

The experimental sample used here was a non-circular homogeneous isotropic tube made of artificial resin (NEUKADUR ProtoCast 113™). Its maximum internal and external diameters, which were measured with a caliper, were 6 and 12 mm, respectively. The density of the resin was \( \rho_1 \approx 1150 \text{ kg/m}^3 \), and the mean velocity of the compressional wave was \( c_1 \approx 2400 \pm 50 \text{ m/s} \).

The surrounding fluid-like medium (and that present in the hollow area) was water at a temperature of \( 18^\circ \) \( (\rho_0 = 1000 \text{ kg/m}^3, c_0 = 1480 \text{ m/s}) \). The transmitter and the receiver were placed 17.5 cm to the right of the center of the bench. The sinogram drawn up contained 180 projections distributed over 360° with an angular increment of 2°. The signals contained 1024 distributed samples occurring every 50 ns (frequency rate of 20 MHz). The nominal frequency of the transducer was 1 MHz.

After performing wavelet-based decomposition on the all cross-sectional signals received, the definition of the outer and inner boundaries of the cylinder was obtained with a much better resolution.

Figure 6. Ultrasonic computed tomography of a non-circular elastic tube. (height) pulse echo (bottom), wavelet \( J = -4 \) (left) sinograms, RF-signals, 180 projections through 360°, 1024 samples; (right) tomogram with 300 x 300 pixels.

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F. Literature