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A new approach is presented for the design of full-diversity multilayer space-time error correcting codes (STECCs), for two transmitters. In addition to the linearity which is exploited to transmit linear combinations of forward error correcting (FEC) codewords, the proposed approach uses space-time threading concepts to obtain high diversity and coding gain on each block-fading multiple-input multiple-output channel corresponding to a space-time codeword. We present in this paper the construction of 2×3 multilayer STECCs, which outperform the previously known ones.

I. Introduction

Space-time block (STB) code designs have recently attracted considerable attention since they improve the reliability of communication systems over fading channels. Tarokh et al. [1] developed some criteria for designing STB codes (for the high SNR regime), in order to minimize the pairwise error probability. Different designs were proposed based on these criteria as orthogonal space-time block (OSTB) codes, introduced by Alamouti [2] and generalized by Tarokh [3], which attracted a lot of interest due to their low optimal decoding complexity. However, OSTB codes have limited rates, thus they do not exploit the full potential of a multiple input multiple output (MIMO) system. Recognizing that OSTB codes do not achieve full-channel capacity in MIMO channels (except for the Alamouti scheme with 2 transmit and 1 receive antennas), Hassibi et al. proposed the linear dispersion codes (LDCs) [4] that maximize the mutual information between transmitted and received signals in order to achieve the full-ergodic capacity of the equivalent MIMO system.

Subsequent work designed full-rate and full-diversity STB codes, using the threaded layering concept, like the threaded algebraic space-time (TAST) codes [5]. The main idea of these codes is that, they are constructed such that threads in the structure are lied in different algebraic subspace to be "transparent" to each others, and each one exploits all the channel spatial diversity in the absence of the others. Further work [6], [7] produced full-rate and full-diversity algebraic STB codes with threaded/multilayer structure with nonvanishing determinant, in order to achieve the diversity-multiplexing gain (D-MG) tradeoff [8]. Another family of full-rate, full-diversity STB codes with nonvanishing determinant, based on the cyclic division algebra, was proposed in [9], [10], [11]. These codes achieve an optimum compromise between the transmission quality and its throughput. It is shown how to construct such codes for any number of transmit antennas.

However, the entire above schemes do not take conjointly into account the FEC techniques, which are added to significantly increase the transmission diversity (transmission quality). A joint design of error control coding, modulation and space-time scheme has been considered in [1] in order to construct the space-time trellis codes (STTCs) that provide a substantial coding gain and a high diversity. These codes have a better performance than STB codes, but since STTCs are based on trellis codes they have a high decoding complexity. In [12], a new family of space-time codes, with low decoding complexity, that tend to incorporate the FEC technique in the design of space-time codes has been proposed. It was shown for this family that using the FEC linearity to transmit linear combinations of FEC codewords, gives a good performance with respect to structures applying the space-time coding and error correcting codes separately, which means that this family of space-time codes seems more adapted than other ones to be concatenated with FEC codes. In this paper, definitions of [12] are slightly modified and we use the term STECCs to represent only the STB codes based on linear combinations of bits, without taking into acount the FEC code applied to binary data, in order to reveal more the ability of this family to correct errors due to the transmission. We refer also to the serial concatenation of both STECCs and FEC codes as concatenated STECCs. We show that STECCs proposed in [12] does not benefit from space-time techniques, to obtain high diversity and coding gain on a block fading channel.

The main purpose of this paper is to combine the threaded layering concept with the STECCs proposed in [12]. In addition to linear combinations used in the construction of these structures to create space-time redundancy, this approach uses the rank and determinant criteria [1], [13], to optimize the space-time design, in order to increase the diversity and the coding gain of the STECCs without increasing the complexity at the receiver. This paper is organized as follows. In section II, we recall the space-time code design criteria for coherent block fading channels, where the channel state information (CSI) is available only at the receiver. Moreover, we demonstrate that the STECCs presented in [12] do not verify the space-time code design criteria. In section III, we present the principle of threaded layering concept to construct a 2×3 full diversity rectangular space-time structure. Furthermore, we apply this concept to the STECCs, and we explain how one can relax

This work was supported in part by Orange Labs.

constraints on space-time code constructions, thanks to the redundancy generated by linear combinations. In section IV, we present the performance of the resulting full-diversity multilayer STECC in comparison with the STECC. Finally, section V presents our conclusions.

II. Coherent Space Time Coding

A. System Model and Notations

In the following, boldface lower case letters will denote vectors and boldface capital letters will denote matrices. Letters $\mathbf{A}_{j,.}$ and $\mathbf{A}_{.,j}$ represent the *jth* row and the *jth* column of the matrix $\mathbf{A}$, respectively. $\mathbf{I}$ represents the identity matrix, $\mathbb{Z}$ and $\mathbb{C}$ denote, respectively, the ring of rational integers and the field of complex numbers. The imaginary number is denoted by $i \triangleq \sqrt{-1}$, and $\mathbb{Z}[\mathrm{i}]$ denotes the ring of complex integers. Finally, $\mathrm{diag}(a_1, \ldots, a_n)$ represents a diagonal matrix where a_j, $j = 1, \ldots, n$, are its diagonal elements.

We consider a coherent system over a $n_t \times n_r$ non frequency-selective block fading channel, where the CSI is perfectly estimated at the receiver. The $n_r \times T$ received signal is

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N} \tag{1}$$

where $\mathbf{X}$ is the $n_t \times T$ transmitted codeword, $\mathbf{H}$ is the $n_r \times n_t$ channel matrix with independent and identically distributed (i.i.d) zero-mean complex Gaussian entries and $\mathbf{N}$ is assumed to be the $n_r \times T$ i.i.d zero-mean complex Gaussian noise.

B. Space-Time Code Design Criteria

It has been shown in [13] that in a coherent scenario using a maximum likelihood (ML) decoding, the space-time code design criteria to minimize the maximum pairwise error probability (PEP) of estimating a codeword $\hat{\mathbf{X}} \neq \mathbf{X}$ at the receiver while $\mathbf{X}$ has been sent, can be summarized as

- *The rank criterion:* Maximize the minimum rank r of matrix $\mathbf{A}(\mathbf{X}, \hat{\mathbf{X}}) = (\mathbf{X} - \hat{\mathbf{X}})(\mathbf{X} - \hat{\mathbf{X}})^H$.
- *The determinant criterion:* Maximize the minimum product of the non-zero eigenvalues, $(\prod_{j=1}^{r} \lambda_j)$, of matrix $\mathbf{A}(\mathbf{X}, \hat{\mathbf{X}})$. This criterion maximizes the coding gain.

It clearly appears that the maximum diversity advantage in this context is $n_t \times n_r$. Space-time codes that achieve such a diversity are called full-diversity codes [1], [13].

C. 2×3 STECCs [12] : Theoretical Analysis

A 2×3 STECC is a 2×3 rectangular STB code based on linear combinations between information bits to create a space-time redundancy. The goal of this subsection is to verify that the STECC extracted from [12] does not satisfy the space-time design criteria.

By considering a M-ary quadrature amplitude modulation (M-QAM), where $M = 2^{m_b}$ so that m_b is the modulation efficiency, the 2×3 STECC is defined as

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & x_3 \\ x_{2\oplus 3} & x_{1\oplus 3} & x_{1\oplus 2} \end{bmatrix} \tag{2}$$

where x_j, $1 \leq j \leq 3$ represents the M-QAM symbol associated to the jth binary m_b-uplet denoted by $\mathbf{c}_j$ and $x_{l\oplus j}$, $1 \leq j \leq 3, 1 \leq l \leq 3, j \neq l$ represents the M-QAM symbol associated to the binary m_b-uplet denoted by $\mathbf{c}_{l\oplus j} = \mathbf{c}_j \oplus \mathbf{c}_l$, $\oplus$ stands for the mod-2 addition. This 2×3 STECC does not satisfy the rank criterion as it is possible to define another space-time error correcting codeword $\hat{\mathbf{X}}$ different from $\mathbf{X}$ by

$$\hat{\mathbf{X}} = \begin{bmatrix} x_1 & x_3 & x_2 \\ x_{2\oplus 3} & x_{1\oplus 2} & x_{1\oplus 3} \end{bmatrix}$$

such that the difference codeword matrix $\mathbf{B}(\mathbf{X}, \hat{\mathbf{X}}) = \mathbf{X} - \hat{\mathbf{X}}$, represented below, has a rank of 1 inferior to the maximum possible rank $r = n_t = 2$.

$$\mathbf{B}(\mathbf{X}, \hat{\mathbf{X}}) = \begin{bmatrix} 0 & x_2 - x_3 & x_3 - x_2 \\ 0 & x_{1\oplus 3} - x_{1\oplus 2} & x_{1\oplus 2} - x_{1\oplus 3} \end{bmatrix}$$

III. Multilayer STECCs

Our multilayer STECC is based on the threaded layering concept, developed by Gamal et al. [5], to construct full-diversity coherent space-time codes. For the sake of self-completeness, the next subsection will describe in details how to construct a 2×3 rectangular space-time structure to achieve full diversity and maximize the coding gain using this concept.

A. Construction of a 2×3 Rectangular Space-Time Code Based on the Threaded Layering Concept

The space-time threading [5] consists in designing a layered structure where each layer is active during all of the available symbol transmission intervals, and uses each of the n_t antennas equally often over time. The key principle of this universal framework is that each layer achieves a full diversity, when other thread elements are set to zero. Furthermore, threads must be "transparent" to each others which can be realized by introducing numbers for making each thread in a different algebraic subspace. These numbers are referred to as "Diophantine numbers" [5]. In general, the number of threads is less than or equal to the number of transmit antennas. In our case, we take two layers. The threaded layering set $\mathbb{L} = \{\ell_1, \ell_2\}$ is defined as
$\ell_j = \left\{ \left(\lfloor t + j - 1 \rfloor_{n_t} + 1, t \right) : 0 \leq t < T = 3 \right\}$ $1 \leq j \leq 2$,
where $\lfloor . \rfloor_{n_t}$ denotes the mod-n_t operation. Table.1 shows the threaded layering for a 2×3 structure using 2 threads.

Each layer can be represented by a 3×1 vector, where each entry is a linear combination of QAM symbols. Denote by $\mathbf{y}$ and $\mathbf{z}$ two 3×1 vectors representing the first and the second layer, respectively.

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \text{ and } \mathbf{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \mathbf{M} \begin{bmatrix} u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

where $\mathbf{u}^T = [u_1, \ldots, u_6]$ is the information symbol vector such that, each component u_j, $1 \leq j \leq 6$ belongs to a QAM constellation. $\mathbf{M}$ is a 3×3 complex rotation matrix. Then, the space-time codeword takes the following form $\mathbf{X} = \begin{bmatrix} y_1 & \phi z_2 & y_3 \\ \phi z_1 & y_2 & \phi z_3 \end{bmatrix}$ where ϕ is a complex number chosen to ensure full diversity and maximize the coding gain for the composite space-time code. ϕ is a Diophantine number.

TABLE I

THE THREADED LAYERING IN COHERENT SCENARIO USING 2 LAYERS. THE NUMBERS REFER TO THREAD INDEXES. THE VERTICAL AND HORIZONTAL AXES CORRESPOND TO THE SPATIAL AND TEMPORAL DIMENSIONS, RESPECTIVELY.

1	2	1
2	1	2

Denote by $\mathbf{R} = \begin{bmatrix} \mathbf{M}_{1,.} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{1\times 3} & \mathbf{M}_{2,.} \\ \mathbf{M}_{3,.} & \mathbf{0}_{1\times 3} \end{bmatrix}$ and $\acute{\mathbf{R}} = \begin{bmatrix} \mathbf{0}_{1\times 3} & \mathbf{M}_{1,.} \\ \mathbf{M}_{2,.} & \mathbf{0}_{1\times 3} \\ \mathbf{0}_{1\times 3} & \mathbf{M}_{3,.} \end{bmatrix}$ two 3×6 matrices, then the space-time codeword $\mathbf{X}$ can be represented by $\mathbf{X} = \begin{bmatrix} \left(\mathrm{diag}(1,\phi,1)\mathbf{Ru}\right)^T \\ \left(\mathrm{diag}(\phi,1,\phi)\acute{\mathbf{R}}\mathbf{u}\right)^T \end{bmatrix}$.

Thanks to the linearity of STB codes, we can reformulate the rank criterion saying that the code is fully diverse if $\left|\det(\mathbf{BB}^H)\right| \neq 0$, where $\mathbf{B} = \mathbf{X}\text{-}\hat{\mathbf{X}} \neq \mathbf{0}$ represents a codeword difference matrix. Let $\mathbf{s} = \mathbf{u} - \hat{\mathbf{u}}$ be the difference information vector, where $\hat{\mathbf{u}}$ is an information symbol vector different from $\mathbf{u}$ ($\hat{\mathbf{u}} \neq \mathbf{u}$), thus each component s_j, $1 \leq j \leq 6$, belongs to $\mathbb{Z}[i]$-lattice, since QAM symbols are finite subsets of $\mathbb{Z}[i]$. Then $\mathbf{B}$ can be written as $\mathbf{B} = \begin{bmatrix} (\mathbf{w}_1)^T \\ (\mathbf{w}_2)^T \end{bmatrix}$ where $\mathbf{w}_1 = \mathrm{diag}(1,\phi,1)\mathbf{Rs}$ and $\mathbf{w}_2 = \mathrm{diag}(\phi,1,\phi)\acute{\mathbf{R}}\mathbf{s}$.
Therefore, by calculating the determinant of $\mathbf{BB}^H$, we obtain

$$\det(\mathbf{BB}^H) = \|\mathbf{w}_1\|^2 \|\mathbf{w}_2\|^2 - |<\mathbf{w}_1|\mathbf{w}_2>|^2 \geq 0 \qquad (3)$$

and a necessary and sufficient condition to obtain a full diversity is that $\left(\mathrm{diag}(1,\phi,1)\mathbf{R} - \alpha\,\mathrm{diag}(\phi,1,\phi)\acute{\mathbf{R}}\right)\mathbf{s} \neq \mathbf{0}$ $\forall\ \alpha \in \mathbb{C}, \forall\ \mathbf{s} \neq \mathbf{0} \in \mathbb{Z}[i]^6$-lattice. Denote $\mathbf{s}_1$ and $\mathbf{s}_2$ two 3×1 vectors belonging to $\mathbb{Z}[i]^3$ where $\mathbf{s}^T = [\mathbf{s}_1\ \mathbf{s}_2] \neq \mathbf{0}$. We can demonstrate that a full diversity is equivalent to

$$\begin{cases} \mathbf{M}_{1,.}\mathbf{s}_1 \neq \mathbf{0} & \forall\ \ \mathbf{s}_1 \neq \mathbf{0} \\ \mathbf{M}_{2,.}\mathbf{s}_2 \neq \mathbf{0} & \forall\ \ \mathbf{s}_2 \neq \mathbf{0} \end{cases} \text{ and } \frac{\phi\mathbf{M}_{1,.}\mathbf{s}_2}{\mathbf{M}_{1,.}\mathbf{s}_1} \neq \frac{\mathbf{M}_{2,.}\mathbf{s}_1}{\phi\mathbf{M}_{2,.}\mathbf{s}_2} \qquad (4)$$

Or

$$\begin{cases} \mathbf{M}_{2,.}\mathbf{s}_2 \neq \mathbf{0} & \forall\ \ \mathbf{s}_2 \neq \mathbf{0} \\ \mathbf{M}_{3,.}\mathbf{s}_1 \neq \mathbf{0} & \forall\ \ \mathbf{s}_1 \neq \mathbf{0} \end{cases} \text{ and } \frac{\phi\mathbf{M}_{3,.}\mathbf{s}_2}{\mathbf{M}_{3,.}\mathbf{s}_1} \neq \frac{\mathbf{M}_{2,.}\mathbf{s}_1}{\phi\mathbf{M}_{2,.}\mathbf{s}_2} \qquad (5)$$

One can see that equation (4) (resp. (5)) implies that the first and the second column (resp. the second and the third column) of a difference codeword matrix are linearly independent. We note that, by using the threaded layering concept we can not guarantee that the first and the third column are linearly independent. When the full diversity is ensured, we maximize the coding gain by applying the determinant criterion.

B. Full-diversity Multilayer STECC

In this subsection, we apply the threaded layering concept to the STECC extracted from [12] and defined in equation (2) in order to increase the diversity and maximize the coding gain. The key principle of our approach is to benefit from the linear combinations of $\{\mathbf{c}_j\}_{1\leq j\leq 3}$, to relax constraints on the rotation matrix and the Diophantine number. Moreover, we keep properties of STECC defined in equation (2), *i.e*, each entry of a space-time codeword matrix is composed of one M-QAM symbol. In addition, we assign the information symbols to the first layer ℓ_1 and the redundancy symbols to the second one ℓ_2. In this case, we have an identity rotation matrix and constraints imposed by equations (4) and (5) are greatly reduced since the two layers are dependent.

A proper arrangement of redundancy symbols associated to ℓ_2 with respect to information symbols, and a judicious choice of ϕ must be done in order to ensure that we have always in a difference codeword matrix 2 columns that are linearly independent (full diversity), and also in order to maximize the coding gain. Thanks to linear combinations, difference space-time symbols verify these 2 properties

1) if $s_j = 0$, $s_l \neq 0 \Rightarrow s_{j\oplus l} \neq 0$.
2) if $s_j = 0$, $s_l = 0 \Rightarrow s_{j\oplus l} = 0$.

where $j,l \in \{1,2,3\}$, and $j \neq l$. Thus, the first row of the codeword matrix must include a redundancy symbol $x_{j\oplus l}$ such that, x_j and x_l are assigned to 2 different layers. As the information and the redundancy symbols belong to a subset of $\mathbb{Z}[i]$, then it is easy to verify that $\phi^2 \notin \mathbb{Z}[i]$ is a sufficient condition to ensure a full diversity taking into account a proper arrangement of redundancy symbols. Therefore, we obtain 2 different arrangement possibilities to construct a full diversity space-time code. We can thus define the full-diversity multilayer STECC as

$$\mathbf{X} = \begin{bmatrix} x_1 & \phi x_5 & x_3 \\ \phi x_4 & x_2 & \phi x_6 \end{bmatrix} \qquad (6)$$

where $(x_4, x_5, x_6) \in \left\{(x_{1\oplus 3}, x_{1\oplus 2}, x_{2\oplus 3}), (x_{2\oplus 3}, x_{1\oplus 2}, x_{1\oplus 3})\right\}$, whatever $\phi^2 \notin \mathbb{Z}[i]$. Additionally, to ensure an energy efficiency we introduce the shaping constraint [10] that can be achieved by taking $|\phi| = 1 \Rightarrow \phi = e^{i\theta}$. This constraint ensures that both layers consumes the same energy and both transmit antennas radiate the same energy. Moreover, to maximize the coding gain, the determinant criterion must be realized, we can demonstrate in our case that maximizing the coding gain is equivalent to find θ_{optimal} such that

$$\theta_{\text{optimal}} = \arg \max_{\substack{0<\theta<2\pi \\ \theta\neq\frac{\pi}{2}\left[\frac{\pi}{2}\right]}} \min_{\substack{(s_1,s_2,s_3)\in S^3 \\ (s_1,s_2,s_3)\neq(0,0,0)}} \det(\mathbf{BB}^H), \qquad (7)$$

where $\det(\mathbf{BB}^H) = \left|s_1 s_2 - \phi^2 s_4 s_5\right|^2 + \left|s_1 s_6 - s_4 s_3\right|^2 + \left|\phi^2 s_5 s_6 - s_2 s_3\right|^2$, and S, a finite subset of $\mathbb{Z}[i]$, contains the difference modulation symbols. One can see that the coding gain for the 2 space-time possibilities are equal. In addition, for Gray-mapped 4-QAM constellation, we demonstrate (see the Appendix A) that the minimum of the determinant of $\mathbf{BB}^H$ is equal to 16 and is independent of the choice of θ, $0 \leq \theta < 2\pi$. We verify also by computer search that $\theta_{\text{optimal}} = 0$ maximize the coding gain for Gray-mapped 16-QAM constellation.

C. 2×3 Concatenated Multilayer STECCs

Let *C(n,L)* be a linear FEC code, where *n* denotes the code length, *L* its dimension and $\mathcal{C}$ the codeword set. The 2×3 concatenated multilayer STECC (resp. 2×3 concatenated STECC) is the 2×3 multilayer STECC (resp. 2×3 STECC) defined in equation (2) (resp. (6)) where $\mathbf{c}_j \in \mathcal{C}$, $1 \leq j \leq 3$ is an information codeword provided by a FEC encoder and $\mathbf{c}_{j \oplus l} = \mathbf{c}_j \oplus \mathbf{c}_l$, $j \neq l$, $1 \leq l \leq 3$ the linear combinations from these information codewords. Therefore, using a 2^{m_b}-QAM for the concatenated multilayer STECC, we obtain $\dfrac{n}{m_b}$ multilayer space-time error correcting codewords. At the receiver side, a lower-complexity turbo-like receiver such as the one described in [14], [15] has to be preferred, due to the high performance of such a receiver and its low complexity in comparison with the exponential one of the optimal receiver. This turbo-like receiver with an interference canceller, optimized according to the minimum mean square error (MMSE) criterion [15], is based on the cooperation of two entities: a MMSE equalizer that exchanges reliable information with a soft input soft output (SISO) channel decoder according to the turbo principle.

In the case of an iterative decoding we can demonstrate that the asymptotic coding gain [16] of the 2×3 concatenated multilayer STECC is independent of the parameter ϕ, and thus ϕ must only satisfy $\phi^2 \notin \mathbb{Z}[i]$. We note that this demonstration is not provided here due to the limited number of pages.

Additionally, the concatenated multilayer STECC is flexible enough, like the concatenated STECC [12], to use puncturing upon it. In order to increase the spectral efficiency, on the price of reducing the transmission diversity, 2 entire FEC codewords can be erased from the proposed structure without any loss of useful information. The puncturing technique must be smartly done so that the receiver can recover the erased FEC codewords by linear combinations of the remaining ones.

IV. Simulation Results

In this section, we present simulation results for two receive antennas using the Gray-mapped 2^{m_b}-QAM constellations. The channel is assumed to be Rayleigh block fading, constant over τ modulation symbol durations.

A. Performance of Multilayer STECC

In this subsection, we compare the proposed multilayer STECC with the STECC presented in [12]. In that case, a ML detection is considered at the receiver. Fig.1 shows that the slop of the multilayer STECC is equal to $n_t.n_r = 4$ for high SNR, which confirms the full diversity of this structure. On the other hand, one can see that the STECC does not ensure a full diversity as was theoretically proved in section II. Note that the computation cost remains the same as for the STECC. For $\tau = 3$ a gain of 5.3 dB is achieved at a BER of 10^{-4}. Moreover, the multilayer STECC performs 0.5 dB worse compared to the Alamouti scheme which is satisfactory as it is obtained without taking into account the FEC coding. Simulation results verify also that for $\tau = 2$, this structure benefits from the diversity of the channel and outperforms by 0.7 dB, at a BER=10^{-5}, the Alamouti scheme.

B. Performance of Concatenated Multilayer STECC

In this case, we assume that $\tau = 3$ and we consider both 4-QAM and 16-QAM Gray-mapped constellations. Fig.2 shows the performance of the concatenated multilayer STECC with respect to the concatenated STECC using the convolutional codes $CC(7,5)_{\mathrm{oct}}$ and $CC(13,15)_{\mathrm{oct}}$. At the reception, the lower complexity turbo-like receiver with an interference canceller optimized according to the MMSE criterion is adopted and convolutional codes are decoded using the SISO BCJR algorithm [17]. Fig.2 shows that the concatenated multilayer STECC performs better than the concatenated STECC for the same complexity. At a BER of 10^{-4} and for Gray-mapped 4-QAM and 16-QAM constellations the gain of the proposed structure is roughly equal to 0.7 dB and 0.5 dB using $CC(7,5)_{\mathrm{oct}}$ and $CC(13,15)_{\mathrm{oct}}$ respectively. By comparison to the 5.3 dB gain obtained when no FEC was applied, the gain is less significant which is due to the diversity improvement resulting from the FEC application.

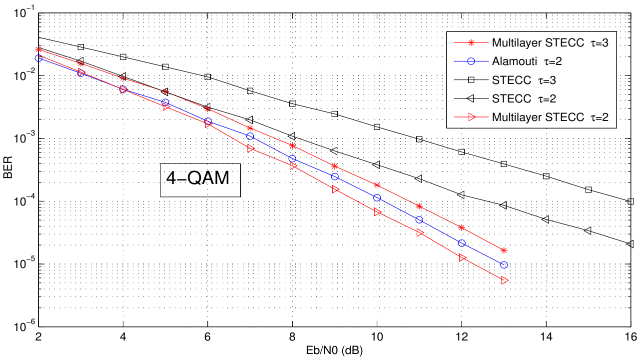

Fig. 1. Performance of the new structure compared to those of [2], [12].

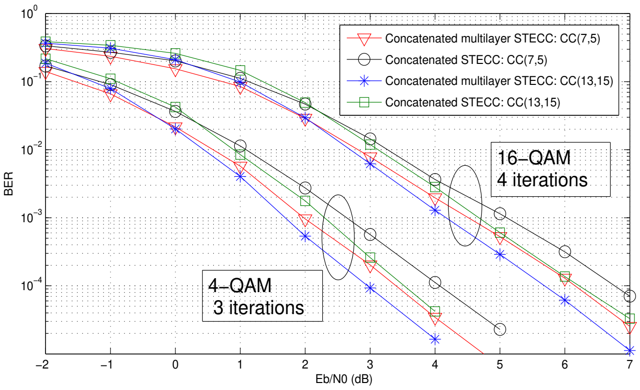

Fig. 2. Turbo Equalizer. L=510. $n_r = 2$. $\tau = 3$.

V. Conclusions

In this paper, we presented the application of threaded layering concept to modify the STECC, presented in [12], in order to increase its diversity. The multilayer STECC proposed for 2 transmit antennas benefits from the space-time code designs in order to increase the diversity and the coding gain of the STECC. We demonstrated that the proposed multilayer STECC achieves the maximum diversity. Simulation results that demonstrate the gain offered by the concatenated multilayer STECC with respect to the concatenated STECC were also presented. As a result constructing a full-diversity space-time code, which has an error protection at the center of its design, improves the transmission quality since the space-time code will be more adapted to be concatenated with a FEC code without increasing the receiver complexity.

Further works will thus consider a generalization of this approach to n_t transmit antennas ($n_t > 2$), and will try to reduce the redundancy in order to increase the code rate.

APPENDIX A

We demonstrate that for Gray-mapped 4-QAM constellation, the minimum determinant of $\mathbf{BB}^H$ is independent of the choice of $\phi = e^{i\theta}$. We have

$$\mathbf{B} = \begin{bmatrix} s_1 & \phi s_5 = \phi s_{1\oplus 2} & s_3 \\ \phi s_4 = \phi s_{1\oplus 3} & s_2 & \phi s_6 = \phi s_{2\oplus 3} \end{bmatrix}$$

where $\mathbf{s} = [s_1, \dots, s_6]^T$ and $s_j \in \{0, \pm 2, \pm 2i, \pm 2 \pm 2i\}$, $1 \leq j \leq 6$, thus $|s_j| \in \{0, m = 2, M = 2\sqrt{2}\}$. By developping the determinant of $\mathbf{BB}^H$, we obtain

$$\begin{aligned} \det(\mathbf{BB}^H) &= |s_1|^2 (|s_2|^2 + |s_6|^2) + |s_5|^2 (|s_4|^2 + |s_6|^2) \\ &+ |s_3|^2 (|s_4|^2 + |s_2|^2) - 2 \operatorname{Re}\{s_1 s_3^* s_4^* s_6\} \\ &- 2 \operatorname{Re}\{(\phi^2)^* s_2 s_5^* (s_1 s_4^* + s_3 s_6^*)\} \end{aligned}$$

where $\operatorname{Re}\{z\}$ denotes the real part of the complex number z. We consider that $(s_1, s_2, s_3) \neq (0,0,0)$. Remind that if $s_j = 0,\ s_l \neq 0 \Rightarrow\ s_{j\oplus l} \neq 0,\ j,l \in \{1,2,3\},\ j \neq l$, thus the minimum of $\det(\mathbf{BB}^H)$ can be determined by considering 2 cases :

1) $\prod_{j=1}^{6} |s_j| = 0,\ \exists\ k, 1 \leq k \leq 3, |s_k| \neq 0$

   If $s_2 s_5 = 0$
   $$\begin{aligned} \det(\mathbf{BB}^H) &\geq |s_2|^2 (|s_1|^2 + |s_3|^2) + |s_5|^2 |s_4|^2 \\ &+ |s_6|^2 |s_5|^2 + (|s_1| |s_6| - |s_3| |s_4|)^2 \end{aligned}$$
   If $s_1 s_3^* s_4^* s_6 = 0$
   $$\begin{aligned} \Rightarrow \det(\mathbf{BB}^H) &\geq (|s_4| |s_5| - |s_1| |s_2|)^2 + |s_3|^2 |s_4|^2 \\ &+ |s_1|^2 |s_6|^2 + (|s_2| |s_3| - |s_5| |s_6|)^2 \end{aligned}$$
   $\Rightarrow$ in this case $\det(\mathbf{BB}^H) \geq m^4$.

2) $\prod_{j=1}^{6} |s_j| \neq 0$

For a 4-QAM, $s_j,\ 1 \leq j \leq 6$ is constructed from 4 binary numbers, $\{c_{jl}\}, \{\acute{c}_{jl}\},\ 1 \leq l \leq 2$. Taking into account the following binary relation

$c_{jl} \oplus c_{kl} = \acute{c}_{jl} \oplus \acute{c}_{kl} \Leftrightarrow\ \forall\, \epsilon \in \{0,1\}, c_{jl} = \acute{c}_{jl} \oplus \epsilon,\ c_{kl} = \acute{c}_{kl} \oplus \epsilon$

We demonstrate that for Gray-mapped symbols

If $s_j s_k \neq 0,\ s_{j\oplus k} \neq 0 \Leftrightarrow \exists\, l \in \{1,2\},\ \exists!\, p \in \{j,k\},\ c_{pl} \neq \acute{c}_{pl}$

As a consequence, we conclude that

- $|s_j| |s_k| |s_{j\oplus k}| = m^2 M\ j,k \in \{1,2,3\}$ and $j \neq k$, each couple constructed from $\{s_j, s_k, s_{j\oplus k}\}$ is constituted of 2 elements which are linearly independent.
- $\prod_{j=1}^{3} |s_j| = m^2 M$

Thus we conclude that $\prod_{j=1}^{6} |s_j| \neq 0 \iff \exists\ j,k,p \in \{1,2,3\}\ j \neq k \neq p\ |s_j| = M, |s_k| |s_p| = m^2,\ |s_{k\oplus p}| = M, |s_{j\oplus k}| |s_{j\oplus p}| = m^2$. Therefore three possibilities are taken into account to lower bound the minimum determinant of $\mathbf{BB}^H$. Note that $\operatorname{Re}\{(\phi^2)^* s_2 s_5^* (s_1 s_4^* + s_3 s_6^*)\} \leq |s_2| |s_5| (|s_1| |s_4| + |s_3| |s_6|)$ and $M = \sqrt{2}m$.

- $|s_1| = M$, $|\operatorname{Re}\{s_1 s_3^* s_4^* s_6\}| \leq |s_1| |s_3| |s_4| |s_6|$
  $\Rightarrow \det(\mathbf{BB}^H) \geq (7 - 4\sqrt{2}) m^4 > m^4$
- $|s_2| = M$, $|\operatorname{Re}\{s_1 s_3^* s_4^* s_6\}| = \dfrac{\sqrt{2}}{2} |s_1| |s_3| |s_4| |s_6|$
  $\Rightarrow \det(\mathbf{BB}^H) \geq (4 - 2\sqrt{2}) m^4 > m^4$
- $|s_3| = M$, $|\operatorname{Re}\{s_1 s_3^* s_4^* s_6\}| = \dfrac{\sqrt{2}}{2} |s_1| |s_3| |s_4| |s_6|$
  $\Rightarrow \det(\mathbf{BB}^H) \geq (4 - 2\sqrt{2}) m^4 > m^4$

As a result from the whole above discusion we demonstrate that the minimum determinant of $\mathbf{BB}^H$ is equal to m^4 and is attained for $(s_1, s_2, s_3) = (0,0,2)$.

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