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## MAGNETIC FIELD PRODUCED BY A PARALLELEPIPEDIC MAGNET OF VARIOUS AND UNIFORM POLARIZATION

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**Abstract**—This paper deals with the modeling of parallelepipedic magnets of various polarization directions. For this purpose, we use the coulombian model of a magnet for calculating the magnetic potential in all points in space. Then, we determine the three components of the magnetic field created by a parallelepiped magnet of various polarization direction. These three components and the scalar magnetic potential are also expressed in terms of fully analytical terms. It is to be noted that the formulas determined in this paper are more general than the ones established in the literature and can be used for optimization purposes. Moreover, our study is carried out without using any simplifying assumptions. Consequently, these expressions are accurate whatever the magnet dimensions. This analytical formulation is suitable for the design of unconventional magnetic couplings, electric machines and wigglers.

### 1. INTRODUCTION

Permanent magnets are widely used in many engineering and industrial applications. Their utilization requires calculation methods based on the fundamental laws of the magnetostatics [1, 2]. Two great kinds of applications can be identified. The first ones use parallelepipedic permanent magnets while the second ones use arc-shaped permanent magnets. This paper deals only with parallelepipedic permanent magnets. However, this study can also be extended to the case of cylindrical permanent magnet topologies.

The first analytical studies dealing with the modeling of parallelepiped magnets were studied by Akoun [3] and Yonnet [4].

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Then, several analytical studies were carried out by using the coulombian model of a magnet [5, 6]. The interest of using fully analytical approaches lies in the fact that they have generally a lower computational cost than finite element methods [7–10]. Moreover, analytical approaches are suitable for parametric optimizations using permanent magnets [10], or coils carrying producing magnetic fields [14, 15]. The magnetic field created by parallelepipedic magnets can be expressed in fully analytical parts whereas the magnetic field produced by arc-shaped permanent magnets is generally based on special functions [16–25].

This paper presents 3D analytical expressions of the magnetic field created by a parallelepipedic permanent magnet of various polarization direction. Indeed, its polarization can be along the  $x$ ,  $y$  and  $z$  direction, in the  $(x-y)$ ,  $(x-z)$  and  $(y-z)$  planes but also in any direction in the coordinate system. Such a study is clearly justified by the progress in manufacturing permanent magnets with more complicated magnetizations. Moreover, permanent magnets with various polarization directions allow us to confine the magnetic flux in ironless structures [26, 27] and to optimize the magnetic field shape in electric machines [28].

We present first the 3D analytical expression of the magnetic scalar potential created by a parallelepipedic magnet of various polarization direction: Such an expression is useful for the study of ferrofluids used with permanent magnets [29, 30]. Indeed, the magnetic pressure of the ferrofluid seal requires the accurate knowledge of the magnetic potential in all points in space.

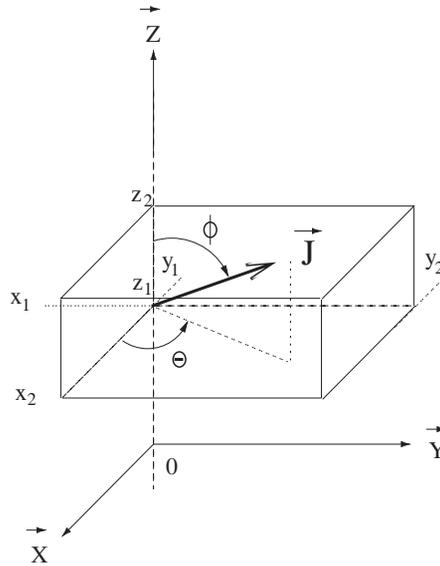
The second part of this paper presents the analytical expressions of the three components of the magnetic field created by a parallelepipedic magnet of various polarization direction.

## 2. ANALYTICAL EXPRESSION OF THE MAGNETIC POTENTIAL PRODUCED BY A PARALLELEPIPED MAGNET WITH A UNIFORM AND ARBITRARY POLARIZATION

### 2.1. Notation and Geometry

We present in this section the 3D analytical expression of the magnetic potential created by a parallelepiped magnet of various polarization direction. To do so, let us first consider the representation shown in Fig. 1.

Its dimensions are given by  $(x_2 - x_1)$ ,  $(y_2 - y_1)$ ,  $(z_2 - z_1)$  and its polarization is denoted  $\vec{J}$ . By using the notations shown in Fig. 1, this



**Figure 1.** Representation of a parallelepiped magnet of various polarization direction.

polarization vector is expressed as follows:

$$\vec{J} = J \cos(\theta) \sin(\phi) \vec{u}_x + J \sin(\theta) \sin(\phi) \vec{u}_y + J \cos(\phi) \vec{u}_z \quad (1)$$

We define the permanent magnet surface as follows:

$$\begin{aligned} d\vec{S}_1 &= +dydz\vec{u}_x \\ d\vec{S}_2 &= -dx dz\vec{u}_y \\ d\vec{S}_3 &= -dydz\vec{u}_x \\ d\vec{S}_4 &= +dx dz\vec{u}_y \\ d\vec{S}_5 &= -dx dy\vec{u}_z \\ d\vec{S}_6 &= +dx dy\vec{u}_z \end{aligned} \quad (2)$$

### 2.2. Analytical Formulation

In the coulombian approach, the magnetic potential created by the parallelepipedic magnet is given by:

$$\Phi(x, y, z) = \sum_{i=1}^6 \left( \frac{1}{4\pi\mu_0} \iint_{S_i} \frac{\vec{J} \cdot d\vec{S}_i}{|\vec{r} - \vec{r}_i|} \right) \quad (3)$$

For the rest of this paper, we adopt the following notation:

$$\vartheta^{(ijk)} \{\bullet\} = \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 (-1)^{i+j+k} (\bullet) \quad (4)$$

By using the equality  $\xi_{ijk} = \sqrt{(x-x_i)^2 + (y-y_j)^2 + (z-z_k)^2}$ , the magnetic potential created by a parallelepiped magnet of various polarization direction is expressed as follows:

$$\Phi(x, y, z) = \frac{J}{4\pi\mu_0} \vartheta^{(ijk)} \{(\sin(\phi) \cos(\theta)\Phi_1 + \sin(\phi) \sin(\theta)\Phi_2 + \cos(\phi)\Phi_3)\} \quad (5)$$

with

$$\begin{aligned} \Phi_1 &= z_k + (x-x_i) \arctan \left[ \frac{z-z_k}{x-x_i} \right] + (y-y_j) \log [z-z_k + \xi_{ijk}] \\ &\quad - (x-x_i) \arctan \left[ \frac{(y-y_j)(z-z_k)}{(x-x_i)\xi_{i,j,k}} \right] + (z-z_k) \log [y-y_j + \xi_{ijk}] \\ \Phi_2 &= z_k + (y-y_j) \arctan \left[ \frac{z-z_k}{y-y_j} \right] + (x-x_i) \log [z-z_k + \xi_{ijk}] \\ &\quad - (y-y_j) \arctan \left[ \frac{(x-x_i)(z-z_k)}{(y-y_j)\xi_{ijk}} \right] + (z-z_k) \log [x-x_i + \xi_{ijk}] \\ \Phi_3 &= y_j + (z-z_k) \arctan \left[ \frac{y-y_j}{z-z_k} \right] + (x-x_i) \log [y-y_j + \xi_{ijk}] \\ &\quad - (z-z_k) \arctan \left[ \frac{(x-x_i)(y-y_j)}{(z-z_k)\xi_{ijk}} \right] + (y-y_j) \log [x-x_i + \xi_{ijk}] \end{aligned} \quad (6)$$

As stated previously, the magnetic scalar potential  $\Phi(x, y, z)$  is fully analytical and does not require any numerical treatment for its determination. Its expression is suitable for representing its isopotentials inside the magnet as well as outside it.

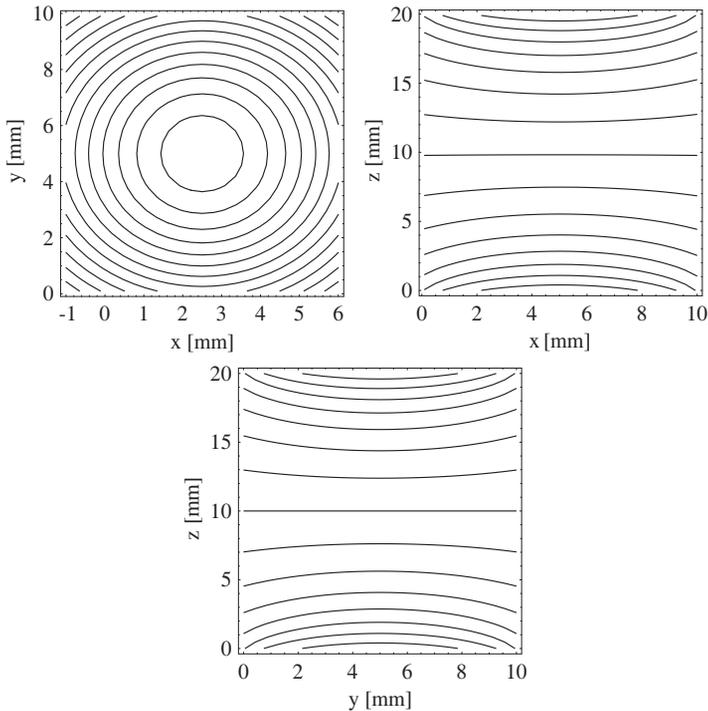
### 2.3. Representation of the Magnetic Scalar Potential

We illustrate our previous analytical expression with two configurations. The first one allows us to verify the accuracy of our expression and allows us to compare it with the ones published in the literature. The second configuration is less usual as the first one: It is an academic illustration of the usefulness of our 3D analytical expression.

*2.3.1. Case of a Parallelepipedic Magnet Whose Polarization Is Directed along the z Direction*

The first configuration is well known and corresponds to the case when the polarization is directed along the  $z$ -direction. We take the following dimensions:  $x_2 - x_1 = 0.005$  m,  $y_2 - y_1 = 0.01$  m,  $z_2 - z_1 = 0.02$  m,  $J = 1$  T. We represent in Fig. 2 three 2D cross-sections of the isopotentials created by the parallelepipedic magnet whose polarization is directed along the  $z$  direction.

Figures 2 show that the iso-potentials are circles in the  $(x-y)$  plane, which is consistent with the polarization direction of the parallelepipedic permanent magnet. Moreover, we see the isopotentials in the  $(x-y)$  and  $(y-z)$  planes are the same: It is still consistent with the polarization direction of the parallelepipedic permanent magnet.



**Figure 2.** 2D representation of the iso-potentials created by a parallelepiped magnet whose polarization is directed along the  $z$  direction;  $(x-y)$ -plane:  $z = 15$  mm,  $(x-z)$ -plane:  $y = 5$  mm,  $(y-z)$ -plane:  $x = 5$  mm.

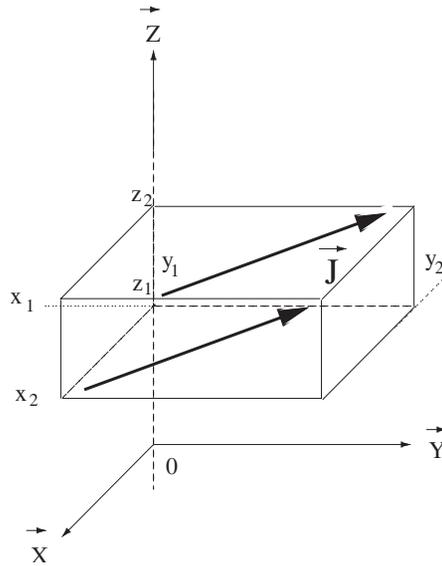
### 2.3.2. Case of a Parallelepipedic Magnet of Various Polarization Direction

The second configuration we consider is a parallelepipedic permanent magnet with the polarization shown in Fig. 3: This polarization is expressed as follows:

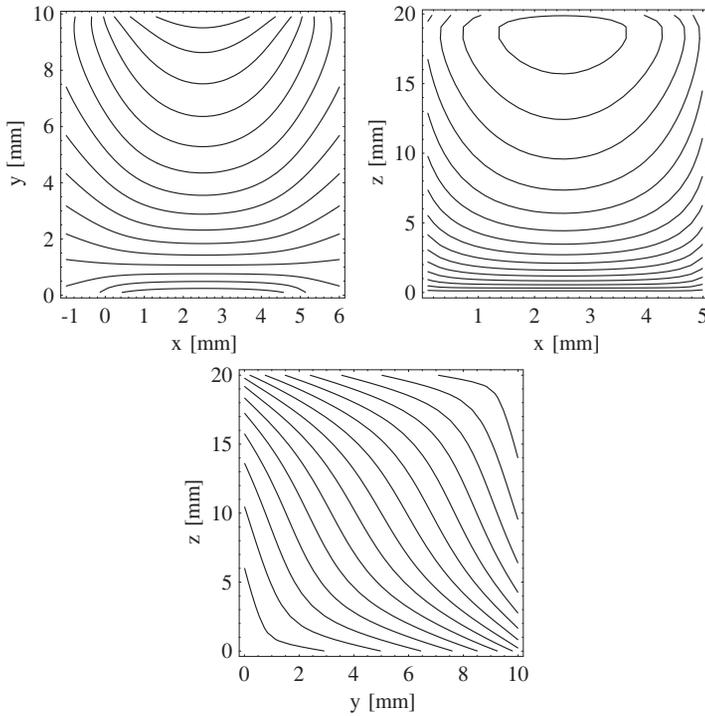
$$\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z \quad (7)$$

We take the following dimensions:  $x_2 - x_1 = 0.005$  m,  $y_2 - y_1 = 0.01$  m,  $z_2 - z_1 = 0.02$  m,  $J = 1$  T. We represent in Fig. 4 three 2D cross-sections of the iso-potentials created by the parallelepiped magnet with the polarization  $\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z$ . In this configuration, we take  $\theta = \frac{\pi}{2}$  rad and  $\phi = \frac{\pi}{4}$  rad.

The computational cost is 1 s for representing the magnetic potential in the three previous illustrations.



**Figure 3.** Representation of a parallelepiped magnet with the following polarization vector:  $\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z$ .



**Figure 4.** 2D representation of the iso-potentials created by a parallelepiped magnet whose polarization is  $\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z$ ;  $(x-y)$ -plane:  $z = 19$  mm,  $(x-z)$ -plane:  $y = 10$  mm,  $(y-z)$ -plane:  $x = 5$  mm.

### 3. ANALYTICAL EXPRESSION OF THE MAGNETIC FIELD PRODUCED BY A PARALLELEPIPED MAGNET WITH A UNIFORM AND ARBITRARY POLARIZATION

The three components of the magnetic field created by one parallelepipedic magnet of various polarization direction can be determined by using the following expression:

$$\begin{aligned}
 H_x(x, y, z) &= -\vec{\nabla}(\phi(x, y, z)) \cdot \vec{u}_x \\
 H_y(x, y, z) &= -\vec{\nabla}(\phi(x, y, z)) \cdot \vec{u}_y \\
 H_z(x, y, z) &= -\vec{\nabla}(\phi(x, y, z)) \cdot \vec{u}_z
 \end{aligned}
 \tag{8}$$

After mathematical manipulations, we obtain the three components  $H_x(x, y, z)$ ,  $H_y(x, y, z)$  and  $H_z(x, y, z)$  that are expressed as fol-

lows:

$$\begin{aligned}
 H_x(x, y, z) &= \frac{J \sin(\phi) \cos(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \left\{ \arctan \left[ \frac{(y - y_j)(z - z_k)}{(x - x_i)\xi_{ijk}} \right] \right\} \\
 &\quad + \frac{J \sin(\phi) \sin(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [z - z_k + \xi_{ijk}]\} \\
 &\quad + \frac{J \cos(\phi)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [y - y_j + \xi_{ijk}]\} \\
 H_y(x, y, z) &= \frac{J \sin(\phi) \cos(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [z - z_k + \xi_{ijk}]\} \\
 &\quad + \frac{J \sin(\phi) \sin(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \left\{ \arctan \left[ \frac{(x - x_i)(z - z_k)}{(y - y_j)\xi_{ijk}} \right] \right\} \quad (9) \\
 &\quad + \frac{J \cos(\phi)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [x - x_i + \xi_{ijk}]\} \\
 H_z(x, y, z) &= \frac{J \sin(\phi) \cos(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [y - y_j + \xi_{ijk}]\} \\
 &\quad + \frac{J \sin(\phi) \sin(\theta)}{4\pi\mu_0} \vartheta^{(ijk)} \{-\log [x - x_i + \xi_{ijk}]\} \\
 &\quad + \frac{J \cos(\phi)}{4\pi\mu_0} \vartheta^{(ijk)} \left\{ \arctan \left[ \frac{(x - x_i)(y - y_j)}{(z - z_k)\xi_{ijk}} \right] \right\}
 \end{aligned}$$

If we take  $\theta = 0$  and  $\phi = 0$ , we obtain the same expression as Bancel [5].

### 3.1. Representation of the Magnetic Field

We illustrate now the use of our three-dimensional analytical expression by calculating the magnetic field modulus  $\mathbf{H}$ . Then, we represent the iso-lines in two configurations corresponding to the previous ones.

#### 3.1.1. Case of a Parallelepipedic Magnet Whose Polarization Is Directed along the $z$ Direction

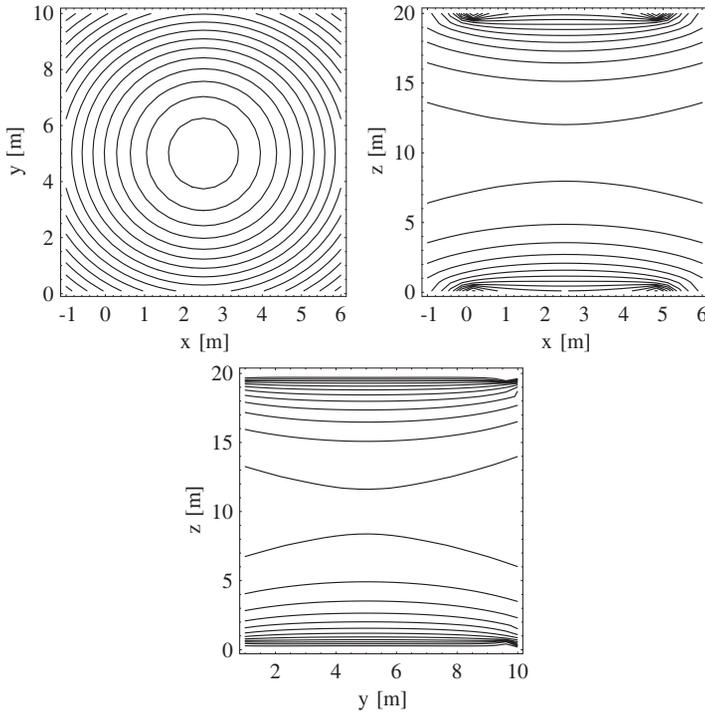
We take the following dimensions:  $x_2 - x_1 = 0.005$  m,  $y_2 - y_1 = 0.01$  m,  $z_2 - z_1 = 0.02$  m,  $J = 1$  T. We represent in Fig. 5 three 2D cross-sections of the iso-lines created by the parallelepiped magnet whose polarization is directed along the axial direction. In this configuration, we take  $\theta = 0$  rad and  $0$  rad.

The computational cost for representing the iso-lines is lower than 1 s: This shows the interest of using a fully analytical approach for

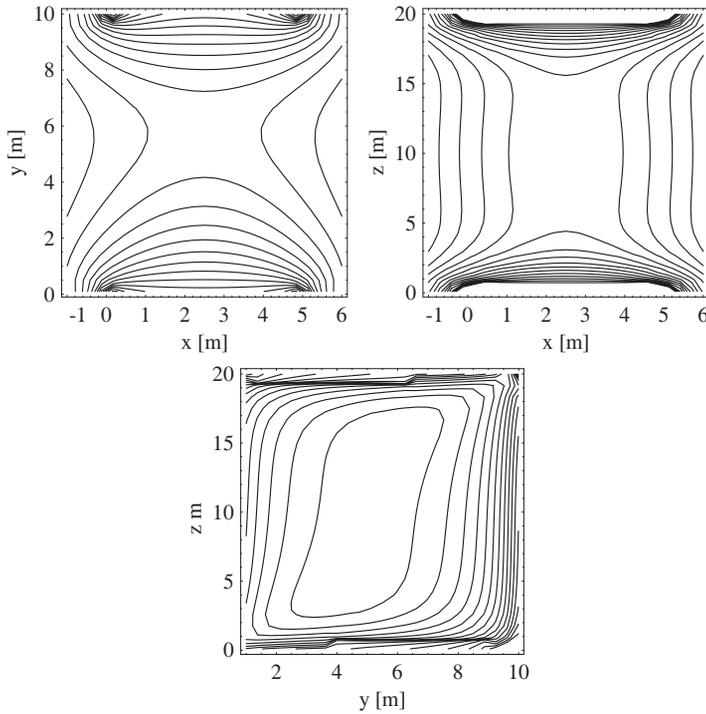
calculating the magnetic field produced by a parallelepipedic magnet of various and uniform polarization.

*3.1.2. Case of a Parallelepipedic Magnet of Various Polarization Direction*

We take the following dimensions:  $x_2 - x_1 = 0.005$  m,  $y_2 - y_1 = 0.01$  m,  $z_2 - z_1 = 0.02$  m,  $J = 1$  T. We represent in Fig. 6 three 2D cross-sections of the iso-lines created by the parallelepiped magnet with the polarization  $\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z$ . In this configuration, we take  $\theta = \frac{\pi}{2}$  rad and  $\phi = \frac{\pi}{4}$  rad.



**Figure 5.** 2D representation of the iso-lines created by a parallelepiped magnet whose polarization is directed along the  $z$  direction;  $(x-y)$ -plane:  $z = 19.9$  mm,  $(x-z)$ -plane:  $y = 5$  mm,  $(y-z)$ -plane:  $x = 4.5$  mm.



**Figure 6.** 2D representation of the iso-potentials created by a parallelepiped magnet whose polarization is  $\vec{J} = \frac{J}{\sqrt{2}}\vec{u}_y + \frac{J}{\sqrt{2}}\vec{u}_z$ ;  $(x-y)$ -plane:  $z = 19$  mm,  $(x-z)$ -plane:  $y = 5$  mm,  $(y-z)$ -plane:  $x = 4.5$  mm.

#### 4. CONCLUSION

This paper has presented three-dimensional analytical expressions for calculating the magnetic scalar potential and the magnetic field produced by a parallelepiped magnet of various polarization direction. In particular, the polarizations considered are entirely uniform, as it is generally the case in practice. By using the coulombian model of a magnet, we have expressed the the three components of the magnetic field in terms of fully analytical parts whose computational cost is very low. Such expressions have been compared to the ones published in the literature when the magnet polarization is directed along the axial direction. From an academic point of view, these expressions are an extension to the case of parallelepiped permanent magnet whose polarizations are directed along the  $x$ ,  $y$  or  $z$  directions.

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