Optimal Routing and Call Scheduling in Wireless Mesh Networks with Localized Information

Christelle Molle, Fabrice Peix, Stéphane Pérennes, Hervé Rivano

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Abstract. Wireless mesh network performance issues have been modeled by the Joint Routing and Scheduling Problem (JRSP) in which a maximum per-flow throughput is computed.
A classical relaxation of JRSP, denoted as the Round Weighting Problem (RWP), consists in assigning enough weight to sets of compatible simultaneous transmissions (rounds), while minimizing the sum of them, thus maximizing the relative weight of each round, which model the throughput.
In this work, we present a new linear formulation of RWP focused on the transport capacity over the network cuts, thus eliminating the routing. We prove its equivalence with existing formulations with flows and formalize a primal-dual algorithm that quickly solves this problem using a cross line and column generations process.
An asset of this formulation is to point out a bounded region, a "bottleneck" of the network, that is enough to optimize in order to get the optimal RWP of the whole network. The size and location of this area is experimentally made through simulations, highlighting a few hop distant neighborhood of the mesh gateways. One would then apply approximated methods outside this zone to route the traffic without degrading the achieved capacity.

1 Introduction

Wireless mesh network is a promising technology for broadband access networking and ubiquitous high-speed services [1]. Wireless mesh networks (WMNs) are dynamically self-organized and self-configured networks in which nodes automatically establish and maintain mesh connectivity among themselves. Each node, called *mesh router*, operates not only as a host but also as a router for the traffic. Mesh routers thus form a fixed infrastructure offering connectivity to mesh clients and gateway functionality for connections to Internet. The WMN integration with Internet is provided through special routers called *mesh gateways* (Fig. 1).

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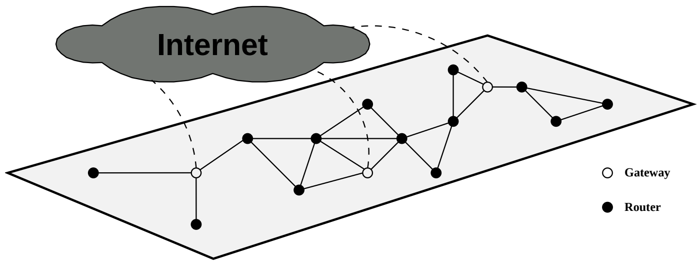


Fig. 1. Example of a wireless mesh network topology.

These multi-hop networks are expected to carry high throughput. The capacity of WMNs, that is, the throughput offered to each flow, is however affected by many factors such as network topology, traffic pattern, resource sharing and radio interferences [2, 3]. Several analytical studies on the capacity of wireless ad-hoc networks have all shown that the capacity decreases as the network size increases [4, 5]. Unlike ad-hoc networks, WMNs are stationary networks in which traffic is mainly router-to-gateway (respectively gateway-to-router) oriented. This special feature makes a bottleneck appear around the gateways, leading to a more constrained available capacity per node [6]. Optimization-based approaches have been investigated trying to maximize the network capacity [7].

A key issue in wireless networking is to cope with the interferences produced by concurrent transmissions. If many concurrent transmissions are successful, they have to be pairwise non interfering. Consequently, MAC protocols achieving conflict-free link scheduling have been developed to avoid interferences [2, 8]. The evolution of a network can thus be seen as the sequential activation of conflict-free sets of links, called *rounds* in the following.

In order to optimize the performances of WMNs further and guaranty a better quality of service (QoS) to the clients, cross-layer approaches have been investigated. We are interested in the *Joint Routing and Scheduling Problem* (JRSP), that computes jointly the router-to-gateway routes and the round scheduling in order to achieve the maximum transport capacity. Linear programming formulations have been developed to give theoretical bounds on the network capacity [9–11]. They are relying on fundamental combinatorial optimization issues such as the multi-commodity flow problem and the fractional colouring problem. A reciprocal optimization problem is the *Gathering Problem* [12, 13] in which the time needed to gather a given traffic demand to a sink node is minimized.

In the case of a steady state operating networks, a relaxation of the JRSP has been introduced. This relaxation exploits the periodicity of the network in order to avoid a costly computation of the round scheduling. Indeed, the capacity of a periodic network can be defined by a combination of the "activation duration" of the rounds, as explained in Section 3. Computing the optimal capacity of the network is then a *Round Weighting Problem* (RWP) [14]. Besides, this relaxation if quite effective since, given a round weighting, one can easily build an actual link scheduling inducing the same activation duration, hence providing the same capacity.

We concentrate on router-to-gateway traffic pattern in WMNs, for which the routing problem can be transformed into a single-commodity flow problem that we want to maximize. In graph theory, the maximum flow problem is known to be the dual of the minimum cut problem. The strong theorem of duality says that the optimal solutions of the both problem are equal [15]. A new representation of the RWP allows us to consider the activation duration of rounds in such a way that traffic can cross the network cuts. In other words, we investigate the RWP in which we forget about the routing and focus on the transport capacity available on the network cuts. Using previous works on JRSP, RWP and column generation:

- we develop a new linear programming formulation that computes the theoretical optimum of RWP in a WMN,
- and we propose practical solutions to be as closer to this bound as possible.

As detailed later in this paper, the RWP formulation involves an exponential number of variables and constraints. In order to cope with large instances, sophisticated process of Operational Research are useful. Column or line generation and primal/dual approaches greatly improve the computational cost of RWP [16]. In the following we extend this approach for solving the cut formulation of RWP efficiently.

The rest of the paper is organized as follows. In Section 3, we present a linear formulation whose optimal solutions are proved to be equivalent to the existing formulations of RWP. A primal-dual algorithm is described in Section 4, using a cross line and column generation process in order to efficiently compute optimal solutions. We analyse the simulation results in Section 5, highlighting the presence of a contention area in the network that restricts the available capacity.

In the next Section, we describe the network model chosen.

2 Network model

The wireless mesh network (WMN) is modeled by a directed and symmetric transmission graph $G = (V, E)$, where V is the disjoint union of the set of mesh routers V_r and mesh gateways V_g: $V = V_r \cup V_g$, $V_r \cap V_g = \emptyset$, and E is the set of possible transmissions between any pair of nodes of V.

We consider a synchronous, periodic network in steady-state. During a network period, each mesh router r of V_r sends a traffic $t(v)$ to the gateways trough multi-hop transmissions. A network period is divided into time slots. During each time slot a set of pairwise non-interfering one-hop transmission is activated to forward the traffic. Such a set of transmissions is called a *round* and is made of a subset of E.

This generic definition allows to consider any interference model like binary models [12, 14] or other models based on the Signal-to-Noise-and-Interference-

Ratio (SINR) [9]. Indeed, the interference model is captured by the structure of the rounds.

Computing link activation is therefore related to assigning an activation duration on a set of rounds during the period. As only one round is activated at a time, the sum of the round activation durations is equal to the network period length which is to be minimized.

3 Routing and Round Weighting

The goal of JRSP is to route these given traffic demands from the mesh routers of V_r to the mesh gateways of V_g on a set of multi-hop activated paths. A path is activated if all its links are selected enough to carry flow on them during the network period: a link capacity is proportional to its activation frequency during the period. Minimizing the network period length ensures to route the total traffic at maximum rate.

Given traffic $t(v)$ for all mesh routers v of V_r, the routing problem in WMNs consists in selecting multi-hop paths from each source node v to at least one gateway g of V_g. To include collision avoidance, link scheduling allows to select, at each time slot of the network period, a set of pairwise non-interfering transmissions.

Computing an actual link scheduling is not necessary when considering periodic networks. Since the throughput is the amount of traffic it carries divided by the length of the period, one can define the capacity as the inverse of the sum of the round activation durations. The goal becomes to route the given traffic demand on multi-hop paths that can be activated with a minimum number of rounds.

This relaxation leads to the round weighting problem described in the following.

3.1 The Round Weighting Problem

Given the transmission graph $G = (V, E)$ described in Section 2, the *round weighting problem* seeks to find a weight function $w : \mathcal{R} \to \mathbb{R}^+$ defined on the *round* set $\mathcal{R}$ which is a subset of 2^E. The constraint is that this weight function must enable some flow to be carried over the network. Each edge e of E inherit a capacity $C_w(e)$ from w defined as $\sum_{R \in \mathcal{R}} w(R)$, and one wishes the flow to be feasible in the network equipped with the capacity C_w.

Let $\mathcal{P}_s$ be the set of paths going from router s to the gateways and $\Phi(P)$ the amount of flow on path $P \in \mathcal{P}_s$. $\mathcal{P} = \cup_{s \in V_r} \mathcal{P}_s$ denotes the set of all paths.

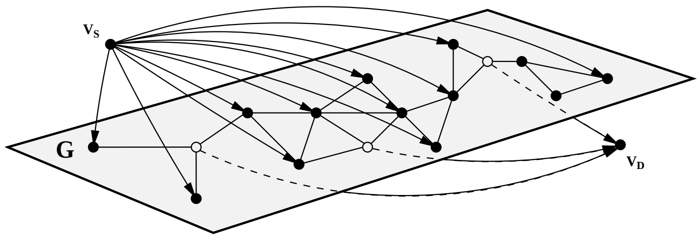


Fig. 2. Associated graph G' extends transmission graph G.

The objective is to minimize the total weight $\sum_{R\in\mathcal{R}} w(R)$ while satisfying the flow (i.e. capacity constraints) as described in the following linear formulation:

$$(1)\begin{cases} Min \displaystyle\sum_{R\in\mathcal{R}} w(R) \\ C_w(e) \geq \displaystyle\sum_{P\in\mathcal{P}, e\in P} \Phi(P) \qquad \forall e \in E \\ \displaystyle\sum_{P\in\mathcal{P}_v} \Phi(P) = t(v) \qquad \forall v \in V_r \\ w(R) \geq 0, \Phi(P) \geq 0 \qquad \forall R \in \mathcal{R}, P \in \mathcal{P} \end{cases}$$

One can remark that, given a round weighting w, the routing problem can be reduced to a maximum flow problem with an only pair (source, destination) by a transformation of the transmission graph G into a modified graph (G', w) depicted in Figure 2 and defined in the following:

Definition 1 (Associated Graph (G', w)**).** *Let $G' = (V', E')$ be the graph constructed from transmission graph G and induced link capacity C_w in the following way:*

- *A* super source *v_S is added with incident links (v_S, r) for every router r of V_r with capacity $t(r)$.*
- *A* sink node *v_D is added with incident links (g, v_D) for every gateway g of V_g with infinite capacity.*
- *Every link e of E has capacity $C_w(e)$.*

Thus $V' = V \cup \{v_S, v_D\}$ and $E' = E \cup \{(v_S, r), \forall r \in V_r\} \cup \{(g, v_D), \forall g \in V_g\}$.

This modification allows to use the well-known *max flow/min cut theorem* from graph theory, linear programming and duality theory. Recalling that the maximum flow of a graph equals the capacity of its minimum cut gives us the opportunity to consider only a cut covering problem instead of the routing.

Constraints of formulation (1) are thus respected if and only if the minimum (v_S, v_D)-cut in G' has a weight greater than $\sum_{v\in V_r} t(v)$, the total traffic that has to be carried to the gateways. This leads to a new linear programming formulation described in the next subsection.

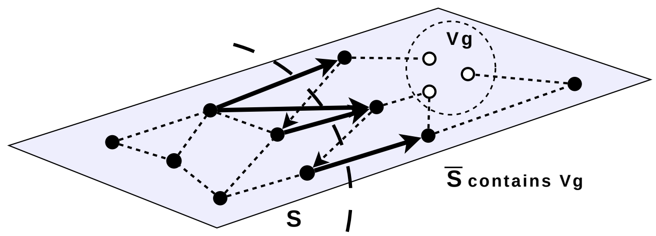


Fig. 3. A cut S in the transmission graph G is a subset of nodes that does not contain the gateways.

3.2 The New Cut/Round Formulation

The duality theory of linear programming has shown that one can formulate the maximum flow problem as a cut covering problem. In the following, we extend this formulation to the RWP and develop a new linear formulation of the problem focusing on the network transport capacity.

In the following, we call $\mathcal{S} \subset 2^V$ the set of cuts of G isolating the gateways: a cut $S \in \mathcal{S}$ is a subset of nodes of V excluding the gateways (see Fig. 3). The border of S, denoted $(S, \overline{S})$, is the set of links connecting a node of S to a node of the complementary set $\overline{S} = V \setminus S$. We thus define the cut traffic $t(S) = \sum_{v \in S} t(v)$ to be the traffic that must cross its border. In the same way, a cut capacity $C_w(S)$ is induced by the weight function w on the links and is defined by $C_w(S) = \sum_{e \in (S,\overline{S})} C_w(e)$.

If we recall the definition of the induced capacity $C_w(e)$, one can obtain the following relation: $C_w(S) = \sum_{R \in \mathcal{R}} \delta(R,S)w(R)$, where $\delta(R,S) = |R \cap (S,\overline{S})|$ corresponds to how many times a round R covers S's border.

From that point, ensuring a sufficient network capacity to carry the flow consists in covering the network cuts of $\mathcal{S}$ by the rounds. Then, the max flow/min cut theorem ensures an optimal solution to correspond to a feasible routing in the network satisfying router demands (cf theorem and proof in the next section).

Now, one can derive from these statements a formulation of the cut covering problem:

$$(2) \begin{cases} \quad Min \sum_{R \in \mathcal{R}} w(R) \\ \sum_{R \in \mathcal{R}} \delta(R,S)w(R) \geq t(S) \qquad \forall S \in \mathcal{S} \\ \quad w(R) \geq 0 \qquad \forall R \in \mathcal{R} \end{cases}$$

3.3 Equivalence

Linear program (2) computes an optimal round weighting such that every cut capacity is greater than the traffic of the cut that has to cross its border. The following Theorem 1 ensures that the induced cut capacities are necessary and sufficient to validate the existence of a feasible routing in the network.

Theorem 1. *Formulations (1) and (2) computes equivalent round weightings.*

Proof. Let w_1 be a feasible weighting of program (1), and Φ a feasible flow in G according to the link capacities induced by w_1. Since Φ is feasible, we know that $\sum_{P\in\mathcal{P}_v}\Phi(P)=t(v)$ for each router v. Thus, if we pick a cut S in G that isolates the gateways, flow conservation on each path P of P_r going from r to a gateway ensures that P contains a link of the border $(S,\overline{S})$. This gives:

$$C_{w_1}(S)=\sum_{e\in(S,\bar{S})}C_{w_1}(e)\geq\sum_{e\in(S,\bar{S})}\sum_{P\in\mathcal{P},e\in P}\Phi(P)\geq\sum_{v\in S}t(v)$$

using satisfied capacity constraints of program (1). By injecting w_1 in the program (2), we obtain a feasible solution since

$$\forall S,\ \sum_{R\in\mathcal{R}}\delta(R,S)w_1(R)=C_{w_1}(S)\geq t(S).$$

That is, the cut capacity is large enough to allow its traffic to cross its border.

In particular, an optimal solution of the first program is an upper bound of the solutions of the second program.

Conversely, let w_2 be a feasible solution of program (2). Then a minimum (v_S,v_D)-cut S^* (according to its capacity) in the graph (G',w_2) can be linked to a unique cut S in G such that $S^*=\{v_S\}\cup S$ and $(S^*,\overline{S^*})=\{(v_S,v),v\in\overline{S}\}\cup(S,\overline{S})$.

One can remark that the gateways cannot be in S since there would be some links (g,v_D) in S^*. As S^* is a minimum cut, it cannot contain links with infinite capacity.

S^* capacity is now expressed by:

$$C_{w_2}(S^*)=\sum_{v\in\overline{S}}C_{w_2}((v_S,v))+\sum_{e\in(S,\overline{S})}C_{w_2}(e)=\sum_{v\in\overline{S}}t(v)+C_{w_2}(S).$$

The corresponding constraint in program (2) thus ensures that $C_{w_2}(S)\geq t(S)=\sum_{v\in S}t(v)$. Then $C_{w_2}(S^*)\geq\sum_{v\in V_r}t(v)$ and the max flow/min cut theorem guaranty the presence of a feasible flow in (G',w_2) with value $\sum_{v\in V_r}t(v)$, that is, a feasible solution of program (1).

In particular, optimal solutions of program (2) upper bound the solutions of program (1), which completes our proof.

From a round weighting solution w, one can construct the associated graph (G',w) and compute a maximum flow of value $\sum_{v\in V_r}t(v)$ from the super source v_S to the sink node v_D. The Ford and Fulkerson algorithm, or the push/relabel algorithm introduced by Goldberg and Tarjan [17] allows to find the set of paths from each router to the gateways in polynomial time.

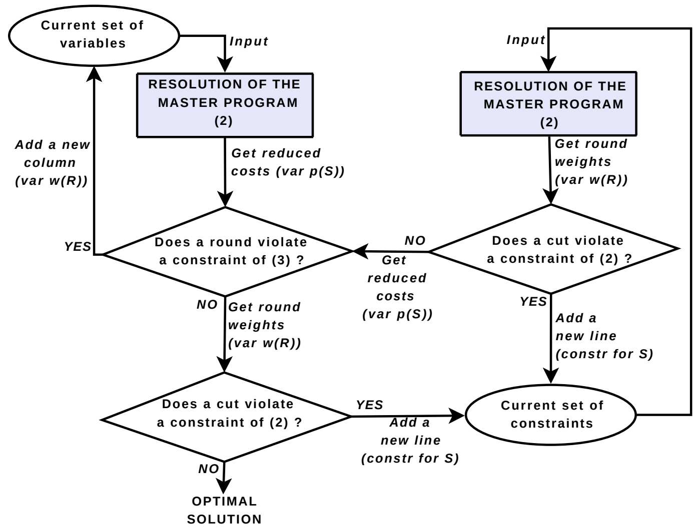


Fig. 4. The cross line and column generation process.

4 The Resolution Method

Column generation is a prominent approach to deal with the exponential size of variables of a linear programming problem. It has already been applied for JRSP and RWP and has proved its efficiency [18, 9, 11]. Actually, these problems suffer from the exponential number of their variables, i.e. rounds. The set of rounds is a subset of 2^E, it is therefore impossible to enumerate all of them in a graph.

The column generation process, described in [11], allows to quickly compute the optimal solution of the problem only with a subset of rounds (generating only those that improve the objective function).

In our case, we deal not only with an exponential number of variables (one for each round), but also with an exponential number of constraints (one for each cut). The corresponding method to avoid the complete enumeration of the constraints is the line generation.

To solve the new cut/round formulation (2), we therefore have to combine cross line and column generation. This leads to a primal-dual algorithm described in the following.

4.1 Cross Line and Column Generation

From an optimal solution of the problem with restricted sets of rounds and cuts, the process seeks to generate new lines (cuts) or columns (rounds) to add to the formulation in order to improve the solution. A new line is found when a constraint is violated, and a new column corresponds to a variable equal to zero that we want to change (Fig. 4).

In formulation (2), a constraint is violated if a cut capacity is too weak, i.e. if $\sum_{R\in\mathcal{R}}\delta(R,S)w(R) < t(S)$ for a given solution w. Line generation is actually computed by a minimum cut algorithm (according to the induced capacities). If the capacity of the minimum cut is greater than its traffic, then we are sure that no cuts violate constraints of program (2), otherwise, the minimum cut algorithm gives a candidate to add to the current set of constraints.

This minimum cut algorithm, called *auxiliary program*, aim to compute new lines improving the solution of the problem. Several minimum cut algorithms exist in the literature. We choose to compute an integer linear program whose formulation is the following. Given the actual optimal weights w, $y(e)$ is a binary variable saying if link e is in the cut's border, and $x(v)$ is another binary variable representing the node's selection to be in the cut.

$$\begin{cases} Min \sum_{e\in E} C_w(e)y(e) - \sum_{v\in V_r} t(v)y(v) \\ x(u) - x(v) \leq y((u,v)) \qquad \forall (u,v)\in E \\ x(v) = 0 \qquad \forall v\in V_g \end{cases}$$

In order to find violated constraints of program (2), the objective seeks to minimize the difference between the capacity of the cut, i.e. the sum of the capacity of the links in its border, and the traffic of the cut, i.e. the sum of the traffic of each node in the cut. Constraints say that the gateways cannot be selected in the cut, and that a link has to be counted in the border if its source node is in the cut and its destination node is not.

Surprisingly, this ILP runs fast and gives the optimal solution quasi instantly. We will see later that simulations give a hint that complexity is not a major issue in the specific case of the round weighting.

Linear program (3) presented below is the dual formulation of formulation (2). It corresponds to a round packing by cuts balanced with $p(S)$. Each round capacity is less than or equal to 1 and the objective is to maximize a profit based on cut traffic. In other words, a constraint is not satisfied if a round has an induced cost greater than 1. Generate a column of program (2) is done by identifying a violated constraint of program (3) when $p(S)$ is given by the reduced costs of the current solution.

$$(3)\begin{cases} Max \sum_{S\in\mathcal{S}} p(S)t(S) \\ \sum_{S\in\mathcal{S}} \delta(R,S)p(S) \leq 1 \qquad \forall R\in\mathcal{R} \\ p(S)\geq 0 \qquad \forall S\in\mathcal{S} \end{cases}$$

Thus, a maximum weighted round generation either gives a good candidate to add to the set of variables, or certify that no such column exists. The second auxiliary program associated to the round generation can also be formulated as an integer linear program with binary variables $y(e)$ in which we try to maximize the round weight, i.e. the sum of the weights of the links in the round, given the actual induced costs p: $Max \sum_{e\in E}\left(\sum_{S\in\mathcal{S},\ e\in(S,\bar{S})} p(S)\right) y(e)$. If this cost is strictly greater than one, the generated round is added to the set of variables. Constraints of this second auxiliary program define the structure of the rounds, therefore it fully depends on the interference model chosen.

The cross line and column generation process is translated into a primal-dual algorithm described in the Algorithm 1, that works as follows.

4.2 The Primal-Dual Algorithm Description

The algorithm starts with the simplest cut of the transmission graph G isolating the gateways, i.e. the set $\mathcal{S}_0 = \{S_0 = V_r\}$ containing all the mesh routers, and a set of rounds containing the singletons $\mathcal{R}_0 = \{\{e\}, e \in E\}$.

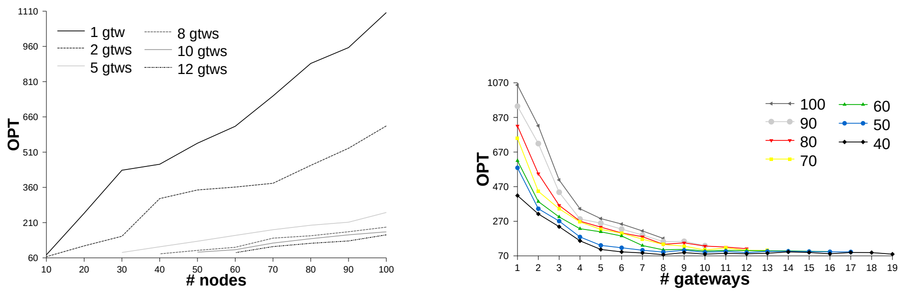
Program (1) computes the cut covering by the rounds, leading to a current local optimal solution. We can see that a feasible solution always exists with $\mathcal{S}_0$ and $\mathcal{R}_0$ since all links (r, g) between each router $r \in V_r$ and a gateway $g \in V_g$ forming the border of S_0 can be activated $t(r)$ times.

Then one checks if all the cuts of G are covered by the rounds, otherwise the line generation is processed to add the non-covered cuts to the set of constraints.

This process is repeated until no such rounds and cuts exist. Then, the separation theorem ensures that we have found the optimal solution.

Algorithm 1 Primal-Dual Algorithm for the RWP in WMNs.

Require: network graph G
Ensure: a round weighting in G
  $\mathcal{S} \leftarrow \{S_0 = V_r\}$
  $\mathcal{R} \leftarrow \{\{e\}, \forall e \in E\}$
  Solve formulation (2)
  $\mathcal{R}_{new} \leftarrow$ Get violating rounds: $\{R,\ s.t.\ 1 < \sum_{S \in \mathcal{S}} \delta(R, S) p(S)\}$
  $\mathcal{S}_{new} \leftarrow$ Get violating cuts: $\{S,\ s.t.\ t(S) > \sum_{R \in \mathcal{R}} \delta(R, S) w(R)\}$
  while $(\mathcal{R}_{new} \neq \emptyset)$ || $(\mathcal{S}_new \neq \emptyset)$ **do**
    while $(\mathcal{R}_{new} \neq \emptyset)$ **do**
      $\mathcal{R} \leftarrow \mathcal{R} \cup \{\mathcal{R}_{new}\}$
      Solve formulation (2)
      $\mathcal{R}_{new} \leftarrow$ Get violating rounds
    end while
    $\mathcal{S}_{new} \leftarrow$ Get violating cuts
    while $(\mathcal{S}_{new} \neq \emptyset)$ **do**
      $\mathcal{S} \leftarrow \mathcal{S} \cup \{\mathcal{S}_{new}\}$
      Solve formulation (2)
      $\mathcal{S}_{new} \leftarrow$ Get violating cuts
    end while
    $\mathcal{R}_{new} \leftarrow$ Get violating rounds
  end while
  return w.



(a) The sum of the round weights increases (i.e. the throughput decreases) linearly with network size increases.

(b) The optimal value decreases (i.e. throughput increases) when adding new gateways.

Fig. 5. Optimal solution versus network size and gateway density.

5 Simulations and Empirical Studies

Our approach has been validated through extensive simulations. We present the results obtained on regular grid topologies (Manhattan-like networks) as well as random topologies following a Poisson distribution as described in the following. We then use the structural properties of the cut formulation to empirically highlight a *bottleneck area* which is conjectured by many researches.

5.1 Simulaton settings

To create a random network, a set of N points has been deployed on a rectangular plane following a Poisson law. We set a link between two nodes if they are within the connection range of each other.

Mesh gateways have been either placed regularly on the grid (in the center, at the corners), or chosen randomly among the nodes. Traffic requirements are either uniform with value 1 or randomly assigned with value between 1 and 20.

For the sake of simplicity and to compare with existing works, we consider the classical binary distance-2 interference model in which a transmission (u, v) competes with all transmissions within a two-hop distance in the transmission graph G. A round is thus a subset of links such that two links are at distance at least 3 in G.

Tests have been realized using the Mascopt library [19]developed by team members, and Ilog Cplex solver on a Intel Core 2 2.4 GHz with 2Gb of memory.

5.2 Computational Results

The simulation run validate our approach as it confirms existing results on the wireless network capacity (Fig. 5). Moreover, the line-column generation primal-dual algorithm quickly solves our round weighting instances with 9 to

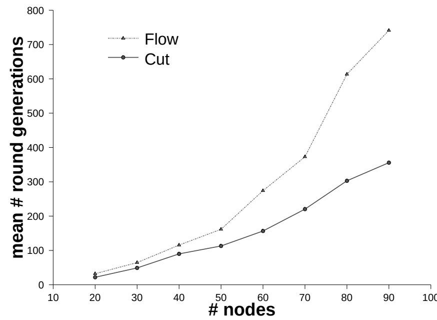


Fig. 6. Number of rounds generated in the optimization of cut and flow formulations.

225 nodes: from tenth of seconds to a few minutes for topologies with more than 100 nodes. On one hand, it allows to solve large-scale instances to optimality. On the other hand, the computational time is roughly the same as the formulation with flows [11]: sub-linear in the network size and linear in the gateways density.

Moreover, one can see in Figure 6 that the number of generated rounds is decreased in comparison to the existing formulation with flows. This is better since the auxiliary program to generate new rounds is an ILP, and is related to the *maximum independent set problem* which is known to be NP-hard in general graphs. Indeed, if we consider a binary interference model, then a round is an independent set of the conflict graph, i.e. the graph where each node is a possible transmission in the network, and there exists a link between two of them if the corresponding transmissions are interfering.

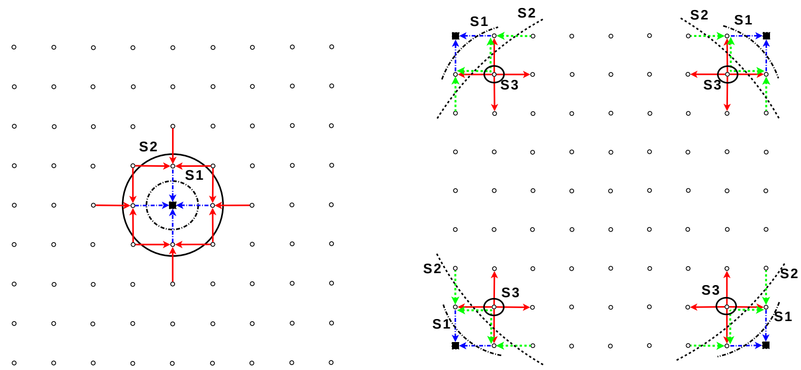
We have however remarked that, in the tested topologies, the computation of new rounds does not take so much time. We think that the particularity of the dual values, as well as some topology characteristics, make the round computation faster than in usual cases. Thus, complexity is not a major issue in the specific case of the weight functions induced by the dual of a concentration flow on the gateways.

In particular, we make a deeper study on the dual values in the next subsection and show that the optimal solution is bounded in a special area, called *bottleneck or contention area*, located within a few hop distance of the gateways.

5.3 Highlighting the Bottleneck Area

In grid topologies, previous researches have shown exact bounds of RWP in the case of one gateway located at the center or at the corner of a grid [20]. In the proof, authors use the primal and dual values to show that only the 2-neighborhood of the gateway matters.

In our study, we say that a cut is active if it has a strictly positive reduced cost, following that its corresponding constraint in the program (2) is tight. A first result with our method on the grid show that all the active cuts are located in the 2-neighborhood of the gateways (Fig. 7). Our results really highlight the



(a) 1 gateway at the center.

(b) 4 gateways at corners.

Fig. 7. A small number of cuts is enough to find the optimal solution of grid networks.

area bounding the solution, confirming the work of [20] and generalizing it on other topologies and with several gateways.

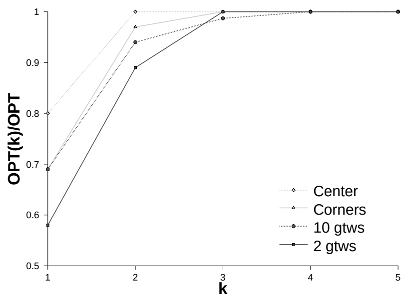
In order to identify this bottleneck area, we do the following process. Given an integer k as input, the algorithm only changes in the auxiliary problems of the cross line and column generation process in which new cuts and rounds are generated. We forces the program to only compute rounds whose links are located in the k-neighborhood of a gateway. Similarly, cuts must have the following property: their border, i.e. the set of links going from a vertex in the cut to a vertex outside it, is in the k-neighborhood of the gateways.

Depending on the value of k, the optimum obtained is lower or equal to the optimal one found on the entire graph: $OPT_1 \leq OPT_2 \leq \cdots \leq OPT_k \leq \cdots \leq OPT$. Indeed, the set of cuts considered, i.e. the constraints of the problem, is limited. Therefore, either we have enough constraints and we find the same result as in the original problem, or the problem is sub-constrained and the optimal is less than the optimal solution of the original minimization problem.

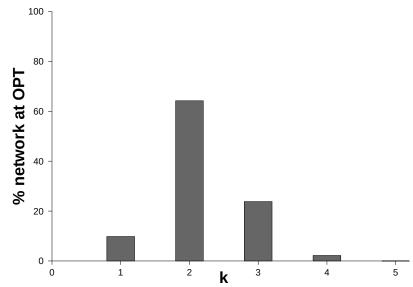
We solved the problem to optimality for $k = 1, \ldots, 5$ on hundreds of topologies. Some results are depicted in Figure 8(a) for grid topologies. More generally, Figure 8(b) presents the percentage of networks solved to optimality in function of the value of k. One can remark that optimal solutions are mostly solved when $k = 2$ or 3, and all optimal solutions have been reached when $k = 4$. Actually, the cases when $OPT_1 = OPT$ happen when the number of gateways is big in comparison to the network size, e.g. the 1-neighborhood of the gateways contains all the nodes. On the contrary, networks where $OPT_3 < OPT$ have a size bigger than 70 nodes in which the node connectivity is really weak around the gateways.

6 Conclusion

This paper has presented a new formulation for the round weighting problem yielding a primal-dual algorithm for wireless mesh networks. The algorithm uses



(a) Optimality gap in function of k for grid topologies for different gateway placements.



(b) Percentage of networks that reach the optimal value in function of k.

Fig. 8. Highlighting the contention area.

a joint line and column generation process useful to deal with large scale instances. The cut covering problem introduced has been validated from the known round weighting formulation with flows by proving the equality of the optimal solutions. Moreover, we conjecture that the proposed algorithm generates only a polynomial number of cuts and rounds.

An asset of this formulation is to point out a bounded region, a "bottleneck" of the network, that is enough to optimize in order to get the optimal RWP of the whole network.

This new approach is useful for practical use. Actually, one can deploy a network that is carefully optimized in a bounded area containing the gateways. In this area, a conflict-free link scheduling is carried out optimally by each gateway for its neighborhood. Then, one can combine approximation algorithms like distributed routing algorithms outside the area that spread the traffic among the mesh routers and bring it correctly to the contention area without degrading the achieved capacity.

Another point for further research concerns the gateway placement problem (GPP) in wireless mesh networks. We think that it is not necessary to have a precise placement since the only constraints to reach the available throughput is to place the gateways in such a way that their respective contention areas have an empty intersection. By showing that the network achievable throughput is bounded by the contention area around the gateways, one can develop algorithms for the GPP that seek to optimize the placement considering only local constraints in the network.

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