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▶ To cite this version:

J. Fontdecaba, G. Métris, P. Gamet, P. Exertier. Solar radiation pressure effects on very high-eccentric formation flying. Proceedings of the 3rd International Symposium on Formation Flying, Missions and Technologies, Apr 2008, Noordwijk, Netherlands. hal-00416588

HAL Id: hal-00416588

https://hal.science/hal-00416588

Submitted on 14 Sep 2009

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SOLAR RADIATION PRESSURE EFFECTS ON VERY HIGH-ECCENTRIC FORMATION FLYING

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ABSTRACT

A real alternative to Lagrange point very low perturbed orbits, for universe observation missions, is high eccentric Earth orbits. Combination of high eccentricity and very large semi-major axis leads to orbits with an important part of flight time far from Earth and its perturbations. Modeling this particular relative motion is the scoop of this paper.

Main perturbation in HEO orbits are solar radiation pressure (SRP) and lunisolar effects, but formations are mainly affected by SRP effects. The modellization of its effects is done in two ways. First we introduce the SRP effects in the equations of the relative acceleration. Second, we obtain explicit analytical expressions of the temporal evolution of the relative motion. Resulting expressions enable very fast computations.

These models are used to study HEO missions. We focus on two different problems: estimation of thrust for station keeping and evaluation of collision risk. We also consider the influence of the difference of ratio surface/mass between satellites.

Key words: formation flying, HEO orbits, solar radiation pressure.

1. INTRODUCTION

Formation flying has become one of the most interesting techniques for space observations. Multiple satellites systems enables large base interferometry and telescopes with high focal length. As these systems need a very high precision on control orbit and determination, Lagrange point orbits are very well-suited. A real alternative to these orbits are HEO (high eccentricity orbit) orbits with very large semi-major axis. This kind of orbits has a short very perturbed passage on the perigee, which is not well-adapted for observations, and a very long low perturbed passage on the apogee, where the conditions for observation are particulary good. Observations are possible since the formation is above Van Allen radiation belt. In some cases, more than 85% of the orbit can be used for observation. During the observation, the

dynamics of the formation, even if is a central body dynamics, is quite particular. First, because of the so high eccentricity (bigger than 0.7), and second because main perturbations are not the same as in LEO (low Earth orbits) orbits. While in LEO orbits drag and J2 effects have important effects, in HEO main perturbation are solar radiation pressure and lunisolar effects. Lunisolar differential perturbations on the formation keep small because of the small distance between satellites, but SRP effects might have big consequences if the satellites do not have the same ratio between their surface and their masse $(\frac{S}{m})$, as its the case in many formations. In order to study the effects of this perturbation, this paper presents a modellization of relative motion taking into account SRP perturbations

Modelling relative motion has been done classically through its cartesian coordinates, with Clohessy-Wilthsire equations [6] and its extension to elliptical reference orbit, known as Lawden equations [11]. This approach is well-suited for the computation of relative accelerations, but it is not so useful to obtain closed form solutions of perturbed problem. More recently, the introduction of differential orbital elements has been an issue to the problem [8], [9]. This method takes use of the orbital elements to extrapolate the orbit. It has been extensely used to study main perturbations on LEO orbits [2], and it is also useful for SRP perturbations. The authors have not found any paper modelling SRP effects on formation flying.

Effects of SRP on orbital elements has been largely studied [10], [1], [4]. Here, we use a similar approach, but we introduce the mean anomaly as independent variable. Moreover, we compute its differential effects on the formation using differential orbital elements. We also use classical approach with Lawden equations perturbed with a differential SRP.

We use our analytical model to study two problems related with formation flying. For numerical simulations, we use the parameters of Italian-French formation flying SIMBOL-X. The first handled problem is estimation of necessary maneuvers. As control method, we have used an ideal open loop system with no errors. We focus on the dynamics of the problem. The system supplies the necessary acceleration to force non-propelled relative orbit to

$$\delta a(t) = -\frac{2\sigma}{n^2} \frac{S}{m} \sum_{1}^{\infty} \left(\frac{2}{s} J_s'(se) \cos sM\mathcal{F} + \frac{2\eta}{es} J_s(se) \sin sM\mathcal{G} \right)$$

$$\delta e(t) = -\frac{\eta\sigma}{nae} \frac{S}{m} \left[\frac{3}{2} e \mathcal{F}_{\omega}(t - t_0) + \sum_{1}^{\infty} \left(\frac{\cos sM}{sn} \left(2\eta J_s'(se)\mathcal{F} + \frac{2\eta}{es} J_s(se)\mathcal{G}_{\omega} \right) + \frac{\sin sM}{sn} \left(\frac{2\eta^2}{e} J_s(se)\mathcal{G} - \frac{2}{s} J_s'(se)\mathcal{F}_{\omega} \right) \right) \right]$$

$$\delta i(t) = -\frac{\sigma}{na\eta \sin i} \frac{S}{m} \left[-\frac{3}{2} e \left(\cos i\mathcal{F}_{\omega} - \mathcal{F}_{\Omega} \right) (t - t_0) + \sum_{1}^{\infty} \left(\frac{2}{s} J_s'(se) \frac{\sin sM}{sn} \left(\cos i\mathcal{F}_{\omega} - \mathcal{F}_{\Omega} \right) - \frac{2\eta}{es} J_s(se) \frac{\cos sM}{sn} \left(\cos i\mathcal{G}_{\omega} - \mathcal{G}_{\Omega} \right) \right) \right]$$

$$\delta \Omega(t) = -\frac{\sigma}{na\eta \sin i} \frac{S}{m} \left[-\frac{3}{2} e \mathcal{F}_i(t - t_0) + \sum_{1}^{\infty} \left(\frac{2}{ns^2} J_s'(se) \sin sM\mathcal{F}_i - \frac{2a\eta}{ens^2} J_s(se) \cos sM\mathcal{G}_i \right) \right]$$

$$\delta \omega(t) = -\frac{\sigma}{nan} \frac{S}{m} \left[-\frac{3}{2} \left(\frac{\eta}{e} \mathcal{F} - \frac{e \cot i}{\eta} \mathcal{F}_i \right) (t - t_0) + \sum_{1}^{\infty} \left(2 \frac{\sin sM}{sn} \left(\frac{\eta}{e} J_s''(se) \mathcal{F} - \frac{\cot i}{\eta} \frac{1}{s} J_s'(se) \mathcal{F}_i \right) - 2 \frac{\cos sM}{sn} \left(\frac{\eta}{e} \left(\frac{\eta}{e} J_s'(se) - \frac{1}{\eta e^2} \frac{1}{s} J_s(se) \right) \mathcal{G} - \frac{\cot i}{\eta} \frac{\eta}{es} J_s(se) \mathcal{G}_i \right) \right) \right]$$

$$\delta M(t) = \frac{\sigma}{na} \frac{S}{na} \left[-\frac{3}{2} \mathcal{F} \left(2e + \frac{\eta^2}{e} \right) (t - t_0) + \sum_{1}^{\infty} \left(2 \frac{\sin sM}{sn} \mathcal{F} \left(\frac{2}{s} J_s'(se) + \frac{\eta^2}{e} J_s''(se) \right) - 2 \frac{\cos sM}{sn} \mathcal{G} \left(\frac{2\eta}{es} J_s(se) + \frac{\eta^3}{e^2} J_s'(se) - \frac{\eta}{e^3} J_s(se) \right) \right) \right]$$

describe the necessary relative trajectory for observation. These simulations have shown the influence of the solar radiation pressure over the keplerian motion.

Second problem deals with the collision risk. It appears in case of propulsion system failure. Once the failure is produced, relative trajectory is no more the necessary trajectory for observation, but natural non-propelled trajectory. The natural trajectory can lead to a collision between satellites. It is interesting to evaluate the risk of each observation in order to classify them and avoid the most dangerous ones.

Paper is structured as follows. First section is the introduction, second section is devoted to the modellization of the solar radiation pressure. Third section is dedicated to the effects of SRP on FF. Four section deals with station keeping and collision risk. We finish with some conclusions.

2. MODELLING SOLAR RADIATION PRES-SURE

First, we compute the effects of SRP in a generic orbit defined through its classical orbital elements \overrightarrow{EO} : $(a,e,i,\Omega,\omega,M)^T$. We also use classical variables $\eta=\sqrt{1-e^2}$ and $n=\sqrt{\frac{\mu}{a^3}}$. Solar radiation pressure produces a force in all satellites which writes:

$$\overrightarrow{f}_{SRP} = -\sigma \frac{S}{m} \overrightarrow{u}_{Sun} \tag{2}$$

where S is the surface of the satellite perpendicular to the direction of the Sun, m is its masse, and σ is a constant which includes the effects of the distance to the Sun, and the reflectivity of the satellite. We take $\sigma=7.10^{-6}$. The value of this variable as function of the reflectivity of the satellite. We have taken a conservative value. Vector \overrightarrow{u}_{Sun} is an unitary vector pointing to the Earth from the Sun, so we neglect the distance between the satellite and the Earth, with respect to the distance Sun-satellite.

This force does not act when Earth shadows the Sun. As we work in eccentric orbits with very high semi-major axis, shadows regions are negligible. For example, for a=100000~km, and e=0.7, shadow region never is higher than 4% of the orbit. So, supposing that there is no shadow, SRP force can be derived from a potential:

$$U_{SRP} = -\sigma \frac{S}{m} r_{sat} \overrightarrow{u}_{sun}. \overrightarrow{u}_{sat}$$
 (3)

Vector \overrightarrow{u}_{sat} is the position of the satellite with respect the Earth, and r_{sat} the distance between the satellite and the Earth. Integration of this potential can be done through Lagrange planetary equations as it is done in [10]. Our method differs from Kozai [10] since we introduce mean anomaly instead of use eccentric anomaly. Approximated integration of Lagrange equations gives temporal evolution of SRP perturbations $(\delta \overrightarrow{EO}|_{SRP}(t))$, which we copy in equation (1).

In these equations, we use Bessel functions $J_s(x)$, and its derivatives with respect the argument $J'_s(x)$

We also use intermediary geometrical functions \mathcal{F} and \mathcal{G} and its derivatives with respect the angles $(\mathcal{F}_i, \mathcal{F}_{\Omega}, \mathcal{F}_{\omega}, \mathcal{G}_i, \mathcal{G}_{\Omega}, \mathcal{G}_{\omega})$.

$$\mathcal{F} = A_s (\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i)$$

$$+B_s (\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i)$$

$$+C_s \sin \omega \sin i$$

$$\mathcal{G} = -A_s (\cos \Omega \sin \omega + \sin \Omega \cos \omega \cos i)$$

$$+B_s (-\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i)$$

$$+C_s \cos \omega \sin i$$
(5)

 A_s, B_s, C_s are the cosinus directors of the position of the Sun with respect to the Earth. It is:

$$\overrightarrow{u}_{Sun} = (A_s, B_s, C_s)^T \tag{6}$$

3. EFFECTS OF SOLAR RADIATION PRES-SURE ON FORMATION FLYING

Thereafter, we use two different representations of relative motion: i) relative position and velocity $(\overrightarrow{\rho}, \overrightarrow{\rho})$ with respect to a rotating reference frame linked to a reference orbit, and ii) the differential orbital elements $(\Delta \overrightarrow{EO})$, it is, the difference of orbital elements between the reference orbit and the studied orbit. In both cases, we use a reference orbit defined by its orbitals elements $\overrightarrow{EO}_{ref}$. We also use following notation for relative position and velocity:

$$\overrightarrow{\rho} = (\rho_R, \rho_T, \rho_N)^T$$

$$\overrightarrow{\rho} = (\dot{\rho}_R, \dot{\rho}_T, \dot{\rho}_N)^T$$
(7)

There are two different ways to introduce the effects of SRP in relative motion: i) writing the equations of the relative acceleration, ii) introducing the perturbation on the differential orbital elements.

3.1. Relative acceleration

Using relative dynamics, we can get the acceleration relative to a rotating reference frame linked to an eccentric orbit which correspond to the orbit of the free satellite. After some computation and the linearization of the equations, we get (8):

$$\frac{\ddot{\rho}}{\dot{\rho}} = -\mu \frac{\overrightarrow{\rho}}{r^3} + \frac{3\mu}{r^3} (\rho_R, 0, 0) + \ddot{\theta}(\rho_T, -\rho_R, 0)
- \dot{\theta}^2(\rho_R, \rho_T, 0) + 2\dot{\theta}(-\dot{\rho}_R, \dot{\rho}_T, 0) + \Delta \overrightarrow{f}_{SRP}|_{RTN}$$
(8)

which are equivalent to Lawden's equations [11] perturbed with SRP differential force. $\dot{\theta}$ its the rotational velocity of the reference frame and $\ddot{\theta}$ its derivative. They are given in [3] as:

$$\dot{\theta} = \frac{n}{(1 - e^2)^{3/2}} (1 + e \cos f)^2 \tag{9}$$

$$\ddot{\theta} = -2n^2 e \sin f \left(\frac{1 + e \cos f}{1 - e^2}\right)^3$$

 $\Delta \overrightarrow{f}_{SRP}$ is the difference of force due to the solar radiation pressure between the two satellites, projected in the local orbital frame RTN. The difference of force is given by a difference of $\frac{S}{m}$ between satellites:

$$\Delta \overrightarrow{f}_{SRP} = -\sigma \Delta \frac{S}{m} \overrightarrow{u}_{Sun} \tag{10}$$

Direction of the force depends on the epoch and on the reference orbital elements.

3.2. Differential orbital elements

Formation flying is also completely characterized using the differential orbital elements between two orbits. Since we neglect the distance Earth-satellite and the effects of the SRP are linear with the parameter $\frac{S}{m}$, differential effects of SRP are given just in case the parameter $\frac{S}{m}$ is not the same for both satellites. In this case, differential effects due to the SRP read:

$$\Delta \overrightarrow{EO}(t)|_{SRP} = \frac{\Delta \frac{S}{m}}{\frac{S}{m}|_{ref}} \delta \overrightarrow{EO_{ref}}(t)|_{SRP}$$
 (11)

where $\Delta \frac{S}{m}$ is the difference of $\frac{S}{m}$ between satellites. Conversion between differential orbital elements and cartesian coordinates can be easily done using matrices given in [5].

4. APPLICATION TO UNIVERSE OBSERVA-TION MISSIONS

In this section we study a Universe observation mission, placed in a HEO orbit, using a single telescope with a big focal length. Telescope would be distributed on two satellites. One satellite would be in free flight while the position of the second satellite would be forced in order to keep focal length and observation direction. The orbit of the first satellite will be used as reference orbit, and we would compute necessary maneuvers to be realized into second satellite. For our simulations, we use orbital parameters of Italian-French mission SIMBOL-X [7]. It

$$a_{ref} = 1064267 \; km \quad e_{ref} = 0,75 \quad i_{ref} = 6^{\circ}$$

$$\Omega_{ref} = 90^{\circ} \qquad \qquad \omega_{ref} = 0^{\circ}$$

Observations are taken while the satellite is above Van Allen radiation belt, and different sources should be observed during a single orbit. When satellites are not ready for observation, when they are behind Van Allen belt, formation must not be kept, but for operability reasons it seems simpler to keep configuration all along the orbit. The most defining variable of the formation is the distance between satellites d, which must be kept during all the mission. Relative position is defined by this distance, and by the position of the observed source on the sky, which is given through its longitude (α) and latitude (β) with respect to an inertial equatorial Earth-centered reference frame. In this inertial reference frame, relative position reads:

$$\overrightarrow{x}|_{IJK} = d(\cos\alpha\cos\beta, \sin\alpha\cos\beta, \sin\beta)$$
 (12)

Transforming this motion in the local orbital frame, we get:

$$\overrightarrow{x}|_{RTN} = \left(K_1 \cos u_{ref} + K_2 \sin u_{ref}, K_3 \cos u_{ref} + K_4 \sin u_{ref}, K_5\right)^T$$
(13)

where $u=\omega+f,\,f$ is the true anomaly, and with the coefficients:

$$K_{1} = d(\cos \Omega \cos \alpha \cos \beta + \sin \Omega \sin \alpha \cos \beta)$$

$$K_{2} = d(-\sin \Omega \cos i \cos \alpha \cos \beta + \cos \Omega \cos i \sin \alpha \cos \beta + \sin i \sin \beta)$$

$$K_{3} = d(-\sin \Omega \cos i \cos \alpha \cos \beta + \sin i \sin \beta)$$

$$+\cos \Omega \cos i \sin \alpha \cos \beta + \sin i \sin \beta)$$

$$K_4 = d(-\cos\Omega\cos\alpha\cos\beta - \sin\Omega\sin\alpha\cos\beta)$$

$$K_5 = d(\sin\Omega\sin i\cos\alpha\cos\alpha\cos\beta - \cos\Omega\sin i\sin\alpha\cos\beta + \cos\sin\beta)$$

4.1. Station keeping

Precedent motion, which will be referenced as \overrightarrow{x}_{obs} , necessary for taking observations, does not correspond in any

case with the natural relative orbit of the satellite \overrightarrow{x}_{nat} , and it is forced through a certain number of maneuvers. The frequency of the maneuvers depends on the necessary precision of the formation. In order to simplify, we suppose a continuous thrust, $\Delta \overrightarrow{a}$, (which is not far from the reality). Necessary thrust can be estimated as the difference between the natural acceleration and observation acceleration. Natural acceleration is computed using equations (8) and observation acceleration is computed using the derivatives of equation (13):

$$\Delta \overrightarrow{a} = |\overrightarrow{a}_{nat} - \overrightarrow{a}_{obs}| \tag{14}$$

The magnitude of impulsions depends on a certain number of variables: position of observation source, epoch of observation, position along the orbit, difference of $\frac{S}{m}$ between satellites. By the following we analyze most relevant ones.

Thrust along the orbit For all simulations, we use the parameters of SIMBOL-X mission:

$$d = 20m \qquad \qquad \Delta \frac{S}{m} = 3,52.10^{-3} m^2/kg$$

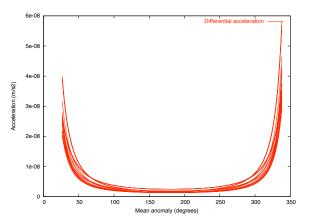


Figure 1. Thrust along orbit for Keplerian motion

Figures (1) and (2) show the variation of the thrust along the orbit. Different curves correspond to different observation directions. Simulations have been done with initial time 20th february 2012 at midnight. Figure (1) corresponds to necessary thrust for keplerian motion, while (2) corresponds to motion perturbed with SRP. Both figures show that main thrust is concentrated in the passage of the perigee, while in the apogee thrust remains nearly constant. In both cases, observation direction does not play a major role. Mean value of non-perturbed motion is $3.10^{-9} m/s^2$, while in perturbed case is $3.10^{-8} m/s^2$. As we will see later, this difference is function of parameter $\Delta \frac{S}{m}$.

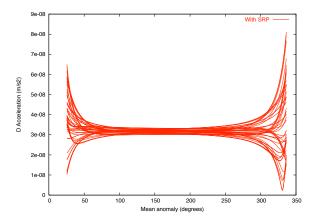


Figure 2. Thrust along orbit for SRP perturbed motion

Influence of the epoch of the year Figures (3) and (4) show the mean thrust along an orbit for the different directions of observations on the sky, and for two different dates: in summer (21st june), and winter (21st december).

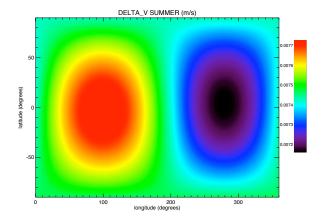


Figure 3. Thrust as function of observation position in summer

Figures show how there are privileged regions where observations are less fuel consumers than others. These regions move along the year. Constraints on observations (observation direction must be perpendicular to the position of the Sun) prevents of taking advantage of these observation regions.

Influence of the difference of SRP From a practical point of view, nowadays the only free parameter is $\Delta \frac{S}{m}$. The reference orbit is fixed for station keeping considerations, and the direction of observation sources must sweep all the sky. Moreover, $\Delta \frac{S}{m}$ is not fixed only by the design, since it changes with the orientation of satellites with respect to the Sun.

Next figure (5), show the effect of changing the value of $\Delta \frac{S}{m}$ on the necessary thrust. We have computed a mean

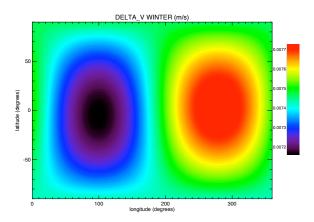


Figure 4. Thrust as function of observation position in winter

value for all the possible sources and over the orbit. Mean value of thrust has been computed along the orbit and for different observation directions. Different curves correspond to different epochs of observation.

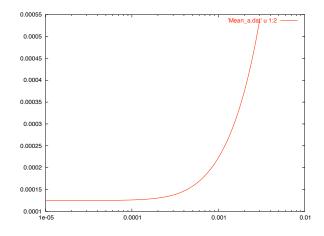


Figure 5. Influence of the difference of $\frac{S}{7}m$ on the thrust

The figure presents three zone: i) keplerian zone, ii) transition zone, iii) SRP zone. In the first zone SRP effects are negligible with respect to the keplerian effects. It means that any changing in the value of $\Delta \frac{S}{m}$ change the thrust. In the third zone, SRP effects dominate keplerian ones. Since the perturbation is linear with $\Delta \frac{S}{m}$, so it is the thrust. The second zone is the transition between two regimes. It may be interesting to design spacecraft in order to operate in first zone.

4.2. Collision risk

Collision risk appears in case of propulsion system failure. In this case, relative motion is no more the described trajectory for observations, but the non-propulsed. Supposing that the failure may be recoverable after a while,

it is necessary to verify that during this time satellites do not collide. In order to do so, we have analyzed different observation trajectories and the influence of different parameters.

We have used our closed analytical solution for the extrapolation of the orbit. It enables very fast computation and computation time is much more small than integration relative accelerations.

For each observation trajectory we have defined two parameters:

- minimum distance: It is defined as follows. For each instant of the observation trajectory, we supposed a failure of the system and we propagate resulting non-propulsed motion during a security time. We determine the minimum distance for this extrapolation. Minimum distance is defined as the minimum of all the extrapolations. If minimum distance is bigger than a safety radius, observation trajectory is safe. If not, the trajectory presents a risk. Second parameter evaluates the risk.
- percentage of orbit with collision risk: In the case
 where observation trajectory is not safe, this parameter is used to evaluate how risky it is. It measures
 the percentage of the observation trajectory where a
 failure of the system leads to a no-propulsed trajectory violating safety radius.

Safety radius (R_S) and security time (T_S) must be specified in mission requirements. In our simulations we use: $R_S=3\ m.,\,T_S=1$ orbit

Following two figures (6), (7) have been obtained for Simbol-x mission, at the 21th february 2012. First figure (6) shows the minimum distance of the observation trajectory as function of the direction of observation. A large part of observations presents no risk, while risky regions are concentrated in the poles. Second figure (7) show the most risky regions, where a failure of propulsion system leads unavoidably to a collision.

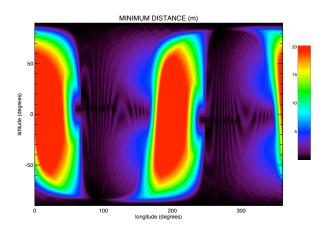


Figure 6. Minimum distance in case of propulsion failure

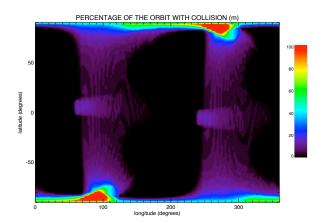


Figure 7. Percentage of orbit with collision risk

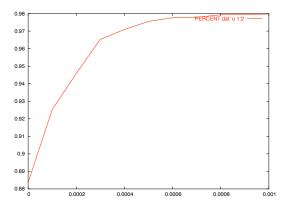


Figure 8. Influence of $\Delta \frac{S}{m}$ in the collision risk

Role of the $\Delta \frac{S}{m}$ Last figure (8) shows the influence of the parameter on the collision risk. We have computed the number of risky observation trajectories with respect to all the possible observation directions as function of the difference of $\frac{S}{m}$ between satellites. Figure shows that bigger values of $\Delta \frac{S}{m}$ are less risky than smaller ones. This phenomenon has a physical explanation since the difference of SRP tends to separate the satellites. At the sight of the figure, parameter $\Delta \frac{S}{m}$ should be, at least, 6.10^{-4} . In the figure, we have plot the same curve for different epochs in order to prove that it has few influence on the result.

5. CONCLUSIONS

This paper presents the necessity of using different approaches to solve different problems. Cartesian coordinates are well-adapted for control problems, but, as we have shown they do not enable easy integration. For fast

extrapolation of orbits, it is better to use differential orbital elements.

So, in this paper we have presented two analytical ways to take into account solar radiation pressure in formation flying. First, is devoted to obtain relative accelerations. Second, we have introduced the perturbation on the extrapolation of the orbit. This method obtains good results and is much faster than direct integration of equations of motion. In the second part of the paper, we have studied the effects of the perturbation on HEO orbits for Universe observation. We have focused on the role of parameter $\Delta \frac{S}{m}$. We have showed how it plays an important role on the size of maneuvers and how it can also be used to minimize collision risk. Precedent figures show the parameter should have a value near 5.10^{-4} . Bigger values lead to more fuel consumer configurations, while smaller values increase collision risk.

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