# User-friendly power management algorithms

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**Abstract**

This article addresses the optimal choice of the waiting period (or *timeout*) that a device should respect before entering sleep mode, so as to optimize a tradeoff between power consumption and user impact. The optimal timeout is inferred by appropriate statistical modeling of the times between user requests. In a test approach, these times are supposed independent, and a constant optimal timeout is inferred accordingly. In a second approach, some dependency is introduced through a hidden Markov chain, which also models specific activity states, like business hours or night periods. This model leads to a statistical framework for computing adaptive optimal timeout values. Different strategies are assessed using real datasets, on the basis of the power consumption, user impact and the frequency of wrong decisions.

## 1 Introduction

The goal of this study is to determine a policy based on the analysis of user behavior achieving a compromise between low power consumption of devices and limited user impact. We primarily describe this with respect to the behavior of printers, however similar policies could also be applied to other devices such as disk drives and displays. Currently, in most printers the time period to wait before entering sleep mode is either set by the administrator or predefined by the device manufacturer according to Energy Star[1] environmental standards. Today, Energy Star criteria do not take into account observed printer usage patterns. Those criteria rather set power consumption requirements depending on the device features (*e.g.* functionalities, estimated volume) and marking technology type (*e.g.* laser, solid ink, inkjet). In this paper, observed printer usage patterns are taken into account through the sequence of print job submissions (referred to as the print process). We model a device having several modes with different power consumptions. For a printer, these might correspond to:

- *Print mode*: The device activates its marking engine, print path and controller and completes any print requests. Power consumption is typically the highest in this mode.
- *Idle mode*: The device is ready to print immediately and therefore a certain power consumption is required to maintain the device in a state of readiness.

[1] http://www.energystar.gov

- *Sleep modes*: The device is not ready to print immediately which induces a delay between the user request and the actual beginning of the print job. These modes are sometimes referred to as *standby* or *power-save modes*. Depending on the printer, one or several such modes are available. Power consumption is typically the lowest in one of these sleep modes.

The difference in power consumption between idle mode and sleep modes is often as large as 40%. Power consumption due to transitions between modes depends on the printing technology (*e.g.* laser, solid ink) and is usually larger than the power consumption in idle mode. In the sequel, the transition from sleep modes to idle mode is referred to as *wakeup* while the transition from idle mode to sleep modes is referred to as *shutdown*. From the consumption point of view, the device features are summarized by the power consumption in each of these modes as well as the energy required to switch between these modes. Therefore, our goal amounts to inferring the optimal inactivity interval (or *timeout period*) before entering into sleep modes, given both the device power consumption model and observed usage patterns.

## 1.1 Consumption model and notations

Our approach relies on the following assumptions. Firstly, assuming that each print request is processed as soon as possible,the power consumption during print jobs is independent from the timeout period. Secondly, power consumptions during idle and sleep modes are supposed to be constant. Finally, printing, shutdown and wakeup transitions are supposed to be instantaneous. Therefore, the optimization focuses on the power consumption in idle and sleep modes and on the energy consumption of the associated transitions. Two kinds of transition are assumed on the printer:

- Transitions from one mode to the sleep mode with closest lower consumption.
- Transitions from one sleep mode to idle mode.

In reality, power consumption often takes the form of a series of pulses, as it is typically thermostatically controlled. Since the timing of these pulses is hard to predict, we assume a time-average power consumption for each mode. Figure 2 illustrates a real printer consumption (Xerox *Phaser 4500* with a single sleep mode) during 5 minutes of use (between 11 am and 11:05 am) and the corresponding power consumption model.

In this paper, we denote by:

- $m$ the number of sleep modes,
- $a$ the power consumption in idle mode (Watts),
- $b_j$ the power consumption in sleep mode $j$ (Watts),
- $c_j$ the energy required to switch from sleep mode $j-1$ to sleep mode $j$ (Joules),
- $d_j$ the wakeup energy required to switch from sleep mode $j$ to print mode (Joules),

with $j = 1, \dots, m$. Note that, in the above notations, sleep mode 0 corresponds to idle mode, and thus one can define $b_0 = a$. These notations are illustrated in Figure 1.

We limit ourselves to timeout strategies consisting in waiting a duration $\tau^{(j)}$ from the latest print onward, before switching into mode $j$. Since each print request must be processed immediately, the actual switch only occurs if the time between latest print job completion and the following print request is larger than $\tau^{(j)}$. This requires that the sequence $(\tau^{(1)}, \dots, \tau^{(m)})$ is increasing. It is also assumed that the sequence $(b_0, \dots, b_m)$ is decreasing, which implies that mode $j$ is only reachable from mode $j-1$.

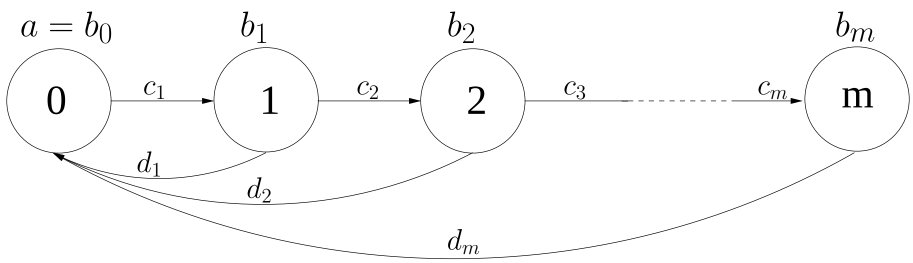


Figure 1: Possible transitions between idle and sleep modes.

### 1.2 Related work

The issue of power saving strategies has already been addressed in several works. Although most of them present a very general framework for power management, their applications mainly focus on hardware device (*e.g.* CPU, monitors, hard disk drives). A wide range of approaches are compared in [15], using the following typology of methods:

- Timeout. A timeout period is fixed either using a quantile of the residual time before next request, or using a parametric function of times between the last two requests and / or request and timeout [6, 11, 15].
- L-shape. This is a variant of timeout approaches dedicated to request patterns where short busy periods tend to be followed by a long idle period [19].
- Exponential average. This approach relies on a prediction of next idle period, based on an average of the previous idle periods with exponential weights [13].
- Stochastic model. These methods aim at finding an optimal probability distribution for the different actions to perform, given the past actions, states of the system and the expected power consumption for each action. The different levels of consumption are related to the notion of state. These approaches mainly rely on the theory of Markov decision processes [20] or their different variants (continuous time, semi-Markov or piecewise homogeneous Markov processes).
- Competitive algorithm. A $c$-competitive power saving algorithm is such that the power consumption is less than $c$ times that of an oracle algorithm [14]. An oracle algorithm considers all random variables, including future observations, as known, and achieves the minimal possible power consumption.
- Learning tree. Adaptive learning trees transform sequences of idle periods into discrete events and store them in tree nodes. They predict idle periods using finite-state machines. This is similar to branch prediction used in microprocessors and selects a path which resembles previous idle periods. At the beginning of an idle period, a learning tree determines an appropriate sleeping state; this algorithm is capable of controlling multiple sleeping states [5].

Our method belongs to the category of stochastic models, and combines the principles of continuous time modeling, piecewise identically distributed times between requests and Markov decision processes.

In [2], a function of a homogeneous Markov process with discrete time and discrete state space is used to model the sequences of requests. The states represent different levels of requests of the device. Markov processes are directly used to model completions of those requests (or *service process*), request queues, and decisions. The system performance is also assessed, based on the waiting time before request completion, and on the number of requests in the queue. In this context, a strategy is optimal if and only if it minimizes a function of the future expected power consumption under a performance constraint (where the notions of power consumption and performance can easily be exchanged). This optimization problem can be seen as a linear programming optimization problem. Thus, an exact optimal solution can be found in polynomial time with respect to input length (the input includes the number of actions, states, the number of requests per time and the queue length).

An extension of this work was proposed in [4] to take into account possible violation of the homogeneity assumption. This is addressed by a piecewise homogeneous Markov request process. This work was further extended in [3] using semi-Markov processes for modeling the dynamics of the states and events (typically the requests). These extensions are still discrete-time approaches. Continuous-time models were proposed in [17] to represent the requests process (by a homogeneous Poisson process), the service process and its queue (by Markov processes with discrete state-space). This paper does not provide details about an algorithm to find a solution to the optimization problem.

In [21], the decision is based on classification algorithms (logistic regression, k-nearest-neighbors or classification trees). A sample of vectors is used to train the system. This vector is composed of characteristics that reflect the state of activity of the user, and in the learning set, the action to perform (turn the device on or off). The time since last request of the device was also added to the vector of characteristics. This approach is compared with a so-called "timeout-based strategy" as defined in [15]. All those approaches take into account the system performance to assess the quality of the method for a given value of power saving, except [17]. In the context of power management for printers, the duration of a CPU cycle is negligible compared to the time between requests. Therefore, a continuous-time model is a natural way to model the request process.

In Section 2, we present a stochastic model for the requests that extends the piecewise homogeneous Markov process presented in [4]. A method is derived for optimally updating the timeout period after each request in the case of one single sleep mode. The results are extended to an arbitrary number of sleep modes. Then a model is proposed for assessing user impact.

In Section 3, particular processes are considered to model the request process: independent models, or hidden Markov models to account for possible temporal heterogeneity. We pay particular attention to parametric models which allow fast update of the timeout. For independent Weibull-distributed times between requests, an explicit formula is provided for the optimal timeout. More generally, the optimal timeout is the solution of a non-linear equation that depends on the model parameters, as estimated by maximum likelihood. In this case, the optimal timeout has no closed form and has to be approximated by numerical methods. In Section 4, two methods are proposed for the comparison of power management strategies. The former is based on out-of-sample prediction of power consumption under different policies. The latter is based on the number of wrong decisions of both types, and on the numbers of shutdowns. The results highlight the good performance of the method based of a fixed timeout period, computed from a parametric statistical model. Possible extensions or alternatives to our approach are provided in the discussion. Proofs are postponed to the Appendices A and B. In Appendix C, we show how our approach is related to the theory of Markov decision processes.

# 2 Stochastic model

The print process model is a particular case of a point process, similar to some reliability models, see for instance [18], Chapter 7. In our framework, the failure sequence is replaced by the print request sequence $\{T_i\}_{i\geq 1}$, with the convention $T_0 = 0$ and where $i$ denotes the index of the print request. Equivalently, the print process can be described by

- $\{X_i\}_{i\geq 1}$, where $X_i = T_i - T_{i-1}$ is the time between the $(i-1)^{\text{th}}$ and the $i^{\text{th}}$ print request,
- $\{N_t\}_{t\geq 0}$, the counting process of print requests, where $\forall t \in \mathbb{R}^+$, $N_t = \max\{i \in \mathbb{N}; T_i \leq t\}$ is the cumulative number of print requests between 0 and $t$.

As a consequence of the previous assumptions, the process $\{N_t\}_{t\geq 0}$ is *simple*: there cannot be more than one print request at a time with probability 1. The print process is depicted in Figure 3. In the next section, we limit ourselves to a single-sleep-mode printer. This approach is then extended to several sleep modes in Section 2.2 and to the quantification of the user impact in Section 2.3.

## 2.1 Single sleep mode

We consider a framework with one sleep mode, so that $m = 1$, where the timeout period $\tau^{(1)}$ may be updated after each print request $i$. Consequently, this timeout period will be denoted by $\tau_i^{(1)}$. Given a probabilistic model for the print process $\{X_i\}_{i\geq 1}$, we aim at optimizing over $\tau_i^{(1)}$ the expectation of the energy consumption, given the past of the print process $X_{1:i-1} := (X_1, \ldots, X_{i-1})$, between two successive print jobs $i-1$ and $i$. To compute this energy consumption, two cases arise:

- Either the time $X_i$ between two successive printings is larger than $\tau_i^{(1)}$. Then the printer stays in idle mode for $\tau_i^{(1)}$ before switching into sleep mode. After a delay $X_i - \tau_i^{(1)}$, the print job is processed and the printer returns to idle mode $j = 0$. Consequently, the energy consumption in this case is $a\tau_i^{(1)} + c_1 + b_1(X_i - \tau_i^{(1)}) + d_1$.
- Or $X_i$ is smaller than or equal to $\tau_i^{(1)}$. Then the printer stays in idle mode for $X_i$ before processing the job. Consequently, the energy consumption in this case is $aX_i$.

These two cases are illustrated in Figures 4(a) and 4(b). As a conclusion, the consumption $h$ between two successive printings is given by:

$$h(X_i, \tau_i^{(1)}) = (a\tau_i^{(1)} + c_1 + b_1(X_i - \tau_i^{(1)}) + d_1)\mathbb{1}_{\{X_i > \tau_i^{(1)}\}} + (aX_i)\mathbb{1}_{\{X_i \leq \tau_i^{(1)}\}}. \tag{1}$$

Let $f_{X_i|X_{1:i-1}}$ be the density function of $X_i$ given $X_{1:i-1}$, $F_{X_i|X_{1:i-1}}$ be its cumulative distribution function, and

$$z_{X_i|X_{1:i-1}}(x) = \lim_{\varepsilon\to 0} \frac{\mathbb{P}(x \leq X_i \leq x + \varepsilon | X_{1:i-1}, X_i > x)}{\varepsilon} = \frac{f_{X_i|X_{1:i-1}}(x)}{1 - F_{X_i|X_{1:i-1}}(x)} \tag{2}$$

be the failure rate function in reliability theory [1], Chapter 2. In our case, it can be interpreted as a *printing rate function*. We also define $\Delta t_1 = (c_1 + d_1)/(a - b_1)$. In a static analysis of the printer energy consumption, $\Delta t_1$ is the time after which switching into sleep mode is less expensive than staying in idle mode (see Figure 5).

**Lemma 1** *The expected consumption between two successive printings given $X_{1:i-1}$ is:*

$$
\begin{aligned}
&\mathbb{E}\left(h(X_i, \tau_i^{(1)})|X_{1:i-1}\right) \\
&= (a-b_1)\left[\left(1 - F_{X_i|X_{1:i-1}}(\tau_i^{(1)})\right)\left(\Delta t_1 + \tau_i^{(1)}\right) - \int_{\tau_i^{(1)}}^{+\infty} x f_{X_i|X_{1:i-1}}(x)dx\right] + a\mathbb{E}(X_i|X_{1:i-1}).
\end{aligned}
\tag{3}
$$

Formally, an optimal timeout period is defined by:

$$\hat{\tau}_i^{(1)} \in \arg\min_{\tau} \mathbb{E}(h(X_i, \tau)|X_{1:i-1}) \tag{4}$$

and can be computed based on the following result.

**Proposition 1** *Two situations are examined, depending on the behavior of the printing rate function.*

- *Suppose that the printing rate function $z_{X_i|X_{1:i-1}}(x)$ is decreasing in $x$. Three cases occur:*
  - *If* $\dfrac{1}{\Delta t_1} < \lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x)$, *then* $\hat{\tau}_i^{(1)} = +\infty$.
  - *If* $\lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x) \leq \dfrac{1}{\Delta t_1} \leq z_{X_i|X_{1:i-1}}(0)$, *then* $\hat{\tau}_i^{(1)}$ *is the unique solution of* $z_{X_i|X_{1:i-1}}(\hat{\tau}_i^{(1)}) = \dfrac{1}{\Delta t_1}$.
  - *If* $z_{X_i|X_{1:i-1}}(0) < \dfrac{1}{\Delta t_1}$, *then* $\hat{\tau}_i^{(1)} = 0$.
- *Suppose that $z_{X_i|X_{1:i-1}}$ is increasing or constant. Four cases occur:*
  - *If* $\dfrac{1}{\Delta t_1} < z_{X_i|X_{1:i-1}}(0)$, *then* $\hat{\tau}_i^{(1)} = +\infty$.
  - *If* $z_{X_i|X_{1:i-1}}(0) \leq \dfrac{1}{\Delta t_1} \leq \min\left(\lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x), \dfrac{1}{\mathbb{E}(X_i|X_{1:i-1})}\right)$, *then* $\hat{\tau}_i^{(1)} = +\infty$.
  - *If* $\max\left(z_{X_i|X_{1:i-1}}(0), \dfrac{1}{\mathbb{E}(X_i|X_{1:i-1})}\right) < \dfrac{1}{\Delta t_1} \leq \lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x)$, *then* $\hat{\tau}_i^{(1)} = 0$.
  - *If* $\lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x) < \dfrac{1}{\Delta t_1}$, *then* $\hat{\tau}_i^{(1)} = 0$.

It appears that three situations are possible. Either the times between printings are so small on average that the printer should not enter sleep mode ($\hat{\tau}_i^{(1)} = \infty$), or they are large on average, and the best strategy is to enter sleep mode immediately ($\hat{\tau}_i^{(1)} = 0$). The intermediate case provides non-degenerate optimal timeouts defined by the equation $z_{X_i|X_{1:i-1}}(\hat{\tau}_i^{(1)}) = 1/\Delta t_1$. This result highlights the separate roles of the printer characteristics (summarized by $\Delta t_1$) and the user behavior (modeled through the printing rate function $z_{X_i|X_{1:i-1}}$).

## 2.2 Multiple sleep modes

In this section, the results derived for single-mode printers are extended to multiple sleep mode printers. As in the previous paragraph, each timeout period $\tau^{(j)}$ may be updated after each print request $i$ and will be denoted by $\tau_i^{(j)}$. To compute the energy consumption, three cases are considered:

- If the time $X_i$ between two successive printings is smaller than $\tau_i^{(1)}$, then the printer stays in idle mode for $X_i$ before processing the job. Consequently, the energy consumption in this case is $aX_i$.
- If $X_i$ is larger than $\tau_i^{(1)}$ and smaller than $\tau_i^{(m)}$, then the printer stays in idle mode for $\tau_i^{(1)}$ before switching into the first sleep mode. Afterward, the printer successively switches into the $r$ following sleep modes where $r$ is such that $\tau_i^{(r)} < X_i < \tau_i^{(r+1)}$. The time spent in each sleep mode is $\tau_i^{(j)} - \tau_i^{(j-1)}$ if $2 \leq j \leq r-1$ and $X_i - \tau_i^{(j)}$ if $j = r$. Consequently, the energy consumption in this case is

$$a\tau_i^{(1)} + \sum_{j=1}^{r-1} \left( c_j + b_j(\tau_i^{(j+1)} - \tau_i^{(j)}) \right) + c_r + b_r(X_i - \tau_i^{(r)}) + d_r.$$

- Finally, if $X_i$ is larger than $\tau_i^{(m)}$, then the printer stays in idle mode for $\tau_i^{(1)}$ before switching into the first sleep mode. Afterward, the printer successively switches into the $m$ sleep modes. As in the previous case, the time spent in each sleep mode is $\tau_i^{(j)} - \tau_i^{(j-1)}$ if $2 \leq j \leq m-1$ and $X_i - \tau_i^{(j)}$ if $j = m$. Consequently, the energy consumption is

$$a\tau_i^{(1)} + \sum_{j=1}^{m-1} \left( c_j + b_j(\tau_i^{(j+1)} - \tau_i^{(j)}) \right) + c_m + b_m(X_i - \tau_i^{(m)}) + d_m.$$

To summarize, the consumption $h$ between two successive print requests is given by:

$$\begin{aligned} h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)}) &= aX_i \mathbb{1}_{\{X_i \leq \tau_i^{(1)}\}} \\ &+ \sum_{r=1}^{m-1} \left( a\tau_i^{(1)} + \sum_{j=1}^{r-1} \left( c_j + b_j(\tau_i^{(j+1)} - \tau_i^{(j)}) \right) + c_r + b_r(X_i - \tau_i^{(r)}) + d_r \right) \mathbb{1}_{\{\tau_i^{(r)} < X_i \leq \tau_i^{(r+1)}\}} \\ &+ \left( a\tau_i^{(1)} + \sum_{j=1}^{m-1} \left( c_j + b_j(\tau_i^{(j+1)} - \tau_i^{(j)}) \right) + c_m + b_m(X_i - \tau_i^{(m)}) + d_m \right) \mathbb{1}_{\{X_i > \tau_i^{(m)}\}}. \end{aligned} \tag{5}$$

Introducing $\Delta t_j = (c_j + d_j - d_{j-1})/(b_{j-1} - b_j)$ for $j = 1, \ldots, m$ with the conventions $b_0 = a$ and $d_0 = 0$, we have:

**Lemma 2** *The expected consumption between two successive print requests given $X_{1:i-1}$ is:*

$$\begin{aligned} &\mathbb{E}\left( h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)}) | X_{1:i-1} \right) \\ &= \sum_{j=1}^{m} (b_{j-1} - b_j) \left[ \left( 1 - F_{X_i|X_{1:i-1}}(\tau_i^{(j)}) \right) \left( \Delta t_j + \tau_i^{(j)} \right) - \int_{\tau_i^{(j)}}^{+\infty} x f_{X_i|X_{1:i-1}}(x) dx \right] \\ &+ a\mathbb{E}(X_i | X_{1:i-1}). \end{aligned} \tag{6}$$

It is remarquable that the expected energy consumption is expanded as the sum of $m$ terms, each of them depending on one and only one timeout. Thus, the optimization problem

$$(\hat{\tau}_i^{(1)}, \ldots, \hat{\tau}_i^{(m)}) = \underset{(\tau_i^{(1)}, \ldots, \tau_i^{(m)})}{\arg\min} \mathbb{E}(h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)})|X_{1:i-1}) \tag{7}$$

can be split into $m$ optimization problems leading to explicit optimal timeouts.

**Proposition 2** *Two situations are examined, depending on the behavior of the printing rate function.*

- *Suppose that the printing rate function $z_{X_i|X_{1:i-1}}(x)$ is decreasing in $x$. For each $j = 1, \ldots, m$ three cases occur:*
  - *If $\frac{1}{\Delta t_j} < \lim_{x \to +\infty} z_{X_i|X_{1:i-1}}(x)$, then $\hat{\tau}_i^{(j)} = +\infty$.*
  - *If $\lim_{x \to +\infty} z_{X_i|X_{1:i-1}}(x) \leq \frac{1}{\Delta t_j} \leq z_{X_i|X_{1:i-1}}(0)$, then $\hat{\tau}_i^{(j)}$ is the unique solution of $z_{X_i|X_{1:i-1}}(\hat{\tau}_i^{(j)}) = \frac{1}{\Delta t_j}$.*
  - *If $z_{X_i|X_{1:i-1}}(0) < \frac{1}{\Delta t_j}$, then $\hat{\tau}_i^{(j)} = 0$.*
- *Suppose that $z_{X_i|X_{1:i-1}}$ is increasing or constant. For each $j = 1, \ldots, m$ four cases occur:*
  - *If $\frac{1}{\Delta t_j} < z_{X_i|X_{1:i-1}}(0)$, then $\hat{\tau}_i^{(j)} = +\infty$.*
  - *If $z_{X_i|X_{1:i-1}}(0) \leq \frac{1}{\Delta t_j} \leq \min\left(\lim_{x \to +\infty} z_{X_i|X_{1:i-1}}(x), \frac{1}{\mathbb{E}(X_i|X_{1:i-1})}\right)$, then $\hat{\tau}_i^{(j)} = +\infty$.*
  - *If $\max\left(z_{X_i|X_{1:i-1}}(0), \frac{1}{\mathbb{E}(X_i|X_{1:i-1})}\right) < \frac{1}{\Delta t_j} \leq \lim_{x \to +\infty} z_{X_i|X_{1:i-1}}(x)$, then $\hat{\tau}_i^{(j)} = 0$.*
  - *If $\lim_{x \to +\infty} z_{X_i|X_{1:i-1}}(x) < \frac{1}{\Delta t_j}$, then $\hat{\tau}_j^{(1)} = 0$.*

## 2.3 Modeling user impact

In reality, transitions between sleep and idle modes may delay printing. The more frequently the system switches between sleep and idle modes, the more the user will be impacted. We thus propose to model this impact by a penalty term in the energy consumption. For the sake of simplicity, let us consider the case of a single-sleep-mode printer. We further assume that the user impact is proportional to the number of shutdown transitions. With such a model, the consumption between two successive print requests (1) is replaced by the cost

$$g(X_i, \tau_i^{(1)}) = h(X_i, \tau_i^{(1)}) + \delta \mathbb{1}_{\{X_i > \tau_i^{(1)}\}}, \tag{8}$$

where $\delta > 0$ is the weight assigned to the user impact in the energy consumption. The expected consumption including this user impact is given in the next Lemma.

**Lemma 3** *The expected penalized consumption between two successive print requests given $X_{1:i-1}$ is:*

$$\mathbb{E}\left(g(X_i,\tau_i^{(1)})|X_{1:i-1}\right)$$
$$= (a-b_1)\left[\left(1-F_{X_i|X_{1:i-1}}(\tau_i^{(1)})\right)\left(\tilde{\Delta}t_1+\tau_i^{(1)}\right)-\int_{\tau_i^{(1)}}^{+\infty} x f_{X_i|X_{1:i-1}}(x)dx\right]+a\mathbb{E}(X_i|X_{1:i-1}) \tag{9}$$

*with* $\tilde{\Delta}t_1 = (c_1+d_1+\delta)/(a-b_1)$.

It turns out that penalizing the consumption by the number of shutdowns can be interpreted as increasing the transition consumption $c_1+d_1$ by $\delta$. As a consequence, Proposition 1 still holds with $\Delta t_1$ replaced by $\tilde{\Delta}t_1$. In particular, when there is a non-degenerate optimal timeout period such that $z_{X_i|X_{1:i-1}}(\hat{\tau}_i^{(1)}) = 1/\tilde{\Delta}t_1$, the optimal timeout period is an increasing function of $\delta$.

# 3 Modeling the print process

According to the previous Section, the optimal timeout period depends on the model for the print process through the printing rate function. In this Section, three different print process models are proposed. In the first two approaches, times between printings are supposed independent. In the last approach, a hidden Markov chain (HMC) is used to model dependencies between printing times. The HMC states can be interpreted as specific states of activity like business hours or night periods. While we only consider a single sleep mode, extension to multiple sleep modes is straightforward.

## 3.1 Renewal process

In this paragraph, the times between print requests are supposed independent. The point process $\{N_t\}_{t\geq 0}$ is then a particular case of renewal process (see [18], Chapter 7). The random variable modeling the times between printings is denoted by $X$, since its distribution does not depend on the index $i$ of the print job. Similarly, the optimal timeout period is denoted by $\hat{\tau}^{(1)}$. In the following, the optimal timeout is studied under the assumptions that $X$ is Weibull or Gamma distributed.

### 3.1.1 Weibull distribution

The probability density function of the two-parameter Weibull distribution is parameterized as

$$f_X(x) = \alpha\lambda^\alpha x^{\alpha-1}e^{-(\lambda x)^\alpha}$$

for $x \geq 0$, where $\lambda > 0$ is a scale parameter and $\alpha > 0$ is referred to as the shape parameter. Let us recall that, in this case the mean time between printings is $\mathbb{E}(X) = \Gamma(1+1/\alpha)/\lambda$ and the printing rate function is $z_X(x) = \alpha\lambda^\alpha x^{\alpha-1}$ for $x \geq 0$. Note that this function can be decreasing if $\alpha \in (0,1)$, increasing if $\alpha > 1$ or constant if $\alpha = 1$ (exponential distribution). As a consequence of Proposition 1, we have

$$\hat{\tau}^{(1)} = \left| \begin{array}{ll} (\alpha\lambda^\alpha\Delta t_1)^{1/(1-\alpha)} & \text{if } \alpha \in (0,1) \\ 0 & \text{if } \alpha \geq 1 \text{ and } \Delta t_1 < \Gamma(1+1/\alpha)/\lambda \\ +\infty & \text{if } \alpha \geq 1 \text{ and } \Delta t_1 > \Gamma(1+1/\alpha)/\lambda. \end{array} \right. \tag{10}$$

Of course, in practical situations, the parameters $\alpha$ and $\lambda$ appearing in (10) are replaced by their maximum-likelihood estimates, see Section 4 for examples.

### 3.1.2 Gamma distribution

The probability density function of the two-parameter Gamma distribution is parameterized as

$$f_X(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}$$

for $x \geq 0$, where $\beta > 0$ is a scale parameter and $\alpha > 0$ is the shape parameter. In this case, we have $\mathbb{E}(X) = \alpha\beta$ but no closed-form expression for the printing rate function is available for non-integral $\alpha$. Nevertheless, it can be shown [1] that, similarly to the Weibull case, the printing rate function is decreasing if $\alpha \in (0, 1)$, increasing if $\alpha > 1$ or constant if $\alpha = 1$ (exponential distribution). Thus,

$$\hat{\tau}^{(1)} = \left| \begin{array}{ll} z_X^{-1}(1/\Delta t_1) & \text{if } \alpha \in (0,1) \\ 0 & \text{if } \alpha \geq 1 \text{ and } \Delta t_1 < \alpha\beta \\ +\infty & \text{if } \alpha \geq 1 \text{ and } \Delta t_1 > \alpha\beta, \end{array} \right. \tag{11}$$

the printing rate function $z_X(x)$ being evaluated numerically through the use of the incomplete gamma function. The computation of $\hat{\tau}^{(1)}$ is achieved with a dichotomy procedure.

## 3.2 Hidden Markov model

The assumption of a constant printing frequency throughout day and night does not seem realistic *a priori*. It can be expected that during given periods, users will tend to print more often or less often than average. Such periods can be interpreted in terms of activity levels, corresponding to different levels of printing rates:

- rush hours with short times between printings, mainly due to users printing a sequence of documents in a short period of time;
- normal hours with medium times between printings;
- off-peak times with long times between printings, due to night or weekends.

This corresponds to a heterogeneous distribution of the times between printings, such that there exist some homogeneous periods $(i_1, \ldots, i_k)$ where $(X_{i_1}, ..., X_{i_k})$ have the same distribution. Those characteristics can be modeled by a Hidden Markov Chain (HMC) print process. In this model, activity periods are defined by non-visible factors, such as the amount of users at a given time in the printer network (which is related to working hours and can vary with the company), the type of users, country or even site specificities. In HMC modeling, these non-visible factors must be deduced from the observed variables. In our case, one might imagine that periods with comparable times between printings tend to correspond to the same level of activity.

### 3.2.1 Definition

The existence of different levels of activity is modeled through a discrete state-space process $(S_1, ..., S_n) = S_{1:n}$, which is assumed Markovian. Here, $S_i$ represents the state of the process at $i$th printing request. Since no direct information is available in the data about the precise level of activity at a given time $t$, this Markov chain must be considered hidden. The hidden chain is related to the print process $(X_1, ..., X_n) = X_{1:n}$ through the conditional distributions of $X_i$ given $S_i = k$ for each possible value $k$ of the Markov chain. Those distributions are specified by a parametric family $(f_\theta)_{\theta \in \Theta}$ of probability density functions.

Formally, an HMC is defined by two processes $X_{1:n}$ (observed process) and $S_{1:n}$ (hidden process) such that:

- $S_{1:n}$ is a homogeneous Markov chain with finite state space $\{1, \ldots, K\}$, with transition matrix $A$ and a distribution $\pi = (\pi_1, ..., \pi_K)$ for the initial state $S_1$. Here, $S_{1:n}$ is assumed ergodic, with stationary distribution $\pi$.
- given $S_{1:n} = s_{1:n}$, the $X_i$ are mutually independent, and independent of the $(S_{i'})_{i' \neq i}$, with conditional probability density functions $f_{\theta_{s_i}}$ (called *emission distributions*).

The set of parameters of this model is $\eta = (\pi, A, \theta_1, ..., \theta_K)$.

Since stationarity is assumed for the hidden process, the observed process is also stationary. However, the observed process is non-stationary given $S_{1:n} = s_{1:n}$; this is why the above HMC can be used to model changes in the printing rate. Other models can also achieve this purpose, *e.g.* the non-stationary (piecewise-stationary) Markovian model based on sliding windows in [4].

#### 3.2.2 Parameter estimation

We use the general notation $\mathbb{P}()$ to denote either a probability mass function or a probability density function, the true nature of $\mathbb{P}()$ being obvious from the context. The parameter is estimated by likelihood maximization, using the Expectation Maximization (EM) algorithm for hidden Markov chains [8]. This iterative algorithm starts from an initial value $\eta^{(0)}$ of the parameter and creates a sequence $(\eta^{(m)})_{m \geq 0}$ whose likelihood grows. The sequence $(\eta^{(m)})_{m \geq 0}$ converges to a consistent solution of the likelihood equations when $\eta^{(0)}$ is close to the optimal solution. At each iteration $m$, it proceeds as follows:

- Expectation (E) step: determination of the $\mathbb{Q}$ function defined by:

$$\begin{aligned}\mathbb{Q}(\eta, \eta^{(m)}) &= \mathbb{E}_\eta \left[\log \mathbb{P}_{\eta^{(m)}}(S_1^n = s_1^n, X_1^n = x_1^n) | X_1^n = x_1^n\right] \\ &= \sum_{k=1}^{K} \log \pi_k \mathbb{P}_{\eta^{(m)}}(S_1 = k | X_1^n = x_1^n) \\ &\quad + \sum_{i=1}^{n-1} \sum_{k,l} \log A_{k,l} \mathbb{P}_{\eta^{(m)}}(S_i = k, S_{i+1} = l | X_1^n = x_1^n) \\ &\quad + \sum_{i=1}^{n} \sum_{k=1}^{K} \log f_{\theta_k}(x) \mathbb{P}_{\eta^{(m)}}(S_i = k | X_1^n = x_1^n) \qquad (12)\end{aligned}$$

- Maximization (M) step: maximization of $\mathbb{Q}(\eta, \eta^{(m)})$ with respect to $\eta$:

$$\eta^{(m+1)} = \arg\max_{\eta} \mathbb{Q}(\eta, \eta^{(m)}) \qquad (13)$$

#### 3.2.3 Adaptive timeout period using HMCs

Since the case of printers with multiple sleep modes is not considered, the timeout period will be denoted by $\tau$ rather than $\tau^{(j)}$. This Section details how to exploit the hidden state values to propose adaptive timeout periods $\hat{\tau}_i$ that are updated after each printing job $i$. Those strategies basically consist in predicting the time to the next print request $X_i$, from the past observed values $X_{1:i-1}$. We propose three approaches to dynamically re-estimate $\tau_i$.

**a) Viterbi-based approach.** This approach consists in deriving a prediction $\hat{S}_i$ for the next the state value as follows:

$$\begin{aligned}\hat{S}_i &= \arg\max_k \left( \max_{s_1,\ldots,s_{i-1}} \mathbb{P}(S_{1:i-1} = s_{1:i-1}, S_i = k | X_{1:i-1}) \right) \\ &= \arg\max_k \left( \max_j \left( \max_{s_1,\ldots,s_{i-2}} A_{jk} \mathbb{P}(S_{1:i-2} = s_{1:i-2}, S_{i-1} = j | X_{1:i-1}) \right) \right) \end{aligned} \quad (14)$$

This value is computed by the Viterbi algorithm [8]. The predicted distribution for $X_i$ is then $f_{\theta_{\hat{S}_i}}$.

**b) Filtering-based approach.** This approach also consists in predicting next state value $S_i$, and in deriving a prediction for the time to next print request $X_i$ from its conditional distribution given $S_i$. It differs from the Viterbi-based approach in the prediction method for $S_i$, characterized by

$$\tilde{S}_i = \arg\max_k \mathbb{P}(S_i = k | X_{1:i-1}) = \arg\max_k \sum_{j=1}^{K} A_{jk} \mathbb{P}(S_{i-1} = j | X_{1:i-1}) \quad (15)$$

where the algorithm for computing $\mathbb{P}(S_{i-1} = j | X_{1:i-1})$ is the same than in the E step of the EM algorithm (forward recursion in [8]). The predicted distribution for $X_i$ is also $f_{\theta_{\tilde{S}_i}}$.

**c) Approach based on full conditional distribution.** This approach consists in computing the printing rate function of $X_i$ given $X_{1:i-1}$. The density function of this distribution is given by:

$$f_{X_i|X_{1:i-1}}(x) = \sum_{k=1}^{K} f_{\theta_k}(x) \beta_i(k) \quad (16)$$

where $\beta_i(k) = \mathbb{P}(S_i = k | X_{1:i-1})$ is the filtered probability, computed in the forward recursion. Thus, the cumulative distribution function is:

$$F_{X_i|X_{1:i-1}}(x) = \sum_{k=1}^{K} F_{\theta_k}(x) \beta_i(k) \quad (17)$$

and the printing rate function is:

$$z_{X_i|X_1,\ldots,X_{i-1}}(x) = \frac{f_{X_i|X_{1:i-1}}(x)}{1 - F_{X_i|X_{1:i-1}}(x)} \quad (18)$$

Each of the three approaches results into an estimated density $f_i$ for the predictive distribution of $X_i$, associated with a printing rate function $z_i$. The optimal timeout period $\hat{\tau}_i$ is given by Proposition 1, replacing $\hat{\tau}_i^{(1)}$ by $\hat{\tau}_i$ and $z_{X_i|X_{1:i-1}}$ by $z_i$.

In the approach based on full conditional distributions, even in the case of Weibull or Gamma observation distribution families $(f_\theta)_{\theta \in \Theta}$, we could not derive general conditions on the parameters $(\theta_k)_{k=1,\ldots,K}$, under which equation

$$z_{X_i|X_{1:i-1}}(\hat{\tau}_i^{(1)}) = \frac{1}{\Delta t_1}$$

has a unique solution. Thus, numerical methods have to be used, to determine whether the optimal timeout period is null, positive or infinite.

# 4 Experiments

In this section, our methodology is illustrated by experiments on two real datasets. The efficiencies of the timeout strategies introduced in Section 3 are compared in terms of energy consumption. These strategies are also compared with three alternatives called *Energy star method*, *oracle method* and *exhaustive search method*. In the *Energy star method*, the timeout period is fixed to $1800s$ in order to comply with the Energy Star standard. In the *oracle method*, the future of the print process is supposed to be known. The printer switches into sleep mode $j$ before print job $i$ if $X_i > \Delta t_j$. Let us highlight that this reference method provides a lower bound on the consumption but cannot be used in practice. Finally, the *exhaustive search method* consists in finding the timeout that minimizes the actual consumption thanks to an exhaustive search. Moreover, the method mentioned in Section 3.1 will be called *static method*, while the methods described in Section 3.2.3 will be referred to as the *Viterbi method*, *filtering method* and *conditional method*.

In both datasets, times between printings are deduced from the print logs, recorded during the whole of the year 2006 on XRCE print infrastructure which is composed of 14 printers and involves 155 users. These data have been collected by the *Xerox Job Tracking Agent*[2]. It is an office print tracking tool that allows the capture and recording of information about end users'printing behaviors. Additionally, real power consumption has been measured on two different printer models with a specific power metering unit[3]. The first printer is a *Xerox WorkCentre 238* model with two sleep modes. Its power consumption is $a_1 = 270W$ in *idle mode*, $b_1 = 150W$ in the *first sleep mode* and $b_2 = 50W$ in the *second sleep mode*. The energy required to switch from the first and second sleep modes to idle mode are respectively $d_1 = 40kJ$ and $d_2 = 200kJ$. Energies to enter sleep modes are negligible, $c_1 = c_2 = 0$. The second printer is a *Phaser 4500* model with one single sleep mode. Its power consumption is $a_1 = 80W$ in *idle mode*, $b_1 = 16W$ in *sleep mode*. The energy required to switch from sleep mode to idle mode is $d_1 = 25.3kJ$ and the energy to enter sleep mode is negligible, $c_1 = 0$.

Static, Viterbi, filtering and conditional methods require the selection of a distribution for the times between print requests. This choice has been made using a goodness-of-fit $\chi^2$ test, whose results are summed up in Table 1.

| Distribution | P-value | |
|---|---|---|
| | Phaser 4500 | WorkCentre 238 |
| Gamma | 8e-02 | 8e-04 |
| Weibull | 1e-07 | 7e-04 |
| Lognormal | 2e-32 | 2e-04 |
| Normal | 5e-75 | 2e-11 |
| Exponential | 1e-89 | 8e-13 |

Table 1: P-values obtained with the $\chi^2$ goodness of fit test for the selection of a distribution for the times between printings.

It appears that Weibull and Gamma are the most appropriate distributions. In the following, a Weibull distribution is adopted to model the distribution of the times between printings, when those are assumed independent. A Gamma distribution would also be suitable but would not allow the derivation of an explicit timeout (see paragraph 3.1.2). The considered HMC model has

[2] http://www.consulting.xerox.com/print-tracking/
[3] Fluke 43B Power Quality Analyzer (http://us.fluke.com)

also Weibull emission distributions, and three states, which can be interpreted as rush, normal and calm periods, from the point of view of the print requests. Note that the number of states could also be selected using penalized likelihood criteria [7] or cross-validation [10]. The M step for parameter estimation by the EM algorithm is given in Appendix B. Paragraph 4.1 is dedicated to the analysis of the out-of-sample performance of the above-mentioned methods without taking user impact into account. In paragraph 4.2, the influence of the penalty term is examined under several points of view: consumption, number of shutdowns, number of false wakeups and false shutdowns.

## 4.1 Cross-validated assessment of different strategies

The goal of this experiment is to investigate the methods' performance on future data, and thus to assess their generalization capacities. We focus on the *Xerox WorkCentre 238* dataset ($n = 3910$ print jobs) and user impact is not considered. Its predefined timeouts according to Energy Star environmental standards is 15 minutes for the first sleep mode and 30 minutes for the second. The test procedure is multi-fold cross-validation [22], as follows: the dataset is divided into $L$ contiguous subsamples of equal size. Then for each subsample $\ell \leq L-1$, the method parameters are estimated on this subsample while the consumption is computed on subsample $\ell + 1$. Three cases are considered: $L = 10$ subsamples of size 391, $L = 30$ subsamples of size 121 and $L = 60$ subsamples of size 61. Results are summarized in Tables 2 and 3.

It appears on Table 2 that exhaustive search, static, filtering and conditional methods are the most efficient ones in terms of consumption. The consumption associated to these methods is about 4.5% larger than the lower bound given by the Oracle method. This slight increase of the optimal consumption indicates that the Weibull distribution fits the inter-print distribution well. Besides, exhaustive search, static and conditional methods are quite robust since they yield a constant consumption whatever the subdivision is. Moreover, the standard deviation of the consumption represents less than 2% of the total consumption. Among these three methods, the static one is a thousand time faster than the other ones. Experiments were conducted in Matlab on an Intel Pentium 4 running at 3GHz.

Focussing on Table 3, it appears that the timeout periods provided by the static and exhaustive search methods are approximatively independent from the subdivision of the sample, for both methods. Let us emphasize that static timeouts benefit from small standard deviation whereas exhaustive search timeouts suffer from a high variability. As a conclusion, static method seems to be an accurate, reliable and fast method to select the optimal timeouts. A decrease of about 20 % of power consumption can be achieved with regard to the Energy Star method.

| | Total consumption ($kWh$) | | | Standard deviation of consumption | | | Mean computation time by sample ($ms$) | | |
|---|---|---|---|---|---|---|---|---|---|
| Sample size | 391 | 121 | 61 | 391 | 121 | 61 | 391 | 121 | 61 |
| Energy Star | 493 | 493 | 493 | 7.82 | 4.63 | 2.99 | 1.0e–01 | 1.2e–01 | 1.4e–01 |
| $\tau^{(1)} = \tau^{(2)} = 0$ | 427 | 427 | 427 | 6.74 | 3.74 | 2.54 | 1.1e–01 | 1.3e–01 | 1.4e–01 |
| Oracle | 373 | 373 | 373 | 6.92 | 3.97 | 2.65 | 3.8e–01 | 3.4e–01 | 2.8e–01 |
| Exhaustive search | 404 | 406 | 405 | 6.81 | 3.97 | 2.65 | 8.6e+03 | 6.9e+03 | 6.6e+04 |
| Static | 404 | 404 | 405 | 6.76 | 3.96 | 2.66 | 1.2e+01 | 1.6e+01 | 1.9e+01 |
| Viterbi | 444 | 426 | 574 | 11.18 | 4.31 | 9.30 | 2.0e+05 | 7.6e+03 | 3.9e+03 |
| Filtering | 411 | 430 | 456 | 6.96 | 5.03 | 3.37 | 1.8e+04 | 7.3e+04 | 3.7e+03 |
| Conditional | 408 | 409 | 410 | 6.69 | 3.92 | 2.69 | 3.1e+04 | 1.2e+04 | 6.1e+03 |

Table 2: Energy consumption and mean computation time associated to the different strategies.

| | | Mean timeouts ($s$) | | | Standard deviation of timeouts | | |
|---|---|---|---|---|---|---|---|
| Sample size | | 391 | 121 | 61 | 391 | 121 | 61 |
| Exhaustive search | $\tau^{(1)}$ | 26 | 25 | 24 | 13 | 17 | 21 |
| | $\tau^{(2)}$ | 203 | 208 | 218 | 55 | 108 | 121 |
| Static | $\tau^{(1)}$ | 11 | 12 | 12 | 2 | 5 | 7 |
| | $\tau^{(2)}$ | 179 | 188 | 192 | 33 | 64 | 84 |

Table 3: Timeout associated to the different strategies.

## 4.2 Assessment of user impact

In this Section, the behavior of the methods is compared when taking user impact into account on the Phaser 4500 printer. Our test procedure is the following: The dataset ($n = 2320$) is divided into 2 subsamples with the same size. Parameters of each method are estimated on the first subsample, while the total consumption is computed on the second one as the penalty $\delta$ varies (see Paragraph 2.3).

The variations of the number of shutdowns (or equivalently wakeups) as a function of the penalty $\delta$ are depicted in Figure 6. Clearly, the larger $\delta$ is, the smaller the number of shutdown is. Unsurprisingly, small numbers of shutdowns correspond to large timeouts (Figure 7) and thus large consumptions (Figure 8). Even though, for a fixed penalty $\delta$, the different methods yield different numbers of shutdowns and different consumptions, it appears on Figure 9 that, for a fixed number of shutdowns, the consumption is about the same whatever the method used. To compare the different methods, we propose a graphical comparison based on an adaption of ROC (Receiver Operating Characteristic) curves [12] to our framework. To this end, let us denote by $\alpha = \mathbb{P}(X_i > \Delta t | X_i < \tau_i)$ the probability of a type I error at $i^{\text{th}}$ print request. In this context, this error occurs when a printer stays in idle mode, whereas it should enter into sleep mode. Similarly, the probability of a type II error is given by $\beta = \mathbb{P}(X_i < \Delta t | X_i > \tau_i)$. In the case of type II errors, the printer enters into sleep mode, whereas it should stay idle. Note that the number $N_{wd}$ of errors of type II is used in [15] as a measure of performance of several algorithms for power management. In practice, $\alpha$ and $\beta$ are estimated by their empirical counterparts. The ROC curve (Figures 10 and 11) is built for each method by drawing $1-\beta$ as a function of $\alpha$ by letting the penalty $\delta$ vary. The best method is the one whose ROC curve is the closest to the vertical line $\alpha = 0$ and to the horizontal line $1-\beta = 0$. At the opposite, the worst method is the one whose ROC curve is the closest to the diagonal line $1-\beta = \alpha$. Here, it appears that static and Viterbi methods are the best ones, since they yield the best compromise between the two risks. Keeping in mind the conclusions of the previous paragraph, it seems that the static method should be preferred since it is the simplest and most robust one.

In the case where no penalty due to user impact is applied ($\delta = 0$), the consumption corresponding to the Energy Star timeout is 20 % higher than that corresponding to the static method. Moreover, the optimal timeout period provided by the static method is $\tau = 10s$. The associated value of the consumption ($78.1kWh$) is only 0.5% lower that the consumption achieved by setting $\tau = 0s$, which corresponds to the so-called *eager* policy in [2]. The eager policy, considered as "often unacceptable" by the authors in their context, is actually nearly optimal for our dataset. However, this result is specific to the considered printer and user behavior: in paragraph 4.1, the eager policy yields a waste of 5.2% in the energy consumption, compared to the static method (see Table 2).

# 5 Conclusion and discussion

In this paper, we have proposed a statistical cost-based analysis to determine optimal timeout period for devices. This method takes into account the real usage patterns in order to optimize power consumption. In a first approach, times between requests were supposed independent and the timeout period inferred accordingly. We also proposed three approaches to dynamically re-estimate an optimal timeout period using a hidden Markov chain to model print events. Finally, a methodology to take into account user impact due to power saving exit transitions is also included. It allows the dynamic timeout period methods to achieve a trade-off between user impact and power consumption, depending on the user's priorities.
From the application of our methods to two real datasets composed of printing requests, it can be remarked that the static method is the fastest and most accurate method to select the optimal timeout periods. Moreover it is rather simple to set up in practice. Indeed, it only learns the parameters of the distribution of the times between printings by the Maximum likelihood estimation method, and deduces the optimal timeout period according to the estimated value of the printing rate.

The theoretical formulation of power consumption in terms of a print process can be considered as a stepping stone for more complex models that will allow the model to progressively gain completeness in the consideration of other several cost factors, as for example device aging due to increased transitions from power saving mode due to a more dynamic power saving policy. We have also established the foundations to develop in the future a power saving strategy capable of performing accurate prediction of power saving entry as described in this article, but also of optimal power saving exit.

Other refinements of our approach may include the use of covariates (*e.g.* hour, day of the week), to adapt the timeout period to different conditions identified *a priori* (contrary to HMCs where the conditions can be interpreted *a posteriori*). The sliding windows approach of [4] is also an alternative for adaptive timeout computation.

Instead of a continuous-time framework where the decision corresponds to the value of a timeout period, a discrete-time framework could be considered. After each time step, the decision would consist in switching or not the printer into sleep mode. This is the approach chosen in [21].

The authors claim that the timeout-based policies "not only waste power during the periods of inactivity, but also needlessly annoy the user when they turn off components at inappropriate times". This is only true when the times of future requests are considered as known (oracle point of view). In the case of unknown times of future requests, the power used to maintain a device into idle mode cannot be considered *a priori* as wasted. However, some generalization of our classical timeout-based approach could be achieved, with potentially better results, by replacing the assumption of a deterministic timeout period by a stochastic one, whose optimal distribution would have to be determined.

A further extension of this work is the challenging issue of optimal redirection of print jobs and power saving policy within a network of printers managed by a server. Given a printing request, this consists in determining on which printer the job has to be processed, and after what delay each printer has to be turned into sleep mode, so as to minimize the global consumption. Modelling this problem should take into account constraints due to user impact, that are partially related to network connectivity.

Finally, our approach deals separately with model identification (parameter estimation from trajectories of user requests) and computation of the optimal timeout periods (in a framework with fixed parameters). As an alternative, a unified model for handling both model identification and decision taking would be provided by the Bayesian Partially-Observed Markov Decision Processes (POMDPs) in [16]. Here the non-observed part of the MDP would consist in, firstly, the unknown parameters, considered as stochastic in a Bayesian framework, and secondly, potential unknown states as in the HMC models. The benefit of Bayesian POMDPs to our application would come from taking into account simultaneously the different sources of uncertainty: states of the printer and of the user, value of the parameter and of the reward.

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# A Appendix: Proofs

**Proof of Lemma 1.** From (1), we have

$$\begin{aligned}
&\mathbb{E}\left(h(X_i,\tau_i^{(1)})|X_{1:i-1}\right)\\
&\quad=\mathbb{E}\left(\left(a\tau_i^{(1)}+c_1+b_1(X_i-\tau_i^{(1)})+d_1\right)\mathbb{1}_{\{X_i>\tau_i^{(1)}\}}|X_{1:i-1}\right)+a\mathbb{E}\left(X_i\mathbb{1}_{\{X_i\le\tau_i^{(1)}\}}|X_{1:i-1}\right)\\
&\quad=\left(a\tau_i^{(1)}+c_1+d_1\right)\mathbb{P}\left(X_i>\tau_i^{(1)}|X_{1:i-1}\right)+b_1\mathbb{E}\left((X_i-\tau_i^{(1)})\mathbb{1}_{\{X_i>\tau_i^{(1)}\}}|X_{1:i-1}\right)+a\mathbb{E}\left(X_i\mathbb{1}_{\{X_i\le\tau_i^{(1)}\}}|X_{1:i-1}\right)\\
&\quad=\left(\tau_i^{(1)}(a-b_1)+c_1+d_1\right)\mathbb{P}\left(X_i>\tau_i^{(1)}|X_{1:i-1}\right)+a\mathbb{E}(X_i|X_{1:i-1})+(b_1-a)\mathbb{E}\left(X_i\mathbb{1}_{\{X_i>\tau_i^{(1)}\}}|X_{1:i-1}\right).
\end{aligned}$$

Letting $\Delta t_1 = (c_1 + d_1)/(a - b_1)$, the expected consumption can be rewritten as

$$\begin{aligned}
&\mathbb{E}\left(h(X_i, \tau_i^{(1)})|X_{1:i-1}\right) \\
&\quad = (a-b_1)\left((\Delta t_1 + \tau_i^{(1)})\mathbb{P}\left(X_i > \tau_i^{(1)}|X_{1:i-1}\right) - \mathbb{E}\left(X_i \mathbb{1}_{\{X_i > \tau_i^{(1)}\}}|X_{1:i-1}\right)\right) + a\mathbb{E}(X_i|X_{1:i-1}) \\
&\quad = (a-b_1)\left(\left(1 - F_{X_i|X_{1:i-1}}(\tau_i^{(1)})\right)(\Delta t_1 + \tau_i^{(1)}) - \int_{\tau_i^{(1)}}^{+\infty} x f_{X_i|X_{1:i-1}}(x)dx\right) + a\mathbb{E}(X_i|X_{1:i-1}),
\end{aligned}$$

and the conclusion follows. ■

**Proof of Proposition 1.** Differentiating $\mathbb{E}(h(X_i, \tau_i^{(1)})|X_{1:i-1})$ with respect to $\tau_i^{(1)}$ yields

$$\frac{d\mathbb{E}\left(h(X_i, \tau_i^{(1)})|X_{1:i-1}\right)}{d\tau_i^{(1)}} = (a-b_1)\left(1 - F_{X_i|X_{1:i-1}}(\tau_i^{(1)})\right)\left(1 - \Delta t_1 z_{X_i|X_{1:i-1}}(\tau_i^{(1)})\right). \quad (19)$$

Let us recall that $a > b_1$. Two main cases are considered.

- Suppose that $z_{X_i|X_{1:i-1}}$ is decreasing. Three situations occur:
  - If $1/\Delta t_1 < \lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x)$, then the derivative (19) is negative, the expected consumption is a decreasing function of $\tau_i^{(1)}$ and thus $\hat{\tau}_i^{(1)} = +\infty$.
  - If $\lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x) \le 1/\Delta t_1 \le z_{X_i|X_{1:i-1}}(0)$, then the following equation

    $$z_{X_i|X_{1:i-1}}(\tau_i^{(1)}) = 1/\Delta t_1 \quad (20)$$

    has an unique root in $(0, +\infty)$ and the expected consumption is a concave function of $\tau_i^{(1)}$. As a consequence, $\hat{\tau}_i^{(1)}$ is the unique solution of (20).
  - Finally, if $z_{X_i|X_{1:i-1}}(0) < 1/\Delta t_1$, then the derivative (19) is positive, the expected consumption is an increasing function of $\tau_i^{(1)}$ and thus $\hat{\tau}_i^{(1)} = 0$.
- Suppose that $z_{X_i|X_{1:i-1}}$ is increasing or constant. Three situations occur:
  - If $1/\Delta t_1 \le z_{X_i|X_{1:i-1}}(0)$, then the derivative (19) is non-positive, the expected consumption is a non-increasing function of $\tau_i^{(1)}$ and thus $\hat{\tau}_i^{(1)} = +\infty$.
  - If $z_{X_i|X_{1:i-1}}(0) < 1/\Delta t_1 < \lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x)$ then equation (20) has an unique root in $(0, +\infty)$ and the expected consumption is a convex function of $\tau_i^{(1)}$. As a consequence, $\hat{\tau}_i^{(1)} = 0$ if $\mathbb{E}(h(X_i, 0)|X_{1:i-1}) < \lim_{x\to\infty} \mathbb{E}(h(X_i, x)|X_{1:i-1})$ and $\hat{\tau}_i^{(1)} = +\infty$ otherwise. Since

    $$\mathbb{E}(h(X_i, 0)|X_{1:i-1}) - \lim_{x\to\infty} \mathbb{E}(h(X_i, x)|X_{1:i-1}) = (a-b_1)(\Delta t_1 - \mathbb{E}(X_i|X_{1:i-1})),$$

    the conclusion follows.
  - Finally, if $\lim_{x\to+\infty} z_{X_i|X_{1:i-1}}(x) \le 1/\Delta t_1$, then the derivative (19) is non-negative, the expected consumption is a non-decreasing function of $\tau_i^{(1)}$ and thus $\hat{\tau}_i^{(1)} = 0$. ■

**Proof of Lemma 2.** Equation (5) and letting $a = b_0$ yield

$$\begin{aligned}
&\mathbb{E}\left(h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)})|X_{1:i-1}\right)\\
&= b_0\mathbb{E}\left(X_i \mathbb{1}_{\{X_i<\tau_i^{(1)}\}}|X_{1:i-1}\right) + \sum_{r=1}^{m-1} b_r\mathbb{E}\left(X_i \mathbb{1}_{\{\tau_i^{(r)}<X_i\leq\tau_i^{(r+1)}\}}|X_{1:i-1}\right) + b_m\mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(m)}\}}|X_{1:i-1}\right)\\
&\quad + \sum_{r=1}^{m-1}\mathbb{P}\left(\tau_i^{(r)} < X_i \leq \tau_i^{(r+1)}|X_{1:i-1}\right)\left(\sum_{j=1}^{r}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_r\right)\\
&\quad + \mathbb{P}\left(X_i > \tau_i^{(m)}|X_{1:i-1}\right)\left(\sum_{j=1}^{m}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_m\right).
\end{aligned}$$

Taking account of

$$\begin{aligned}
\mathbb{E}\left(X_i \mathbb{1}_{\{\tau_i^{(r)}<X_i\leq\tau_i^{(r+1)}\}}|X_{1:i-1}\right) &= \mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(r)}\}}|X_{1:i-1}\right) - \mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(r+1)}\}}|X_{1:i-1}\right),\\
\mathbb{E}\left(X_i \mathbb{1}_{\{X_i<\tau_i^{(1)}\}}|X_{1:i-1}\right) &= \mathbb{E}\left(X_i|X_{1:i-1}\right) - \mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(1)}\}}|X_{1:i-1}\right)
\end{aligned}$$

the expected consumption can be rewritten as

$$\begin{aligned}
&\mathbb{E}\left(h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)})|X_{1:i-1}\right)\\
&= -\sum_{j=1}^{m}(b_{j-1}-b_j)\mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(j)}\}}|X_{1:i-1}\right) + \left(1 - F_{X_i|X_{1:i-1}}(\tau_i^{(m)})\right)\left(\sum_{j=1}^{m}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_m\right)\\
&\quad + \sum_{r=1}^{m-1}\left(\left(F_{X_i|X_{1:i-1}}(\tau_i^{(r+1)}) - F_{X_i|X_{1:i-1}}(\tau_i^{(r)})\right)\left(\sum_{j=1}^{r}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_r\right)\right) + b_0\mathbb{E}(X_i|X_{1:i-1}).
\end{aligned}$$

Splitting the second right-hand term into two parts yields

$$\begin{aligned}
&\mathbb{E}\left(h(X_i, \tau_i^{(1)}, \ldots, \tau_i^{(m)})|X_{1:i-1}\right)\\
&= -\sum_{j=1}^{m}(b_{j-1}-b_j)\mathbb{E}\left(X_i \mathbb{1}_{\{X_i>\tau_i^{(j)}\}}|X_{1:i-1}\right) + \sum_{j=1}^{m}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_m\\
&\quad + b_0\mathbb{E}(X_i|X_{1:i-1}) + \sum_{r=2}^{m} F_{X_i|X_{1:i-1}}(\tau_i^{(r)})\left(\sum_{j=1}^{r-1}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_{r-1}\right)\\
&\quad - \sum_{r=1}^{m} F_{X_i|X_{1:i-1}}(\tau_i^{(r)})\left(\sum_{j=1}^{r}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_r\right)
\end{aligned}$$

and collecting the two last right-hand terms we obtain

$$\begin{aligned}
&\mathbb{E}\left(h(X_i,\tau_i^{(1)},\ldots,\tau_i^{(m)})|X_{1:i-1}\right)\\
&=-\sum_{j=1}^{m}(b_{j-1}-b_j)\mathbb{E}\left(X_i\mathbb{1}_{\{X_i>\tau_i^{(j)}\}}|X_{1:i-1}\right)+b_0\mathbb{E}(X_i|X_{1:i-1})+\left(\sum_{j=1}^{m}\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j\right)+d_m\right)\\
&\quad-F_{X_i|X_{1:i-1}}(\tau_i^{(1)})\left(\tau_i^{(1)}(b_0-b_1)+d_1\right)-\sum_{j=2}^{m}F_{X_i|X_{1:i-1}}(\tau_i^{(j)})\left(\tau_i^{(j)}(b_{j-1}-b_j)+c_j+d_j-d_{j-1}\right).
\end{aligned}$$

Finally, letting $\Delta t_j=(c_j+d_j-d_{j-1})/(b_{j-1}-b_j)$ with the convention $d_0=0$, we have

$$\begin{aligned}
&\mathbb{E}\left(h(X_i,\tau_i^{(1)},\ldots,\tau_i^{(m)})|X_{1:i-1}\right)\\
&=-\sum_{j=1}^{m}(b_{j-1}-b_j)\mathbb{E}\left(X_i\mathbb{1}_{\{X_i>\tau_i^{(j)}\}}|X_{1:i-1}\right)+\sum_{j=1}^{m}(\tau_i^{(j)}+\Delta t_j)(b_{j-1}-b_j)\\
&\quad-\sum_{j=1}^{m}(b_{j-1}-b_j)\,F_{X_i|X_{1:i-1}}(\tau_i^{(r)})(\tau_i^{(j)}-\Delta t_j)+b_0\mathbb{E}(X_i|X_{1:i-1})\\
&=\sum_{j=1}^{m}(b_{j-1}-b_j)\left[\left(1-F_{X_i|X_{1:i-1}}(\tau_i^{(j)})\right)\left(\Delta t_j+\tau_i^{(j)}\right)-\int_{\tau_i^{(j)}}^{+\infty}xf_{X_i|X_{1:i-1}}(x)dx\right]\\
&\quad+b_0\mathbb{E}(X_i|X_{1:i-1}),
\end{aligned}\tag{21}$$

and the conclusion follows. ■

# B Appendix: M step of the EM algorithm for Weibull HMCs.

This paragraph describes the M step of EM algorithm, dedicated to parameter re-estimation in HMCs, in the case of Weibull emission distributions.
It can be seen from equations (13) and (12), that the re-estimation procedure for the $\pi_k$ and $A_{k,l}$ parameters is not specific to Weibull distributions. Consequently the usual formulae (6.14) and (6.15) in [8] hold. For all $k=1,..,K$ the new values of parameters $(\lambda_k^{(m+1)},\alpha_k^{(m+1)})$ after $m$ iterations of the EM algorithm cancel the partial derivatives of the $\mathbb{Q}$ function, and thus satisfy the system:

$$\left\{\begin{array}{l}\sum_t \mathbb{P}_{\eta^{(m)}}(S_t=k|X_1^n=x_1^n)\dfrac{\partial}{\partial\lambda_k}\log f_{\lambda_k,\alpha_k}(x)=0\\ \sum_t \mathbb{P}_{\eta^{(m)}}(S_t=k|X_1^n=x_1^n)\dfrac{\partial}{\partial\alpha_k}\log f_{\lambda_k,\alpha_k}(x)=0.\end{array}\right.\tag{22}$$

Let $\xi_k^{(t)}=\mathbb{P}_{\eta^{(m)}}(S_t=k|X_1^n=x_1^n)$. Since for Weibull emission distributions,

$$\log f_{\lambda_k,\alpha_k}(x)=\log(\alpha_k)+\alpha_k\log(\lambda_k)+(\alpha_k-1)\log(x)-(\lambda_k x)^{\alpha_k},$$

we have

$$\frac{\partial\log f_{\lambda_k,\alpha_k}(x)}{\partial\lambda_k}=\frac{\alpha_k}{\lambda_k}-\alpha_k x^{\alpha_k}\lambda_k^{\alpha_k-1}$$

and

$$\frac{\partial\log f_{\lambda_k,\alpha_k}(x)}{\partial\alpha_k}=\frac{1}{\alpha_k}+\log(\lambda_k)+\log x-(\log(\lambda_k x))(\lambda_k x)^{\alpha_k}.$$

The first equation of the system (22) can be rewritten as

$$\sum_t \xi_k^{(t)} \frac{\partial}{\partial \lambda_k} \log f_{\lambda_k,\alpha_k}(x_t) = \sum_t \xi_k^{(t)} \left[\frac{\alpha_k}{\lambda_k} - \alpha_k x_t^{\alpha_k} \lambda_k^{\alpha_k - 1}\right] = 0$$

$$\Leftrightarrow \frac{\alpha_k}{\lambda_k} \sum_t \xi_k^{(t)} - \alpha_k \lambda_k^{\alpha_k-1} \sum_t \xi_k^{(t)} x_t^{\alpha_k} = 0 \Leftrightarrow \sum_t \xi_k^{(t)} = \lambda_k^{\alpha_k} \sum_t \xi_k^{(t)} x_t^{\alpha_k} \tag{23}$$

$$\Leftrightarrow \lambda_k = \left[\frac{\sum_t \xi_k^{(t)} x_t^{\alpha_k}}{\sum_t \xi_k^{(t)}}\right]^{-\frac{1}{\alpha_k}}. \tag{24}$$

Replacing the expression of $\lambda_k$ obtained in equation (24) into the second equation of the system (22) yields

$$\begin{aligned}
0 &= \sum_t \xi_k^{(t)} \frac{\partial}{\partial \alpha_k} \log f_{\lambda_k,\alpha_k}(x_t) = \sum_t \xi_k^{(t)} \left[\frac{1}{\alpha_k} + \log \lambda_k + \log x_t - (\log(\lambda_k x_t))(\lambda_k x_t)^{\alpha_k}\right] \\
&= \sum_t \xi_k^{(t)} \left(\frac{1}{\alpha_k} + \log x_t + \log \lambda_k\right) - \lambda_k^{\alpha_k} \sum_t \xi_k^{(t)} x_t^{\alpha_k} (\log \lambda_k + \log x_t) \\
&= \sum_t \xi_k^{(t)} \left(\frac{1}{\alpha_k} + \log x_t\right) - \lambda_k^{\alpha_k} \sum_t \xi_k^{(t)} x_t^{\alpha_k} \log x_t \\
&\quad + \log \lambda_k \left[\sum_t \xi_k^{(t)} - \lambda_k^{\alpha_k} \sum_t \xi_k^{(t)} {x_t}^{\alpha_k}\right].
\end{aligned} \tag{25}$$

Using equations (23) and (24) in (25) yields

$$0 = \sum_t \xi_k^{(t)} \log x_t - \left[\frac{\sum\limits_t \xi_k^{(t)}}{\sum\limits_t \xi_k^{(t)} x_t^{\alpha_k}}\right] \sum_t \xi_k^{(t)} x_t^{\alpha_k} \log x_t + \frac{1}{\alpha_k} \sum_t \xi_k^{(t)}$$

$$\Leftrightarrow \quad 0 = \alpha_k \left[\frac{\sum\limits_t \xi_k^{(t)} \log x_t}{\sum\limits_t \xi_k^{(t)}} - \frac{\sum\limits_t \xi_k^{(t)} x_t^{\alpha_k} \log x_t}{\sum\limits_t \xi_k^{(t)} x_t^{\alpha_k}}\right] + 1. \tag{26}$$

Equation (26) has no known solution; hence it has to be solved numerically, by the algorithm described in [9] in the circumstances.

# C Appendix: Markov decision processes

In this Section, a connection between our approach and the theory of Markov decision processes (MDPs) is established. More specifically, the problem of determining the optimal timeout period by minimizing the expected consumption up to following request, defined by equation (4), is shown to be a particular case of an MDP with a continuous action space, if the times between printings are independent random variables. The value function with (finite) horizon 1 of the corresponding MDP is shown to be the opposite of the expected future cost. Moreover, this MDP has a single possible state, which explains why an explicit solution of this problem could be derived in Section 2.1.

## C.1 General principle

Markov decision processes are a class of optimization problems for controlling the temporal evolution of an agent in a given environment characterized by a set of states $\mathcal{S}$. At each time step $t$, given the state $S_t$ of the environment, the agent is allowed to perform an action $A_t$ chosen from a set $\mathcal{A}$. The chosen action may modify the next state $S_{t+1}$ of the environment, and brings a scalar reward $R_{t+1}$ to the agent. All the quantities $A_t, S_t$ and $R_t$ constitute a homogeneous random process. The problem is to determine the distribution for $A_t$ that maximizes the expected future rewards given the current state $S_t$.

The process $(A_t, S_t, R_t)_{t\in\mathbb{N}}$ is supposed to obey the *Markov property*. Moreover, under the three following assumptions:

1. the action $A_{t+1}$ is independent on the past of the three processes up to time $t$, and on $R_{t+1}$, given state $S_{t+1}$;
2. the reward $R_{t+1}$ is independent on the past of the three processes up to time $t$ given the states $S_t$ and $S_{t+1}$, and given $A_t$;
3. $S_{t+1}$ is independent on the past of the three processes up to time $t$ given $S_t$ and $A_t$;

an MDP is totally specified by the following distributions:

- the transition probabilities $\mathcal{P}^a_{ss'} = \mathbb{P}(S_{t+1} = s'|S_t = s, A_t = a)$, which define how next state is affected by current state $s$ and the chosen action $a$;
- the policy function $\pi(s, a) = \mathbb{P}(A_t = a|S_t = s)$, which defines what action to choose given current state $s$;
- the reward distribution, *i.e.* the distribution of $R_{t+1}$ given $S_t = s$, $A_t = a$ and $S_{t+1} = s'$.

The optimization problem associated with this MDP consists in finding the policy $\pi : \mathcal{S} \times \mathcal{A} \to [0, 1]$ that maximizes the expected future rewards (under the constraints $\sum_a \pi(s, a) = 1$ and $\forall(a, s), \pi(a, s) \geq 0$). The future rewards are modelled through the random variable $\mathfrak{R}_t = \sum_{k=0}^{\infty} \gamma_k R_{t+k+1}$, where $\forall k$, $\gamma_k$ represents the weight of the reward after $k + 1$ time steps. The sequence $(\gamma_k)_{k\in\mathbb{N}}$ is referred to the *discount* sequence. The function to be maximized is called the *value function* and is denoted by $V^\pi$; it corresponds to the expectation of $\mathfrak{R}_t$ given the value $s$ of current state. This leads to the following formal definitions:

$$V^\pi(s) = \mathbb{E}(\mathfrak{R}_t|S_t = s) = \sum_{k=0}^{\infty} \gamma_k \mathbb{E}(R_{t+k+1}|S_t = s) \tag{27}$$

$$\text{and } \hat{\pi}(s, .) = \arg\max_{\pi(s,.)} V^\pi(s). \tag{28}$$

The reward distribution $\mathbb{P}(R_{t+1}|S_t = s, A_t = a, S_{t+1} = s')$ is only involved through its expectation $\mathcal{R}^a_{ss'} = \mathbb{E}(R_{t+1}|S_t = s, A_t = a, S_{t+1} = s')$; consequently, only $\mathcal{R}^a_{ss'}$ needs to be defined explicitly.

There are two particular cases of interest for the sequence $(\gamma_k)_{k\in\mathbb{N}}$:

- $\forall k$, $\gamma_k = \gamma^k$, where $0 \leq \gamma < 1$. In this case, $V^\pi(s)$ satisfies a fixed point equation known as the *Bellman equation*. Generally, no closed form is available for the optimal policy.

- $\forall k$, $\gamma_k = \delta_0(k) = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$, where $\delta$ denotes the Kronecker symbol. Then, it is easily shown that:
$$V^\pi(s) = \sum_a \pi(s,a) \sum_{s'} \mathcal{P}^a_{ss'} \mathcal{R}^a_{ss'}. \tag{29}$$

**Lemma 4** *In the case where the sequence $(\gamma_k)_{k\in\mathbb{N}}$ is defined by $\gamma_k = \delta_0(k)$, the optimal policy is:*
$$\hat{\pi}(s,a') = \begin{cases} 1 & \textit{if } a' = \arg\max_a \sum_{s'} \mathcal{R}^a_{ss'} \mathcal{P}^a_{ss'} \\ 0 & \textit{otherwise.} \end{cases}$$
*In the case where the state space is reduced to a singleton $\mathcal{S} = \{1\}$ and where $\forall k$, $\gamma_k = \gamma^k$, the optimal policy is:*
$$\hat{\pi}(1,a') = \begin{cases} 1 & \textit{if } a' = \arg\max_a \mathcal{R}^a_{1,1} \\ 0 & \textit{otherwise.} \end{cases}$$

**Proof of Lemma 4.** In the case where $\gamma_k = \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{else} \end{cases}$, then

$$\begin{aligned} V^\pi(s) &= \mathbb{E}(R_{t+1}|S_t = s) \\ &= \sum_a \pi(s,a) \sum_{s'} \mathbb{E}(R_{t+1}|S_t = s, S_{t+1} = s', A_t = a)\mathbb{P}(S_{t+1} = s'|S_t = s, A_t = a) \\ &= \sum_a \pi(s,a) \sum_{s'} \mathcal{R}^a_{ss'} \mathcal{P}^a_{ss'}. \end{aligned} \tag{30}$$

The optimal policy is the policy $\pi$ maximizing the value function $V^\pi$:
$$\hat{\pi} = \arg\max_\pi \left( \sum_a \pi(s,a) \sum_{s'} \mathcal{R}^a_{ss'} \mathcal{P}^a_{ss'} \right)$$
with $\sum_a \pi(s,a) = 1$ and $\pi(s,a) \geq 0 \ \forall (s,a)$.

In the case where the state space is reduced to a singleton $\mathcal{S} = \{1\}$, and if $\gamma_k = \gamma^k \quad \forall k$, the optimal policy is, from Bellman equation (see [20]):
$$\begin{aligned} V^\pi(1) &= \sum_a \pi(s,a) \sum_{s'} \mathcal{P}^a_{ss'} \left[\mathcal{R}^a_{ss'} + \gamma V^\pi(s')\right] \\ &= \sum_a \pi(1,a)\mathcal{R}^a_{1,1} + \gamma V^\pi(1) \sum_a \pi(1,a) \end{aligned}$$
since $\mathcal{S} = \{1\}$ and $\forall a$, $\mathcal{P}^a_{ss'} = 1$. Remarking that $\sum_a \pi(1,a) = 1$, we have
$$\begin{aligned} (1-\gamma)V^\pi(1) &= \sum_a \pi(1,a)\mathcal{R}^a_{1,1} \\ \Leftrightarrow \quad V^\pi(1) &= \frac{1}{(1-\gamma)} \sum_a \pi(1,a)\mathcal{R}^a_{1,1}. \end{aligned} \tag{31}$$

In both equations (30) and (31), for each $s$, $V^\pi(s)$ is a linear function with respect to $\pi(s,a)$. Thus, a classical result of the optimization theory states that the maximum of $V^\pi(s)$ is achieved on an endpoint of an edge of the simplex $\left\{\pi(s,a)|a \in \mathbb{R}, \pi(s,a) \geq 0 \text{ and } \sum_a \pi(s,a) = 1\right\}$.
As a consequence, in the case where the sequence $(\gamma_k)_{k\in\mathbb{N}}$ is defined by $\gamma_k = \delta_0(k)$, the optimal policy is:

$$\hat{\pi}(s,a') = \begin{cases} 1 & \text{if } a' = \arg\max_a \sum_{s'} \mathcal{R}^a_{ss'}\mathcal{P}^a_{ss'} \\ 0 & \text{otherwise.} \end{cases}$$

In the case where the state space is reduced to a singleton $\mathcal{S} = \{1\}$ and where $\forall k$, $\gamma_k = \gamma^k$, the optimal policy is:

$$\hat{\pi}(1,a') = \begin{cases} 1 & \text{if } a' = \arg\max_a \mathcal{R}^a_{1,1} \\ 0 & \text{otherwise.} \end{cases}$$

■

Thus, the optimal policy corresponds to a deterministic strategy. Given current state $s$, the chosen action systematically is the one that maximizes $\sum_{s'} \mathcal{R}^a_{ss'}\mathcal{P}^a_{ss'}$ or $\mathcal{R}^a_{1,1}$ (depending on the sequence $(\gamma_k)_{k\in\mathbb{N}}$), with respect to $a$.

## C.2 Connection of our approach with MDPs

In this paragraph, our approach is shown to be a particular case of MDP. The proof is derived in the case of one single sleep mode printer for the sake of simplicity, but can easily be extended to an arbitrary number of sleep modes.

In our context, the set of actions for the printer is the timeout period $a \in \mathcal{A} = \mathbb{R}^+$, which corresponds to $\tau^{(1)}$ in previous paragraphs. The decision is taken after each print job, after which the printer is necessarily in *idle* mode. Consequently, the state space is $\mathcal{S} = \{idle\}$. In our problem, the reward is minus the cost between two successive printings. It only depends on the time between printings $X_i$ and on the action $a$, *i.e.* the timeout period.

The time index $i \in \mathbb{N}$ represents the number of past printings requests. Hence, even if the times of requests $T_i$ take continuous values, the MDP is essentially a discrete-time problem, where decisions are taken after each print job only.

Let the expected reward be defined as $\mathcal{R}^a = -\mathbb{E}(h(X_i,a))$, where $h$ is the cost between two successive print jobs defined by equation (1) (or (5) in the case of multiple sleep modes). The transition probabilities are $\mathcal{P}^a_{ss'} = 1$, $\forall\ a$, $s$ and $s'$, since the printer is always in *idle* state when a decision is taken.
As a consequence, from equation (29), the value function $V^\pi(s)$ for $\gamma_k = \delta_0(k)$ is:

$$V^\pi(s) = -\sum_a \pi(s,a)\mathbb{E}(h(X_i,a)),$$

and according to Lemma 4, the optimal policy is:

$$\pi(s,a') = \begin{cases} 1 & \text{if } a' = \arg\max_a -\mathbb{E}(h(X_i,a)) = \arg\min_a \mathbb{E}(h(X_i,a)) \\ 0 & \text{otherwise,} \end{cases}$$

which corresponds to the optimization problem in equation (4) in the case where the times between printings $X_i$ are independent random variables.

To conclude, our approach is a degenerate case of an MDP problem with a continuous action space and with one single state. Using the particular discount sequence $\gamma_k = \delta_0(k)$, the expected future cost coincides with the value function of the MDP with (finite) horizon 1. Since the state space is reduced to a single state, an explicit solution of this problem can be derived. This solution is given in Proposition 1.

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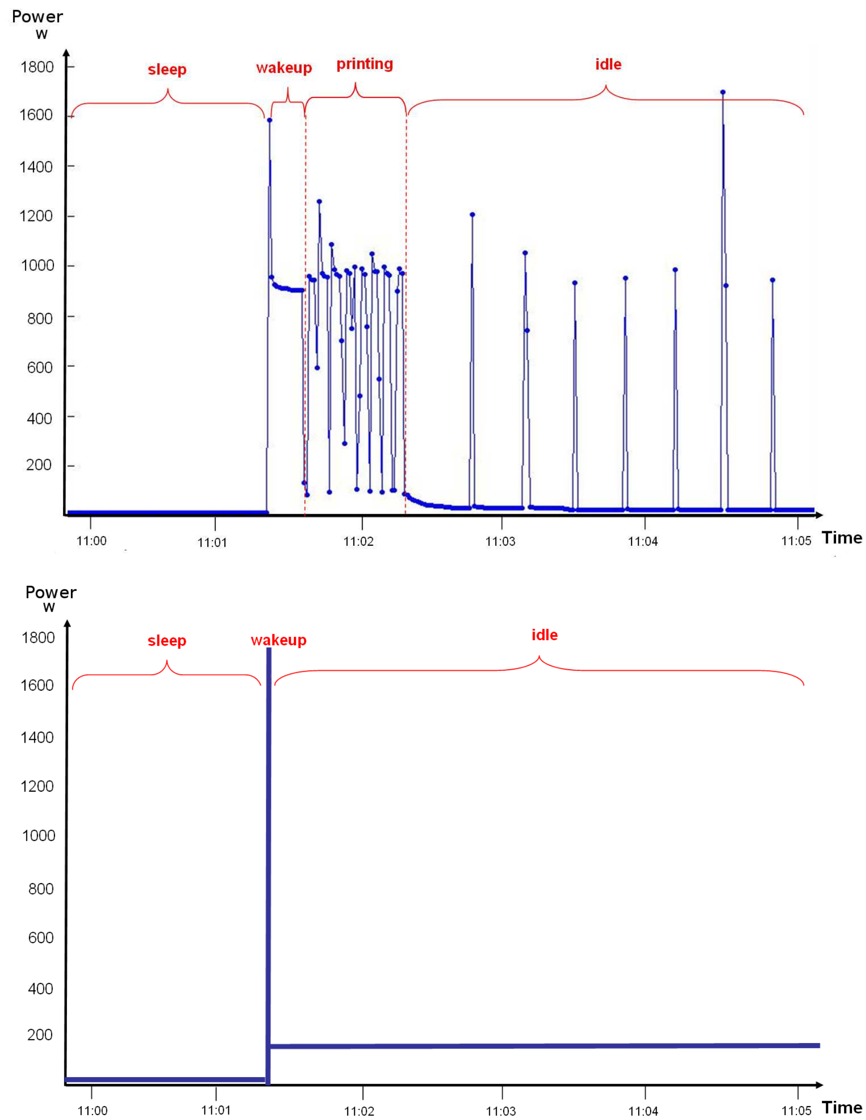



Figure 2: Top: Real power consumption of a printer (between 11 am and 11:05 am). During this period, the device switches between 4 different states: sleep, wakeup, print and idle modes. Bottom: simplified consumption model.

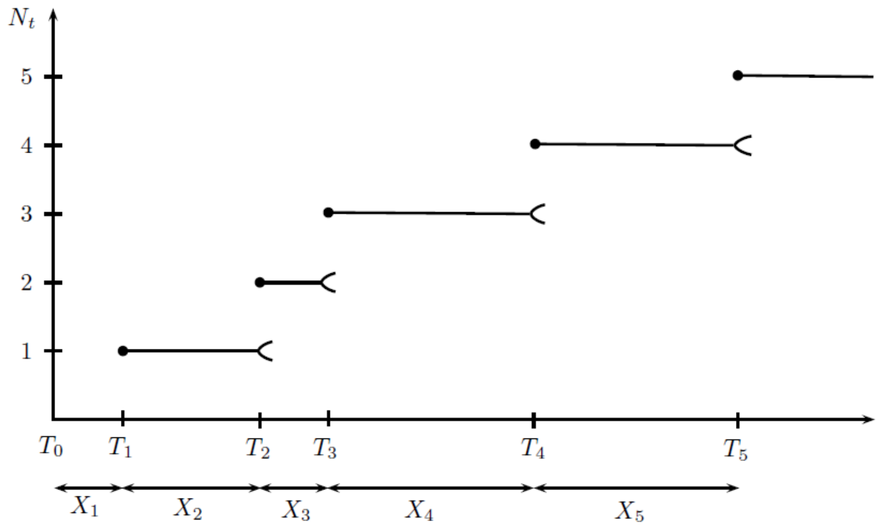
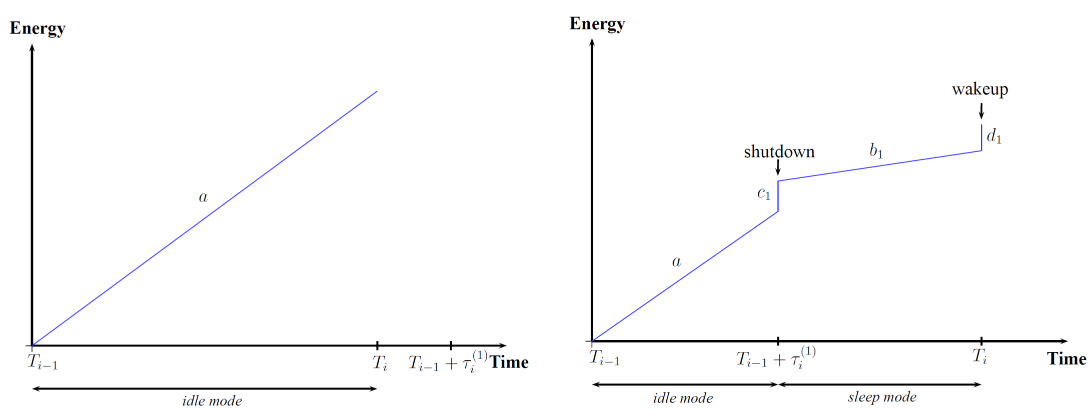



Figure 3: Print Process




(a) When $X_i \leq \tau_i^{(1)}$ (b) When $X_i > \tau_i^{(1)}$

Figure 4: Energy consumption between $T_{i-1}$ and $T_i$ according to the position of $T_{i-1}$, $T_i$ and $T_{i-1} + \tau_i^{(1)}$

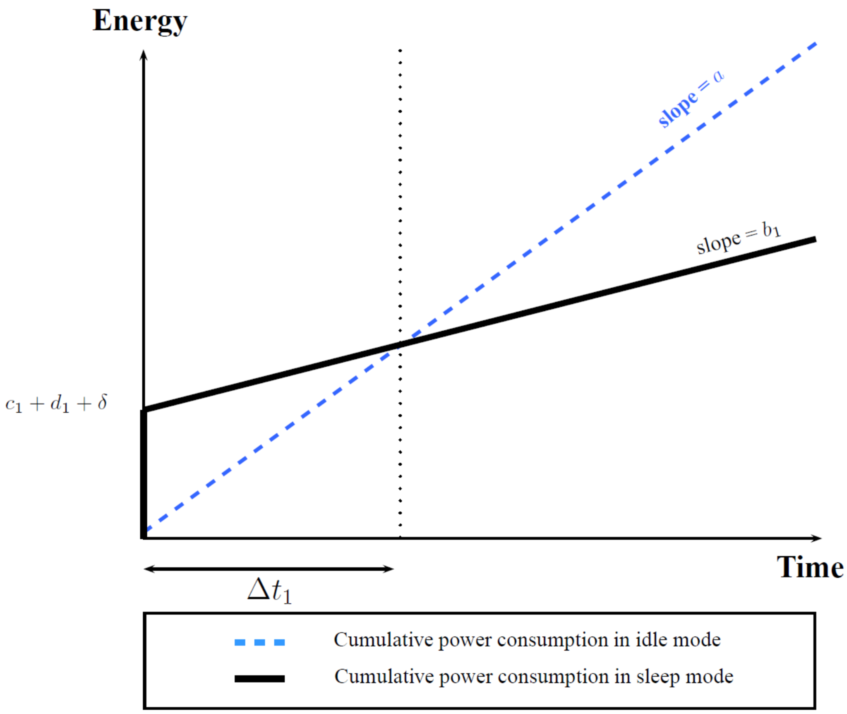



Figure 5: Graphical interpretation of $\Delta t_1$

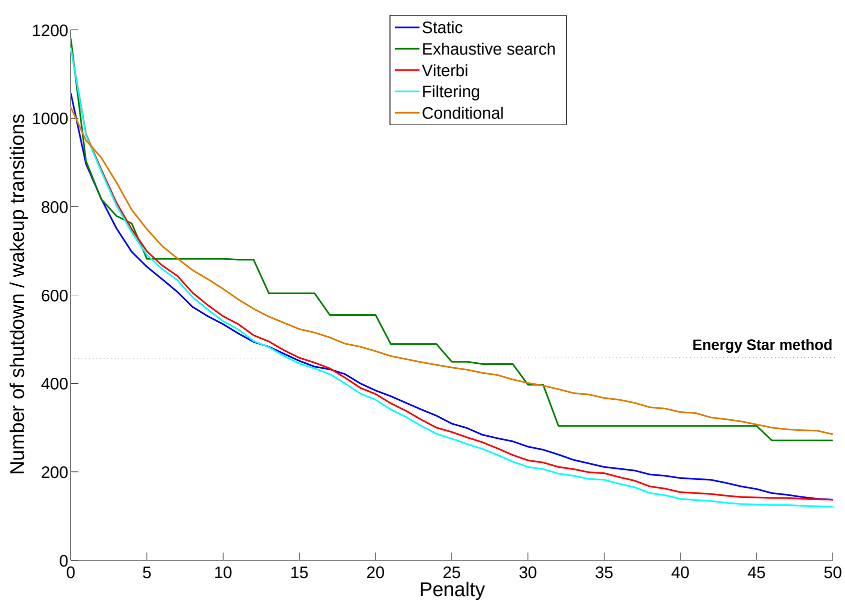



Figure 6: Number of shutdown (or equivalently wakeup) transitions as the penalty $\delta$ increases.

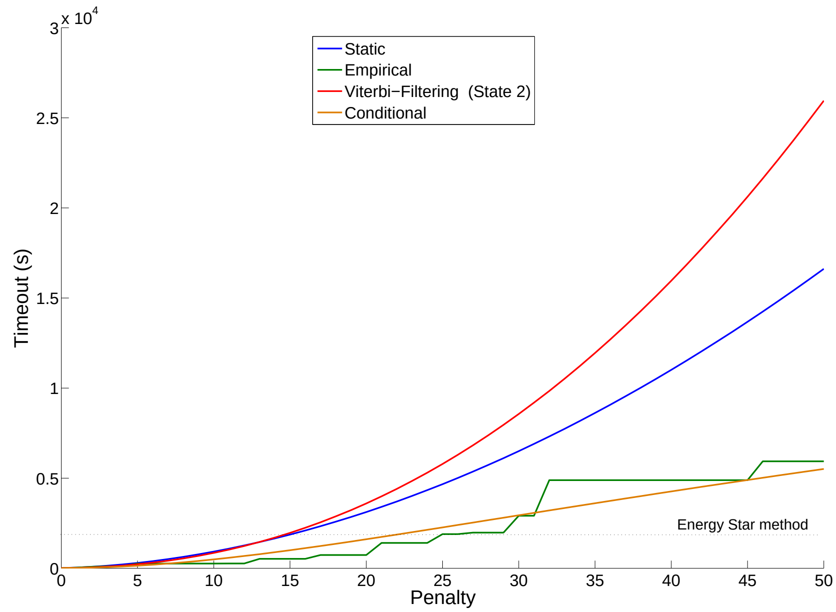



Figure 7: Timeout as the penalty $\delta$ increases.

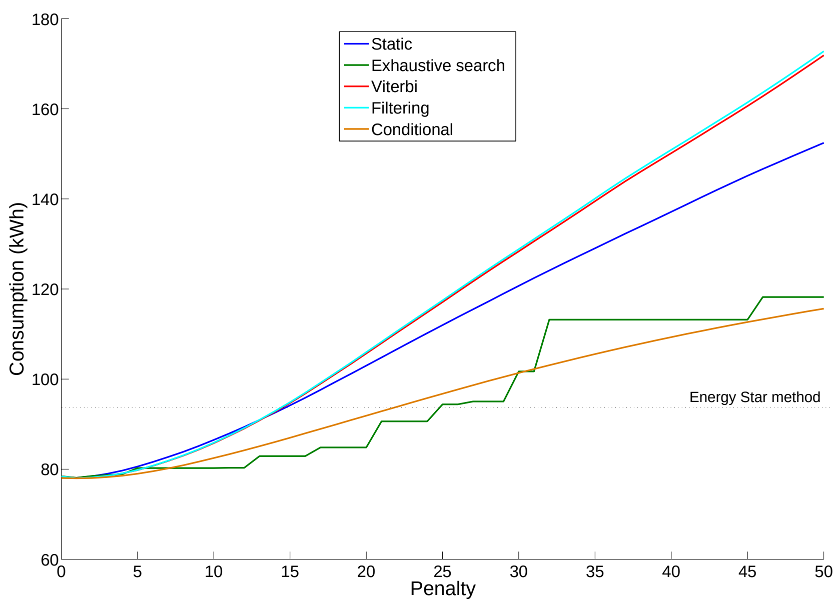



Figure 8: Consumption as the penalty $\delta$ increases.

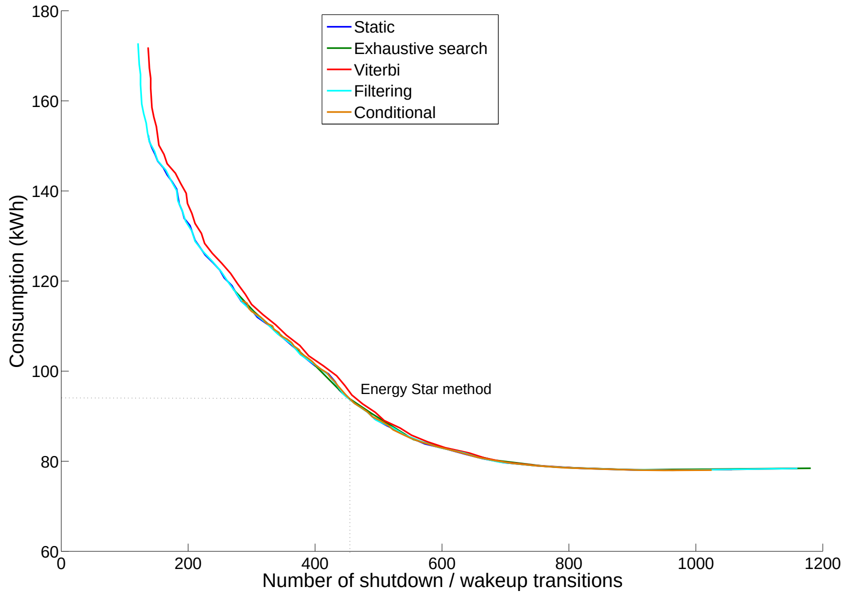



Figure 9: Consumption as a function of the number of shutdown transitions.

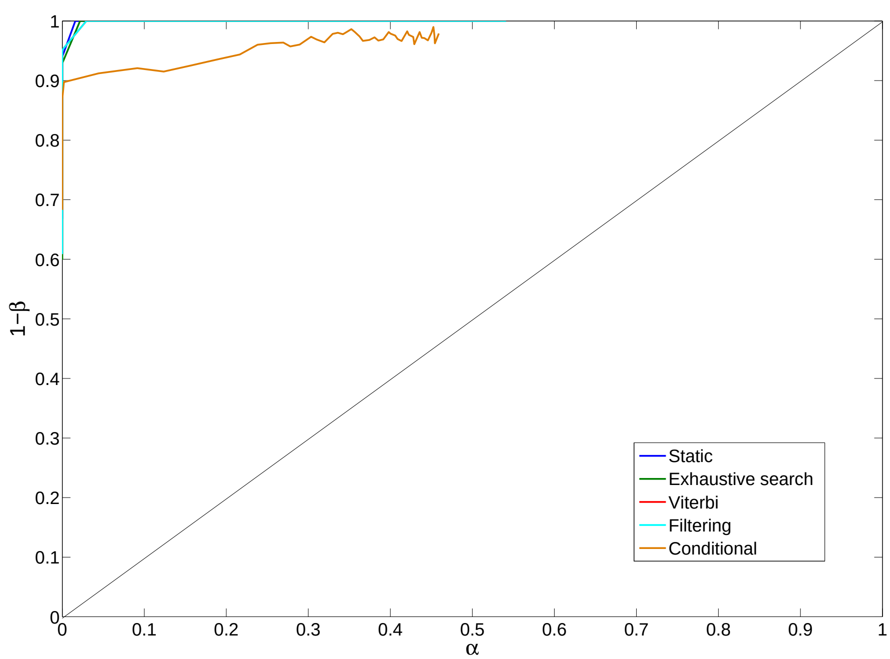



Figure 10: ROC curves obtained by varying the penalty $\delta$.

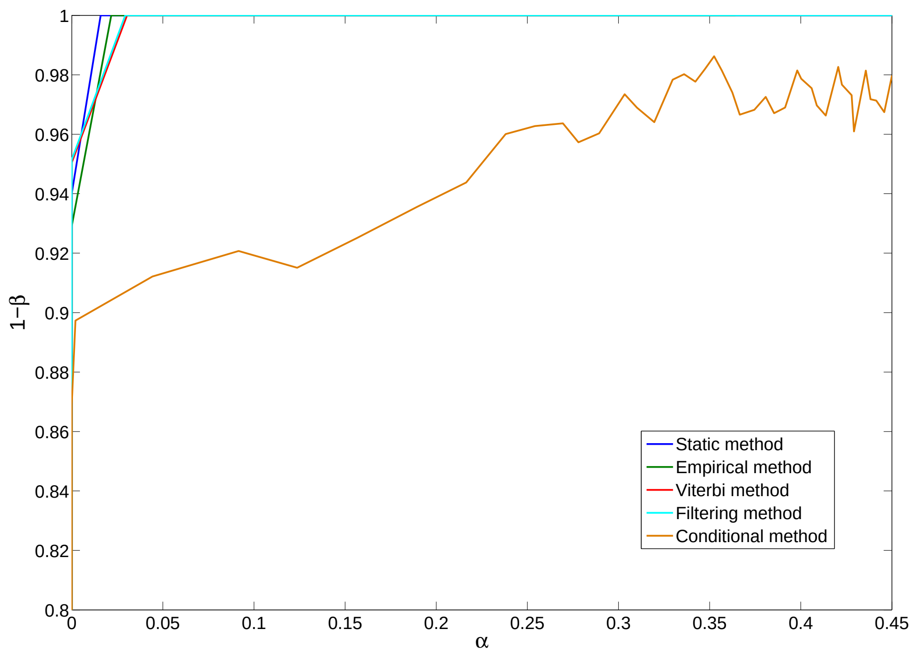



Figure 11: Close-up of upper left part of Figure 10
(ROC curves obtained by varying the penalty $\delta$).