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The CONTSID toolbox for Matlab: extensions and latest developments

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Abstract: This paper describes the latest developments for the CONtinuous-Time System IDentification (CONTSID) toolbox to be run with MATLAB which includes time-domain identification methods for estimating continuous-time models directly from sampled data. The main additions to the new version aim at extending the available methods to handle wider practical situations in order to enhance the application field of the CONTSID toolbox. The toolbox now includes routines to solve errors-in-variables and closed-loop identification problems, as well as non-linear continuous-time model identification techniques.

Keywords: continuous-time model, discrete-time data, Matlab toolbox, software tools, system identification

1. INTRODUCTION

This paper describes the latest developments for the continuous-time system identification (CONTSID) toolbox for MATLAB®, which supports continuous-time (CT) transfer function and state-space model identification directly from regularly or irregularly time-domain sampled data, without requiring the determination of a discrete-time (DT) model. The motivation for developing the CONTSID toolbox was first to fill in a gap, since no software support was available to serve the cause of direct time-domain identification of continuous-time linear models but also to provide the potential user with a platform for testing and evaluating these data-based modelling techniques. The CONTSID toolbox was first released in 1999 (Garnier and Mensler [1999]). It has gone through several updates, some of which have been reported at recent symposia (Garnier and Mensler [2000], Garnier et al. [2003a, 2006]). The key features of the CONTSID toolbox are:

- it supports most of the time-domain methods developed over the last thirty years (Garnier et al. [2003b]) for identifying linear dynamic continuous-time parametric models from measured input/output sampled data;
- it provides transfer function and state-space model identification methods for single-input single-output (SISO) and multiple-input multiple-output (MIMO) systems, including both traditional and more recent approaches;
- it can handle mild irregularly sampled data in a straightforward way;
- it may be seen as an add-on to the system identification (SID) toolbox for MATLAB®. To facilitate its use, it has been given a similar setup to the SID toolbox;
- it provides a flexible graphical user interface (GUI) that lets the user analyse the experimental data, identify and evaluate models in an easy way.

The latest version of the CONTSID toolbox has the following three major additions:

- it supports errors-in-variables CT transfer function model identification;
- it provides routines to estimate linear CT transfer function model in closed loop;
- it includes methods to identify nonlinear CT Hammerstein models.

The paper is organised in the following way. An overview of the standard linear CT models and methods available in the toolbox is first presented in Section 2. A brief description of the toolbox is given in Section 3. Recent developments to solve continuous errors-in-variables, closed-loop and nonlinear model identification problems are finally described in Section 4.

2. STANDARD CONTINUOUS-TIME MODELS AND METHODS

The CONTSID toolbox includes routines to identify CT linear transfer function and state-space models directly from regularly or irregularly time-domain sampled data (Garnier et al. [2008]).

2.1 Linear time-invariant transfer function models

The toolbox supports transfer function models of the following forms

\[ y(t_k) = \sum_{i=1}^{m} G^i(p)u^i(t_k) + H(q)e(t_k) \]  

(1)

where \( p \) denotes the differential operator and \( q \) is the standard forward shift operator; \( u^i(t_k) \) and \( y(t_k) \) represent the deterministic inputs and noisy output at time instant \( t_k \), respectively. \( e(t_k) \) is a zero-mean DT white Gaussian sequence. Here, the model of the basic dynamic system is in continuous time, while the associated additive noise
Continuous-time state-space models considered in the CONTSID toolbox take the form
\[\begin{align*}
\dot{x}(t_k) &= Ax(t_k) + Bu(t_k) \\
y(t_k) &= Cx(t_k) + Du(t_k) + \xi(t_k)
\end{align*}\]
where \(u(t_k) \in \mathbb{R}^{n_u}\) is the input vector and \(y(t_k) \in \mathbb{R}^{n_y}\) the output vector and \(x(t_k) \in \mathbb{R}^n\) is the state vector at time \(t_k\), \(\xi(t_k) \in \mathbb{R}^{n_x}\) is the possibly coloured output noise vector.

Two types of approaches for CT state-space model identification are available in the CONTSID toolbox. A first family of techniques relies on the a priori knowledge of structural indices, and considers the estimation of CT canonical state-space models. From the knowledge of the observability indices, the canonical state-space model can, in a straightforward way, be first transformed into an equivalent input–output polynomial description that is linear-in-its-parameters and therefore more suitable for the parameter estimation problem. A preprocessing method may then be used to convert the differential equation into a set of linear algebraic equations in a similar way to that for CT ARX type of models. The unknown model parameters can finally be estimated by LS or IV-based algorithms. This scheme has been implemented for the GPMF approach only.

A second class of state-space model identification schemes is based on the subspace-estimation techniques. Most efficient data-based modelling methods, discussed so far, rely on iterative, non-linear optimisation or IV-type methods to fit parameters in a preselected model structure, so as to best fit the observed data. Subspace methods are an alternative class of identification methods that are ‘one-shot’ rather than iterative, and rely on linear algebra.

Moreover, these subspace methods are attractive since canonical forms are not required, while fully parameterised state-space models are estimated directly from sampled I/O data. Most commonly known subspace methods were developed for DT model identification. The association of the more efficient preprocessing methods with subspace methods of the 4SID family has been implemented in the toolbox.

Table 2 summarises the methods available in the CONTSID toolbox for CT state-space model identification.

2.3 Linear time varying transfer function models

In many situations, there is a need to estimate the model at the same time as the data is collected during the measurement. The model is then ‘updated’ at each time instant some new data become available. The updating is performed by a recursive algorithm. Recursive versions RLSSVF, RIVSVF and RSRIVC of the LS, IV-based SVF methods and optimal IV technique for CT hybrid OE models are available in the CONTSID toolbox.

2.4 Identification from mild irregularly sampled data

The problem of system identification from non-uniformly sampled data is of importance as this case occurs in
several applications. The case of irregularly sampled data is not easily handled by discrete-time model identification techniques while mild irregularity can be easily handled by some of the CONTSID toolbox methods. This is because the differential-equation model is valid whatever the time instants considered and, in particular, it does not assume regularly sampled data, as required in the case of the standard difference-equation model.

Table 3 lists the functions available for CT model identification from irregularly sampled data.

3. THE CONTSID TOOLBOX: A BRIEF DESCRIPTION

The CONTSID toolbox is compatible with MATLAB® versions 6.x and 7.x. Two external commercial toolboxes are required: the Control toolbox and the SID toolbox. The current version can be considered as an add-on to the SID toolbox and makes use of the iddata, idpoly and idss objects used in the SID toolbox. It can be downloaded from http://www.cran.uhp-nancy.fr/contsid/.

All available parametric model estimation functions share the same command structure

\[ m = \text{function(data,modstruc)} \]

The input argument data is an iddata object that contains the output- and input-data sequences along with the sampling time and inter-sample behaviour for the input, while modstruc specifies the particular structure of the model to be estimated. The specific parameters depend on the preprocessing method used. The resulting estimated model is contained in m, which is a model object that stores the various usual information. The function name is defined by the abbreviation for the estimation method and the abbreviation for the associated preprocessing technique, as for example, IVSVF for the instrumental variable-based state-variable filter approach or SIDGPMF for subspace-based state-space model identification GPMF approach.

Note that help on any CONTSID toolbox function may be obtained from the command window by invoking classically help name_function.

In addition to the parameter estimation routines, the toolbox also includes tools for generating excitation signals, selecting the model orders as well as for evaluating the estimated CT model properties.

The main demonstration program called idcdemo provides several examples illustrating the use and the relevance of the CONTSID toolbox approaches. These demos also illustrate what might be typical sessions with the CONTSID toolbox.

4. LATEST DEVELOPMENTS FOR THE CONTSID TOOLBOX

Recent developments aimed at extending the available methods to handle wider practical situations in order to enhance the application field of the CONTSID toolbox. The latest version of the toolbox includes routines to solve errors-in-variables (Mahata and Garnier [2006], Thil et al. [2008]) and closed-loop identification (Gilson et al. [2008], Young et al. [2009]) problems, as well as non-linear continuous-time model identification techniques (Laurain et al. [2008]).

4.1 Linear transfer function model identification in closed loop

The toolbox now supports routines to estimate CT linear time-invariant transfer function in a closed loop framework of the form shown in Figure 1. The data generating system is assumed to be given by the relations

\[
\begin{aligned}
\{ y(t) = G(p)u(t) + H(p)e(t) \\
\{ u(t) = r(t) - C_c(p)y(t) 
\end{aligned}
\]  

(8)

The process is denoted by \( G(p) = B(p)/A(p) \) and the controller by \( C_c(p) \). \( u(t) \) describes the process input signal, \( y(t) \) the process output signal. For ease of notation we introduce an external signal \( r(t) = r_1(t) + C_c(p)r_2(t) \). Moreover, it is also assumed that the CT signals \( u(t) \) and \( y(t) \) are uniformly sampled at \( T_s \). A coloured disturbance is assumed to affect the closed-loop: bearing in mind the spectral factorisation theorem, this noise term, \( \xi(t) = H(p)e(t) \), is modelled as linearly filtered white noise. The external signal \( r(t) \) is assumed to be uncorrelated with the noise disturbance \( \xi(t) \).

The CT model identification problem is to find estimates of \( G(p) \) from finite sequences \( \{ r(t_k) \}_{k=1}^N \), \( \{ u(t_k) \}_{k=1}^N \), \( \{ y(t_k) \}_{k=1}^N \), of, respectively, the external signal, the process input and output DT data.

![Fig. 1. Closed-loop configuration](image-url)
Two methods based on optimal IV may handle closed-loop system identification in the CONTSID toolbox. The first one, named as CLRIVC aims at estimating the following hybrid Box-Jenkins model structure:

\[
\begin{align*}
    y(t_k) &= \frac{B(p)}{F(p)} u(t_k) + C(q^{-1}) \epsilon(t_k) \\
    \text{with } u(t_k) &= r(t_k) - C_c(p) y(t_k)
\end{align*}
\]  

(9)

It involves an iterative (or relaxation) algorithm in which, at each iteration, the auxiliary model used to generate the instrumental variables, as well as the associated prefilters, are updated, based on the parameter estimates obtained at the previous iteration (see Gilson et al. [2008] for a full description of this method).

The above formulation of the CLRIVC estimation problem is considerably simplified if it is assumed in the CT BJ model structure that the additive noise is white, i.e. \( H(p) = 1 \). In this case, the assumed model structure is a CT hybrid OE model given as:

\[
\begin{align*}
    y(t_k) &= \frac{B(p)}{F(p)} u(t_k) + \epsilon(t_k) \\
    \text{with } u(t_k) &= r(t_k) - C_c(p) y(t_k)
\end{align*}
\]  

(10)

Although appealing, the above optimal IV approaches are quite complex. Therefore, rather simpler but sub-optimal two-stage SRIVC-based approaches have been very recently developed Young et al. [2009]. These are in the same spirit as the two-stage algorithm suggested by Van den Hof and Schrama [1993] for discrete-time systems and they have the same advantage of not requiring prior knowledge of the control system. However, the two-stage algorithms are more sophisticated than the Van den Hof and Schrama approach because each stage exploits the appropriate open loop SRIVC/RIVC algorithms: namely, the SRIVC algorithm (rather than the FIR model estimation used by Van den Hof) for estimating the control input signal, followed by the estimation of the encosed, controlled system, based on this estimated control input, using either the SRIVC or the full RIVC algorithm. Conveniently, all of these algorithms are already available as computational routines in the CONTSID toolbox. Although the two-stage algorithms perform well and can even work well if the enclosed system is unstable, they are not statistically efficient. However, they point the way to the addition of a third stage that can induce optimality and so provide parameter estimates that have similar properties to those obtained from the RIVC algorithm applied in the equivalent open loop situation.

4.2 Linear errors-in-variables transfer function model identification

The toolbox now supports routines to estimate CT linear time-invariant transfer function in an errors-in-variables (EIV) framework as represented in Figure 2. The noise-free input and output signals are related by

\[
y_0(t) = G(p) u_0(t) = \frac{B(p)}{A(p)} u_0(t)
\]  

(11)

where \( G(p) \) is the CT transfer function of the system. \( u_0(t) \) and \( y_0(t) \) are sampled at time-instants \( \{t_k\}_{k=1}^N \), not necessarily uniformly spaced. The sampled signals are both contaminated by discrete-time noise sequences, denoted as \( \tilde{u}(t_k) \) and \( \tilde{y}(t_k) \) respectively. The measured input and output signals are therefore given by

\[
\begin{align*}
    u(t_k) &= u_0(t_k) + \tilde{u}(t_k) \\
    y(t_k) &= y_0(t_k) + \tilde{y}(t_k)
\end{align*}
\]  

(12)

with

\[
\begin{align*}
    A(p) &= a_0 + a_1 p + \ldots + a_{n_a} p^{n_a} + p^{n_a} \\
    B(p) &= b_0 + b_1 p + \ldots + b_{n_b} p^{n_b}
\end{align*}
\]  

(14)

Continuous-time model identification in an EIV framework is a relatively unexplored area. An attempt based on the use of second order statistics combined with state-variable filtering has been first implemented Mahata and Garnier [2006], when the noises contaminating the data are assumed to be white. The whiteness of the noises allows not only to simplify the algorithms, but to rule out identifiability problems as well. Indeed, without any further assumptions on the signal and noise models, it is well-known that the general EIV model is not uniquely identifiable from second order statistics. EIV systems suffer from this lack of identifiability, and it is thus of interest to study alternative approaches based on higher-order statistics. Two simple estimators have been recently developed. The cornerstones of the proposed solution are the use of third or fourth-order cumulants and state-variable filtering (Thil et al. [2007, 2008]. They can be applied in various noise situations, including the case of coloured and/or correlated noises on input and output of the system.

<table>
<thead>
<tr>
<th>Program</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS/SVF</td>
<td>LS-based state-variable filter method for CT ARX models</td>
</tr>
<tr>
<td>IV/SVF</td>
<td>IV-based state-variable filter method for CT ARX models</td>
</tr>
<tr>
<td>COE</td>
<td>non-linear optimisation method for CT hybrid OE models</td>
</tr>
<tr>
<td>SRIVC</td>
<td>optimal instrumental variable method for CT hybrid OE models</td>
</tr>
<tr>
<td>SIDGPMF</td>
<td>subspace-based generalised Poisson moment functionals method for CT state-space models</td>
</tr>
</tbody>
</table>

Table 3. Available functions for CT model identification from irregularly sampled data

Fig. 2. CT system in an errors-in-variables (EIV) framework
4.3 Nonlinear Hammerstein model identification

This section presents the extension of the CONTSID for the identification of Hammerstein CT Box–Jenkins models as presented in Figure 3. The non-linear function \( f(.) \) is assumed to be a sum of known basis functions \( \gamma_1, \gamma_2, \ldots, \gamma_n \) as given as:

\[
\bar{u}(t) = \sum_{i=1}^{l} \alpha_i \gamma_i(u(t)) \quad \text{with} \quad \alpha_1 = 1. \tag{15}
\]

\( u(t) \) and \( y(t) \) are assumed to be uniformly sampled at a constant sampling time \( T_s \).

The Hammerstein system is described by the following input-output relationship:

\[
\begin{aligned}
x(t) &= G(p)\bar{u}(t) \\
\xi(t_k) &= H(q)e(t_k) \\
y(t_k) &= x(t_k) + \xi(t_k),
\end{aligned}
\tag{16}
\]

where

\[
G(p) = \frac{B(p)}{F(p)}. \tag{17}
\]

\( B(p) \) and \( F(p) \) are polynomials in differential operator \( p \) with \( p^i x(t) = \frac{d^i x(t)}{dt^i} \). The developed method is based on the identification of a hybrid Box–Jenkins model, where the linear and the noise models are not constrained to have common polynomials. The coloured noise associated with the sampled output measurement \( y(t_k) \) has rational spectral density and can be represented by a discrete-time autoregressive moving average ARMA model:

\[
\xi(t_k) = H(q)e(t_k) = \frac{C(q^{-1})}{D(q^{-1})} e(t_k) \tag{18}
\]

where \( C(q^{-1}) \) and \( D(q^{-1}) \) are polynomials in the backward shift operator. \( e(t_k) \) is a zero-mean, normally distributed, discrete-time white noise sequence: \( e(t_k) \sim N(0, \sigma_e^2) \).

All details of the recently developed refined instrumental variable-based identification method can be found in Lau- rain et al. [2008].

The implemented IV-based routine is illustrated below by an example of how it works. This example considers a second-order SISO Hammerstein CT system without delay. The complete equation for the data-generating system has the following form:

\[
\begin{aligned}
\bar{u}(t_k) &= u(t_k) + 0.5u^2(t_k) + 0.25u^3(t_k) \\
y(t_k) &= \frac{10p + 30}{p^2 + p + 3} u(t) + e(t_k),
\end{aligned} \tag{19}
\]

\( e(t_k) \) is a zero-mean DT white Gaussian noise sequence. Let us first create an \texttt{idpoly} model structure object describing the model. The polynomials are entered in descending powers of the differential operator

\[
m0 = \text{idpoly}(1, [10 30], 1, [1 1 5], ‘Ts’, 0);
\]

\( ‘Ts’ \) and 0 indicate here that the system is time continuous.

\( u \) is generated using a uniform distribution centered on 0 with values between -2 and 2 and is of length 2000. The sampling period is chosen to be 0.5

\[
\begin{aligned}
N &= 2000; \\
u &= 4*(\text{rand}(N,1))-2; \\
Ts &= 0.5;
\end{aligned}
\]

The output of the nonlinear function is then calculated:

\[
\begin{aligned}
f &= [1 0.5 0.25]; \\
\text{for} \ ik = 1:length(f) \\
\quad \hat{u}bar &= ubar + f(ik)*u: ik;
\end{aligned}
\]

We then create an \texttt{iddata} object for the input signal with no output, the input \texttt{ubar} and sampling interval \texttt{Ts}:

\[
\text{datau} = \text{iddata}([], \text{ubar}, \text{Ts});
\]

We then create an \texttt{iddata} object with output \texttt{ydet}, inputs \texttt{u}, \texttt{u^2}, \texttt{u^3} and sampling interval \texttt{Ts}:

\[
\text{upow} = [u u.^2 u.^3]
\]

Let us now consider the case when a white Gaussian noise is added to the output samples. The variance of \( e(t_k) \) is adjusted to obtain a signal-to-noise ratio (SNR) of 10 dB. The SNR is defined as

\[
\text{SNR} = 10 \log \frac{P_{\text{ydet}}}{P_e}
\tag{20}
\]

where \( P_e \) represents the average power of the zero-mean additive noise on the system output (e.g., the variance) while \( P_{\text{ydet}} \) denotes the average power of the noise-free output fluctuations.

\[
\begin{aligned}
\text{snr} &= 10; \\
y &= \text{simc}(m, \text{datau}, \text{snr}); \\
\text{data} &= \text{iddata}(y, \text{upow}, \text{Ts});
\end{aligned}
\]

We then identify a CT Hammerstein model for this system from the \texttt{iddata} object \texttt{data} with the NLSRIVC method. The extra pieces of information required are:

- the number of denominator and numerator parameters and number of samples for the delay of the model
- the number of basis function (see (15)) is directly given by the number of columns in \texttt{upow}.

The NLSRIVC routine can now be used as follows:

\[
mH = \text{NLSRIVC(data, [2 2 0]);}
\]

Let us now compare the model output for the input signal with the measured noisy output. This can be done easily by using the \texttt{compare} CONTSID routine:

\[
\text{compare}(\text{data}, \text{mH}, 1:200); \quad \text{which plots the noisy and the simulated model outputs as shown in Figure 4.}
\]
5. CONCLUSION

This paper has outlined the main features of the latest developments for the Matlab CONTSID toolbox and illustrated the recent extensions. The toolbox, which provides access to most of the time-domain continuous-time model identification techniques that allow for the direct identification of continuous-time models from discrete-time data, is in continual development. Planned new release will include more techniques to solve the non-linear continuous-time model identification problems.

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