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Dynamics of Jupiter Trojans during the 2:1 mean motion resonance crossing of Jupiter and Saturn

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ABSTRACT

In the early phase of the Solar system evolution, while the outer planets migrated due to their interaction with a planetesimal disc, Jupiter may have crossed the 2:1 mean motion resonance with Saturn. It is well known that this dynamical event has profound consequences on the evolution of an alleged initial Trojan population of Jupiter. In this paper, we analyse in details the dynamics of Jupiter Trojans during the resonance crossing. We find that orbital instability is not confined to the central 2:1 resonance region but occurs in a more extended region where secular and secondary resonances perturb the Trojan orbits while the planets approach, cross and leave the 2:1 resonance. In addition, Jupiter and Saturn are locked after the resonance crossing in an apsidal corotation which has an additional destabilizing effect on Trojans. The synergy of the secular resonance, secondary resonances and apsidal corotation is needed to fully remove an initial Trojan population. New Trojans can be temporarily captured from the planetesimal disc while Jupiter crosses this extended instability region. After the disappearance of major secondary resonances, the secular resonance and the break of the apsidal corotation, the temporarily captured Trojans are locked and can remain stable over long time-scales.

Key words: celestial mechanics – minor planets, asteroids.

1 INTRODUCTION

According to a widely accepted scenario, the outer planets of the Solar system are embedded in a gas-free disc of planetesimals in the last stage of planetary formation. Gravitational interactions between planets and planetesimals dominate the dynamical evolution of the disc. Planetesimals are scattered by the planets in a chaotic manner. Orbital angular momentum and energy are exchanged resulting in planetary migration eventually with significant orbital changes. This phenomenon, described extensively for the first time by Fernandez & Ip (1984), was invoked then to suggest Pluto's capture in a 3/2 resonance by Neptune during its outward migration (Malhotra 1993).

At what heliocentric distances did the planets form and how far did they migrate? Two major models are proposed: in the first model, investigated by Fernandez & Ip (1984) and applied by Malhotra (1993), initial planetary orbits are widely spaced between 5.2 and about 25 au. A recently proposed model, the NICE model (Tsiganis et al. 2005), assumes an initially closely spaced distribution between 5.3 and 17 au. In the latter model, Uranus and Neptune exchange their orbits during migration.

The two migration models differ mainly in the assumption of the initial semimajor axes of the planets. The driving mechanism for planetary migration is the same: planets scatter planetesimals

inwards and outwards. Inwards scattering moves a planet outwards while it moves inwards when a planet scatters a planetesimal outwards. In a closed system in equilibrium without loss of planetesimals where planets scatter planetesimals inwards and outwards, no significant migration would occur. In an open system with loss of planetesimals, on the other hand, significant planetary migration is possible. Jupiter plays a crucial role since it ejects easily planetesimals received from the other planets out of the Solar system. As a consequence, Jupiter migrates towards the Sun while the other three planets migrate outwards. Migration is halted when the outermost planet reaches the edge of the planetesimal disc and when most of the planetesimals scattered between the planets are removed.

Jupiter and Saturn cross in the NICE model the 2:1 mean motion resonance (MMR) soon after planetary migration has started. As a consequence, the eccentricities of both planetary orbits increase. Saturn approaches the orbit of the third planet which is excited and which, therefore, has close approaches with the fourth planet. The third and fourth planet may exchange orbits which moves the third planet rapidly towards 20 au deep inside the planetesimal disc surrounding in the beginning the four planets. Dynamical friction with planetesimals damps rapidly enough planetary eccentricities to avoid close encounters between the third and fourth planet which would result eventually in a destabilization of the outer planetary system. The planetary orbits separate due to migration and their eccentricities are damped to present values due to dynamical friction.

In a scenario where Jupiter and Saturn cross the 2:1 MMR, Jupiter Trojans are destabilized. The destabilization was first attributed to

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the particular perturbations of the 2:1 MMR solely (Michtchenko, Beaugé & Roig 2001). Later, Morbidelli et al. (2005) attributed the destabilization to the 3:1 secondary resonance between harmonics of the libration frequency of Trojan orbits and a critical argument of the 2:1 MMR. This secondary resonance is very effective to remove Trojans in case of a very low migration speed. Within a frozen model without migration, all Trojans are removed (Morbidelli et al. 2005) on a time-scale of about 1 Myr. One cannot exclude, however, that a considerable fraction of Trojans survives since each secondary resonance is quite narrow and Trojans may pass through. In this paper, we will show that due to the presence of a major secular resonance on both sides of the 2:1 MMR, original Trojans are removed due to the synergy between secondary and secular resonances independently of the planet migration rate. In addition, the lock of Jupiter's and Saturn's orbits into apsidal corotation after the 2:1 MMR crossing significantly contributes to the destabilization until the locking is broken.

While primordial Trojans are destabilized before, during and after the crossing of the 2:1 MMR, nearby planetesimals can be temporarily trapped on Trojan orbits via the reverse chaotic path. As soon as Jupiter leaves the extended instability region, the latest captured Trojans remain locked on tadpole orbits for long time-scales comparable to the age of the planetary system. Morbidelli et al. (2005) have shown that the orbital distribution of the observed Trojans corresponds to the orbital distribution of the captured Trojans.

Temporary trapping in coorbital motion appears to occur still at present. Everhart (1973) described temporary captures in horseshoe orbits and Karlsson (2004) identified about 20 transitional objects in a sample of about 1200 Trojans. Candidates are Centaurs that can be trapped as Trojans for short periods of $10^4 - 10^5$ yr (Horner & Evans 2006). This shows that the stable region for Jupiter Trojans is surrounded by a chaotic layer (Marzari, Tricarico & Scholl 2003b) where a population of temporary Trojans resides. At present, the stable and unstable regions are well separated and an object residing in the transient population cannot become a permanent Jupiter Trojan without the help of a non-conservative process. There are some slow diffusion gates from the stable to the unstable region like those identified by Robutel & Gabern (2006) related to commensurabilities between the secular frequency of the Trojan perihelion longitude and the frequency of the Great Inequality (2:5 almost resonance between the present Jupiter and Saturn). However, it is very unlikely that a transient Trojan can follow in the reverse sense these paths to become a permanent Trojan.

We describe in Section 2 the major perturbations acting near the 2:1 MMR on Jupiter Trojans in the early phase of the NICE migration model. Section 3 is devoted to the synergy of these major perturbations leading to a total loss of a possible initial Trojan population independent of migration rates. In Section 4, we show that perturbations in the central Jupiter–Saturn 2:1 MMR region, where at least one of the resonant arguments librates, do not lead to global instability even in a frozen model as suggested by Michtchenko et al. (2001). A Frequency Map Analysis (FMA) reveals extended stable regions.

2 SOURCES OF INSTABILITY FOR JUPITER TROJANS BEFORE, DURING AND AFTER THE 2:1 MMR CROSSING

In this section, we describe the sweeping of resonances through the Trojan region before and after the 2:1 MMR crossing of Jupiter and Saturn.

2.1 The numerical models

The goal of our numerical modelling is to explore the stability of Trojan orbits during the migration of Jupiter and Saturn through the 2:1 MMR. The migration rates of the two planets have to be computed within a model that includes all the outer planets and a disc of massive planetesimals as described in Tsiganis et al. (2005). For this reason, we have first reproduced the dynamical evolution of the outer planets using the same model of Tsiganis et al. (2005) and adopting the same SYMBA5 numerical algorithm (Duncan, Levison & Lee 1998). It is a symplectic integrator that models the gravitational interactions among planets, the gravitational forces exerted by the planets on planetesimals and vice versa. The gravitational interactions among planetesimals are omitted in order to gain computing time. SYMBA5 is particularly designed to handle close encounters among planetesimals and planets, the main mechanism driving the migration of the outer planets. Using the starting conditions for the planets described in Tsiganis et al. (2005) for relatively slow migration, we performed a numerical simulation that matches closely that shown in Tsiganis et al. (2005). Hereafter, we refer to this simulation as Planets and Planetesimals Simulation (PPS). The four outer planets are started on closely packed, almost circular and coplanar orbits. The semimajor axes a of Jupiter and Saturn are 5.45 and 8.50, respectively, so that they will cross the 2:1 MMR during their migration. Orbital eccentricities e and inclinations i are equal to 0.001 at start. Following Tsiganis et al. (2005), we use 4500 massive planetesimals to produce the migration of the four planets.

In Fig. 1, we show the semimajor axis and eccentricity of Jupiter as obtained in our PPS simulation. Before the 2:1 MMR crossing, Jupiter's eccentricity is equal on average to 0.01 in spite of its small starting value. This is due to the forced component of Saturn which

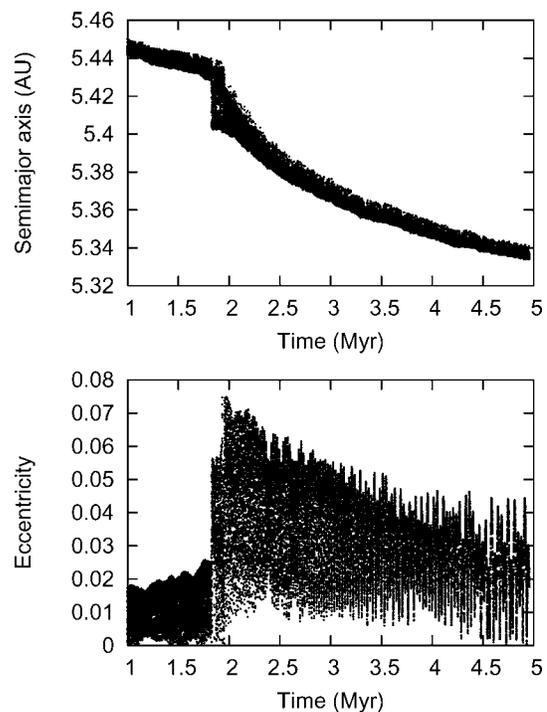


Figure 1. Orbital evolution of Jupiter semimajor axis and eccentricity during the migration of the planets driven by planetesimal scattering (PPS simulation). The outcome is very similar to that shown in Tsiganis et al. (2005). We focus on the resonance crossing that occurs after about 1.8 Myr from the beginning of the simulation.

grows while approaching the resonance location. The 2:1 MMR crossing is marked by a sudden jump in eccentricity related to the separatrix crossing and by large oscillations in semimajor axis. After the crossing, the eccentricity is slowly damped down while the planet continues to migrate towards its present location.

From the time series of the orbital elements of both Jupiter and Saturn, computed within the PPS simulation, we can derive the planet migration rate da/dt of the semimajor axis and the eccentricity damping rate de/dt and produce a *synthetic* model. In this model, the effect of the planetesimal scattering is simulated by adding analytically the da/dt and de/dt terms to the equations of motion of the planets. Such an approach was exploited to model the effect of circumstellar discs on exoplanets, for instance by Lee & Peale (2002) and Kley, Peitz & Bryden (2004). The authors used analytic expressions to estimate the changes in a and e due to the interactions with the disc when advancing the planets from time t_i to t_{i+1} . We follow the formalism outlined in the appendix of the paper by Lee & Peale (2002) and, to model the migration of planets, we introduce da/dt and de/dt in the SYMBA5 integrator to produce the migration and neglect all the massive planetesimals. We concentrate on the orbital evolution of Jupiter and Saturn since they are responsible for the stability or instability of Jupiter Trojans. Uranus and Neptune are needed in the PPS model in order to transport the planetesimals responsible for the migration of the outer planetary system. However, by a series of numerical tests, we have verified that their influence on the Trojan orbits of Jupiter is negligible compared to that of Jupiter itself and Saturn.

In the synthetic model, we must account for the fact that the migration of Jupiter and Saturn caused by planetesimal encounters is linear only over a limited amount of time and not over the whole migration period. The number of planetesimals in planet crossing orbits is in fact declining causing a slow decrease of da/dt and de/dt . We therefore tune the synthetic integrator to the PPS run by using values of da/dt and de/dt that are derived from PPS at different times during the evolution of Jupiter and Saturn. In this way, the synthetic model accurately reproduces the evolution of the planets during the 2:1 MMR and even after. Moreover, it retains all the dynamical features needed to analyse the stability of Jupiter Trojans. The initial values of da/dt and de/dt for Jupiter are -7.39×10^{-9} au yr $^{-1}$ and -3.76×10^{-10} yr $^{-1}$, respectively. After 10 Myr, these values have decreased to -4.05×10^{-9} au yr $^{-1}$ and -2.05×10^{-10} yr $^{-1}$. For Saturn, the da/dt ranges from 2.23×10^{-8} to 1.24×10^{-8} au yr $^{-1}$ in 10 Myr while the de/dt goes from -2.80×10^{-9} to -1.65×10^{-9} yr $^{-1}$.

The main advantage of using the synthetic integrator is its speed. We can compute the orbital evolution of Jupiter and Saturn and of massless Trojans on a time-scale of at least 100 times shorter than that required by a full model that includes the massive planetesimals (PPS-type model). CPU time is a critical issue since we have to explore the stability of Trojans in the phase space for different intervals of time during migration and in different dynamical configurations. In addition, with the synthetic model there is the possibility of easily changing the values of da/dt and de/dt which are strongly model dependent. We also tested a synthetic model based on the RADAU integrator and the results were in agreement with the SYMBA5 synthetic model.

To identify possible resonances between the motion of the planets and that of Trojans, we have to evaluate the major orbital frequencies of these bodies. However, the dynamical system evolves because of planetary migration, and the frequencies change with time. To compute the value of these frequencies at a given instant of time, we use *frozen* models. We extract the osculating orbital elements of

the planets and Trojans at the required time and start a numerical integration of the trajectories with the migration switched off (both $d a/dt$ and $d e/dt$ are set to 0). In this way, we compute a time series of orbital elements for the non-migrating planets and Trojans long enough to derive precise values of the frequencies.

To compute initial orbital elements for Trojans at different times during the evolution of the planetary system, we select random initial conditions within a ring surrounding the orbit of Jupiter. The semimajor axis of any putative Trojan is selected in between $0.9a_J$ and $1.1a_J$ where a_J is the semimajor axis of Jupiter. The eccentricity can be as large as 0.5 and the inclination extends up to 50° . The other orbital angles are selected at random between 0° and 360° . Each set of initial conditions is integrated for 10^4 yr and if the critical argument $\lambda - \lambda_J$ librates in this time period, a body with that set of initial conditions is included in the sample of virtual Jupiter Trojans. The choice of wide ranges in eccentricity and libration amplitude, somewhat wider than the present ones, is dictated by the chaotic evolution of the orbital elements before, during and after the 2:1 MMR crossing. This chaotic evolution can drive a given Trojan orbit from a high eccentric orbit into an almost circular one, and it can strongly reduce its libration amplitude. We cannot neglect at this stage orbits that are unstable on the long term since they might be turned into stable ones during the dynamical evolution caused by the planetary migration. A body is considered to be ejected out of the swarm during its evolution when its critical argument no longer librates.

2.2 Secular resonance with Jupiter

The secular evolution of eccentricities and perihelion longitudes of the Jupiter–Saturn system, as described by the Lagrange–Laplace averaged theory, is characterized by two major frequencies that we call g_1 and g_2 following Murray & Dermott (1999). These frequencies are not constant during planetary migration since their values depend on the semimajor axes of the two planets through the Laplace coefficients. The linear Lagrange–Laplace theory has an analytical solution that allows to compute both g_1 and g_2 as a function of planetary orbital elements. However, this solution fails in proximity of the 2:1 MMR and we resort to a full numerical approach to compute the two frequencies during planetary migration. We ‘freeze’ the dynamical system at different stages of migration (frozen model) and we estimate both g_1 and g_2 from the time series of the non-singular variables h and k of Jupiter over 1×10^6 yr. As usual, we define these variables by $h = e * \cos(\varpi)$ and $k = e * \sin(\varpi)$. For the computation of precise values for the two frequencies, we use the so-called Modified Fourier Transform (MFT) analysis (Laskar, Froeschlè & Celletti 1992; Laskar 1993a,b), which we had already applied to study the stability properties of the present Jupiter Trojan population (Marzari et al. 2003b). g_1 and g_2 are by far the frequencies with the largest amplitude computed from the MFT.

One of the two frequencies sweeps through the Trojan region during the migration of the planets reaching values typical of the proper frequency g of Jupiter Trojans. We call this frequency g_1 while the other frequency, g_2 , has a longer period and does not influence the Trojan motion. When g_1 is equal or very close to g , a secular resonance is established. Figs 2 and 3 show the behaviour of g_1 as a function of Jupiter’s semimajor axis and of time during migration, respectively. The period corresponding to g_1 decreases while Jupiter and Saturn approach the 2:1 MMR and it rises back after the 2:1 MMR. Fig. 4 shows for comparison of the ratio of the orbital periods between Saturn, P_S , and Jupiter, P_J . When Jupiter and Saturn approach, cross and leave the 2:1 MMR, g_1 sweeps through the

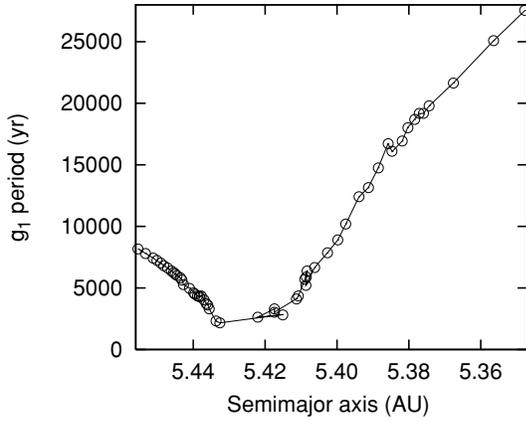


Figure 2. Period corresponding to frequency g_1 as a function of Jupiter’s semimajor axis.

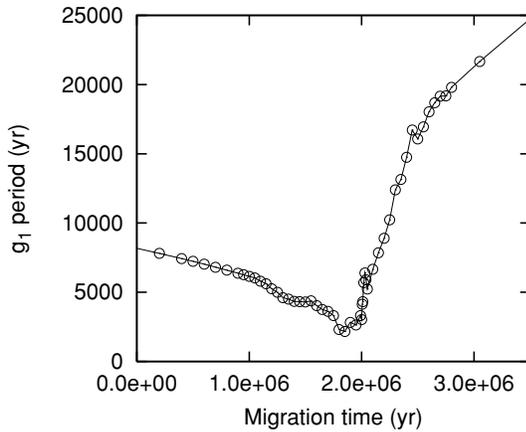


Figure 3. Period corresponding to frequency g_1 as a function of time.

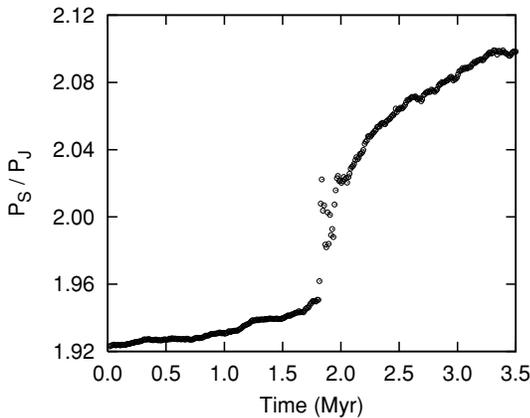


Figure 4. Ratio between Saturn’s (P_S) and Jupiter’s (P_J) orbital periods as a function of time.

Trojan phase space causing strong perturbations that lead mostly to instability. Libration amplitudes and/or eccentricities of Trojans are increased resulting in close encounters with Jupiter. Due to the functional dependence of g on the proper elements of the Trojan orbits (Marzari et al. 2003b), the secular resonance appears first at high inclinations, moves then down to low inclinations when the planets

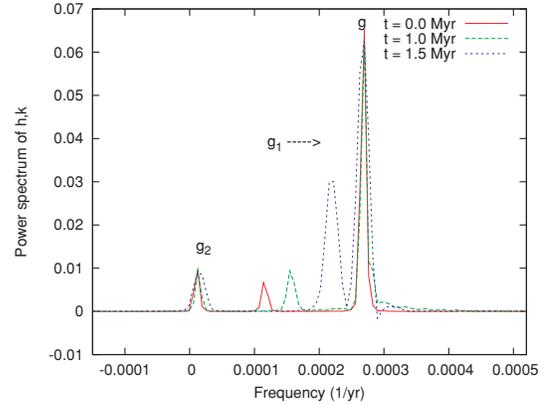


Figure 5. Power spectrum of h and k for a Trojan while Jupiter and Saturn approach the 2:1 MMR. The secular frequencies g_1 and g_2 of the Jupiter–Saturn system and the Trojan’s proper frequency g are given at three different times. Frequency g_1 moves towards g destabilizing the Trojan’s orbit at 1.5 Myr.

reach the 2:1 MMR and finally climbs back to high inclinations after resonance crossing. This behaviour will be described in more detail in Section 3.1. Fig. 5 shows a power spectrum of the complex signal $h + ik$ for a Trojan orbit. The frequency g_1 approaches g when the planets migrate towards the 2:1 MMR leading to resonant perturbations. The frequency g_2 does not change much and remains far from g . The Trojan becomes unstable just after the third instant of time as shown in Fig. 5 before the 2:1 MMR is crossed ($t = 1.5$ Myr). When it falls inside the $g = g_1$ resonance, its orbit is in fact destabilized on a short time-scale by a fast change in eccentricity and libration amplitude.

However, the delicate dynamical equilibrium of the Trojan motion is perturbed even when g_1 is only close to g , outside the secular resonance borders. The term proportional to $g - g_1$ in the disturbing function is dynamically important generating a chaotic evolution of Trojan orbits even if on a longer time-scale compared to those cases falling into the resonance. A similar effect was observed for Uranus Trojans whose diffusion speed in the phase space is strongly increased, leading to chaotic motion, in proximity of the fundamental frequencies g_5 and g_7 of the Solar system (Marzari, Tricarico & Scholl 2003a). When g_1 leaves the Trojan region after the 2:1 MMR, it remains anyway close to g for a long time persisting as a source of instability. Moreover, after the 2:1 MMR, Jupiter and Saturn are locked in an apsidal resonance that enhances the strength of the $g - g_1$ term by coupling the perturbations of Jupiter to those of Saturn (see next section).

A change in the initial values of Jupiter and Saturn in the migration model would move the location of the 2:1 MMR and the corresponding values of the semimajor axes of both Jupiter and Saturn at the crossing. However, this does not alter the effect of the secular resonance on the stability of Trojans. The resonance sweeping occurs anyway since g_1 and g_2 depend on the semimajor axis of Jupiter a_J , according to the Lagrange–Laplace averaged theory, in the same way as the frequency g depends on a_J following Erdi’s theory of Trojan motion (Erdi 1979).

2.3 Secondary resonances with harmonics of the ‘2:1 Great Inequality’

There are two independent critical resonance arguments for the 2:1 MMR of Jupiter and Saturn: $\theta_1 = \lambda_J - 2\lambda_S + \varpi_J$ and $\theta_2 =$

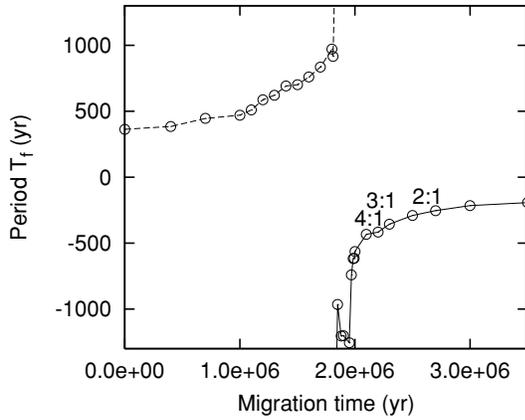


Figure 6. Evolution of the circulation period T_f of θ_1 as a function of time. The resonance crossing is marked by a discontinuity in the period of θ_1 .

$\lambda_J - 2\lambda_S + \varpi_S$, where λ and ϖ denote, respectively, mean longitude and longitude of perihelion. Either of the two critical arguments librates while the other circulates or both critical arguments librate simultaneously. In the latter case, the difference between the two critical arguments $\theta_2 - \theta_1 = \Delta\varpi$ also librates. This means that Jupiter and Saturn are in apsidal corotation.

While Jupiter migrates towards the Sun and Saturn in opposite direction, both θ_1 and θ_2 circulate prograde before the 2:1 MMR and retrograde after. The frequency of θ_1 and θ_2 may become commensurable with the libration frequency of the critical argument of Jupiter Trojans. This is the case of a secondary resonance which was investigated by Kortenkamp, Malhotra & Michtchenko (2004) for Neptune Trojans. The authors found that a Neptune Trojan in a secondary resonance can significantly enhance its libration amplitude possibly leading in some cases to instability. The importance of secondary resonances for Jupiter Trojans in the frame of the NICE model was recognized by Morbidelli et al. (2005). Secondary resonances can be encountered before and after the crossing of the 2:1 MMR. In a frozen model without migration, the 3:1 secondary resonance after the 2:1 MMR removes all Trojans on a time-scale of 1 Myr while the 2:1 secondary resonance removes 70 per cent of them. In a migration model, these removal rates can be significantly less if the secondary resonances are crossed rapidly.

In Fig. 6, we show the period T_f of the frequency f of circulation of θ_1 as a function of the semimajor axis of Jupiter during migration. Different secondary resonances are crossed. Crossing, however, is fast, in particular after the 2:1 MMR. In proximity of the 4:1 secondary resonance, for example, the period of θ_1 changes by approximately 20 per cent in only 3×10^4 yr. In Fig. 7, we illustrate with a shaded stripe the frequency interval of f (translated into periods) for which there is a 4:1 (lower shaded stripe), 3:1 (middle shaded stripe) and 2:1 commensurability (upper shaded stripe) with the libration frequency of a Trojan swarm. We consider Trojans up to 50° in inclination and up to 0.35 in eccentricity corresponding to libration periods roughly ranging from 145 and 190 yr. The sweeping appears to be fast, in particular for the 4:1 and 3:1 secondary resonances, taking also into account that any individual Trojan will be affected only by a fraction of the time spent by f to cover the entire shaded region. It is worthy to note here that the migration speed is relatively low within the different NICE models (Morbidelli et al. 2005; Tsiganis et al. 2005). A faster migration would further reduce the relevance of secondary resonances in the destabilization of Jupiter Trojans during the 2:1 MMR.

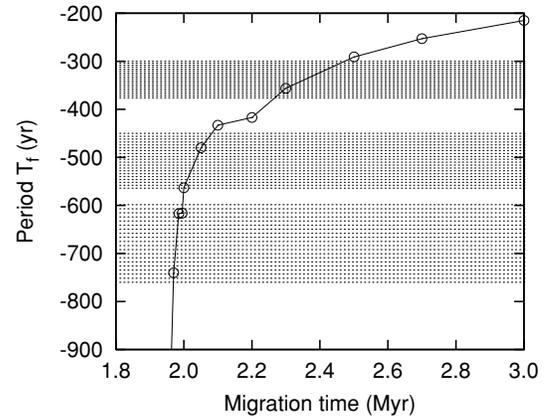


Figure 7. Crossing of the 4:1 (lower shaded stripe), 3:1 (middle stripe) and 2:1 (upper stripe) secondary resonances.

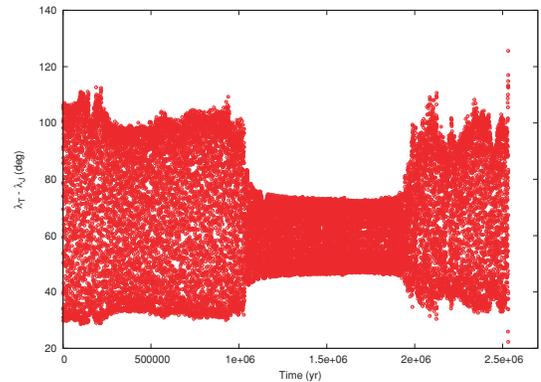


Figure 8. Jumps in libration amplitude of a Trojan approaching the 2:1 MMR. At 1 Myr it crosses the 2:1 secondary resonance, at 2 Myr the 3:1. Finally, before Jupiter and Saturn cross the 2:1 MMR, the Trojan enters the g_1 secular resonance and is destabilized.

As for the secular resonance, the crossing of the secondary resonances occurs before and after the 2:1 MMR. However, there is a substantial difference between the two dynamical configurations. Before the 2:1 MMR, the secondary resonance sweeping causes sharp jumps in libration amplitude and eccentricity that in most cases do not fully destabilize the Trojan orbit. As shown in Fig. 8, the crossing of the 2:1 secondary resonance at $t \sim 1$ Myr reduces the libration amplitude increasing the stability of the orbit. When the 3:1 secondary resonance is encountered later, the initial libration amplitude is restored. The Trojan orbit becomes finally unstable when it crosses the secular resonance with g_1 . Of course, for librators with large amplitudes, the perturbations of the secondary resonances may lead to a destabilization of the Trojan orbit.

Totally different is the dynamical behaviour after the 2:1 MMR. The secondary resonances are much more effective in destabilizing Trojan orbits independent of their libration amplitude. The reason for the different efficiency of secondary resonances before and after the 2:1 MMR is due to two independent causes.

(i) Immediately after the 2:1 MMR crossing, the eccentricity of Jupiter is on average higher. This reinforces presumably secondary resonances and the secular resonance. We tested this hypothesis before the 2:1 MMR by numerically integrating the same Trojan orbits in a model with the eccentricity of Jupiter set to an average value 2.5 times higher compared to that of the reference model which

approximately corresponds to the average increase observed in simulations. Trojans are started in between the 2:1 and 3:1 secondary resonance. Trojans surviving at least 2×10^4 yr decreased by 33 per cent with respect to the Jovian low-eccentricity case. On average, the lifetime was reduced by 22 per cent in the higher eccentric case.

(ii) After the 2:1 MMR crossing, the planets are always locked in apsidal corotation, according to our simulations. This additional dynamical effect contributes to destabilize Trojan orbits. To estimate the effects of apsidal corotation, we have used the same model described in the previous item (that with higher eccentricity) and forced apsidal corotation of the planets before the 2:1 MMR crossing by a convenient choice of the orbital angles of the planets. A comparison between apsidal and non-apsidal corotation after resonance crossing is not possible since the system finds always rapidly the apsidal corotation state. In the apsidal corotation model, the number of surviving Trojans drops by about 23 per cent, compared to that without apsidal corotation, and the Trojan lifetime is shortened by 42 per cent.

2.4 Effect of apsidal corotation between Jupiter and Saturn on the dynamics of Trojans

After the 2:1 MMR crossing, Jupiter and Saturn are locked in apsidal corotation in all our simulations. In most cases, apses are anti-aligned with $\Delta\varpi = \varpi_J - \varpi_S$ librating about 180° . This apsidal corotation is broken much later. The presence of apsidal corotation, as stated in the previous section, has significant consequences for the instability of Trojans.

(i) It enhances the effects of the $g - g_1$ secular term since the frequencies of the precessional rates for the perihelia longitudes of Jupiter and Saturn are commensurable. In Fig. 9, we show the power spectrum of a Trojan started in between the 2:1 MMR and the 3:1 secondary resonance. The two peaks corresponding to the g_1 and g_2 frequencies of the Jupiter and Saturn system are clearly visible and g_1 is close to the proper frequency g . The peaks are much higher as compared to the power spectrum in Fig. 5 which is obtained before the 2:1 MMR where $\Delta\varpi$ circulates.

(ii) The secondary resonances after the 2:1 MMR become very effective in destabilizing Trojan orbits due to the increased eccentricity of Jupiter as pointed out above. The coupling between the apsidal corotation and secondary resonances causes a fast growth of the eccentricity and a corresponding shift in the libration centre of

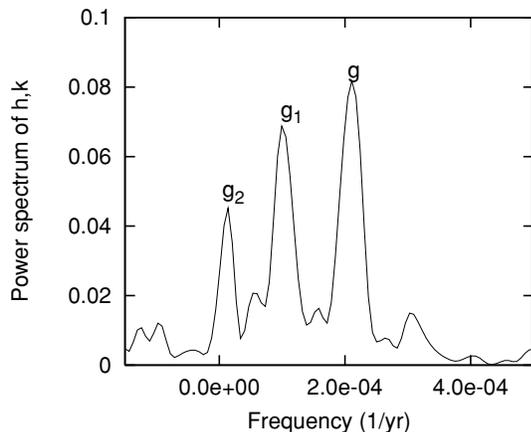


Figure 9. Power spectrum of a Trojan trajectory after the 2:1 MMR crossing. The apsidal corotation increases the strength of the secular term g_1 that leads to chaotic evolution.

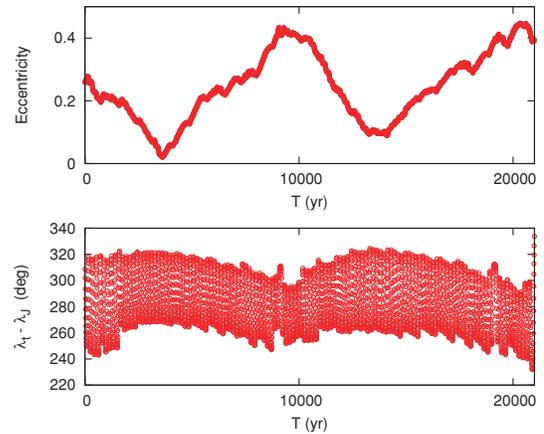


Figure 10. Orbital evolution of a Trojan after the 2:1 MMR crossing perturbed by the 4:1 secondary resonance. The libration centre is shifted from 300° to a lower value due to the apsidal corotation between Jupiter and Saturn.

the Trojan tadpole motion away from the Lagrangian points L4 and L5 (Namouni, Christou & Murray 1999). In Fig. 10, we show an example for this shift and eccentricity increase in the 4:1 secondary resonance after the 2:1 MMR crossing. The Trojan is destabilized at the end by a close encounter with Jupiter. Note that this behaviour is never observed *before* the 2:1 MMR crossing where there is no apsidal corotation and the eccentricity of Jupiter is lower.

3 SYNERGY BETWEEN SECONDARY RESONANCES, THE SECULAR RESONANCE g_1 AND APSIDAL COROTATION

In order to investigate the combined effects of the three identified major perturbations, we start Trojan populations at different migration stages before and after the 2:1 MMR crossing. Simulations begin in between the major secondary resonances 4:1, 3:1, 2:1, 1:1 and right before and after the onset of apsidal corotation.

The starting values are produced by randomly generating the initial orbital elements and checking for the critical libration angle. Maximal starting inclinations and eccentricities are taken somewhat larger than in the presently observed Trojan population of Jupiter because, as already anticipated in Section 2.1, the chaotic evolution in the proximity of the resonance crossings may reduce their values and lead to a stable tadpole orbit.

3.1 Fate of Trojans before the 2:1 MMR crossing

When Jupiter and Saturn approach the 2:1 MMR, Trojans cross secondary resonances and, in particular, the g_1 secular resonance. Secondary resonances before the 2:1 MMR are a very weak instability source and destabilize solely tadpole orbits with large libration amplitude. The secular resonance g_1 , on the other hand, may remove Trojans with any libration amplitude when it sweeps through the region. Even when a body is not exactly within the g_1 resonance but close by, it feels the perturbations of the $g - g_1$ term and it may be destabilized, even if on a longer time-scale. Fig. 11 illustrates the erosion of two initial Trojan populations starting at different times, 2.3 and, respectively, 0.6 Myr before the 2:1 MMR. The two populations are generated with the same random process described in Section 2.1 and, as a consequence, they are dynamically similar. It

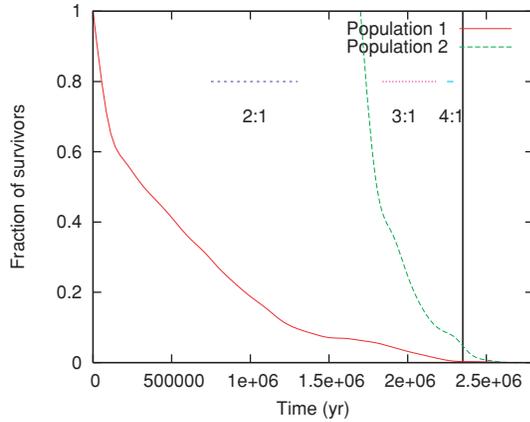


Figure 11. Evolution of two fictitious Trojan populations while Jupiter and Saturn approach the 2:1 MMR. Population 1 is started 2.3 Myr before the resonance crossing. It is eroded mainly by the secular resonance g_1 . Secondary resonances affect solely librators with large amplitudes. Population 2 is started 6×10^5 yr before the 2:1 MMR. Somewhat more bodies survive since the sweeping g_1 resonance affects less high inclined Trojans. Behind the 2:1 MMR, these surviving bodies are removed by effects discussed below. The continuous line marks the 2:1 MMR. The dotted horizontal lines show the location of the secondary resonances over all the range of libration amplitude of the bodies in the two populations. The time-span covered by the secondary resonances is shrinking because the migration is faster in proximity of the 2:1 MMR.

appears that the secular resonance and the secondary resonances are more effective close to the 2:1 MMR where they destabilize more than 90 per cent of the Trojan population. The location of the secondary resonances is shown during the evolution of the populations as horizontal lines in Fig. 11. Before the 2:1 MMR resonance crossing, the sweeping of these resonances is slower than after the 2:1 MMR when the migration of the planets is much faster. However, as discussed above, before the 2:1 MMR secondary resonances are significantly weaker because of the reduced eccentricity of Jupiter. We also recall that each individual Trojan has its own libration period and it is affected by the secondary resonances only during a fraction of the time covered by the resonance sweeping.

The secular resonance sweeping through the Trojan region is illustrated in Figs 12 and 13. Fig. 12 shows the escape time as a function of initial inclination for Population 2. The secular resonance at the beginning of the simulation destabilizes a large number of bodies with inclinations around 25° . They all leave the Trojan region in less than 1×10^5 yr. The critical value of 25° for inclination is determined by the choice of the initial orbits of Jupiter and Saturn before the 2:1 MMR which in turn determines the value of g_1 . A smaller initial distance between the planets, like in case of Population 1, would have destabilized most Trojans at a higher inclination. As the planets move towards the 2:1 MMR during their migration, the frequency g_1 increases and perturbs Trojan orbits at a progressively lower inclination. According to Marzari et al. (2003a), the proper frequency g is higher for low inclined Trojans. In our sample of Trojans, there are naturally unstable orbits since we do not exclude those trajectories with large libration amplitude and high eccentricity. Some may be injected deeper in the stable region after crossing secondary resonances. Most of them, however, escape on a short time-scale and populate the figure at the lower edge of the y-axis.

The high inclined Trojans that survive the 2:1 MMR crossing are destabilized when the frequency g_1 decreases again (its period

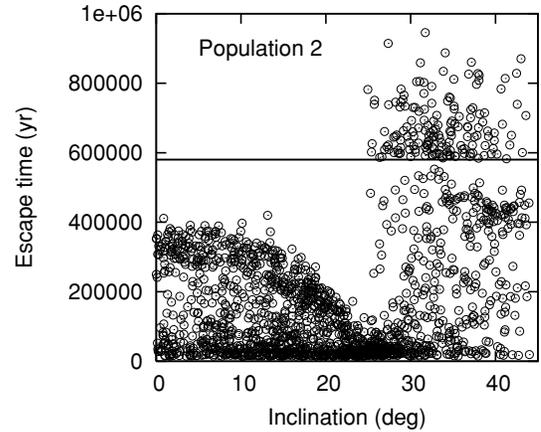


Figure 12. Escape time for Population 2 Trojans of Fig. 11 versus initial orbital inclination. The secular resonance sweeps through the Trojan region starting from an inclination of about 25° down to low inclined orbits. The continuous line marks the 2:1 MMR.

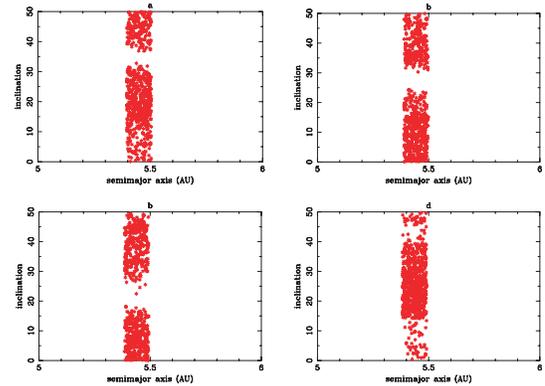


Figure 13. Sweeping of the secular resonance when the planets approach the 2:1 MMR. The orbits of the same initial Trojan swarm are integrated within four different frozen models with the planets progressively closer to the 2:1 MMR.

grows) as shown in Figs 2 and 3. If some high inclination Trojans survive somehow the first sweep of the secular resonance, either by chance or because the planets start their migration close to the resonance location as in Fig. 12, they probably will be destabilized by the second resonance sweeping when the planets move away from the resonance.

In Fig. 13, we illustrate the distribution in inclination and semimajor axis of the same Trojan swarm integrated in frozen models with the planets progressively approaching the 2:1 MMR. The empty stripe corresponds to the secular resonance destabilizing the orbits on a time-scale of 1×10^5 yr in a frozen model. The sweeping proceeds towards lower inclinations while the planets approach the 2:1 MMR, whereas it rises back after the crossing in a symmetric way. We like to emphasize that bodies in the secular resonance are destabilized on a short time-scale. Trojans whose frequency is close to g_1 but are not within the resonance borders are perturbed by the term $g - g_1$ and have a slower chaotic behaviour. This also explains why high inclined Trojans in Fig. 12 are slowly eroded away. It also accounts for the fact that both Populations 1 and 2 are fully destabilized notwithstanding that Population 1 is started much farther away from the 2:1 MMR crossing.

3.2 Evolution of a Trojan population after the 2:1 MMR crossing: from the 4:1 to the 1:1 secondary resonance

The closer Jupiter and Saturn start their migration to the 2:1 MMR, the more initial Trojans may survive due to the dependence of the destabilizing secular resonance on orbital inclination. After the 2:1 MMR, the surviving Trojans would encounter a second time the secular resonance and the secondary resonances. Both the secular and the secondary resonances are reinforced by apsidal corotation and by the larger eccentricity of Jupiter and the destabilization rate is significantly higher. In Fig. 14, we show the evolution of two populations of Trojans after the 2:1 MMR. The population in the upper diagram starts in between the 4:1 and 3:1 secondary resonance. The Trojans of the second population in the lower diagram have libration frequencies which place them in between the 3:1 and 2:1 secondary resonances. To understand the features of the two diagrams, we have to keep in mind that the libration frequency depends on several orbital elements. A Trojan population with about the same semimajor axis is spread over a large range of their other orbital parameters. As a consequence, the Trojans cross the same secondary resonance at different times. Moreover, the proper frequency g depends mostly on the inclination and the effects of the secular resonance appear, as already noted above in the discussion of Fig. 12, at different inclinations during the sweeping. According to Fig. 14, orbits with low inclination are rapidly destabilized while the escape time grows significantly for inclinations higher than 15° . At high inclinations, the secular resonance arrives at a later time and it takes longer to destabilize Trojans. Some of the orbits of the second population survive the 3:1 crossing but are ejected before reaching the 2:1 sec-

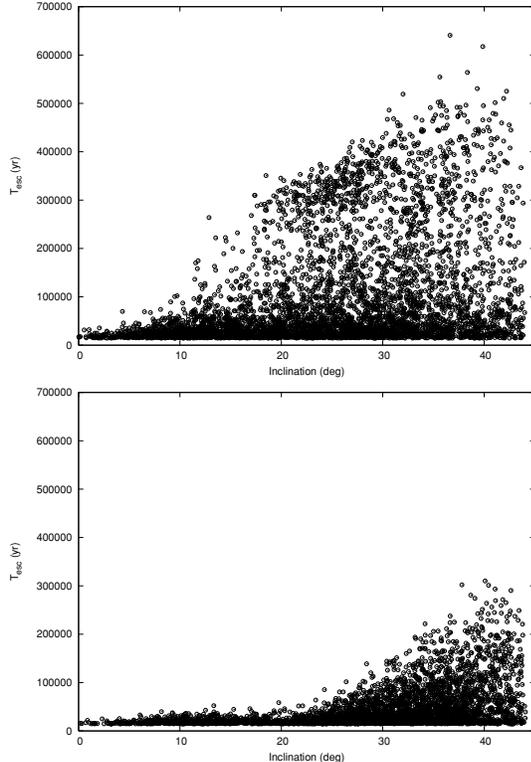


Figure 14. Escape time versus initial inclination for two Trojan populations started after the 2:1 MMR crossing. The first population (top) has libration frequencies encompassed between the 4:1 and 3:1 secondary resonance. The second population (bottom) is started in between the 3:1 and the 2:1 secondary resonance.

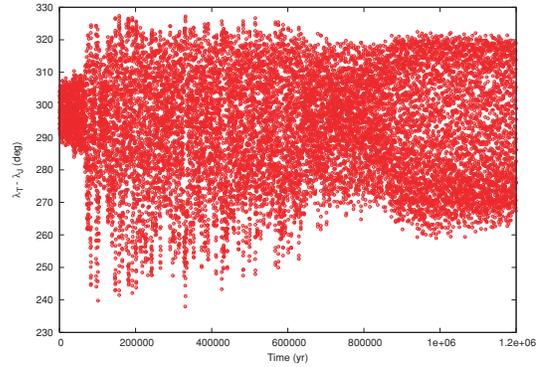


Figure 15. Evolution of the critical argument of a Trojan orbit crossing the 1:1 secondary resonance. The crossing is marked by the first sharp jump in libration amplitude. Subsequently, far from the secondary resonance, the orbit stays chaotic since the dynamics are still influenced by the secular frequency g_1 powered up by the apsidal resonance.

ondary resonance. Only a few Trojans get beyond. At low inclinations where the instability is very fast, we observe a rapid pumping up of eccentricity and a corresponding shift in the libration centre. The power spectrum also shows the vicinity of the secular resonance. We conclude that, after the secondary resonance crossing, no initial Trojan can survive due to the synergy between secondary and secular resonances. Their effects are enhanced by the apsidal corotation of the two planets. The chaotic trapping of new Trojans appears to be difficult at low inclinations where the instability is very fast on time-scales of the order of a few 10^4 yr, while it might be more efficient at higher inclinations where the slow instability might allow the formation of a steady-state transient population of unstable Trojans.

At the 1:1 secondary resonance, significantly farther away from the 2:1 MMR, a sharp jump in libration amplitude D and eccentricity occurs for Trojan orbits. An example is given in Fig. 15 where D changes during the 1:1 crossing. This resonance is weaker as compared to the previously encountered secondary resonances and it does not fully destabilize tadpole orbits but it induces chaotic variations of D . After the 1:1 crossing, when the libration frequency is away from the frequency of either θ_1 or θ_2 , the libration amplitude still shows an irregular behaviour. By inspecting the power spectrum of the h and k variables of the Trojan orbit, we find that the secular frequency g_1 is still relevant with a peak about half the size of the proper one. It is still a source of slow chaotic diffusion for the Trojan orbit.

3.3 Far away from secondary resonances: still chaotic changes of orbital elements

As noted before, when the system gets beyond the 1:1 secondary resonance both the eccentricity and the libration amplitude of tadpole orbits show a slow chaotic evolution which is enhanced when the planets cross mutual higher order MMRs. Fig. 16 shows the evolution of a Trojan trajectory with initially small values of $D \sim 20^\circ$, $e_p \sim 0.03$ and $i_p \sim 19^\circ$. This orbit would lie deeply in the stable region for the present configuration of the planets and it is far from any significant secondary resonance. The secular frequency g_1 appears to be still somehow relevant for the stability of the Trojans causing moderate libration amplitude variations. However, the orbit is finally destabilized during the crossing of a 4:9 MMR between Jupiter and Saturn. The large libration amplitude increase beginning

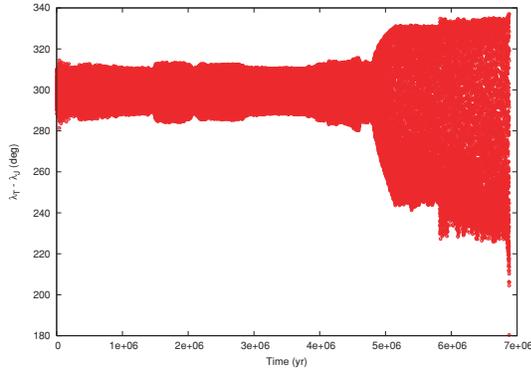


Figure 16. The critical argument of a Trojan orbit started far away from secondary resonances. The orbit is still mildly chaotic because of the presence of the secular frequency g_1 . When Jupiter and Saturn cross the 4:9 MMR, the libration amplitude of the orbit increases until ejection out of the Trojan region.

at $\sim 5 \times 10^6$ yr leads to a fast destabilization of the tadpole orbit. Significant changes of the proper elements are still possible at this stage and the door for chaotic trapping is still open.

After the apsidal corotation is broken, and the weakened secular frequency g_1 has moved farther away from g , Trojan orbits are finally stable on a long time-scale with no detectable variations of the libration amplitude. The door for chaotic capture is closed and the Trojan population approaches its present configurations with no other significant remixing of proper elements.

4 WHEN JUPITER AND SATURN ARE IN THE 2:1 MMR

The instability of Jupiter Trojans with the planets in the 2:1 MMR with Saturn was investigated by Michtchenko et al. (2001) in a frozen model. Jupiter and Saturn are in apsidal corotation. Trojan starting values are confined to inclinations of 5° , $\varpi - \varpi_J = 60^\circ$, $\lambda - \lambda_J = 60^\circ$ and eccentricity lower than 0.3. Using a RADAU integrator, the authors find instability over a very short time-scale of about 10^4 yr. This indicates that if the migration of Jupiter and Saturn was very slow, a temporary capture of the planets in the 2:1 MMR might have led to global instability of Trojans. However, when we performed numerical simulations of Trojan orbits in a frozen model like Michtchenko et al. (2001), we did not find short-term instability. Using their semimajor axes for Jupiter and Saturn and confining Trojans to their starting region, and using also a RADAU integrator, we found a large number of stable Trojans over at least 10^5 yr. Instability for this restricted starting region in phase space usually does not set on before 1 Myr. Similar results were obtained by Nesvorný & Dones (2002) and Marzari & Scholl (2002) in static models where the planets were moving on fixed orbits. We will perform here a more detailed analysis of the stability of Trojans when Jupiter and Saturn are locked in the 2:1 MMR by using the FMA as described in Marzari et al. (2003a). The semimajor axes of Jupiter and Saturn correspond to values of the NICE model. Migration is switched off, so that the planets do not leave the resonance (frozen model). As pointed out above, the two resonance variables θ_1 and θ_2 may both librate (apsidal corotation) or only one may librate while the other circulates (Marzari, Scholl & Tricarico 2006). We applied the FMA analysis for both cases. Our results show that the stability of Trojans depends strongly on their initial conditions and on the behaviour of the two resonance

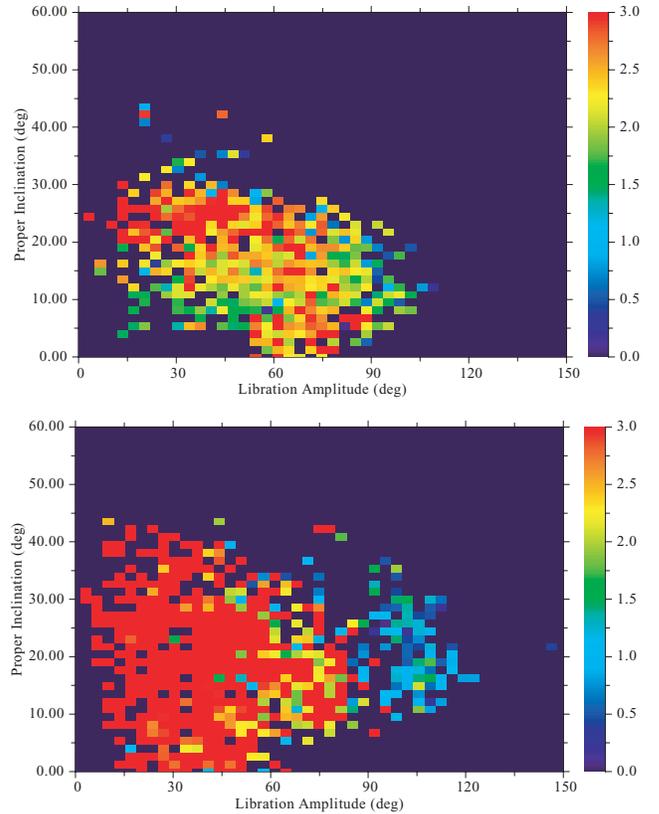


Figure 17. Diffusion portraits of Trojan orbits for the 2:1 MMR. In the upper diagram, both critical resonant arguments librate and the planets are in corotation. In the lower diagram, where a wider region has lower diffusion speeds, marked by red squares, only one critical argument librates.

arguments. The upper diagram in Fig. 17 represents a diffusion portrait for Trojan orbits with Jupiter and Saturn in apsidal corotation. Corotation is possible around 0° or 180° . Since we obtain in most migration models corotation around 180° , we use this alignment mode for producing the diagram. Extended stability regions appear between medium and high inclinations and for a large range of values for libration amplitudes D of Trojans. Empty regions in the plot indicate instability times shorter than 1 Myr. The most stable region (the red one) has values for diffusion speed comparable to those of present Jupiter Trojans (Marzari et al. 2003b) suggesting that bodies can survive for a long interval of time of the order of some Gyr. For bodies with higher diffusion speed, we still expect lifetimes of the order of $10^7 - 10^8$ yr. The stable region extends down to low inclinations with libration amplitudes of about $D \sim 60^\circ$ where we found stability with different integrators, contrary to Michtchenko et al. (2001).

In the lower diagram of Fig. 17, we consider a different dynamical state for the two planets in resonance. Only one of the two critical arguments librates. Consequently, $\Delta\varpi$ circulates. The stability area is more extended in this case, and orbits with low inclination can be found at low values of libration amplitude D . These results reinforce the idea that corotation contributes significantly to reduce dynamical lifetimes of Trojans.

5 CONCLUSIONS

We investigate the depletion of an alleged initial Jupiter Trojan population in the frame of the NICE model describing the early

migration phase of the outer planets during which Jupiter and Saturn may have crossed their mutual 2:1 MMR. The loss of an initial population, possibly trapped during the growth of the planet, is due to the synergy of three different effects:

(i) A secular resonance with the frequency g_1 , one of the two frequencies that, according to the Lagrange–Laplace theory, determine the secular evolution of the eccentricity and perihelion longitude of Jupiter and Saturn.

(ii) Secondary resonances due to commensurabilities between a critical resonance argument of the 2:1 MMR and the libration frequency of the critical argument of the Trojan orbits.

(iii) Jupiter and Saturn’s apsidal corotation after the 2:1 MMR crossing.

While the planets approach the 2:1 MMR, the secular resonance g_1 sweeps through the Trojan region. It appears first at high inclinations and it moves down to almost zero degrees when the planets reach the centre of the 2:1 MMR. It moves up again at higher inclinations after the resonance crossing, sweeping for a second time the Trojan region. Also secondary resonances appear before and after the 2:1 MMR crossing but they sweep across the Trojan region at a faster rate, in particular after the 2:1 MMR crossing. Before the 2:1 MMR crossing, secondary resonances remove very few Trojans while they are more effective after the crossing because of the increase of Jupiter’s eccentricity. Also the secular resonance g_1 is stronger after the 2:1 MMR crossing for the higher eccentricity of Jupiter and also because of the apsidal corotation of Jupiter and Saturn’s orbit. When the frequency g_1 moves out of the Trojan region but is still bordering it, the secular term $g - g_1$ is strong enough to perturb the Trojan motion causing instability on a relatively longer time-scale. While Trojans are removed, new Trojans can be captured by the reverse chaotic path from the surrounding planetesimal population which drives planetary migration. The newly captured Trojans might be lost again until the secular resonance, secondary resonances and higher order MMRs between Jupiter and Saturn disappear.

The centre of the 2:1 MMR, where at least one of the critical resonance arguments librates, is not particularly effective in destabilizing Jupiter Trojans. Its effect is much weaker as compared to

the secular resonance g_1 and the secondary resonances after the 2:1 MMR crossing. When the planets are steadily locked in resonance, we find extended stability regions in the phase space of Trojan orbits.

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REFERENCES

- Duncan M. J., Levison H. F., Lee M. H., 1998, *AJ*, 116, 2067
 Erdi B., 1979, *Cel. Mech.*, 20, 59
 Everhart E., 1973, *AJ*, 78, 316
 Fernandez J. A., Ip W.-H., 1984, *Icarus*, 58, 109
 Fleming H. J., Hamilton D. P., 2000, *Icarus*, 148, 479
 Horner J., Evans N. W., 2006, *MNRAS*, 367, L20
 Karlsson O., 2004, *A&A*, 413, 1153
 Kley W., Peitz J., Bryden G., 2004, *A&A*, 414, 735
 Kortenkamp S., Malhotra R., Michtchenko T., 2004, *Icarus*, 167, 347
 Laskar J., 1993a, *Physica D*, 67, 257
 Laskar J., 1993b, *Cel. Mech. Dyn. Astr.*, 56, 191
 Laskar J., Froeschlè C., Celletti A., 1992, *Physica D*, 56, 253
 Lee M. H., Peale S. J., 2002 *ApJ*, 567, 596
 Malhotra R., 1993, *Nat*, 365, 819
 Marzari F., Scholl H., 2002, *Icarus*, 159, 328
 Marzari F., Tricarico P., Scholl H., 2003a, *A&A*, 410, 725
 Marzari F., Tricarico P., Scholl H., 2003b, *MNRAS*, 345, 1091
 Marzari F. et al. (eds), 2003c, University of Arizona Press, Tucson, p. 725
 Marzari F., Scholl H., Tricarico P., 2006, *A&A*, 453, 341
 Michtchenko T., Beaugé C., Roig F., 2001, *AJ*, 122, 3485
 Morbidelli A., Levison H. F., Tsiganis K., Gomes R., 2005, *Nat*, 435, 462
 Murray C. D., Dermott S. F., 1999, *Solar System Dynamics*. Cambridge Univ. Press, Cambridge
 Namouni F., Christou A. A., Murray C. D., 1999, *Phys. Rev. Lett.*, 83, 2506
 Nesvorný D., Dones L., 2002, *Icarus*, 160, 271
 Robutel P., Gabern F., 2006, *MNRAS*, 372, 1463
 Tsiganis K., Gomes R., Morbidelli A., Levison H. F., 2005, *Nat*, 435, 459

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