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To cite this version:
Alexandre Seuret. Lyapunov-Krasovskii Functionals Parameterized with Polynomials. 6th IFAC Symposium on Robust Control Design (ROCOND09), Jun 2009, Haifa, Israel. 2009. <hal-00386275>

HAL Id: hal-00386275
https://hal.archives-ouvertes.fr/hal-00386275
Submitted on 20 May 2009

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Lyapunov-Krasovskii Functionals
Parameterized with polynomials

Alexandre Seuret

Abstract: A novel method based on Lyapunov-Krasovskii functionals for the stability analysis of linear systems with constant is introduced. The Lyapunov-Krasovskii functionals are provided using polynomial parameters. Stability conditions are derived in the form of linear matrix inequalities. Examples show that these computationally tractable conditions can give tighter stability results than the ones in the literature.

Keywords: Time-delay systems; Stability analysis; Lyapunov-Krasovskii functionals; Linear matrix inequalities

1. INTRODUCTION

Stability analysis of time-delay systems is an important topic in many disciplines of science and engineering Gu et al. [2003], Niculescu [2001], Richard [2003]. Motivating applications are found in diverse areas such as biology, chemistry, telecommunication control engineering, economics, and population dynamics Kolmanovskii et al. [1999]. The introduction of more complex and flexible LKFs is required to overcome these difficulties. The discretization method introduced in Gu [1997], Gu et al. [2003], allows constructing piecewise linear functions as the LKF parameters by dividing the delay interval into several smaller intervals on which the parameters of the LKF are linearly varying. The stability analysis leads to less conservative conditions than in the case of constant parameters but extensions to the time-varying delay case are not straightforward. In Papachristodoulou et al. [Dec. 12-14, 2007], a method to build LKFs with varying parameters based on sum of squares tools is introduced. However the computational complexity is quite high. Another recent approach method is based on Integral Quadratic Constraints (IQC) Kao and Rantzer [2007]. It provides a new interpretation of the LKF and the potential conservatism (see Ariba and Gouaisbaut [2007b]). In Zhabko [2001], complete LKFs, which correspond to necessary and sufficient conditions of stability, are constructed by solving a functional differential equation. It is useful to derive robust stability conditions with respect to delay variations Fridman and Niculescu [2008] or parameter uncertainties Kolmanovskii and Zhabko [2003], Mondie et al. [2005].

In this article, a novel method to parameterize LKFs is provided. The parameters are polynomial functions. The candidate of the LKF is the same as the one defined in Papachristodoulou et al. [Dec. 12-14, 2007]. However in this article, the stability conditions only concerns constant delay and are provided using LMIs but not sum of squares tools. The parameters are defined using a polynomial solution of a particular linear differential equation. The resulting stability condition is expressed in terms of LMIs. The method is able to handle systems with constant delays.

1 The work by A. Seuret is supported by the European Commission through the FP7 Project FeedNetBack.
The paper is organized as follows: Section II is devoted to the formulation of the problem. The form of the LKFs is examined in Section III. Section IV concerns the stability analysis in the case of constant delays. Two examples are provided in Section VI to show the efficiency of the method.

Notations. Given an $n$-dimensional state vector $x$ and a non-negative delay $\tau$, $x(t)$ denotes a function such that $x_{\theta}(\theta) = x(t - \theta)$ for all $\theta \in [-\tau, 0]$. For a matrix $P \in \mathbb{R}^{n \times n}$, $P^T$ denotes matrix transposition and $P > 0$ denotes that $P = P^T$ is positive definite. The $n \times n$ identity matrix is denoted $I_n$. For any matrices $A$ and $B$, diag$(A, B)$ denotes the block diagonal matrix with $A$ and $B$ on the diagonal and zeros elsewhere. The notation $A_{k, ij}$ indicates the $k \times k$ matrix located between rows $ik+1$ and $ik+k$ and columns $jk+1$ and $jk+k$. An asterisk in a symmetric matrix denotes a matrix element that is from the corresponding upper triangular part. The symbol “$\otimes$” represents the classical Kronecker product between two matrices.

2. PROBLEM FORMULATION

Consider a linear system with a single delay
\[
\begin{align*}
x'(t) &= Ax(t) + A_1x(t-\tau) \\
x(\theta) &= \phi(\theta), &\forall \theta \in [-\tau, 0],
\end{align*}
\]

where $x(t) \in \mathbb{R}^n$ is the state variable and $A_0, A_1 \in \mathbb{R}^{n \times n}$ are constant matrices. The delay $\tau > 0$ is assumed to be constant. The function $\phi : [-\tau, 0] \rightarrow \mathbb{R}^n$ represents the function of the initial conditions. We use the general form of LKFs of Gu and Gu [1997]

\[
V(x_t) = x^T(t)P x(t) + 2x^T(t) \int_{-\tau}^{0} Q(\xi)x(t + \xi)d\xi + \int_{-\tau}^{0} x^T(t + \xi)S(\xi)x(t + \xi)d\xi + \int_{-\tau}^{0} \int_{t-\tau}^{0} x^T(t + s)R(s, \xi)dx(t + \xi)d\xi
\]

where $P \in \mathbb{R}^{n \times n}$ is positive definite and $Q(\xi), R(s, \xi) = R^T(\xi, s), S(\xi) \in \mathbb{R}^{n \times n}$ are continuous functions.

The problem considered in this paper is how to design LKFs (2) with polynomial parameters. As noted in the introduction, interesting developments already allow considering variations in $Q$, $R$, and $S$. The developments have been provided to construct piecewise constant [Ariba and Gouaisbaut [2007b], Kao and Rantzer [2007]], piecewise linear parameters [Gu [1997], Gu et al. [2003]] and polynomial parameters (Papachristodoulou et al. [Dec. 12-14, 2007]). This article introduces a novel method which allows an explicit construction of polynomial functions $Q, R$ and $S$ of any order. The method is based on the solutions of a linear differential equation. The stability conditions are given in terms of linear matrix inequalities.

3. PARAMETRIZATION OF LYAPUNOV-KRASOVSKI FUNCTIONALS

As in Seuret and Johansson [2009], we consider some real functions $f^i$ where $i = 1, \ldots, N$ and the following functions $Q, R$ and $S$ such that for all $s$ and $\xi$ in $[-\tau, 0]$

\[
\begin{align*}
Q(\xi) &= \sum_{i=1}^{N} f^i(\xi)Q_i, \\
S(\xi) &= \sum_{i=1}^{N} f^i(\xi)f^i(\xi)S_{ij}, \\
R(s, \xi) &= \sum_{i=1}^{N} f^i(s)f^i(\xi)R_{ij}
\end{align*}
\]

where $Q_i$, $S_{ij}$ and $R_{ij}$ for $i, j = 1, \ldots, N$ are constant matrices. Introducing the vector function $W_f(\xi) = [f^1(\xi) \ldots f^N(\xi)]^T$, a nice expression of the functions can be derived

\[
\begin{align*}
Q(\xi) &= QW_f(\xi), \\
S(\xi) &= W_f^T(\xi)SW_f(\xi), \\
R(s, \xi) &= W_f^T(s)RW_f(\xi)
\end{align*}
\]

where $W_f = W_f \otimes I_n$ and the constant matrices $Q_f, R_f$ and $S_f$ are such that $(Q_f)_{n,11} = Q_i$, $(R_f)_{n,ij} = R_{ij}$ and $(S_f)_{n,ij} = S_{ij}$. The functions which define the LKF are thus expressed in a simple way. A lemma to ensure that the LKF is positive definite is thus formulated.

Lemma 1. (Seuret and Johansson [2009]) Consider a given delay $\tau > 0$. If there exist two positive definite matrices $P \in \mathbb{R}^{n \times n}$ and $S \in \mathbb{R}^{N_n \times N_n}$ and two matrices $Q \in \mathbb{R}^{n \times N_n}$ and $R \in \mathbb{R}^{N_n \times N_n}$ such that

\[
\Xi = \begin{bmatrix} P & Q \\ Q^T & R + S/\tau \end{bmatrix} > 0.
\]

Then the functional $V$ with (3) is positive definite.

Proof. Consider the functional (2) with the functions $Q, R, S$ as in (3). Define the vector $\Phi_f(t) = f^1, W_f(\xi)x(t+\xi)$ The second and the last terms of $V$ can thus be rewritten as $2x^T(t)Q\Phi_f(t)$ and $\Phi_f(t)R\Phi_f(t)^T$. Provided that $S > 0$, Jensen’s inequality ensures that $\int_{-\tau}^{0} x^T(t + \xi)W_f^T(\xi)S(W_f(\xi)x(t + \xi))d\xi \geq \Phi_f^T(t)S/\tau \Phi_f(t)$. Denote $\xi_0(t) = [x^T(t), \Phi_f(t)^T]^T$, the functional satisfies $V(x_t) \geq \xi_0^T(t)\Xi\xi_0(t) \geq \epsilon\|x\|^2$ and $V$ is positive definite if (4) holds.

Remark 1. Inequality (4) which concerns the positive definiteness of the LKF is the same as in Gu [1997]. Lemma 1 is an extension of the result from Gu [1997] to all LKFs defined as (2) with (3).

In Lemma 1, additional information is taken into account to design less conservative conditions. In Ariba and Gouaisbaut [2007a, b], it was proven that increasing the dimension of the state vector (considering $X_1 = [x^T(t) x^T(t+\tau) \ldots ]^T$ or $X_2 = [x^T(t) x^T(t-\tau) \ldots ]^T$) considerably reduces the conservatism. In this article, we introduce the vector $\Phi_f$ with polynomial functions $f^i$.

3.1 A simple method to produce the LKF parameters

As in Seuret and Johansson [2009], for a given integer $N$, consider a matrix $D \in \mathbb{R}^{N \times N}$. Define $W : [-\tau, 0] \rightarrow \mathbb{R}^N$ such that

\[
\begin{align*}
\dot{W}(\xi) &= DW(\xi), \\
W(0) &= W_0
\end{align*}
\]
where \( W_0 \in \mathbb{R}^N \). Consider now the LKF (2) defined by

\[
Q(\xi) = QW(\xi),
S(t) = W^T(t)SW(\xi),
R(s, \xi) = W^T(s)RW(\xi)
\]

where \( W = W \otimes I_n \), \( Q \) is in \( \mathbb{R}^{n \times n} \) and \( S, R \in \mathbb{R}^{n \times N \times n} \) are symmetric constant matrices. In the latter, we will say that the pair \((D, W_0)\) generates the LKF V if the functions \( Q, R \) and \( S \) are given by (5) and (6). There is no restriction on the matrix \( D \). Depending on its eigenvalues, \( Q, R \) and \( S \) can be polynomial, exponential and trigonometric functions. As \( W \) is the solution of the linear differential equations of the type of (5), the functions \( Q, R \) and \( S \) are given by (5) and (6). There is no restriction on the matrix \( D \). Depending on its eigenvalues, \( Q, R \) and \( S \) are polynomial functions. These parameters are defined by \( N \) polynomials of degree \( N \). Consider now the LKF (2) defined by

\[
Q(\xi) = QW(\xi),
S(t) = W^T(t)SW(\xi),
R(s, \xi) = W^T(s)RW(\xi)
\]

and their evaluation at any instant \( a \) and \( b \) in \([-\tau, 0]\) are as follows

\[
Q(a) = Qe^{DA}W_0,
S(a) = W^T_0(e^{DA})^TSe^{DA}W_0,
R(a, b) = W^T_0(e^{DA})^TRe^{BD}W_0.
\]

where \( D = D \otimes I_n \) and \( W_0 = W_0 \otimes I_n \).

To obtain polynomial parameters, the matrix \( D \) should be a nilpotent matrix, i.e. there exists a integer \( N^* \leq N \) such that \( D^{N^*} = 0 \). In the latter, we will only consider the matrix

\[
D = \begin{bmatrix}
1 & 0 & \ldots & 0 & 0 \\
0 & 1 & \ldots & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & 1 & 0 \\
0 & 0 & \ldots & 0 & \ldots
\end{bmatrix}
\quad \text{and} \quad
W_0 = \begin{bmatrix}
1 \\
0 \\
\vdots \\
\xi^{N-1}
\end{bmatrix}
\]

In such situation, the vector function \( W(\xi) \) is given by

\[
W(\xi) = e^{DA}W_0 = \begin{bmatrix}
1 \\
0 \\
\vdots \\
\xi^{N-1}
\end{bmatrix}
\]

It is thus easy to see that the parameters \( Q, R \) and \( S \) are polynomial functions. These parameters are defined by the constant matrices \( Q, R \) and \( S \) which allow defining any polynomials of degree \( N - 1 \) or \( 2N - 2 \). In the next section, a stability criteria is provided based on such parameters.

It is clear that if one considers other nilpotent matrices \( D \) and/or other vectors \( W_0 \), the results will be equivalent since the main objective is to define parameter for the LKF. The use of various and different pairs of \((D, W_0)\) just corresponds to a change of coordinates of the polynomial.

In the sequel, the variable \( \Phi(t) \) is used to represent

\[
\Phi(\xi) = \int_{-\tau}^{0} W(\xi)x(t + \xi)d\xi = \begin{bmatrix}
\int_{-\tau}^{0} \xi^0 x(t + \xi)d\xi \\
\vdots \\
\int_{-\tau}^{0} \xi^{N-1} x(t + \xi)d\xi
\end{bmatrix}
\]

Note that using integrations by parts, it is possible to see the vector \( \Phi(t) \) as an augmented vector of the primitives of the state \( x \).

4. STABILITY ANALYSIS

In this section, the stability analysis of systems with constant delay is provided. Based on functionals of the form (2) with (6), easy computable LMIs are developed. Consider system (1) with a constant delay \( \tau \). The following theorem holds:

Theorem 1. For a given \( N \) and a constant delay \( \tau > 0 \), if there exist \( P = P^T \in \mathbb{R}^{n \times n} \), \( Q \in \mathbb{R}^{n \times N} \) and \( S, T, R = R^T \in \mathbb{R}^{n \times N \times n} \) such that

\[
S > 0, \quad T > 0, \quad \Xi > 0,
\]

\[
\Pi_1 = T + D^T(S + T)D + FT + TF > 0
\]

and

\[
\Pi_2 = \begin{bmatrix}
\pi_{11} & PA_1 - QW(-\tau) & A_1^TQ + W_1^T \Xi - QD \\
* & -W(\tau)^T W(-\tau) & A_1^TQ - W(\tau)^T \Xi R \\
* & * & \pi_{44}
\end{bmatrix} < 0
\]

where

\[
\pi_{11} = PA_1 + A_1^TP + QW_0 + (QW_0)^T + W_0^T S W_0,
\]

\[
\pi_{44} = -\Pi_1/\tau - D^T \Xi R + RD,
\]

\[
\Xi = \text{diag}\{0, 1, \ldots, N - 1\} \otimes I_n
\]

Then system (1) is asymptotically stable.

Proof. Consider an integer \( N \) and the matrix \( D \in \mathbb{R}^{N \times N} \), \( W_0 \in \mathbb{R}^N \) as defined in (7). The functional is rewritten as

\[
V(x_t) = x^T(t)P x(t) + 2x^T(t) \int_{-\tau}^{0} Q(\xi)x(t + \xi)d\xi + \int_{-\tau}^{0} x^T(t + \xi)S(\xi)x(t + \xi)d\xi + \int_{-\tau}^{0} x^T(t + s)R(s, \xi)dx(t + \xi)d\xi
\]

where \( Q, R \) are given in (6) and

\[
S(\xi) = W^T(\xi)SW(\xi) + (\xi + \tau)W^T(\xi)TW(\xi)
\]

where \( S, T, R \in \mathbb{R}^{n \times N \times n} \) are constant matrices. Denote \( \xi_0(t) = [x^T(t), \Phi^T(t)]^T \), it is easy to see that

\[
V(x_t) \geq \xi_0^T(t)\Xi \xi_0(t) + \int_{-\tau}^{0} x^T(t + \xi)(\xi + \tau)W^T(\xi)TW(\xi)x(t + \xi)d\xi
\]

Then if \( \Xi > 0 \) and \( T > 0 \), \( V \) is positive definite. The second part of the proof concerns the analysis of the derivative of the functional \( V \). Using an integration by parts, the derivative of \( \Phi(t) \), denoted as \( \Phi_1(t) = \int_{-\tau}^{0} \xi^0 x(t + \xi)d\xi \), is given by
The number, \(n\), the introduction of the matrices \(\Pi_1\) and \(\Pi_2\) of order negative definite and, consequently, the system is asymptotically stable provided that \(\Pi_2^{T}(t)\Pi_2(t)\) is bounded by 1. Noting that the last term can be rewritten as follows:

\[
\dot{\Phi}(\xi) = \begin{bmatrix} \Phi_0(t) \\
\Phi_1(t) \\
\vdots \\
\Phi_{n-1}(t) \end{bmatrix} = -D\Phi(t) + W(0,\xi)\dot{x}(t) - W(\tau)\xi(t-\tau).
\]

As in Gu et al. [2003], the differentiation of \(V\) leads to

\[
V(x(t)) = 2\xi^T(t)\Phi(t)\dot{x}(t) + x^T(t)(QW(0) + Q^T(0) + S(0))x(t) - 2\xi^T(t)Q(-\tau)x(t-\tau) - x(t-\tau)S(-\tau)x(t-\tau)
\]

\[
-2x^T(t-\tau)\int_{0}^{\tau}R(\tau,\xi)x(\tau+\xi)d\xi + 2\dot{x}^T(t)\int_{0}^{\tau}Q(\xi)(t+\xi)d\xi + 2\dot{x}^T(t)\int_{-\tau}^{0}\left[QD + R(0,0)|x(t+\xi)d\xi - \int_{-\tau}^{0}\int_{-\tau}^{0}x^T(t+s)(\dot{x}^T(t+s)D^T R(s,\xi) + R(s,\xi)D)x(t+\xi)d\xi ds\right] \quad (12)
\]

From the definition of \(S\), its derivative is given by

\[
\dot{S}(\xi) = W^T(\xi)((S +\tau T)D^T + D(S +\tau T) + T)W(\xi) + 2QW(\xi)TDW(\xi)
\]

Noting that the last term can be rewritten as follows:

\[
2\xi^T(t)TDW(\xi) = 2W^T(\xi)TD\left[\xi \ldots \xi N\right]^T = 2W^T(\xi)TD\left[0 \ldots 0\right] = 2W^T(\xi)TFW(\xi)
\]

Replacing \(\dot{x}(t)\) by \(A_0x(t) + A_1x(t-\tau)\) and applying Lemma 2, (12) becomes

\[
V(x(t)) = x^T(t)(PA_0 + A_1^TP + QW_0 + W_0Q^T + W_0^T S W_0)x(t) + 2x^T(t)(PA_1 - QW(-\tau))x(t-\tau) - x^T(t-\tau)W^T(-\tau)SW(-\tau)x(t-\tau) + 2x^T(t)[A_0^T Q - QD + W_0^T R]\Phi(t) - \Phi^T(t)[D^T R^T + RD]\Phi(t) - \int_{-\tau}^{0}x^T(t+\xi)W^T(\xi)\Pi_1 W(\xi)x(t+\xi)d\xi
\]

Since \(\Pi_1 > 0\), Jensen’s inequality ensures that the last term of the previous expression is bounded by \(-\Phi^T(t)\Pi_1/\tau\Phi(t)\). Introducing the vector \(\zeta(t) = [x^T(t), x^T(t-\tau), \Phi^T(t)]^T\), one has \(V(x(t)) \leq \zeta^T(t)\Pi_2\zeta(t)\). Then provided that \(\Pi_2 < 0\), the derivative of order \(N^2\) is negative definite and, consequently, the system is asymptotically stable.

**Remark 2.** The number, \(N^2\), from the discretization method of Gu [1997] and the dimension, \(N\), of the matrix \(D\) do not corresponds to the same level of complexity. In terms of the quantity of variables to determine, a discretization of order \(N^2\) is equivalent to solve Theorem 1 with \(D\) of dimension \(N + 1\).

**Remark 3.** Theorem 1 provides only sufficient but not necessary conditions. The only conservatism introduced in the proof comes from the Jensen’s inequality. However the increase of the dimension of \(N\) reduces the conservatism.

**Remark 4.** The introduction of the matrix \(T\) is required to solve the stability conditions. If \(T\) is taken as the zero matrix, then \(\Pi_1\) can not be definite positive and the last diagonal term of \(\Pi_2\) can not be definite negative. This comes from the matrices \(D\) which is a singular.

**Remark 5.** In Theorem 1, the number of variables to solve is \(n(n+1)/2 + n^2 N + 3nN(nN + 1)/2\). However there are redundancies in the variables. Since for each matrixes matrices \(R, S\) and \(T\), the objective is to provide a polynomial parameters of degree \(2N - 2\). Thus only \(2N - 1\) matrix variables are required instead of \(nN(nN + 1)/2\). Following this idea, it is possible to consider only matrices \(R\) and respectively \(S\) and \(T\) of the form:

\[
\begin{bmatrix}
R_{11} & R_{12} & \ldots & 0 \\
R_{12} & R_{22} & \ldots & 0 \\
0 & 0 & \ldots & 0 \\
0 & 0 & \ldots & 0 \\
\end{bmatrix}
\]

Then the necessary number of variables becomes \(n(n+1)/2 + 3/2(n^2 + n)N + 3n^2(N - 1)\). The number of variable is thus not increasing as \(N^2\) but as \(N\).

There also are another source of redundancy of variable. It comes from the definition of the parameter \(S\). Some terms of the polynomial defined with \(S\) and \(T\) have the same degree. It would be possible to reduce the number of variables to solve but this will not be considered in this article.

**Remark 6.** The stability conditions of Theorem 1 are expressed in a simple way compare to the discretization method. This is due to the use of the high dimensional matrices \(Q, R, S\) and \(T\) in the LMIs. In Gu [1997], the conditions are expressed using each square matrices \(Q_{n,1,j}\) and so on, which requires lots of definitions. Another interesting aspect of Theorem 1 is that the LMIs strictly have the same expression whatever degree, \(N\), of the polynomial, the nilpotent matrices \(D\) and the initial vector condition \(W_0\).

5. SYSTEMS WITH PARAMETER UNCERTAINTIES

Consider now the linear system with constant delay and parameter uncertainties represented as polytopic uncertainties:

\[
\dot{x}(t) = \sum_{i=1}^{M} \lambda_i(t) \left\{A_{0i}x(t) + A_{1i}x(t-\tau)\right\}
\]

where \(M\) is an integer and the scalar functions \(\lambda_i\) are such that

\[
\forall t \geq 0, \quad \sum_{i=1}^{M} \lambda_i(t) \quad \text{and} \quad \lambda_i(t) \geq 0
\]

The following theorem holds

**Theorem 2.** For a given \(N\) and a constant delay \(\tau > 0\), if there exist \(P = P^T \in \mathbb{R}^{n \times n}\), \(Q \in \mathbb{R}^{n \times n}\) and \(S, T, R = R^T \in \mathbb{R}^{n \times n\times n}\) such that for all \(i = 1, \ldots, M\)

\[
S > 0, \quad T > 0, \quad \Xi > 0, \quad \Pi_1 = T + D^T(\dot{S} +\tau T) + (S +\tau T)D + \dot{FT} + TF > 0
\]

and \(\Pi_{2i} =\)
### 6. EXAMPLES

**Example 1** Consider the following system Fridman and Shaked [2006b], Gu [1997]
\[
\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -2 & 0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} x(t - \tau)
\]

An analytical analysis in Gu et al. [2003] indicates that the system is stable if the delay belong to the interval \([0.100168, 1.7178]\). The stability of such a system can not be performed using simple LKFs Gu [1997], Fridman and Niculescu [2008]. Table 1 exposes the results obtained by Theorem 1. The conservatism is reduced as the dimension of the matrix increases. The results are less conservative than the discretization method for the same level of complexity (number of variables to solve). Table 1 also shows that it is not required to increase so much the dimension of \(D\) to provide an accurate estimate of the stability interval.

**Example 2** Consider system (1) Fridman and Shaked [2002], Gu [1997], Kao and Rantzer [2005] or Wu et al. [2004] with
\[
\dot{x}(t) = \begin{bmatrix} 0 \\ -2 \\ -0.9 \end{bmatrix} x(t) + \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} x(t - \tau)
\]

As \(\tau\) is constant, an application of the Nyquist criterion shows that the system is stable has eigenvalues on the imaginary axis for \(\tau = 6.172\). The results from the literature and using Theorem 2 are summarized in Table 2. A first comment concerns the fact that when \(N = 1\), which finally corresponds to the case of constant parameters, the stability conditions are equivalent to the various methods which also consider constant parameters. Then for \(N \geq 0\), the conservatism is reduced by the introduction of polynomials in the LKF’s parameters. For this example, the results obtained by Theorem 1 are less conservative than using most of existing ones.

### 7. CONCLUSION

In this article, the stability of linear systems with constant delays is studied. The parameters which define the LKF are expressed using a solution of a particular linear differential equation which has polynomial solutions. The stability condition is expressed in terms of easy computable LMIs and leads to interesting results in the constant delay case. Even if the discretization method Gu [2001] leads to less conservative conditions, the ones developed in this article are easier to compute. The next step of development would include stability conditions for systems with time-varying delays.

### REFERENCES


